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Effect of the Gauss-Bonnet parameter in the stability of thin-shell wormholes

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We study the stability of thin-shell wormholes in Einstein–Maxwell–Gauss-Bonnet gravity. The equation of state of the thin-shell wormhole is considered first to obey a generalized Chaplygin gas, and then we generalize it to an arbitrary state function that covers all known cases studied so far. In particular, we study the modified Chaplygin gas and give an assessment for a general parotropic fluid. Our study is in d dimensions, and with numerical analysis in $d = 5$, we show the effect of the Gauss-Bonnet parameter in the stability of thin-shell wormholes against the radial perturbations.

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I. INTRODUCTION

In an attempt to minimize the exotic matter of a traversable wormhole, Matt Visser introduced the concept of a thin-shell wormhole (TSW) [1]. More precisely, in Ref. [2], two copies of the Schwarzschild spacetimes are cut and glued to make the TSW. On the other hand, Brady, Louko, and Poisson studied the stability of a thin shell around a black hole in Ref. [3]. In that work, using the Israel's junction conditions [4], the mechanical stability of a static, spherically symmetric massive thin shell was investigated. Following this work, Poisson and Visser in Ref. [5] considered the stability of the TSW against linearized perturbations around some static spherically symmetric solutions of the Einstein equations. In that paper, in particular, the form of equation of state of the matter that supports the TSW was chosen to be $p = p(\sigma)$, and following the calculation, a parameter $\beta^2(\sigma) \equiv \frac{\partial p}{\partial \sigma}$ that plays an important role for having a stable TSW was defined. Irrespective of the form of $p(\sigma)$, it was shown that $\frac{\partial p}{\partial \sigma}$ at the static configuration, which occurs at $a = a_0$, the equilibrium radius of the throat of the TSW, appears in the final condition. The idea of a TSW and its stability have been developed and generalized in many directions. Ishak and Lake, in their work [6], continued along the previous line by adding the cosmological constant into the solution of the bulk spacetime. Eiroa and Simeone [7] developed the cylindrical TSW, and Lobo studied the phantom wormholes and their stability in Ref. [8], while a TSW in dilaton gravity was introduced in Ref. [9]. A generic, dynamic spherically symmetric thin shell and its corresponding stability was discussed in Ref. [10]. Chaplygin gas traversable wormholes and a generalized Chaplygin gas supported spherically symmetric TSW were discussed in Ref. [11], while a higher-dimensional static spherically symmetric TSW in the Einstein–Maxwell theory was studied by Rahaman, Kalam, and Chakraborty in Ref. [12]. Vacuum thin-shell solutions in five-dimensional Lovelock gravity were studied in Ref. [13]. Extension toward the Einstein–Maxwell–Gauss-Bonnet (EMGB) gravity was investigated in Ref. [14], and its stability and existence of a TSW supported by normal matter was discussed in Ref. [15] .The nonasymptotically flat TSW in the higher-dimensional spherically symmetric Einstein–Yang–Mills theory was considered in Ref. [16] and its extension to Einstein–Yang-Mills–Gauss-Bonnet was given in Ref. [17]. A TSW in Hořava–Lifshitz gravity was introduced in Ref. [18], and a TSW in the Lovelock modified theory of gravity was given in Ref. [19]. In Ref. [20], a rotating TSW in Kerr spacetime was found, and a TSW in Brans–Dicke theory and its stability were investigated in Ref. [21]. Furthermore, a TSW in the Dvali, Gabadadze, and Porrati theory was determined in Ref. [22], while a TSW in the Einstein-nonlinear Maxwell theory was found in Ref. [23].

The above list is not complete, and there are some other works that in some senses generalized the idea of the TSW introduced in Refs. [1,2]. Another form of generalization also is going on parallel to the concept of the TSW, which is the Israel junction conditions [4]. In Ref. [24], the generalized Darmois–Israel boundary conditions were worked out, and using it, generalized junction conditions in Einstein–Gauss-Bonnet (EGB) gravity and in third-order Lovelock gravity were found in Refs. [17,19]. For the whole set of Lovelock theories, the Israel junction conditions were generalized by Gravanisa and Willison in Ref. [25].

Among other aspects, the foremost challenging problems related to the TSW [1–23] are, i) positivity of energy density and ii) stability against symmetry-preserving perturbations. To overcome these problems, recently there have been various attempts in EGB gravity with Maxwell and Yang–Mills sources. Specifically, with the negative Gauss-Bonnet (GB) parameter ($\alpha < 0$), we obtained a stable TSW, obeying a linear equation of state, against radial perturbations [15]. By linear equation of state, it is meant that the energy density σ and surface pressure p satisfy a linear relation. To respond to the other challenge,

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however, i.e., the positivity of the energy density ($\sigma > 0$), we maintain still a cautious optimism. To be realistic, only in the case of Einstein–Yang-Mills–Gauss-Bonnet theory and in a finely tuned narrow band of parameters were we able to beat both of the above-stated challenges [15]. Our stability analysis with the negative energy density was extended further to cover nonasymptotically flat (NAF) dilatonic solutions [16].

In this paper, we show that stability analysis of a TSW extends to the case of a generalized Chaplygin gas (GCG), which has already been considered within the context of Einstein–Maxwell TSWs [4]. Because of the accelerated expansion of our Universe, a repulsive effect of a Chaplygin gas (CG) has been considered widely in recent times. By the same token, therefore, it would be interesting to see how a GCG supports a TSW against radial perturbations in GB gravity. For this purpose, we perturb the TSW radially and reduce the equation into a particle in a potential well problem with zero total energy. The stability amounts to the determination of the positive domain for the second derivative of the potential. We obtain plots that provide us such physical regions indicating stable wormholes. Beside the example of a GCG, we consider an equation of state with its general form. Namely, the relation between the pressure p and the energy density σ is given by the parotropic form $p = \psi(\sigma)$, for an arbitrary function $\psi(\sigma)$. The stability criteria for such a wormhole have been derived as well.

The organization of the paper is as follows. In Sec. II, we introduce our formalism of the TSW in EMGB theory. The stability problem of the obtained TSW supported by GCG is considered in Sec. III. In Sec. IV, we generalize our equation of state further and consider cases other than the GCG. The paper ends with our conclusion in Sec. V.

II. TSW IN EMGB GRAVITY

The d-dimensional EMGB action without cosmological constant

$$S = \frac{1}{16\pi G}\int \sqrt{|g|}d^dx\left(R + \alpha\mathcal{L}_{\mathrm{GB}} - \frac{1}{4}\mathcal{F}\right), \tag{1}$$

where G is the d-dimensional Newton constant, $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant, and α is the GB parameter with Lagrangian

$$\mathcal{L}_{\mathrm{GB}} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \tag{2}$$

The variation of S with respect to $g_{\mu\nu}$ yields the EMGB field equations,

$$G_{\mu\nu} + 2\alpha H_{\mu\nu} = T_{\mu\nu}, \tag{3}$$

in which $H_{\mu\nu}$ and $T_{\mu\nu}$ are given by

$$\begin{aligned} H_{\mu\nu} = 2(&-R_{\mu}{}^{\sigma\kappa\tau}R_{\nu\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} \\ &- 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}) - \frac{1}{2}g_{\mu\nu}\mathcal{L}_{\mathrm{GB}}, \end{aligned} \tag{4}$$

$$T_{\mu\nu} = F_{\mu\alpha}F_{\nu}{}^{\alpha} - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{5}$$

Our static spherically symmetric metric ansatz will be

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-2}, \tag{6}$$

in which

$$\begin{gathered} d\Omega^2_{d-2} = d\theta_1^2 + \sum_{i=2}^{d-2}\prod_{j=1}^{i-1}\sin^2\theta_j d\theta_i^2 \\ 0 \le \theta_{d-2} \le 2\pi, 0 \le \theta_i \le \pi, 1 \le i \le d-3 \end{gathered} \tag{7}$$

and $f(r)$ is to be found.

Construction of the thin-shell wormhole in the static spherically symmetric spacetime follows the standard procedure used before [1–3]. In this method, we consider two copies $\mathcal{M}_{1,2}$ of the spacetime

$$\mathcal{M}_{1,2} = \{(t, r, \theta_1, \ldots, \theta_{d-2})| r \ge a, \quad a > r_h\}, \tag{8}$$

which are geodesically incomplete manifolds for which the boundaries are given by the following timelike hypersurface:

$$\Sigma_{1,2} = \{(t, r, \theta_1, \ldots, \theta_{d-2})| F(r) = r - a = 0, \quad a > r_h\}. \tag{9}$$

By identifying the above hypersurfaces on $r = a$, one gets a geodesically complete manifold $\mathcal{M} = \mathcal{M}_1 \cup \mathcal{M}_2$.

We introduce the induced coordinates on the wormhole $\xi^a = (\tau, \theta_1, \theta_2, \ldots)$—with τ the proper time—in terms of the original bulk coordinates $x^\gamma = (t, r, \theta_1, \ldots, \theta_{d-2})$. Further to the Israel junction conditions [4], the generalized Darmois–Israel boundary conditions [24] are chosen for the case of EMGB modified gravity. The latter conditions on Σ take the form

$$2\langle K_{ab} - Kh_{ab}\rangle + 4\alpha\langle 3J_{ab} - Jh_{ab} + 2P_{acdb}K^{cd}\rangle = -\kappa^2 S_{ab}, \tag{10}$$

in which $\langle . \rangle$ stands for a jump across the hypersurface $\Sigma = \Sigma_1 = \Sigma_2$, $h_{ab} = g_{ab} - n_a n_b$ is the induced metric on Σ with normal vector n_a, and $S_a^b = \mathrm{diag}(\sigma, p_{\theta_1}, p_{\theta_2}, \ldots)$ is the energy-momentum tensor on the thin shell. Therein, the extrinsic curvature $K^{\pm}_{ab}$ (with trace K) is defined as

$$K^{\pm}_{ab} = -n^{\pm}_c\left(\frac{\partial^2 x^c}{\partial\xi^a\partial\xi^b} + \Gamma^c_{mn}\frac{\partial x^m}{\partial\xi^a}\frac{\partial x^n}{\partial\xi^b}\right)_{r=a}. \tag{11}$$

The divergence-free part of the Riemann tensor P_{abcd} and the tensor J_{ab} (with trace J) are given also by

$$P_{abcd} = R_{abcd} + (R_{bc}h_{da} - R_{bd}h_{ca}) - (R_{ac}h_{db} - R_{ad}h_{cb}) + \frac{1}{2}R(h_{ac}h_{db} - h_{ad}h_{cb}), \tag{12}$$

$$J_{ab} = \frac{1}{3}[2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{ab} - K^2K_{ab}]. \tag{13}$$

The black hole solution of the EMGB field equations (with $\Lambda = 0$) is given by [26]

$$f_\pm(r) = 1 + \frac{r^2}{2\tilde{\alpha}} \times \left(1 \pm \sqrt{1 + 4\tilde{\alpha}\left(\frac{2M}{8\pi r^{d-1}} - \frac{Q^2}{2(d-2)(d-3)r^{2(d-2)}}\right)}\right), \tag{14}$$

in which $\tilde{\alpha} = (d-3)(d-4)\alpha$, M is an integration constant related to the Arnowitt-Deser-Misner (ADM) mass of the black hole, and Q is the electric charge of the black hole. (We must comment that in the rest of the paper we assume $\alpha \geq 0$, and the calculations are based on the negative branch solution, i.e., $f(r) = f_-(r)$.) The corresponding electric field 2-form is given by

$$\mathbf{F} = \frac{Q}{r^{2(d-2)}} dt \wedge dr. \tag{15}$$

The components of the energy-momentum tensor on the thin shell are

$$\sigma = -S_\tau^\tau = -\frac{\Delta(d-2)}{8\pi}\left[\frac{2}{a} - \frac{4\tilde{\alpha}}{3a^3}(\Delta^2 - 3(1+\dot{a}^2))\right], \tag{16}$$

$$p = S_{\theta_i}^{\theta_i} = \frac{1}{8\pi}\left\{\frac{2(d-3)\Delta}{a} + \frac{2\ell}{\Delta} - \frac{4\tilde{\alpha}}{3a^2}\left[3\ell\Delta - \frac{3\ell}{\Delta}(1+\dot{a}^2) + \frac{\Delta^3}{a}(d-5) - \frac{6\Delta}{a}\left(a\ddot{a} + \frac{d-5}{2}(1+\dot{a}^2)\right)\right]\right\}, \tag{17}$$

in which $\ell = \ddot{a} + f'_\pm(a)/2$, $\Delta = \sqrt{f_\pm(a) + \dot{a}^2}$, and while a "dot" implies a derivative with respect to the proper time τ, a "prime" denotes differentiation with respect to the argument of the function. These expressions pertain to the static configuration if we consider $a = a_0 =$ constant, and therefore

$$\sigma_0 = -\frac{\sqrt{f_\pm(a_0)}(d-2)}{8\pi} \times \left[\frac{2}{a_0} - \frac{4\tilde{\alpha}}{3a_0^3}(f_\pm(a_0) - 3)\right], \tag{18}$$

$$p_0 = \frac{\sqrt{f_\pm(a_0)}}{8\pi}\left\{\frac{2(d-3)}{a_0} + \frac{f'_\pm(a_0)}{f_\pm(a_0)} - \frac{4\tilde{\alpha}}{3a_0^2}\left[\frac{3}{2}f'_\pm(a_0) - \frac{3f'_\pm(a_0)}{2f_\pm(a_0)} + (d-5)\left(\frac{f_\pm(a_0) - 3}{a_0}\right)\right]\right\}. \tag{19}$$

We add also that in the case of a dynamic throat the conservation equation amounts to

$$\frac{d}{d\tau}(\sigma a^{(d-2)}) + p\frac{d}{d\tau}(a^{(d-2)}) = 0. \tag{20}$$

III. STABILITY OF THE EMGB TSW SUPPORTED BY GCG

Our aim in the following is to perturb the throat of the thin-shell wormhole radially around the equilibrium radius a_0. To do this, we assume that the equation of state is in the form of a GCG [11], i.e.,

$$p = \left(\frac{\sigma_0}{\sigma}\right)^\nu p_0, \tag{21}$$

in which $\nu \in (0, 1]$ is a free parameter and σ_0/p_0 correspond to σ/p at the equilibrium radius a_0. We plug the latter expression into the conservation energy equation (20) to find a closed form for the dynamic tension on the thin shell after perturbation as follows:

$$\sigma(a) = \sigma_0\left[\left(\frac{a_0}{a}\right)^{(1+\nu)(d-2)} + \frac{p_0}{\sigma_0}\left(\left(\frac{a_0}{a}\right)^{(1+\nu)(d-2)} - 1\right)\right]^{\frac{1}{1+\nu}}. \tag{22}$$

Equating this with the one found in Eq. (16), one finds a particlelike equation of motion,

$$\dot{a}^2 + V(a) = 0, \tag{23}$$

which describes the behavior of the throat after the perturbation. The intricate potential $V(a)$ satisfies

$$\sigma = -\frac{\sqrt{f_\pm(a) - V(a)}(d-2)}{8\pi} \times \left[\frac{2}{a} - \frac{4\tilde{\alpha}}{3a^3}(f_\pm(a) + 2V(a) - 3)\right], \tag{24}$$

in which σ is given by Eq. (22). At the static configuration at which $a = a_0$, one can show that $V(a_0) = 0$ and $V'(a_0) = 0$. This implies that Eq. (23) can be expanded about $a = a_0$ such that

$$\dot{x}^2 + \frac{1}{2}V''(a_0)x^2 = 0, \tag{25}$$

in which $x = a - a_0$. The derivative of the latter equation with respect to τ yields

$$\ddot{x} + \frac{1}{2}V''(a_0)x = 0, \tag{26}$$

which upon $V''(a_0) \geq 0$ admits an oscillatory motion or stability of the thin-shell wormhole at $a = a_0$. The exact form of $V''(a_0)$ is given by

$$V''(a_0) = \frac{\mathfrak{B}_1 \nu + \mathfrak{B}_2}{2a_0^2 f_0[3a_0^2 - 2\tilde{\alpha}(3 - f_0)][a_0^2 + 2\tilde{\alpha}(1 + f_0)]}, \tag{27}$$

where

$$\begin{aligned}\mathfrak{B}_1 = 6\Big[&-\frac{2\tilde{\alpha}(d-5)f_0^2}{3} \\ &+ [(-f_0' a_0 + 2(d-5))\tilde{\alpha} + a_0^2(d-3)]f_0 \\ &+ \frac{f_0' a_0(a_0^2 + 2\tilde{\alpha})}{2}\Big] \\ &\times [4f_0^2\tilde{\alpha} + (-2\tilde{\alpha}f_0' a_0 - 2a_0^2 - 12\tilde{\alpha})f_0 \\ &+ f_0' a_0(a_0^2 + 2\tilde{\alpha})]\end{aligned} \tag{28}$$

and

$$\begin{aligned}\mathfrak{B}_2 = {}& -16\tilde{\alpha}^2(d-5)f_0^4 + 8\tilde{\alpha}f_0^3[(\tilde{\alpha}f_0'' - 18 + 4d)a_0^2 + \tilde{\alpha}f_0'(d-7)a_0 + 12\tilde{\alpha}(d-5)] \\ & + \{[(4f_0'^2 - 32f_0'')a_0^2 - 32(d-7)f_0' a_0 - 144(d-5)]\tilde{\alpha}^2 - 16[f_0'' a_0^2 + (d-6)f_0' a_0 + 6(d-4) - 3]a_0^2\tilde{\alpha} \\ & - 12a_0^4\tilde{\alpha}(d-3)\}f_0^2 + 2[3a_0^3 f_0'' + 3(d-3)a_0^2 f_0' - 2\tilde{\alpha}(f_0'^2 - 3f_0'')a_0 + 6\tilde{\alpha}f_0'(d-7)](a_0^2 + 2\tilde{\alpha})a_0 f_0 \\ & - 3a_0^2 f_0'^2(a_0^2 + 2\tilde{\alpha})^2.\end{aligned} \tag{29}$$

Figure 1 depicts a five-dimensional plot of the stable region with respect to a_0 and ν with $M = 20$, $Q = 1$, and variable $\tilde{\alpha}$. The stable regions are indicated by the letter S. As it is displayed in Fig. 1, the stability region has two parts in each case, the area in negative ν and positive ν. The former is almost for $\nu < -1$, which is not a physical state. The latter contains partly the interval $\nu \in (0, 1]$, which is in our interest. We observe that by increasing $\tilde{\alpha}$ this physical stable region develops, and therefore the TSW is more stable. In addition to the stable regions in Fig. 1, we plot the metric function to give an estimation of the location of the horizon for the same parameters.

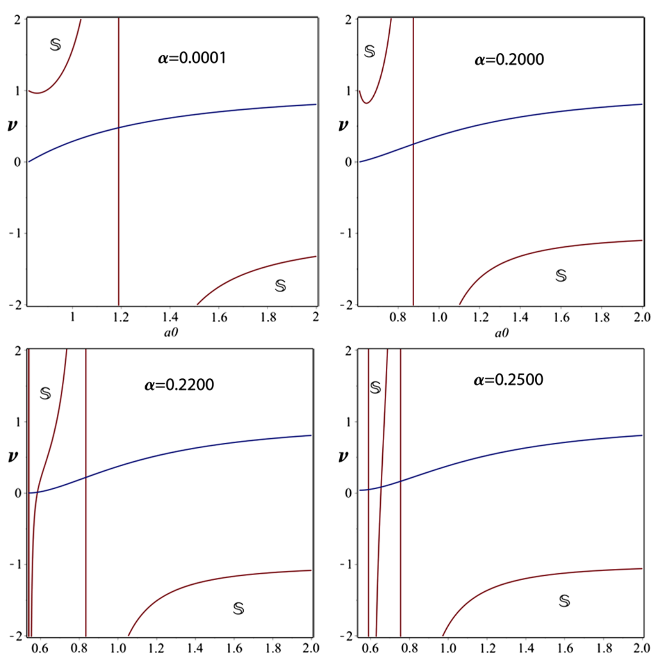

FIG. 1 (color online). Stability region in terms of ν and radius of the throat a_0 for $d = 5$, $M = 20$, $Q = 1$ and various values of α. The stable region is denoted by S. The metric function is also displayed for r larger than the horizon.

IV. STABILITY OF THE EMGB TSW SUPPORTED BY AN ARBITRARY EQUATION OF STATE

In this section, we study the stability of the EMGB TSW, which is supported by an arbitrary gas with the barotropic equation of state

$$p = \psi(\sigma), \tag{30}$$

in which $\psi(\sigma)$ is an arbitrary function of σ. This covers naturally the polytropic equation of state $p \sim \sigma^{1+\frac{1}{n}}$ with the index $0 \leq n < \infty$. As before, we consider the static equilibrium configuration at $a = a_0$, where σ_0 and p_0 are given by Eqs. (18) and (19). Furthermore, the equation of motion of the throat after the perturbation is still given by Eq. (23), where $V(a)$ satisfies the condition (24) in which σ in the left-hand side is the energy density after the perturbation. The form of σ, explicitly, depends on the form of $p = \psi(\sigma)$ and can be found by applying the energy conservation law (20), which is also equivalent with

$$\sigma' = -\frac{d-2}{a}(\sigma + p). \tag{31}$$

Further, one has

$$\sigma'' = -\frac{(d-2)}{a}p' + \frac{(d-1)(d-2)}{a^2}(\sigma + p), \tag{32}$$

in which a prime denotes the derivative with respect to a. Having $p' = \psi'(\sigma)\sigma'$, the latter equation reads

$$\sigma'' = \frac{(d-2)(\sigma+p)}{a^2}[(d-2)\psi'(\sigma) + (d-1)]. \tag{33}$$

Nevertheless, using Eqs. (31) and (33), one can explicitly find the form of $V'(a)$ and $V''(a)$ from Eq. (24) and show that at $a = a_0$, $V(a_0)$ and $V'(a_0)$ vanish while

$$V''(a_0) = \frac{2(d-2)f_0\psi'(\sigma_0)\mathfrak{G}_1 + \mathfrak{G}_2\tilde{\alpha} + 2a_0^2\mathfrak{G}_3}{2a_0^2 f_0[a_0^2 + 2\tilde{\alpha}(1+f_0)]}, \tag{34}$$

in which

$$\mathfrak{G}_1 = 4\tilde{\alpha}f_0^2 - (2\tilde{\alpha}a_0f_0' + 12\tilde{\alpha} + 2a_0^2)f_0 + a_0f_0'(a_0^2 + 2\tilde{\alpha}), \tag{35}$$

$$\begin{aligned}\mathfrak{G}_2 = {}& 8(d-5)f_0^3 + f_0^2[-4a_0^2f_0'' - 4f_0'(d-7)a_0 \\ & - 24(d-5)] + 4a_0\left[\left(f_0'' - \frac{f_0'^2}{2}\right)a_0 + f_0'(d-7)\right]f_0 \\ & - 2a_0^2f_0'^2,\end{aligned} \tag{36}$$

$$\mathfrak{G}_3 = a_0^2\left(f_0f_0'' - \frac{f_0'^2}{2}\right) + (f_0f_0'a_0 - 2f_0^2)(d-3). \tag{37}$$

We note that $\psi'(\sigma_0)(= \frac{p_0'}{\sigma_0'}) = \frac{d\psi}{d\sigma}|_{\sigma=\sigma_0}$, while the other functions are calculated at $a = a_0$. Depending on the form of ψ, we face different TSW. For instance, setting $\frac{d\psi}{d\sigma} = \eta_0 =$ constant reduces to a linear gas supporting TSW with

$$\psi = \eta_0\sigma + C, \tag{38}$$

where C is a constant. Imposing $p(a = a_0) = p_0$ and $\sigma(a = a_0) = \sigma_0$ leads to $C = p_0 - \eta_0\sigma_0$ and therefore

$$\psi = \eta_0(\sigma - \sigma_0) + p_0, \tag{39}$$

which is the case studied in Ref. [27]. Another interesting case is given by $\frac{d\psi}{d\sigma} = -\frac{\eta_0}{\sigma^2}$, giving

$$\psi = \frac{\eta_0}{\sigma} + C, \tag{40}$$

in which C is an integration constant. Again, imposing $p(a = a_0) = p_0$ and $\sigma(a = a_0) = \sigma_0$ dictates that $C = p_0 - \frac{\eta_0}{\sigma_0}$ and therefore

$$\psi = \eta_0\left(\frac{1}{\sigma} - \frac{1}{\sigma_0}\right) + p_0. \tag{41}$$

Setting $p_0 - \frac{\eta_0}{\sigma_0} = 0$ or $\eta_0 = p_0\sigma_0$ implies the well known CG which we have studied in the previous chapter i.e.,

$$\psi = p_0\frac{\sigma_0}{\sigma}. \tag{42}$$

Another important state that has been considered recently is the modified generalized Chaplygin gas (MGCG) obtained by setting

$$\frac{d\psi}{d\sigma} = \xi_0 + \frac{\nu\eta_0}{\sigma^{\nu+1}} \tag{43}$$

$$(\xi_0 = \text{constant}) \tag{44}$$

which implies

$$\psi = \xi_0\sigma - \frac{\eta_0}{\sigma^\nu} + C. \tag{45}$$

Applying $p(a = a_0) = p_0$ and $\sigma(a = a_0) = \sigma_0$ yields $C = p_0 + \frac{\eta_0}{\sigma_0^\nu} - \xi_0\sigma_0$ and consequently

$$\psi = \xi_0(\sigma - \sigma_0) - \eta_0\left(\frac{1}{\sigma^\nu} - \frac{1}{\sigma_0^\nu}\right) + p_0. \tag{46}$$

Setting $C = 0$ or $\eta_0 = \sigma_0^\nu(\xi_0\sigma_0 - p_0)$ simplifies the latter equation as

$$\psi = \xi_0\sigma - \frac{\eta_0}{\sigma^\nu}, \tag{47}$$

which has been studied in Ref. [28]. Figure 2 depicts the effect of the GB parameter on the stability regions of the CG model of the TSW in pure GB gravity (i.e., $Q = 0$). It is observed that increasing the value of the GB parameter decreases the stability areas. Figure 3 displays stability regions as Fig. 2 but with $Q = 1$. Almost the same effect of the GB parameter is seen in this case, too. We note from the standard CG model that $0 < \eta_0$ while the figures are plotted for $-2 < \eta_0 \leq 2$. What we are referring to as the stability region should be understood in this interval.

Figures 4 and 5 are plots of stability regions for TSW in EGB ($Q = 0$) and EMGB ($Q = 1$) supported by MGCG ($\xi_0 \neq 0$, $\eta_0 \neq 0$, $\nu = 1$). Figure 4 should be compared with Fig. 2, and Fig. 5 should be compared with Fig. 3 to see the change of the stability of the TSW in the EGB and EMGB bulk due to MGCG instead of CG. We observe that effects of MGCG become more significant for the regions of stability $r < r_h$ and for the cases that admit no horizon.

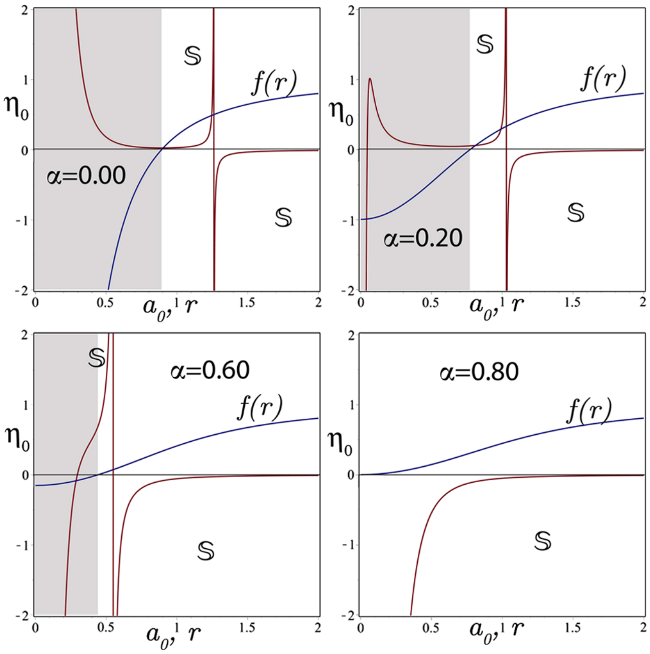

FIG. 2 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for CG ($\nu = 1$, $\xi_0 = 0$), $d = 5$, $M = 20$, $Q = 0$, and various values of α. The stable region is denoted by S, which is identified by $V''(a_0) > 0$, from Eqs. (27)–(29). The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

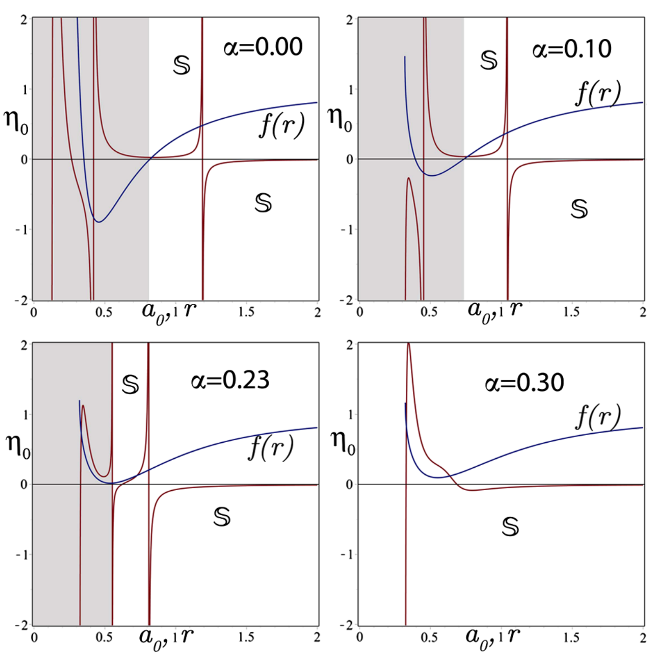

FIG. 3 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for CG ($\nu = 1$, $\xi_0 = 0$), $d = 5$, $M = 20$, $Q = 1$, and various values of α. The stable region is denoted by S. The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

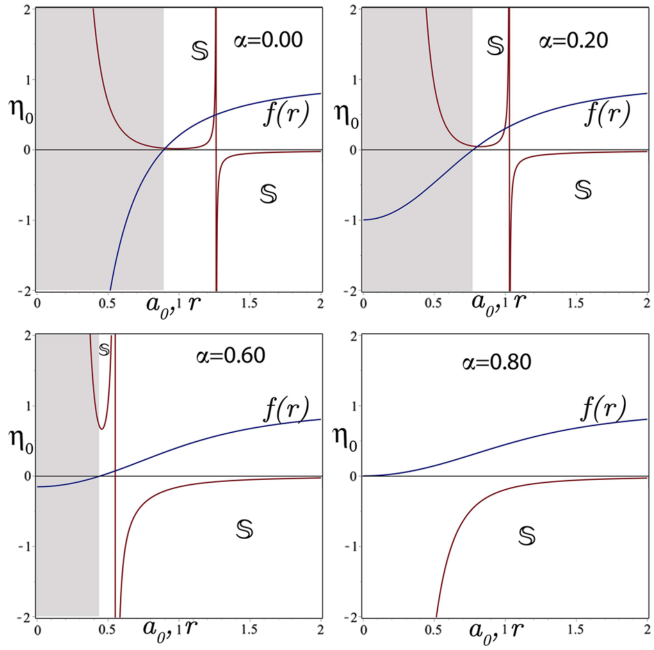

FIG. 4 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for MGCG ($\nu = 1$, $\xi_0 = 1$), $d = 5$, $M = 20$, $Q = 0$, and various values of α. The stable region is denoted by S. The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

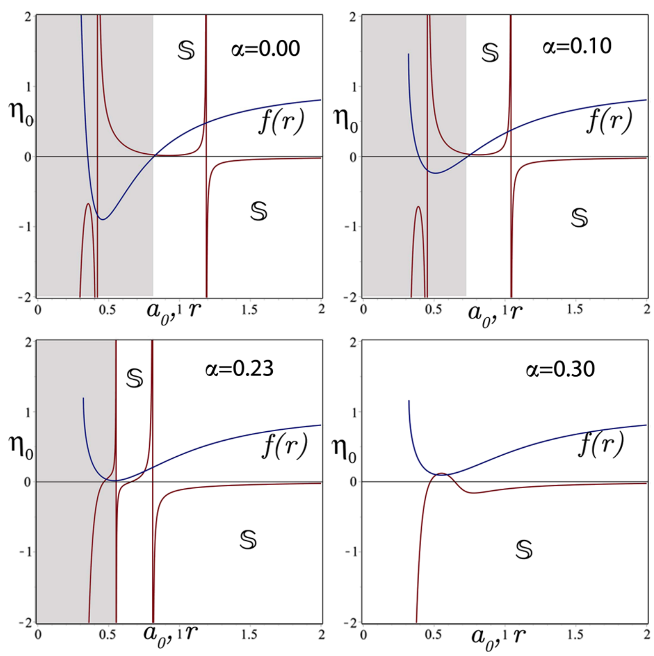

FIG. 5 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for MGCG ($\nu = 1$, $\xi_0 = 1$), $d = 5$, $M = 20$, $Q = 1$, and various values of α. The stable region is denoted by S. The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

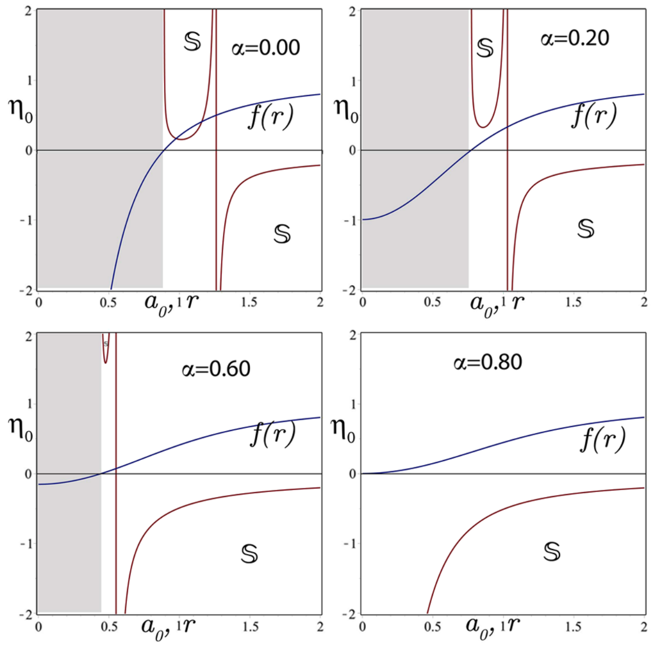

FIG. 6 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for logarithmic gas (LG) for $d = 5$, $M = 20$, $Q = 0$, and various values of α. The stable region is denoted by S. The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

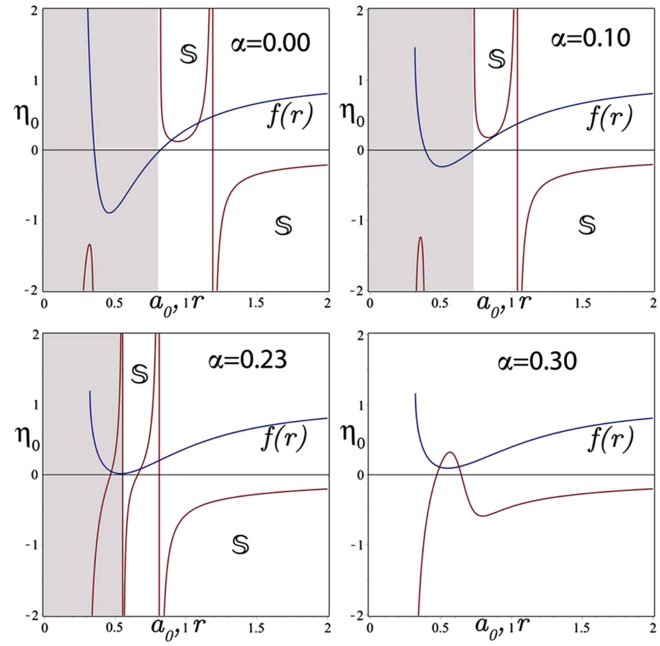

FIG. 7 (color online). Stability region in terms of η_0 and the radius of the throat a_0 for LG for $d = 5$, $M = 20$, $Q = 1$, and various values of α. The stable region is denoted by S. The metric function is also displayed in terms of r. The shaded region is for $r < r_h$ in which r_h is the event horizon.

A. Logarithmic model of gas supporting the TSW in EMGB gravity

As one can see from Eq. (34), in $V''(a_0)$, only $\psi'(\sigma_0)$ appears. In the case of GCG, i.e., $\psi = -\frac{\eta_0}{\sigma^\nu}$ with $0 < \eta_0$ and $0 < \nu \leq 1$, $\psi'(\sigma) = \frac{\nu\eta_0}{\sigma^{\nu+1}}$. We note that the case $\nu = 0$ is excluded; for this reason, separately we consider the case $\nu = 0$ briefly here. When $\nu = 0$, $\psi'(\sigma) = \frac{\eta_0}{\sigma}$, which implies $\psi = \eta_0 \ln|\sigma| + C$. In Figs. 6 and 7, we plot the stability regions of the TSW supported by the logarithmic state equation in EGB and EMGB bulk metrics, respectively.

V. CONCLUSION

In conclusion, for a GCG obeying the equation of state $p = (\frac{\sigma_0}{\sigma})^\nu p_0$, we have found stable regions within a physically acceptable range of parameters in EMGB gravity. The role of GB parameter α in the formation of stable TSW is investigated. It is found that formation of stable regions is highly dependent on the value of α as depicted in our numerical plots. The energy density, however, turns out to be negative to suppress such a TSW as a prominent candidate. Besides, a general equation of state is considered in the form $p = \psi(\sigma)$, which reproduces all known particular cases. It is found that depending on the tuning of the parameters stable regions expand/shrink accordingly. Unfortunately, in all cases tested, one had to be satisfied with a negative energy density as the supporting agent for the TSW in EMGB theory. Finally, we wish to comment that in addition to the classical role played by wormholes their possible quantum roles within the context of the "firewalls paradox" has recently been highlighted [29]. It is speculated that the emitted Hawking particles are entangled through wormholes to the innerhorizon particles of a black hole [30]. Once justified, the subject of wormholes will turn into a hot topic to transcend classical boundaries to occupy a significant role even in quantum gravity.

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