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2+1 dimensional magnetically charged solutions in Einstein-power-Maxwell theory

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We obtain a class of magnetically charged solutions in $2+1$ dimensional Einstein-Power-Maxwell theory. In the linear Maxwell limit, such horizonless solutions are known to exist. We show that in $3D$ geometry, black hole solutions with magnetic charge do not exist even if it is sourced by the power-Maxwell field. Physical properties of the solution with particular power k of the Maxwell field is investigated. The true timelike naked curvature singularity develops when $k > 1$ which constitutes one of the striking effects of the power-Maxwell field. For specific power parameter k, the occurrence of a timelike naked singularity is analyzed in the quantum mechanical point of view. Quantum test fields obeying the Klein-Gordon and the Dirac equations are used to probe the singularity. It is shown that the class of static pure magnetic spacetime in the power-Maxwell theory is quantum-mechanically singular when it is probed with fields obeying Klein-Gordon and Dirac equations in the generic case.

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I. INTRODUCTION

Unlike the case of four-dimensional spacetime, gravitational and electromagnetic fields in $2+1$ -dimensions ($3D$) show significant differences. The absence of a free gravitational field (or Weyl curvature) in $3D$ for instance, is one such noteworthy property as far as gravity is concerned. The addition of extra sources beside the cosmological constant, therefore, becomes indispensable to turn this reduced dimension into an attractive arena for doing physics. We recall the Reissner-Nordström (RN) example in which there is a symmetric duality between the electric and magnetic fields. That is, dual of Maxwell field 2-form in four dimensions is still a 2-form. In $3D$, on the other hand, duality maps a 2-form into 1-form and vice versa. Besides, the interpretation of the sources of the electric fields in $3D$ is not ambiguous, however, considering the magnetic sources the interpretation is not much clear. Yet, for a number of reasons, which can be summarized as—contributing to our understanding of their four-dimensional counterparts—the $3D$ solutions persist to be a center of attraction in general relativity. The prototype example of such $3D$ black hole solutions is known to be the Banados-Teitelboim-Zanelli (BTZ) [1]. This black hole was sourced by a mass, a static electric field, and a negative cosmological constant. The existence of magnetically charged $3D$ solutions was also addressed shortly after BTZ [2–5]. Dias and Lemos have studied magnetic solutions in $3D$ Einstein theory including the rotating version [6] of the works cited in [2–5] and also the magnetic point sources in Brans-Dicke theories [7]. The common result verified, among found solutions, the absence of such magnetic black holes. In other words, $3D$ Einstein-Maxwell (EM) equations do not admit a solution that can be interpreted as a black hole with pure magnetic fields. Furthermore, these solutions are free of curvature singularities. The nonsingular magnetic Melvin universe [8] in four dimensions is well known to provide information about the existence of such solutions in different dimensions as well. As a matter of fact, a magnetic solution has physically radical differences in comparison with its electric counterpart which are related by a duality transformation [5,9,10]. Although pure magnetic black holes in $3D$ are yet to be found, we may anticipate that they are crucial in understanding the global entropic flow and storage / loss of information in such lower dimensions.

In this paper, we wish to go beyond linear Maxwell electromagnetism and to consider the recently-fashionable nonlinear electrodynamics (NED) coupled with gravity in the presence of a negative cosmological constant. This formalism has already found applications [11–16], but to the best of our knowledge in $3D$ pure magnetic version of the power-law nonlinearity remained untouched. From the outset, let us remark that the power (i.e., k) in the power-law Maxwell theory cannot be arbitrary but has to satisfy (at least) some of the energy conditions which are discussed in the Appendix. It is demonstrated that pure magnetically charged black holes do not exist even in this formalism. It is known that the interest in NED aroused long ago during 1930s with the hopes to eliminate divergences due to point charges. However, it is proved in this paper that according to the value of the power-Maxwell parameter in connection with energy conditions, the solutions admit regular and naked singular characteristics. Occurrence of naked singularities is known to violate the cosmic censorship hypothesis. Understanding and the resolution of naked singularities in general relativity remain one of the most challenging problems to be solved. It is widely believed that the scales where this singularity

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forms, classical attempts toward the resolution should be replaced by the quantum theory of gravity. This motivates us to investigate the formation and stability of naked singularities within the framework of quantum mechanics. Our analysis will be based on the criterion of Horowitz and Marolf [17] (HM) in which quantum test particles obey the Klein-Gordon and the Dirac equations are used to probe a naked singularity. The criterion of HM has been used in different spacetimes to investigate such classically naked singular spacetimes, i.e. whether they remain singular or not within the context of quantum mechanics [18–24].

Meanwhile, it must be admitted that the physical interpretation of the magnetic solution, whether it is due to a magnetic monopole or a vortex, remains unclear. Naturally, such interpretations become less clear in the power-Maxwell case as opposed to the case of standard linear Maxwell theory.

The plan of the paper is as follows. In Sec. II, the action of the Einstein-power-Maxwell formalism, solutions to the field equations are given. In Sec. III, the occurrence of naked singularity is analyzed within the framework of quantum mechanics. First, the definition of quantum singularities for general static spacetimes is reviewed and then the Klein-Gordon and the Dirac fields are used to test the quantum singularity. The paper ends with Conclusion in Sec. IV.

II. THE SOLUTION AND SPACETIME STRUCTURE

We start with the three-dimensional action in Einstein-power-Maxwell theory of gravity with a cosmological constant Λ $(8\pi G = 1)$

$$I = \frac{1}{2}\int dx^3\sqrt{-g}\Big(R - \frac{2}{3}\Lambda - \mathcal{F}^k\Big), \tag{1}$$

in which $\mathcal{F}$ is the magnetic Maxwell invariant defined by

$$\mathcal{F} = F_{\mu\nu}F^{\mu\nu}.$$

The field 2-form is given by

$$\mathbf{F} = B(r)dr \wedge d\theta, \tag{2}$$

where $B(r)$ stands for the magnetic field to be determined. Our metric ansatz for three dimensions is chosen as

$$ds^2 = -f_1(r)dt^2 + \frac{dr^2}{f_2(r)} + f_3(r)d\theta^2, \tag{3}$$

in which $f_i(r)$ are some unknown functions to be found. The parameter k in the action is a real constant which is restricted by the energy conditions (see the Appendix). Note that $k = 1$ is a linear Maxwell limit and in our treatments we consider the case $k \neq 1$, so that our treatment does not cover the linear Maxwell limit. The variation with respect to the gauge potential yields the Maxwell equation

$$\mathbf{d}\,({}^\star\mathbf{F}\mathcal{F}^{k-1}) = 0, \tag{4}$$

where $\star$ means duality and $\mathbf{d}(.)$ stands for the exterior derivative. Remaining field equations are

$$G^\nu_\mu + \frac{1}{3}\Lambda\delta^\nu{}_\mu = T^\nu_\mu, \tag{5}$$

in which

$$T^\nu_\mu = -\frac{1}{2}(\delta^\nu{}_\mu\mathcal{F}^k - 4k(F_{\mu\lambda}F^{\nu\lambda})\mathcal{F}^{k-1}) \tag{6}$$

is the energy-momentum tensor due to the NED. It is readily seen that for $k = 1$ all the foregoing expressions reduce to those of the standard linear Maxwell theory. Nonlinear Maxwell Eq. (4) determines the unknown magnetic field in the form

$$B^2 = \frac{f_3(r)}{f_2(r)}\frac{P^2}{f_1(r)^{1/2k-1}}, \tag{7}$$

in which P is interpreted as the magnetic charge. Imposing this into the energy-momentum tensor (6) results in

$$T^\mu_\nu = \frac{1}{2}\mathcal{F}^k\mathrm{diag}(-1, 2k-1, 2k-1), \tag{8}$$

and the explicit form of $\mathcal{F}$ is given by

$$\mathcal{F} = 2\frac{P^2}{f_1(r)^{1/2k-1}}. \tag{9}$$

The exact solution comes after solving the Einstein Eqs. (5), which is expressed by the metric functions

$$f_1(r) \equiv A(r) = -M + \frac{|\Lambda|}{3}r^2 = \frac{|\Lambda|}{3}(r^2 - r_+^2), \tag{10}$$

$$f_2(r) = \frac{1}{r^2}\Big(r^2 + \frac{9\tilde{P}^2(2k-1)^2}{(k-1)\Lambda^2}A(r)^{k-1/2k-1}\Big)A(r), \tag{11}$$

$$f_3(r) = \frac{r^2}{A(r)}f_2(r), \qquad k \neq 1, \tag{12}$$

where M may be interpreted as the mass and $\tilde{P}^2 = 2^{k-1}P^{2k}$. We note that $r_+^2 = |\frac{3M}{\Lambda}|$, and it should not be taken as a horizon radius since our solution does not represent a black hole. One finds the Ricci and Kretschmann scalars as

$$R = -2|\Lambda| - 8\tilde{P}^2\Big(k - \frac{3}{4}\Big)A^{-(k/2k-1)}, \tag{13}$$

$$\mathcal{K} = \frac{4}{3}\Lambda^2 + \frac{32}{3}\tilde{P}^2\Big(k - \frac{3}{4}\Big)|\Lambda|A^{-(k/2k-1)} + 4(8k(k-1)+3)\tilde{P}^4A^{-(2k/2k-1)}. \tag{14}$$

As one observes, depending on k, one can put the solution into three general categories. In the first category, $\frac{1}{4} \leq k < \frac{1}{2}$, and therefore R and $\mathcal{K}$ are regular as the WEC and SEC (see Appendix) are both satisfied. Since we may have $f_3(r_\circ) = 0$ for some $r_\circ$, it suggests that our coordinate patch is not complete and needs to be revised. In such case, we set

$$x^2 = r^2 - r_\circ^2, \tag{15}$$

which leads to the line element

$$ds^2 = -g_1(x)dt^2 + \frac{dx^2}{g_2(x)} + g_3(x)d\theta^2 \tag{16}$$

with the metric functions

$$g_1(x) = \frac{|\Lambda|}{3}(x^2 + r_\circ^2 - r_+^2), \tag{17}$$

$$g_2(x) = \left(x^2 + r_\circ^2 - \frac{9\tilde{P}^2(2k-1)^2}{|k-1|\Lambda^2} g_1(x)^{k-1/2k-1}\right)\frac{g_1(x)}{x^2}, \tag{18}$$

$$g_3(x) = \left(x^2 + r_\circ^2 - \frac{9\tilde{P}^2(2k-1)^2}{|k-1|\Lambda^2} g_1(x)^{k-1/2(k-1)}\right), \quad k \neq 1. \tag{19}$$

Here, one can show that for $x \in [0, \infty)$ then $g_3(x) < 0$, which implies a nonphysical solution and hence the power in this interval $\frac{1}{4} \leq k < \frac{1}{2}$ should be excluded. The second category of solutions can be found by setting $\frac{1}{2} < k < 1$ in which $g_3(x) > 0$ possessing a nonsingular solution. It should be noted that the case for $k = 1$ is already considered in [2–5] and the resulting spacetime has no curvature singularity. The third category of solutions is when $k > 1$ which results in a curvature singularity. Therefore, by shifting the coordinate in accordance with $y^2 = r^2 - r_+^2$ we relocate the singularity to the point $y = 0$ which will be a naked singularity and our interest in this paper will be confined entirely to this third category of solutions. In this new coordinate, the line element reads as

$$ds^2 = -h_1(y)dt^2 + \frac{dy^2}{h_2(y)} + h_3(y)d\theta^2, \tag{20}$$

$$h_1(y) = \frac{1}{3}|\Lambda|y^2, \tag{21}$$

$$h_2(y) = \left(y^2 + r_+^2 + \frac{9\tilde{P}^2(2k-1)^2}{(k-1)\Lambda^2}\left(\frac{1}{3}|\Lambda|y^2\right)^{k-1/2(k-1)}\right)\left(\frac{|\Lambda|}{3}\right), \tag{22}$$

$$h_3(y) = \frac{3}{|\Lambda|}h_2(y), \qquad k \neq 1 \tag{23}$$

with the scalars

$$R = -2|\Lambda| - 8\tilde{P}^2\left(k - \frac{3}{4}\right)\left(\frac{1}{3}|\Lambda|y^2\right)^{-(k/2(k-1))}, \tag{24}$$

$$\mathcal{K} = \frac{4}{3}\Lambda^2 + \frac{32}{3}\tilde{P}^2\left(k - \frac{3}{4}\right)|\Lambda|\left(\frac{1}{3}|\Lambda|y^2\right)^{-(k/2k-1)} + 4(8k(k-1)+3)\tilde{P}^4\left(\frac{1}{3}|\Lambda|y^2\right)^{-(2k/2k-1)}. \tag{25}$$

It can be seen that for $k > 1$, both R and $\mathcal{K}$ are singular at $y = 0$, and this singularity can easily be shown to be timelike.

Finally, we add here that in the same frame but with an electric field matter there exists a black hole solution whose physical properties is considered in a separate study [25].

III. SINGULARITY ANALYSIS

It has been emphasized in Sec. II that the solution admits classical naked singularity if the parameter $k > 1$. This property is in fact one of the most important consequences of the power-Maxwell field, because the previously obtained magnetically charged solution in 2 + 1 dimensional geometry with $k = 1$ is regular [2–5]. Naked singularities are one of the "unlikable" predictions of the classical general relativity. The reason is the cosmic censorship conjecture which forbids the formation of classical naked singularities. Therefore, the resolution of these singularities stand as an extremely important problem to be solved. Since naked singularity occurs at very small scales where classical general relativity is expected to be replaced by quantum theory of gravity, it is worth it to investigate the nature of this singularity with quantum test fields. In probing the singularity, quantum test particles/fields obeying the Klein-Gordon and Dirac equations are used. Our analysis will be based on the pioneering work of Wald [26], which was further developed by Horowitz and Marolf (HM) to probe the classical singularities with quantum test particles obeying the Klein-Gordon equation in static spacetimes having timelike singularities. According to HM, the singular character of the spacetime is defined as the ambiguity in the evolution of the wave functions. That is to say, the singular character is determined in terms of the ambiguity when attempting to find self-adjoint extension of the operator to the entire Hilbert space. If the extension is unique, it is said that the space is quantum mechanically regular. The brief review is as follows:

A. Quantum Singularities

Consider a static spacetime $(M, g_{\mu\nu})$ with a timelike Killing vector field ξ^μ. Let t denote the Killing parameter and Σ denote a static slice. The Klein-Gordon equation in this space is

$$(\nabla^\mu\nabla_\mu - M^2)\psi = 0. \tag{26}$$

This equation can be written in the form

$$\frac{\partial^2\psi}{\partial t^2} = \sqrt{f}D^i(\sqrt{f}D_i\psi) - fM^2\psi = -A\psi, \tag{27}$$

in which $f = -\xi^\mu \xi_\mu$ and D_i is the spatial covariant derivative on Σ. The Hilbert space $\mathcal{H}$, $(L^2(\Sigma))$ is the space of square integrable functions on Σ. The domain of the operator A $D(A)$ is taken in such a way that it does not enclose the spacetime singularities. An appropriate set is $C_0^\infty(\Sigma)$, the set of smooth functions with compact support on Σ. Operator A is real, positive and symmetric therefore its self-adjoint extensions always exist. If it has a unique extension A_E, then A is called essentially self-adjoint [27–29]. Accordingly, the Klein-Gordon equation for a free particle satisfies

$$i\frac{d\psi}{dt} = \sqrt{A_E}\psi, \tag{28}$$

with the solution

$$\psi(t) = \exp[-it\sqrt{A_E}]\psi(0). \tag{29}$$

If A is not essentially self-adjoint, the future time evolution of the wave function (29) is ambiguous. Then, HM criterion defines the spacetime quantum mechanically singular. However, if there is only a single self-adjoint extension, the operator A is said to be essentially self-adjoint and the quantum evolution described by Eq. (29) is uniquely determined by the initial conditions. According to the HM criterion, this spacetime is said to be quantum mechanically nonsingular. In order to determine the number of self-adjoint extensions, the concept of deficiency indices is used. The deficiency subspaces $N_\pm$ are defined by (see Ref. [30] for a detailed mathematical background),

$$\begin{aligned} &N_+ = \{\psi \in D(A^*), \qquad A^*\psi = Z_+\psi, \\ &ImZ_+ > 0\} \quad \text{with} \quad \text{dimension} n_+ \\ &N_- = \{\psi \in D(A^*), \qquad A^*\psi = Z_-\psi, \\ &ImZ_- < 0\} \quad \text{with} \quad \text{dimension} n_-. \end{aligned} \tag{30}$$

The dimensions (n_+, n_-) are the deficiency indices of the operator A. The indices $n_+(n_-)$ are completely independent of the choice of $Z_+(Z_-)$ depending only on whether Z lies in the upper (lower) half complex plane. Generally, one takes $Z_+ = i\lambda$ and $Z_- = -i\lambda$, where λ is an arbitrary positive constant necessary for dimensional reasons. The determination of deficiency indices then reduces to counting the number of solutions of $A^*\psi = Z\psi$; (for $\lambda = 1$),

$$A^*\psi \pm i\psi = 0 \tag{31}$$

that belong to the Hilbert space $\mathcal{H}$. If there is no square integrable solutions (i.e., $n_+ = n_- = 0$), the operator A possesses a unique self-adjoint extension and it is essentially self-adjoint. Consequently, a sufficient condition for the operator A to be essentially self-adjoint is to investigate the solutions satisfying Eq. (31) that do not belong to the Hilbert space.

B. Klein-Gordon Fields

It was previously stated that the obtained solution is naked singular for $k > 1$. Quantum singularity analysis is almost hopeless for technical reasons if the analysis is for any $k > 1$. Therefore, we restrict our analysis to a specific parameter $k = 2$. This specific choice simplifies the metric which is given by

$$ds^2 = -h_1(y)dt^2 + \frac{dy^2}{\tilde{h}_2(y)} + \tilde{h}_3(y)d\theta^2, \tag{32}$$

$$h_1(y) = \frac{1}{3}|\Lambda|y^2, \tag{33}$$

$$\tilde{h}_2(y) = (y^2 + r_+^2 + \alpha y^{2/3})\frac{|\Lambda|}{3}, \tag{34}$$

$$\tilde{h}_3(y) = \frac{3}{|\Lambda|}\tilde{h}_2(y), \tag{35}$$

where $\alpha = \frac{81\tilde{P}^2}{\sqrt{[3]3|\Lambda|^{5/3}}} > 0$ is a constant. The Kretschmann scalar for this particular, $k = 2$, is given by

$$\mathcal{K} = \frac{4}{3}\Lambda^2 - \frac{40\tilde{P}^2|\Lambda|^{1/3}}{\sqrt{[3]3y^{4/3}}} + \frac{(76\tilde{P}^4)3^{4/3}}{|\Lambda|^{4/3}y^{8/3}}. \tag{36}$$

Clearly, $y = 0$ is a true curvature singularity. Upon separation of variables, $\psi = F(y)e^{in\theta}$, we obtain the radial portion of Eq. (31) as

$$\begin{aligned} &\frac{d^2F(y)}{dy^2} + \frac{1}{y}\left\{1 + \frac{y}{\tilde{h}_2(y)}\frac{d(\tilde{h}_2(y))}{dy}\right\}\frac{dF(y)}{dy} \\ &+ \frac{1}{\tilde{h}_2(y)}\left\{\frac{c}{\tilde{h}_3(y)} - M \pm \frac{i}{h_1(y)}\right\}F(y) = 0, \end{aligned} \tag{37}$$

where $c \in \mathbb{R}$ is a separation constant. Since the singularity is at $y = 0$, for small values of y each term in the above equation simplifies for massless ($M = 0$) case to

$$\frac{d^2F(y)}{dy^2} + \frac{1}{y}\frac{dF(y)}{dy} \pm \frac{\nu^2}{y^2}iF(y) = 0, \tag{38}$$

where $\nu^2 = \frac{9}{|\Lambda|^2 r_+^2} > 0$, whose solution is

$$F(y) = C_{1\nu}y^{\sqrt{\pm i\nu}} + C_{2\nu}y^{-\sqrt{\pm i\nu}}, \tag{39}$$

in which $C_{1\nu}$ and $C_{2\nu}$ are arbitrary constants. In order to check the square integrability, we define the function space on each $t =$ constant hypersurface Σ as $\mathcal{H} = \{F \mid \|F\| < \infty\}$ with the following norm given for the metric (32) as

$$\|F\|^2 = \frac{q^2}{2}\int_0^{\text{constant}} \frac{1}{\sqrt{h_1(y)}}\sqrt{\frac{\tilde{h}_3(y)}{\tilde{h}_2(y)}}|F|^2 dy \sim \int_0^{\text{constant}} \frac{|F|^2}{y}dy, \tag{40}$$

where q is a constant parameter. The above solution is checked for the square integrability near $y=0$, for each sign of the solution found in Eq. (39). The solution is square integrable if and only if the constant parameter $C_{2n}=0$, such that for each sign of Eq. (39) we have

$$\|F\|^2 \sim \int_0^{\text{constant}} y^{\sqrt{2}\nu-1}dy = \frac{y^{\sqrt{2}\nu}}{\sqrt{2}\nu}\Bigg|_0^{\text{constant}} < \infty. \tag{41}$$

Therefore,the operator A has deficiency indices $n_+ = n_- = 1$, and is not essentially self-adjoint, so that the spacetime is quantum-mechanically singular.

C. Dirac Fields

The Dirac equation in $3D$ curved spacetime for a free particle with mass m is given by

$$i\sigma^\mu(x)[\partial_\mu - \Gamma_\mu(x)]\Psi(x) = m\Psi(x), \tag{42}$$

where $\Gamma_\mu(x)$ is the spinorial affine connection given by

$$\Gamma_\mu(x) = \frac{1}{4}g_{\lambda\alpha}[e^{(i)}_{\nu,\mu}(x)e^{\alpha}_{(i)}(x) - \Gamma^\alpha_{\nu\mu}(x)]s^{\lambda\nu}(x), \tag{43}$$

$$s^{\lambda\nu}(x) = \frac{1}{2}[\sigma^\lambda(x), \sigma^\nu(x)]. \tag{44}$$

Since the fermions have only one spin polarization in $3D$ [31], the Dirac matrices $\gamma^{(j)}$ can be given in terms of Pauli spin matrices $\sigma^{(i)}$ [32] so that

$$\gamma^{(j)} = (\sigma^{(3)}, i\sigma^{(1)}, i\sigma^{(2)}), \tag{45}$$

where the Latin indices represent internal (local) frame. In this way,

$$\{\gamma^{(i)}, \gamma^{(j)}\} = 2\eta^{(ij)}I_{2\times2}, \tag{46}$$

where $\eta^{(ij)}$ is the Minkowski metric in $3D$ and $I_{2\times2}$ is the identity matrix. The coordinate dependent metric tensor $g_{\mu\nu}(x)$ and matrices $\sigma^\mu(x)$ are related to the triads $e^{(i)}_\mu(x)$ by

$$g_{\mu\nu}(x) = e^{(i)}_\mu(x)e^{(j)}_\nu(x)\eta_{(ij)}, \qquad \sigma^\mu(x) = e^\mu_{(i)}\gamma^{(i)}, \tag{47}$$

where μ and ν stand for the external (global) indices. The suitable triads for the metric (32) are given by

$$e^{(i)}_\mu(t,y,\theta) = \mathrm{diag}\left(y\sqrt{\frac{|\Lambda|}{3}}, \left(\frac{3}{|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}\right)^{1/2}, (y^2+r_+^2+\alpha y^{2/3})^{1/2}\right). \tag{48}$$

The coordinate dependent gamma matrices and the spinorial affine connection are given by

$$\begin{aligned}\sigma^\mu(x) &= \left(\left(\sqrt{\frac{3}{|\Lambda|}}\right)\frac{\sigma^{(3)}}{y}, i\left(\frac{|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}{3}\right)^{1/2}\sigma^{(1)}, \frac{i\sigma^{(2)}}{(y^2+r_+^2+\alpha y^{2/3})^{1/2}}\right),\\ \Gamma_\mu(x) &= \left(\frac{|\Lambda|(y^2+r_+^2+\alpha y^{2/3})^{1/2}\sigma^{(2)}}{6}, 0, \frac{i\sqrt{|\Lambda|}}{6y^{1/3}\sqrt{3}}(3y^{4/3}+\alpha)\sigma^{(3)}\right).\end{aligned} \tag{49}$$

Now, for the spinor

$$\Psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \tag{50}$$

the Dirac equation can be written as

$$\begin{aligned}&\frac{i}{y}\sqrt{\frac{3}{|\Lambda|}}\frac{\partial\psi_1}{\partial t} - \left(\frac{|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}{3}\right)^{1/2}\frac{\partial\psi_2}{\partial y} + \frac{i}{\sqrt{(y^2+r_+^2+\alpha y^{2/3})}}\frac{\partial\psi_2}{\partial\theta}\\ &\quad - \left(\frac{\sqrt{|\Lambda|}(3y^{4/3}+\alpha)}{6y^{1/3}\sqrt{3}(y^2+r_+^2+\alpha y^{2/3})^{1/2}} + \frac{\sqrt{3|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}}{6y}\right)\psi_2 - m\psi_1 = 0,\end{aligned} \tag{51}$$

$$\begin{aligned}&-\frac{i}{y}\sqrt{\frac{3}{|\Lambda|}}\frac{\partial\psi_2}{\partial t} - \left(\frac{|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}{3}\right)^{1/2}\frac{\partial\psi_1}{\partial y} - \frac{i}{\sqrt{(y^2+r_+^2+\alpha y^{2/3})}}\frac{\partial\psi_1}{\partial\theta}\\ &\quad - \left(\frac{\sqrt{|\Lambda|}(3y^{4/3}+\alpha)}{6y^{1/3}\sqrt{3}(y^2+r_+^2+\alpha y^{2/3})^{1/2}} + \frac{\sqrt{3|\Lambda|(y^2+r_+^2+\alpha y^{2/3})}}{6y}\right)\psi_1 - m\psi_2 = 0.\end{aligned} \tag{52}$$

The following ansatz will be employed for the positive frequency solutions:

$$\Psi_{n,E}(t,x) = \begin{pmatrix} Z_{1n}(y) \\ Z_{2n}(y)e^{i\theta} \end{pmatrix} e^{in\theta} e^{-iEt}. \tag{53}$$

The radial part of the Dirac equation becomes

$$\begin{aligned} Z'_{2n}(y) + \Bigg\{ &\frac{\sqrt{3}(n+1)}{\sqrt{|\Lambda|}(y^2 + r_+^2 + \alpha y^{2/3})} \\ &+ \frac{(3y^{4/3}+\alpha)}{6y^{1/3}(y^2+r_+^2+\alpha y^{2/3})} + \frac{1}{2y}\Bigg\} Z_{2n}(y) \\ &+ \frac{1}{\sqrt{(y^2+r_+^2+\alpha y^{2/3})}}\left\{ m\sqrt{\frac{3}{|\Lambda|}} - \frac{3E}{|\Lambda| y}\right\} Z_{1n}(y)e^{-i\theta} = 0 \end{aligned} \tag{54}$$

$$\begin{aligned} Z'_{1n}(y) + \Bigg\{ &-\frac{\sqrt{3}n}{\sqrt{|\Lambda|}(y^2 + r_+^2 + \alpha y^{2/3})} \\ &+ \frac{(3y^{4/3}+\alpha)}{6y^{1/3}(y^2+r_+^2+\alpha y^{2/3})} + \frac{1}{2y}\Bigg\} Z_{1n}(y) \\ &+ \frac{1}{\sqrt{(y^2+r_+^2+\alpha y^{2/3})}}\left\{ m\sqrt{\frac{3}{|\Lambda|}} + \frac{3E}{|\Lambda| y}\right\} Z_{2n}(y)e^{i\theta} = 0. \end{aligned} \tag{55}$$

The behavior of the Dirac equation near $y = 0$ reduces to

$$Z''_j(y) + \frac{2}{y} Z'_j(y) + \frac{\beta^2}{y^2} Z_j(y) = 0, \qquad j = 1, 2 \tag{56}$$

where $\beta^2 = \frac{1}{4} + (\frac{3E}{|\Lambda| r_+})^2$. The solution is given by

$$Z_j(y) = C_{1j} y^{\gamma_1} + C_{2j} y^{\gamma_2}, \tag{57}$$

where C_{1j} and C_{2j} are arbitrary constants and exponents are given by

$$\gamma_1 = -\frac{1}{2} + i\frac{3|E|}{|\Lambda| r_+}, \qquad \gamma_2 = -\frac{1}{2} - i\frac{3|E|}{|\Lambda| r_+}.$$

The condition for the Dirac operator to be quantum mechanically regular requires that both solutions should belong to the Hilbert space $\mathcal{H}$. The squared norm for this solution

$$\sim \int_0^{\text{constant}} \frac{|Z_j(y)|^2}{y} dy \sim \int_0^{\text{constant}} y^{-2} dy \sim \frac{1}{y}\Big|_0^{\text{constant}} \to \infty, \tag{58}$$

diverges. This implies that solution does not belong to the Hilbert space. Consequently, if the classical singularity at $y = 0$ is probed with fermions the spacetime behaves quantum mechanically singular.

IV. CONCLUSION

In this paper, a new class of magnetically charged solutions in $3D$ Einstein-Power-Maxwell theory has been presented. As in the linear Maxwell case, our solutions do not admit black holes but apart from the linear Maxwell case the power-law Maxwell theory admits singular solutions as well. The main contribution of the nonlinear Maxwell field in our solutions is to create timelike naked singularities for specific values of parameter $k > 1$ which is nonexistent in the linear theory. This singularity has been analyzed from the quantum mechanical point of view. Quantum test particles obeying the Klein-Gordon and the Dirac equations are used to probe the singularity.

The analysis of the naked singularity from quantum mechanical point of view has revealed that the considered spacetime is generically quantum singular when it is probed with fields obeying Klein-Gordon and Dirac equations. It is interesting to note that, in contrast to the considered spacetime, the probe of naked singularity with Dirac fields in other $3D$ metrics, namely, BTZ [20] and matter coupled BTZ [23] spacetimes was shown to be quantum mechanically regular. It is also shown in this study that for general modes of spin zero Klein-Gordon fields, the spacetime is still singular.

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APPENDIX: ENERGY CONDITIONS

When a matter field couples to any system, energy conditions must be satisfied for physically acceptable solutions. We follow the steps as given in [33,34] to find the bounds of the power parameter k of the Maxwell field.

1. Weak Energy Condition (WEC)

The WEC states that

$$\rho \geq 0 \quad \text{and} \quad \rho + p_i \geq 0 \quad (i = 1, 2) \tag{A1}$$

in which ρ is the energy density and p_i are the principal pressures given by

$$\rho = -T_t^t = \frac{1}{2}\mathcal{F}^k, \quad p_i = T_i^i = \frac{2k-1}{2}\mathcal{F}^k \,(\text{no sum}). \tag{A2}$$

This condition imposes that $k > 0$.

2. Strong Energy Condition (SEC)

This condition states that

$$\rho + \sum_{i=1}^{2} p_i \geq 0 \quad \text{and} \quad \rho + p_i \geq 0, \tag{A3}$$

which amounts, together with the WEC, to constrain the parameter $k \geq \frac{1}{4}$.

3. Dominant Energy Condition (DEC)

In accordance with DEC, the effective pressure p_{eff} should not be negative, i.e. $p_{\mathrm{eff}} \geq 0$ where

$$p_{\mathrm{eff}} = \frac{1}{2} \sum_{i=1}^{2} T_i^i. \tag{A4}$$

One can show that DEC, together with SEC and WEC, impose the following condition on the parameter k as

$$k > \frac{1}{2}. \tag{A5}$$

4. Causality Condition (CC)

In addition to the energy conditions, one may impose the causality condition (CC)

$$0 \leq \frac{p_{\mathrm{eff}}}{\rho} < 1, \tag{A6}$$

which implies that

$$\frac{1}{2} \leq k < 1. \tag{A7}$$

The CC is clearly violated in our solutions since we abide by the parameter $k > 1$ throughout the paper.

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