UNIVERSITY OF SOUTHAMPTON

Compton Scattering

by

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Quantum Field Theory I
Christmas Problem

Faculty of Physical and Applied Sciences
School of Physics and Astronomy

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"Imagination ... is more important than knowledge. Knowledge is limited. Imagination encircles the world."

Albert Einstein (14 March 1879 , 18 April 1955 (aged 76))

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Abstract

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Doctor of Philosophy in Physics

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This is my calculations of Christmas problem which is Compton scattering [1] . So far I have calculated the diffential cross section with respect to t (square momentum-transfer between the initial-state and final-state photons), $\frac{d\sigma}{dt}$ for the Compton Scattering (the scattering of photons from charged particles ,is called after Arthur Compton who was the first to measure photon-electron scattering in 1922) of a photon and an electron. Then I calculated the diffential cross section with respect to scattering angle $\frac{d\sigma}{d\cos\theta}$ in the rest frame of the incident electron,by two diffent methods and verified that both methods yield the same results.At the end I have plotted and commented the diffential cross section $\frac{d\sigma}{dt}$ against t and the diffential cross section $\frac{d\sigma}{d\cos\theta}$ against $\cos\theta$ for three different values of the centre-of-mass energy.

Contents

Chapter 1

CALCULATION OF THE SQUARE MATRIX ELEMENT

1.1 Introduction

$e^-\gamma \to e^-\gamma$

Another classic experiment in physics is Compton scattering which demonstrates the particle nature of electromagnetic waves. EM waves that interact with matter decrease in energy and cause a shift in wavelength of the incident radiation. This cannot be explained by the classical theory of Thomson scattering and must be explained with quantum mechanics.

Arthur H. Compton observed the scattering of x-rays from electrons in a carbon target and found scattered x-rays with a longer wavelength than those incident upon the target. The shift of the wavelength increased with scattering angle according to the Compton formula:

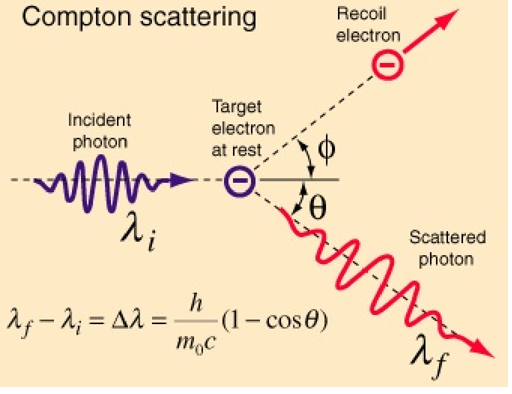

FIGURE 1.1

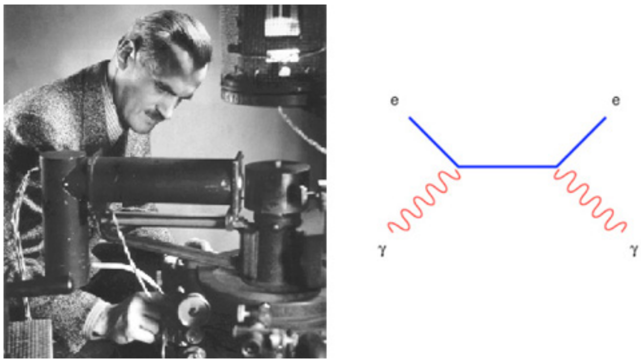

FIGURE 1.2

Compton explained and modeled the data by assuming a particle (photon) nature for light and applying conservation of energy and conservation of momentum to the collision between the photon and the electron. The scattered photon has lower energy and therefore a longer wavelength according to the Planck relationship.

At a time (early 1920's) when the particle (photon) nature of light suggested by the photoelectric effect was still being debated, the Compton experiment gave clear and independent evidence of particle-like behavior. Compton was awarded the Nobel Prize in 1927 for the "discovery of the effect named after him".

One of the greatest scientific revolutions in the history of mankind was the development of Quantum Mechanics. Its birth was a very difficult process, extending from Planck's paper of 1900 to the papers of Einstein, Bohr, Heisenberg, Schroedinger, Dirac and many others. After 1925-1927, a successful theory was in place, explaining many complicated phenomena in atomic spectra. Then attention moved to higher energy phenomena. It was in this period, 1928- 1932, full of great new ideas and equally great confusions, that the Klein-Nishina Formula [2] played a crucial role. It dealt with the famous classical problem of the scattering of light waves by a charged particle. This classical problem had been studied by J. J. Thomson.The cosmic microwave background is thought to be linearly polarized as a result of Thomson scattering. Probes such as WMAP and the current Planck mission attempt to measure this polarization.Furthermore,Inverse-Compton scattering can be viewed as Thomson scattering in the rest frame of the relativistic particle.
Conceptually in classical theory, the scattered waves' frequency must be the same as the incoming frequency, resulting in a total cross-section:

$\rho = \frac{8\pi e^4}{3m^2c^4}$

But in 1923 in an epoch making experiment, Compton found that the scattered waves had a lower frequency than the incoming waves. He further showed that if one adopts

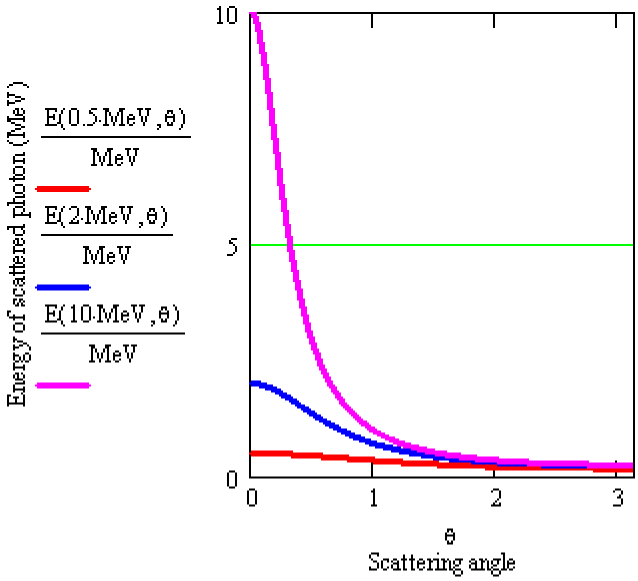

FIGURE 1.3: the ratio E(E',q)/E' as a function of the polar scattering angle q, for specified incident photon energies E'. Scattering kinematics dictate that scattering through large angles q can reduce large incident photon energy values dramatically.

Einstein‘s ideas about the light quanta, then conservation laws of energy and of momenta in fact led quantitatively to the lower frequency of the scattered waves.
Compton also tried to guesstimate the scattering cross-section, using a half-baked classical picture with ad hoc ideas about the frequency change, obtaining:

$\rho = \frac{8\pi e^4}{3m^2c^4}\frac{1}{1+2(h\nu/mc^2)}$

Now, when $h\nu$ is very small compared to mc^2, this formula reduces to Thomson‘s.This Compton theory was one of those magic guess works so typical of the 1920‘s, He knew his theory cannot be entirely correct, so he made the best guess possible.

1.2 Feynman Diagrams

Feynman diagrams are pictorial representations of AMPLITUDES of particle reactions, i.e scatterings or decays. Use of Feynman diagrams can greatly reduce the amount of computation involved in calculating a rate or cross section of a physical process.Like electrical circuit diagrams, every line in the diagram has a strict mathematical interpretation.
Each Feynman diagram represents an AMPLITUDE (M). Quantities such as cross sections and decay rates (lifetimes) are proportional to M^2. In lowest order perturbation theory M is the fourier transform of the potential Born Approximation. The transition

rate for a process can be calculated using time dependent perturbation theory using Fermis Golden Rule: [3]

transition rate= $\frac{2\pi}{\hbar}(M^2) \times (phasespace)$

The differential cross section in the CM frame is:

$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2(E_1+E_2)^2}\frac{p2}{p1}(\Pi_l 2m_l)M^2 = \frac{d\sigma}{2\pi dcos\theta}$ where $\frac{d\sigma}{dcos\theta} = 2p1.k1\frac{d\sigma}{dt}$ [4] , [5]

In most cases M^2 cannot be calculated exactly.Often M is expanded in a power series. Feynman diagrams represent terms in the series expansion of M

QED Rules:

- Solid lines are charged fermions electrons or positrons (spinor wavefunctions).
- Wavy (or dashed) lines are photons.
- Arrow on solid line signifies e- or e+, - arrow in same direction as time, + arrow opposite direction as time.
- At each vertex there is a coupling constant, $\sqrt{\alpha}$, $\alpha = 1/137$ = fine structure constant.
- Quantum numbers are conserved at a vertex, e.g. electric charge, lepton number.
- Virtual Particles do not conserve E, p , virtual particles are internal to diagram(s) , for γs: $E2 - p2 \neq 0$ (off mass shell) in all calculations we integrate over the virtual particles 4-momentum (4d integral) .
- Photons couple to electric charge , no photons only vertices.

We classify diagrams by the order of the coupling constant. Since $\alpha_{QED} = 1/137$ higher order diagrams should be corrections to lower order diagrams.This is just perturbation Theory!! This expansion in the coupling constant works for QED since $\alpha_{QED} = 1/137$ Does not work well for QCD where $\alpha_{QCD} \to 1$

The name Compton scattering refers to the scattering of photons by free electrons. In the language of quantumelectrodynamics an incoming photon with fourmomentum k1 and polarization vector ϵ_μ is absorbed by an electron (or another charged particle) and a second photon with fourmomentum k2 and polarization vector ϵ^*_ν is emitted.The lines marked p1 and p2 represent incoming and outgoing electrons (e^-) with momenta p1 and p2, The corresponding Feynman diagrams are shown in the figure.

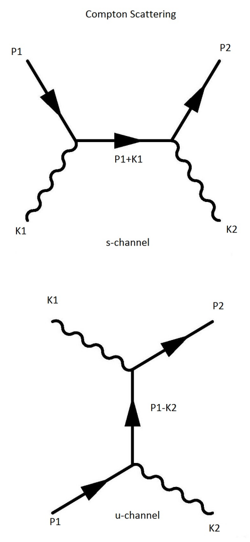

Figure 1.4

For the first diagram we have :

Right-hand vertex : (-ieγ^{μ})

Left-hand vertex : (-ieγ^{ν})

Internal Propagator : $\frac{-i(\not p1+\not k1+m)}{(p1+k1)^2-m^2}$

For the second diagram we have :

Above-hand vertex : (-ieγ^{ν})

Below-hand vertex : (-ieγ^{μ})

Internal Propagator : $\frac{-i(\not p1-\not k2+m)}{(p1-k2)^2-m^2}$

The Feynman rules tell us exactly how to write down an expression for M (Matrix element).

Here I have introduced the Compton Matrix element :

$M_a = \epsilon^*_{\mu}(k2)\epsilon_{\nu}(k1)\bar{u}(p2)(ie\gamma^{\mu})\frac{-i(\not p1+\not k1+m)}{(p1+k1)^2-m^2}(ie\gamma^{\nu})u(p1)$

$M_b = \epsilon_{\nu}(k1)\epsilon^*_{\mu}(k2)\bar{u}(p2)(ie\gamma^{\nu})\frac{-i(\not p1-\not k2+m)}{(p1-k2)^2-m^2}(ie\gamma^{\mu})u(p1)$

In terms of Mandelstam variables I have : (p1+k1)$^2 = s, (p1-k2)^2 = u$

To simplify the numerators,I used a bit of Dirac algeabra:

$(\not{p}1 + m)\gamma^{\nu} u(p1) = (2p1^{\nu} - \gamma^{\nu}\not{p}1 + \gamma^{\nu} m)u(p1)$

$=2p1^{\nu} u(p1) - \gamma^{\nu}(\not{p}1 - m)u(p1)$

$=2p1^{\nu} u(p1)$

After using this trick on the numerator of each propagator, we obtain :

$M_a = \frac{-ie^2}{s^2-m^2}\epsilon^*_{\mu}(k2)\epsilon_{\nu}(k1)\bar{u}(p2)(\gamma^{\mu}\not{k}1\gamma^{\nu} + 2\gamma^{\mu}p1^{\nu}u(p1))$

$M_b = \frac{-ie^2}{u^2-m^2}\epsilon^*_{\mu}(k2)\epsilon_{\nu}(k1)\bar{u}(p2)(\gamma^{\nu}\not{k}2\gamma^{\mu} - 2\gamma^{\nu}p1^{\mu}u(p1))$

$M = M_a + M_b$

The total Matrix element is :

$M = -e^2\epsilon^*_\mu(k2)\epsilon_\nu(k1)\bar{u}(p2)[\frac{(\gamma^\mu \not{k}1\gamma^\nu+2\gamma^\mu p1^\nu}{s-m^2} + \frac{\gamma^\nu \not{k}2\gamma^\mu-2\gamma^\nu p1^\mu}{u-m^2}]u(p1)$ [6]

Then, I calculated the square of this expression for M and sum over electron and photon polarization states.
I used the identity of $\sum u(p1)\bar{u}(p2) = \not{p}1 + m$ for summing over electron polarizations.

and I used the similar identity $\sum \epsilon^*_\mu(k2)\epsilon_\nu(k1) \to -g_{\mu\nu}$

After using those identities and average the squared amplitude over the initial electron and photon polarizations,and sum over the final electron and photon polarizations,I have founded ;

$1/4\sum_{spins}|M|^2 = \frac{e^4}{4}g_{\mu\rho}g_{\nu\sigma}.Tr[(\not{p}2+m)[\frac{(\gamma^\mu \not{k}1\gamma^\nu+2\gamma^\mu p1^\nu)}{s-m^2} + \frac{\gamma^\nu \not{k}2\gamma^\mu-2\gamma^\nu p1^\mu}{u-m^2}]$

$.\ (\not{p}1+m)[\frac{(\gamma^\sigma \not{k}1\gamma^\rho+2\gamma^\rho p1^\sigma)}{s-m^2} + \frac{(\gamma^\rho \not{k}2\gamma^\sigma-2\gamma^\rho p1^\sigma)}{u-m^2}]]$

$= \frac{e^4}{4}[\frac{I}{(s-m^2)} + 2\frac{II}{(s-m^2)(u-m^2)} + \frac{III}{(U-m^2)}]$ where

$I = Tr[(\not{p}2+m)(\gamma^\mu \not{k}1\gamma^\nu + 2\gamma^\mu p1^\nu)(\not{p}1+m)(\gamma_\nu \not{k}1\gamma_\mu + 2\gamma_\mu p1_\nu)]$

II= Tr[$(\not{p}2+m)(\gamma^\mu \not{k}1\gamma^\nu + 2\gamma^\mu p1^\nu)(\not{p}1+m)(\gamma_\mu \not{k}2\gamma_\nu - 2\gamma_\mu p1_\nu)$]

III= Tr[$(\not{p}2+m)(\gamma^\nu \not{k}2\gamma^\mu - 2\gamma^\nu p1^\mu)(\not{p}1+m)(\gamma_\mu \not{k}2\gamma_\nu - 2\gamma_\mu p1_\nu)$]

As you can see that I and III is same if replace k1 with -k2.So only I and II were calculated.

For first of the traces (I) ,there are 16 terms inside the trace,but half of them vanish because of odd number of γ matrices.

After calculating those traces by FORM [7] and using the mandelstam variables $\frac{(s-m^2)}{2} = p1.k1$, $\frac{(m^2-u)}{2} = p2.k1$, $\frac{(2m^2-t)}{2} = \frac{(s+u)}{2} = p1.p2$ I could write that:

$I = 16(2m^4 + m^2(s-m^2) - \frac{1}{2}(s-m^2)(u-m^2))$

II=-16(4m$^4 + m^2(s-m^2) + m^2(u-m^2))$

III=16(2m$^4 + m^2(u-m^2) - \frac{1}{2}(s-m^2)(u-m^2))$

After putting those in our square matrix element,finally obtained the square matrix element in terms of s and u.

$$\frac{1}{4}\sum|M|^2 = 4e^4[\tfrac{(2m^4+m^2(s-m^2)-\frac{1}{2}(s-m^2)(u-m^2))}{(s-m^2)} - \tfrac{4m^4+m^2(s-m^2)+m^2(u-m^2)}{(s-m^2)(u-m^2)} + \tfrac{2m^4+m^2(u-m^2)-\frac{1}{2}(s-m^2)(u-m^2)}{(u-m^2)}]$$

In addition , I could calculate this square matrix element by calculating using this way:

$$M^2 = \frac{1}{4}[(M_1^2) + (M_2^2) + 2\Re(M_1^* M_2)]$$

By using this way I also found ,

$$\mathrm{M}_1^2 = \frac{e^4}{(s-m^2)^2} g^{\mu\rho} g^{\nu\sigma} \mathrm{Tr}\big[\gamma_\rho(\not{p}1 + \not{k}1 + m)\gamma_\sigma \times (\not{p}1' + m)\gamma_\nu(\not{p}1 + \not{k}1 + m)\gamma_\mu(\not{p}2 + m)\big].$$

$$\mathrm{M}_2^2 = \frac{e^4}{(u-m^2)^2} g^{\mu\rho} g^{\nu\sigma} \mathrm{Tr}\big[\gamma_\rho(\not{p}1 - \not{k}2 + m)\gamma_\sigma \times (\not{p}2' + m)\gamma_\nu(\not{p}1 - \not{k}2 + m)\gamma_\mu(\not{p}1 + m)\big].$$

$$(\mathrm{M}_1^* M_2) = \frac{e^4}{(s-m^2)(u-m^2)} g^{\mu\rho} g^{\nu\sigma} \times \mathrm{Tr}[\gamma_\rho(\not{p}1 + \not{k}1 + m)\gamma_\sigma(\not{p}2' + m)\gamma_\mu \times (\not{p}1 - \not{k}2' + m)\gamma_\nu(\not{p}1 + m)]$$

I also calculated those traces by using the FORM [7] and reach the same result with other way.

$$\mathrm{M}^2 = 2e^4\left[m^4\left(\frac{1}{p1\cdot k2} - \frac{1}{p1\cdot k1}\right)^2 - 2m^2\left(\frac{1}{p1\cdot k2} - \frac{1}{p1\cdot k1}\right) + \frac{p1\cdot k2}{p1\cdot k1} + \frac{p1\cdot k1}{p1\cdot k1}\right]$$

and then

$$\mathrm{M}^2 = \frac{1}{4}\frac{8e^4}{(s-m^2)^2}[m^2(3s+u+m^2) - su] + \frac{8e^4}{(u-m^2)^2}[m^2(s+3u+m^2) - su] + \frac{16m^2e^4}{(s-m^2)(u-m^2)}(s+u+2m^2).$$

I checked them and found they are almost equal.

Now, I also calculated traces by using Maple Package [8] :

Compton scattering process γ+e⁻ ->γ+e⁻

```
> vectors(p1,k1,k2);  # define p1,k1,k2 as 4-d vectors
```

$$\{k1, k2, p1\}$$

```
> p2:=p1+k1-k2;       # express p2 from 4-d momentum conservation law
  ### p1,p2 - momentums of electrons, k1,k2 - 4-d impulses of photons
```

$$p2 := p1 + k1 - k2$$

Now let's define 4-dimension scalar product in term of Mandelshtum variables s,u,t (See, [1])

```
> definemore(sc4,
  sc4(p1,p1)=m^2,
  sc4(k1,k1)=0,
  sc4(k2,k2)=0,
  sc4(p2,p2)=m^2,
  sc4(p1,k1)=(s-m^2)/2,
  sc4(p1,k2)=(m^2-u)/2,
  sc4(k1,k2)=(-t)/2);
```

- 'm' is electron mass.

```
> g:=gamma_;          # we will use 'g' for short
```

$$g := gamma\_$$

Let's enter squared matrix element, which was given from [1,2].

```
> M_squared:=(Z(p1)+Z(k1)-Z(k2)+m)&*((g[mu]&*Z(k1)&*g[nu]+2*g[mu]
  &*p1[nu])/2/sc4(p1,k1)+(g[nu]&*Z(k2)&*g[mu]-2*g[nu]&*p1[mu])
  /2/sc4(p1,k2)) &* (Z(p1)+m)&*((g[nu]&*Z(k1)&*g[mu]+2*g[mu]&*p1
  [nu])/2/sc4(p1,k1)+(g[mu]&*Z(k2)&*g[nu]-2*g[nu]&*p1[mu])/2/sc4
  (p1,k2));
```

$$M\_squared := \&*\Bigg(Z(p1) + Z(k1) - Z(k2) + m,$$

$$\frac{1}{2}\,\frac{\&*\left(gamma\_{\mu}, Z(k1), gamma\_{\nu}\right) + 2\,gamma\_{\mu}\,\&*\,p1_{\nu}}{\frac{1}{2}\,s - \frac{1}{2}\,m^2}$$

$$+\frac{1}{2}\,\frac{\&*\left(gamma\_{\nu}, Z(k2), gamma\_{\mu}\right) - 2\,gamma\_{\nu}\,\&*\,p1_{\mu}}{\frac{1}{2}\,m^2 - \frac{1}{2}\,u}, Z(p1) + m,$$

$$\frac{1}{2}\,\frac{\&*\left(gamma\_{\nu}, Z(k1), gamma\_{\mu}\right) + 2\,gamma\_{\mu}\,\&*\,p1_{\nu}}{\frac{1}{2}\,s - \frac{1}{2}\,m^2}$$

$$+\frac{1}{2}\frac{\&*\left(gamma_{\neg\mu}, Z(k2), gamma_{\neg\nu}\right) - 2\, gamma_{\neg\nu}\, \&*\, p1_{\mu}}{\frac{1}{2}m^2 - \frac{1}{2}u}\Bigg)$$

- we neglected a `e' - electron charge. Usual it's presented as factor e^4

Now calculate (at Pentium-700 it takes 11 seconds).

```
> temp:=SP(M_squared);
```

$$temp := \frac{16\,m^2}{\frac{1}{2}s - \frac{1}{2}m^2} - \frac{16\,m^2}{\frac{1}{2}m^2 - \frac{1}{2}u} + \frac{8\,m^4}{\left(\frac{1}{2}s - \frac{1}{2}m^2\right)^2} + \frac{16\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)}{\frac{1}{2}s - \frac{1}{2}m^2}$$
$$+ \frac{16\left(\frac{1}{2}s - \frac{1}{2}m^2\right)}{\frac{1}{2}m^2 - \frac{1}{2}u} + \frac{8\,m^4}{\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)^2} - \frac{16\,m^4}{\left(\frac{1}{2}s - \frac{1}{2}m^2\right)\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)}$$
$$+ \frac{8\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)m^2}{\left(\frac{1}{2}s - \frac{1}{2}m^2\right)^2} - \frac{8\left(\frac{1}{2}s - \frac{1}{2}m^2\right)m^2}{\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)^2} - \frac{4\,m^2\,t}{\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)^2} - 16$$
$$- \frac{4\,t\,m^2}{\left(\frac{1}{2}s - \frac{1}{2}m^2\right)^2} - \frac{4\,t\,m^2}{\left(\frac{1}{2}m^2 - \frac{1}{2}u\right)\left(\frac{1}{2}s - \frac{1}{2}m^2\right)} - \frac{4\,t}{\frac{1}{2}s - \frac{1}{2}m^2}$$
$$+ \frac{4\,t}{\frac{1}{2}m^2 - \frac{1}{2}u}$$

Looks terrible. Let's simplify:

```
> temp:=simplify(temp);
```

$$temp := \frac{1}{(-s+m^2)^2\,(m^2-u)^2}\left(8\left(14\,m^2 s^2 u + 14\,m^2 s\,u^2 + 2\,m^6 s + 2\,m^6 u - 2\,m^4 u^2\right.\right.$$
$$- 2\,m^4 s^2 - 2\,u^3 s - 2\,s^3 u - 2\,s^2 u^2 + 6\,m^8 + 6\,m^2 t\,s\,u - 28\,m^4 s\,u - m^2 t\,s^2 - m^4 t\,s$$
$$\left.\left.- m^4 t\,u - m^2 t\,u^2 - t\,s\,u^2 - t\,s^2 u\right)\right)$$

Now we express answer in term of only two Mandelshtum variables 'u' and 's' (without 't')

```
> ans:=simplify(subs(u=2*m^2-t-s,temp));
```

$$ans := \frac{1}{(-s+m^2)^2\,(m^2-t-s)^2}\left(8\left(-2\,m^2 s\,t^2 - 8\,m^2 s^3 + 3\,m^4 t^2 - 8\,m^6 s + 12\,m^4 s^2 + 2\,m^8\right.\right.$$
$$\left.\left.+ 3\,t^2 s^2 + 4\,t\,s^3 + s\,t^3 + 2\,s^4 - m^2 t^3 - 8\,m^2 t\,s^2 + 4\,m^4 t\,s\right)\right)$$

OK. Let's compare with well-known answer (given from [1,2])

$$> ans\_test := 8\,simplify\left(\frac{sc4(p1,k2)}{sc4(p1,k1)} + \frac{sc4(p1,k1)}{sc4(p1,k2)} + 2\,m^2\left(\frac{1}{sc4(p1,k1)} - \frac{1}{sc4(p1,k2)}\right)\right.$$
$$\left.+ m^4\left(\frac{1}{sc4(p1,k1)} - \frac{1}{sc4(p1,k2)}\right)^2\right)$$

$ans\_test :=$

$$\frac{1}{\left(-s+m^2\right)^2\left(m^2-u\right)^2}\left(8\left(7\,m^2 s^2 u+7\,m^2 s\,u^2+m^2 s^3-3\,m^4 u^2-3\,m^4 s^2-u^3 s\right.\right.$$
$$\left.\left.-s^3 u+6\,m^8-14\,m^4 s\,u+u^3 m^2\right)\right)$$

```
> ans_test:=simplify(subs(u=2*m^2-t-s,temp));
```

$$ans\_test := \frac{1}{\left(-s+m^2\right)^2\left(m^2-t-s\right)^2}\left(8\left(-2\,m^2 s\,t^2-8\,m^2 s^3+3\,m^4 t^2-8\,m^6 s+12\,m^4 s^2\right.\right.$$
$$\left.\left.+2\,m^8+3\,t^2 s^2+4\,t\,s^3+s\,t^3+2\,s^4-m^2 t^3-8\,m^2 t\,s^2+4\,m^4 t\,s\right)\right)$$

```
>
> simplify(ans-ans_test);
```

$$0 \tag{1}$$

```
>
>
>
>
>
```

- that's ok.

```
Microsoft Windows [Version 6.1.7600]

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C:\Users\phys2015>form comp2

Starting Form

FORM by J.Vermaseren,version 3.3(Jul  9 2009) Run at: Thu Jan 20 23:14:38 2011

   symbol e,m,s,t,u;

   vector p1,p2,k1,k2;

   index mu,nu;

   local sterm = (g_(1,p2)+m*gi_(1))*g_(1,mu)*(g_(1,p1)+g_(1,k1)+m*gi_(1))*g_(1,nu)   *(g_
(1,p1)+m*gi_(1))*g_(1,nu)*(g_(1,p1)+g_(1,k1)+m*gi_(1))*g_(1,mu);

   local uterm = (g_(1,p2)+m*gi_(1))*g_(1,mu)*(g_(1,p1)-g_(1,k2)+m*gi_(1))*g_(1,nu)   *(g_
(1,p1)+m*gi_(1))*g_(1,nu)*(g_(1,p1)-g_(1,k2)+m*gi_(1))*g_(1,mu);

   local interterm = (g_(1,p1)+m*gi_(1))*g_(1,mu)*(g_(1,p1)+g_(1,k1)+m*gi_(1))*g_(1   ,nu)
*(g_(1,p2)+m*gi_(1))*g_(1,mu)*(g_(1,p1)-g_(1,k2)+m*gi_(1))*g_(1,nu);

   trace4, 1;

   .sort

Time =     0.00 sec    Generated terms =      19

        sterm       Terms in output =       9

                    Bytes used      =     202

Time =     0.00 sec    Generated terms =      19

        uterm       Terms in output =       9

                    Bytes used      =     202
```

```
Time =     0.00 sec   Generated terms =      18

   interterm       Terms in output =      12

               Bytes used     =      262

  repeat;

  ** These identities follow since all external particles are on mass shell

   id p1.p1 = m^2;

   id p2.p2 = m^2;

   id k1.k1 = 0;

   id k2.k2 = 0;

  ** These identities follow due the conservation of momentum: p1+k1=p2+k2

   id p2.k2 = p1.k1;

   id k1.p2 = p1.k2;

  ** These identities follow from the definition of the Mandlestam variables

   id p1.k1 = (s-p1.p1-k1.k1)/2;

   id p2.k2 = (s-p2.p2-k2.k2)/2;

   id p1.p2 = -(t-p1.p1-p2.p2)/2;

   id k1.k2 = -(t-k1.k1-k2.k2)/2;

   id p1.k2 = -(u-p1.p1-k2.k2)/2;

   id k1.p2 = -(u-k1.k1-p2.p2)/2;

   id t = 2*m^2-s-u;

  endrepeat;

  print sterm;

  print uterm;

  print interterm;

  .end;

Time =     0.00 sec   Generated terms =      21

     sterm       Terms in output =       4

               Bytes used     =      70

Time =     0.00 sec   Generated terms =      21

     uterm       Terms in output =       4

               Bytes used     =      70

Time =     0.00 sec   Generated terms =      55

   interterm       Terms in output =       3

               Bytes used     =      52

  sterm =

    - 8*s*u + 8*m^2*u + 24*m^2*s + 8*m^4;

  uterm =

    - 8*s*u + 24*m^2*u + 8*m^2*s + 8*m^4;

  interterm =

   8*m^2*u + 8*m^2*s + 16*m^4;

  0.00 sec out of 0.05 sec

C:\Users\phys2015>
```

Chapter 2

DIFFERENTIAL CROSS SECTION FOR COMPTON SCATTERING

2.1 DCS with respect to t, $\frac{d\sigma}{dt}$

Now I calculated the differential cross section with respect to t($\frac{d\sigma}{dt}$) for the Compton Scattering by using the total cross section for a scattering event equation:

$\sigma = \int \frac{d^3p2}{(2\pi)^3 2E_{p2}} \frac{d^3k2}{(2\pi)^3 2E_{k2}} (2\pi)^4 \delta^{(4)}(p1 + k1 - p2 - k2) \frac{M^2}{2(s-m^2)}$ (2.1)

After write k2 integral in a Lorentz invariant form ,taking this integral , using polar coordinates for p2 and lastly differentiate wrt t as similar way in our lecture notes.I have:

$\frac{d\sigma}{dt} = \frac{1}{2(s-m^2)} \int \frac{dE_{p2}}{8\pi p1} \delta[(p1 + k1 - p2)^2] M^2$ [5]

Then calculating the delta function integral gives the below equation:

$\frac{d\sigma}{dt} = \frac{|M|^2}{2(s-m^2)} \times \frac{1}{16\pi p1\sqrt{s}}$ where $F = 2(s - m^2)$

Before writing the result of this equation , I defined the $\vec{p} = \frac{s-m^2}{2\sqrt{s}}$.Then the final result in the center of mass frame is:

$\frac{d\sigma}{dt} = \frac{1}{64\pi(s-m^2)^2} \times [\frac{8e^4}{(s-m^2)^2}[m^2(3s + u + m^2) - su] + \frac{8e^4}{(u-m^2)^2}[m^2(s + 3u + m^2) - su] + \frac{16m^2e^4(s+u+2m^2)}{(s-m^2)(u-m^2)}]$

2.2 DCS with respect to θ, $\frac{d\sigma}{d\cos\theta}$

As we know that in the real life ,compton scattering most probably occurs with some angle.So it is better to define our differential scattering cross section with respect to θ instead of t. There are two ways for doing this.

1. By direct conversion of dt into $d\cos\theta$ by using the chain rule
2. By returning to the phase-space integral and computing in the lab frame.

2.2.1 By direct conversion of dt into $d\cos\theta$

In this direct coversion, I used the chain rule :

$\frac{d\sigma}{d\cos\theta} = \frac{dt}{d\cos\theta} \cdot \frac{d\sigma}{dt}$

As I know that from Modern Physics course, the compton scattering formula without some factors is:

$\frac{1}{E_{k2}} - \frac{1}{E_{k1}} = \frac{1}{m}(1 - cos\theta)$

then derive the E_{k2} :

$E_{k2} = \frac{m.E_{k1}}{E_{k1}(1-cos\theta)+m)}$

Then using in the Mandelstam variable t,

$t = (k1 - k2)^2 = -2E_{k1}E_{k2}(1 - cos\theta)$

I rewrote t in terms of cosθ :

$t = \frac{-2mE_{k1}^2(1-cos\theta)}{E_{k1}(1-cos\theta)+m)}$

after that :

$\frac{dt}{dcos\theta} = 2\frac{s-m^2}{2m}$ [5]

and after changing M^2 in terms of ω by using those identities $p1.k1 = m.E_k1$ $p1.k2 = m.E_k2$

$\omega = \frac{E_k2}{E_k1}$:

Finally I reached the solution :

$\frac{d\sigma}{dcos\theta} = \frac{\pi(\alpha\omega)^2}{m^2}(\omega + \frac{1}{\omega} - \sin^2\theta)$ where $M^2 = 2e^4(\omega + \frac{1}{\omega} - \sin^2\theta)$ This is Klein-Nishina Formula

Before I plot the graph, I change my mass variable into compton wavelength

$m^2 = \frac{\hbar}{\lambda \times c}$

2.2.2 By returning to the phase-space integral in Lab Frame

Compton scattering is mostly calculated in the lab frame,in which the electron is initially at rest

- FourMomentum of initially Photon is $k1(E_{k1}, \mathbf{k1})$
- FourMomentum of initially Electron is $p1(m, 0)$
- FourMomentum of final Photon is $k2(E_{k2}, E_{k2}sin\theta, 0, E_{k2}cos\theta)$
- FourMomentum of final Electron is $p2(E_{p2}, \mathbf{p2})$

Express the cross section in terms of $E_{k1}, E_{k2}\theta$

$m^2 = (p2)^2 = (p1 + k1 - k2)^2 = p1^2 + 2p.(k1 - k2) - 2k1.k2$

$=\mathrm{m}^2 + 2m(E_{k1} - E_{k2}) - 2E_{k1}E_{k2}(1 - cos\theta)$ [6]

found similarly :

$\frac{1}{E_{k2}} - \frac{1}{E_{k1}} = \frac{1}{m}(1 - cos\theta)$

then derived the E_{k2} :

$E_{k2} = \frac{m.E_{k1}}{E_{k1}(1-cos\theta)+m)}$

and used in the total cross section integral (2.1) and evaluated to find result.

$$\sigma = \frac{1}{F}\int \frac{d^3k2}{(2\pi)^3 2E_{k2}} \frac{d^3p2}{(2\pi)^3 2E_{p2}} (2\pi)^3 \delta^{(3)}(k1 - k2 - p2) \times (2\pi)\delta(m + E_{k1} - E_{p2} - E_2)M^2$$

$$= \frac{1}{(2\pi)^2 \cdot 4F}\int \frac{E_{k2}^2 dE_{k2} d(\cos\theta) d\phi}{E_{k1}E_{k2}} \delta(m + E_{k1} - (m^2 + E_{k1}^2 + E_{k2}^2 - 2E_{k1}E_{k2}\cos\theta - E_{k2})^{\frac{1}{2}})M^2$$

then differentiate with respect to $\cos(\theta)$:

In the lab frame, $F = 4mE_{k1}$,
so $\frac{d\sigma}{d\cos\theta} = \frac{\omega^2 M^2}{32\pi m^2}$

In the limit $w \to 1$ the cross section becomes :

$\frac{d\sigma}{dcos(\theta)} = \frac{\pi\alpha^2}{m^2}(1 + cos^2(\theta))$;$\sigma = 8\pi\alpha^2/3m^2$

This is Thomson cross section for scattering of classical electromagnetic radiation by a free electron.

Chapter 3

GRAPHICS AND COMMENTS

Differential cross section wrt t plotted against t and differential cross section wrt $\cos\theta$ plotted against $\cos\theta$ (has exactly shape Compton observed in his experiment) using various limiting values of $\sqrt{s}$ for $s = 1.0000099\,m^2$, $s = 2m^2$ and $s = 99999\,m^2$

Hence, If the particles are scattered without any angle $\theta = 0$,differential cross section takes the minimum value so dispersion is small.

In addition,as we know that energy is inversely proportional to wavelength, those graphs is looking similar with CMBR spectrum.It says that there is a minimun point for cross section at a special energy of photon .Note the low-energy limit of Thomson scattering(the elastic scattering of electromagnetic radiation by a free charged particle, as described by classical electromagnetism.The particle kinetic energy and photon frequency are the same before and after the scattering).If the particles have high energy ,they behave under quantum laws ,however if they have low-energy,they behave classically.

$\frac{d\sigma}{dcos\theta} = \frac{\pi(\alpha)^2}{m^2}(\frac{E_{k2}}{E_{k1}})^2(\frac{E_{k2}}{E_{k1}} + \frac{E_{k1}}{E_{k2}} - \sin^2\theta)$ where $M^2 = 2e^4(\omega + \frac{1}{\omega} - \sin^2\theta)$ and $\omega = \frac{E_{k2}}{E_{k1}}$

This is Klein-Nishina Formula

where: $m^2 = \frac{\hbar}{\lambda \times c}$

When $E_{k2} << m$ one obtains the well-known Thomson formula :

$\frac{d\sigma}{dcos(\theta)} = \frac{\pi\alpha^2}{m^2}(1 + cos^2(\theta))$

It is clear that the differential scattering cross section for Thomson Scattering is independent of the frequency of the incident wave whereas Compton is dependent of the frequency of the incident wave, and is also symmetric with respect to forward and backward scattering.

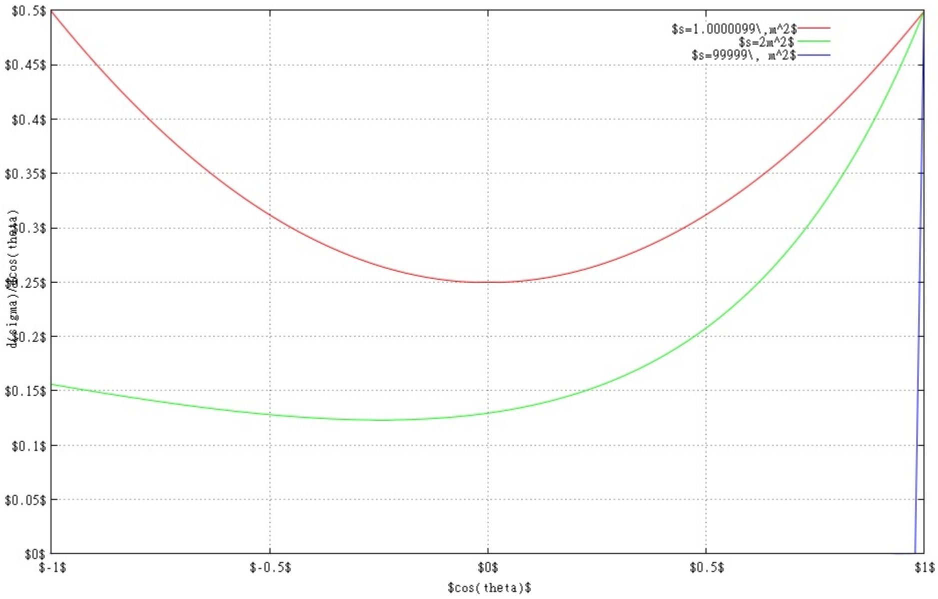

FIGURE 3.1: DCS with respect to $\cos(\theta)$

The classical scattering cross section is modified by quantum effects when the energy of the incident photons, $\hbar\omega$, becomes comparable with the rest mass of the scattering particle, mc^2 . The scattering of a photon by a charged particle is called Compton scattering, and the quantum mechanical version of the Compton scattering cross section is known as the Klein-Nishina formula. As the photon energy increases, and eventually becomes comparable with the rest mass energy of the particle, the Klein-Nishina formula predicts that forward scattering of photons becomes increasingly favored with respect to backward scattering. The Klein-Nishina cross section does, in general, depend on the frequency of the incident photons. Furthermore, energy and momentum conservation demand a shift in the frequency of scattered photons with respect to that of the incident photons.

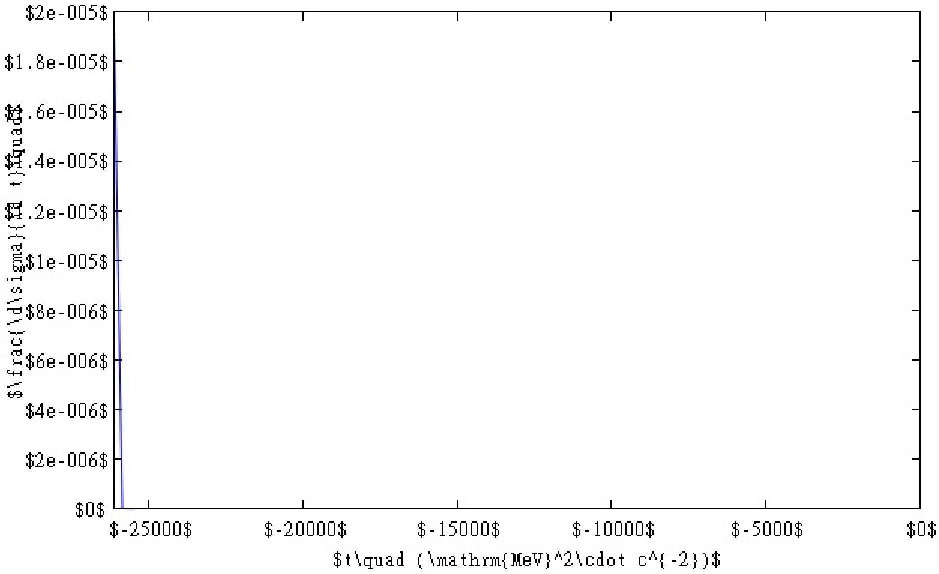

FIGURE 3.2: DCS with respect to t_max

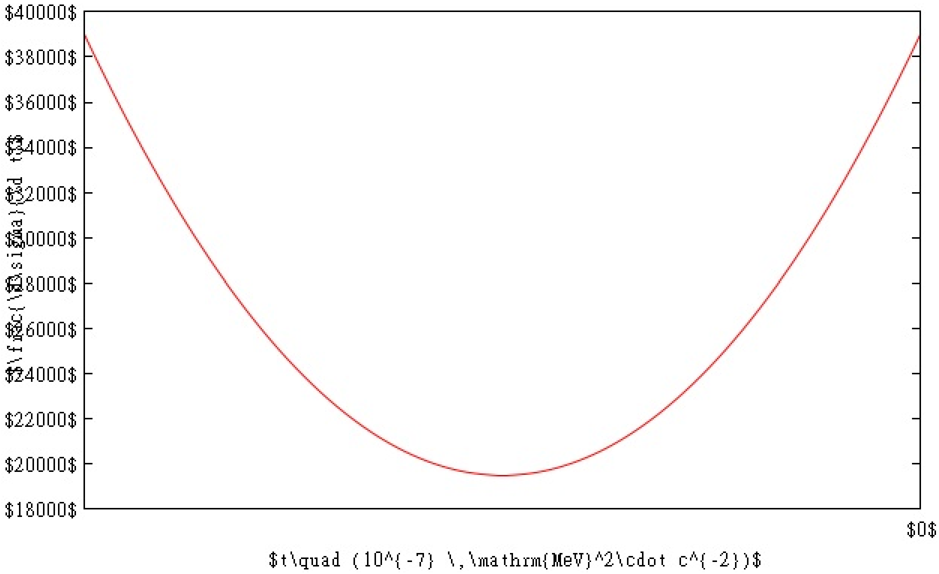

FIGURE 3.3: [DCS with respect to t_min

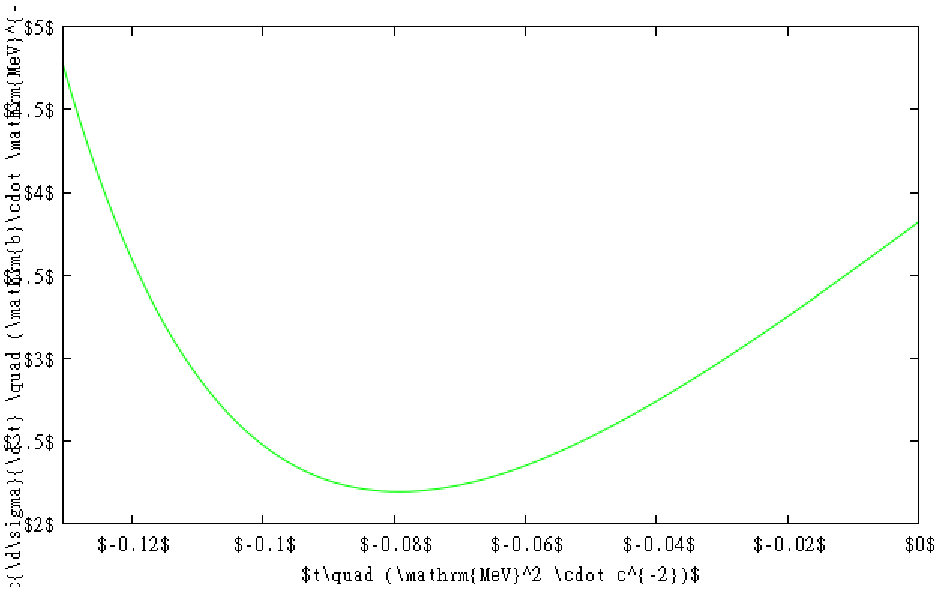

Figure 3.4: DCS with respect to t$_m id$

Bibliography

[1] Arthur H. Compton. A quantum theory of the scattering of x-rays by light elements. *Physical Review*, 21(05), 1923.

[2] Y Klein, O; Nishina. Uber die streuung von strahlung durch freie elektronen nach der neuen relativistischen quantendynamik von dirac. *Z. F. Phys.*, 52(853 869), 1929.

[3] David Griffiths. *Introduction to Elementary Particles*. Number 2. 2008.

[4] F.Mandl G.Shaw. *Quantum Field Theory*. Number 2. 2010.

[5] D.A.Ross. *Quantum Field Theory 1 Lecture Notes*.

[6] D. V. Schroeder M. E. Peskin. *An Introduction to Quantum Field Theory*. Number 5.74. 1995.

[7] http://www.nikhef.nl/ t68/ Jos Vermaseren. *FORM 3.3*. 2009.

[8] http://a-siver.chat.ru/ Andrea Siver, Physics Department of Moscow State University Russia. *For the calculation of trace of Dirac gamma matrices*. 2002.