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Higher dimensional Yang–Mills black holes in third order Lovelock gravity

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ABSTRACT

By employing the higher ($N > 5$)-dimensional version of the Wu–Yang ansatz we obtain magnetically charged new black hole solutions in the Einstein–Yang–Mills–Lovelock (EYML) theory with second (α_2) and third (α_3) order parameters. These parameters, where α_2 is also known as the Gauss–Bonnet parameter, modify the horizons (and the resulting thermodynamical properties) of the black holes. It is shown also that asymptotically ($r \to \infty$), these parameters contribute to an effective cosmological constant—without cosmological constant—so that the solution behaves de-Sitter (anti de-Sitter) like.

1. Introduction

As the requirements of string theory/brane world cosmology, higher dimensional ($N > 4$) spacetimes have been extensively investigated during the recent decades. Extensions of $N = 4$ Einstein's gravity has already gained enough momentum from different perspectives. General relativity admits local black hole solutions as well as global cosmological solutions such as de-Sitter (dS) and Anti de-Sitter (AdS), which are important from the field theory (i.e., AdS/CFT) correspondence point of view. Once a pure gravity solution (with or without a cosmological constant) is found the next (routine) step has been to search for the corresponding Einstein–Maxwell (EM) solution with the inclusion of electromagnetic fields. As a result EM dS (AdS) spacetimes have all been obtained and investigated to great extend. One more extension from a different viewpoint, which is fashionable nowadays, is to consider extra terms in the Einstein–Hillbert action such as the ones considered by Lovelock in higher dimensions to obtain solutions, both black holes and cosmological [1]. The added extra terms in the action have the advantage, as they should, that they do not give rise to higher order field equations. It is known that in $N = 4$, the second order (also known as Gauss–Bonnet) Lovelock Lagrangian becomes trivial unless coupled with non-trivial sources such as non-minimal scalar fields [9]. In higher dimensions ($N \geqslant 5$), however coupling with electromagnetic fields proved fruitful and gave rise to interesting black hole/cosmological solutions [2]. In this regard, Brihaye, et al., have worked on particle-like solutions of EYM fields coupled with GB gravity in N-dimensional spherically symmetric spacetime [5]. To have a non-trivial theory with the higher order Lovelock Lagrangian with third order parameter on the other hand, we need the dimensionality of our spacetime to be $N \geqslant 7$. In this Letter we shall follow similar steps, to extend the results of electromagnetic fields to the Yang–Mills (YM) fields with gauge group $SO(N-1)$. As expected, going from Maxwell to YM constitutes a highly non-trivial process originating from the inherent non-linearity of the latter. To cope with this difficulty we employ a particular YM Ansatz solution which was familiar for a long time to the high energy physics community. This is the Wu–Yang ansatz, which was originally introduced in $N = 4$ field theory [3,4]. Recently we have generalized this ansatz to $N = 5$, in the Einstein–Gauss–Bonnet (EGB) theory and obtained a new Einstein–Yang–Mills–Gauss–Bonnet (EYMGB) black hole [5]. By a similar line of thought we wish to extend those results further to $N > 5$ and also within the context (for $N \geqslant 7$) of third order Lovelock gravity. Our results show that both the second (α_2) and third (α_3) order parameters modify the EYM black holes as well as their formation significantly. For instance in the g_{tt} term the gauge charge term comes with the opposite sign and fixed power of $\frac{1}{r^2}$, which is unprecedented in the realm of EM black holes of higher dimensions. This makes construction of black hole types from pure YM charge (with negligible mass, for example) possible and enriches our list of black holes with new properties. What follows for $N \geqslant 7$, for technical reasons, we assume a relation between α_2 and α_3 which are completely free otherwise. Not only black holes but the asymptotical behaviors and properties of our spacetimes are determined by these parameters as well. It is not difficult to anticipate that by the same token the same problem can further be generalized to cover the fourth (α_4), fifth (α_5), etc., order terms in the action to be superimposed to the Einstein–Hillbert (EH) Lagrangian. By studying the relative weight of contribution from higher Lovelock terms it is not difficult to anticipate that the EH and GB terms dominate over the higher order, much more tedious terms. For this reason we restrict ourselves in this Letter to maximum third order (α_3) terms in the Lagrangian.

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2. Action and field equations

The action which describes the third order Lovelock gravity coupled with Yang–Mills field without a cosmological constant in N dimensions reads [1]

$$I_G = \frac{1}{2}\int_{\mathcal{M}} dx^N \sqrt{-g}\left[\mathcal{L}_{\text{EH}} + \alpha_2\mathcal{L}_{\text{GB}} + \alpha_3\mathcal{L}_{(3)} - \operatorname{tr}\left(F^{(a)}_{\mu\nu}F^{(a)\mu\nu}\right)\right], \tag{1}$$

where $\operatorname{tr}(.) = \sum_{a=1}^{(N-1)(N-2)/2}(.)$, $\mathcal{L}_{\text{EH}} = R$ is the Einstein–Hilbert Lagrangian, $\mathcal{L}_{\text{GB}} = R_{\mu\nu\gamma\delta}R^{\mu\nu\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss–Bonnet (GB) Lagrangian, and

$$\begin{aligned}\mathcal{L}_{(3)} = {} & 2R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\rho\tau}R^{\rho\tau}{}_{\mu\nu} + 8R^{\mu\nu}{}_{\sigma\rho}R^{\sigma\kappa}{}_{\nu\tau}R^{\rho\tau}{}_{\mu\kappa} \\ & + 24R^{\mu\nu\sigma\kappa}R_{\sigma\kappa\nu\rho}R^{\rho}{}_{\mu} + 3RR^{\mu\nu\sigma\kappa}R_{\sigma\kappa\mu\nu} \\ & + 24R^{\mu\nu\sigma\kappa}R_{\sigma\mu}R_{\kappa\nu} + 16R^{\mu\nu}R_{\nu\sigma}R^{\sigma}{}_{\mu} \\ & - 12RR^{\mu\nu}R_{\mu\nu} + R^3,\end{aligned} \tag{2}$$

is the third order Lovelock Lagrangian. Here R, $R_{\mu\nu\gamma\delta}$ and $R_{\mu\nu}$ are the Ricci Scalar, Riemann and Ricci tensors respectively, while the gauge fields $F^{(a)}_{\mu\nu}$ are

$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu + \frac{1}{2\sigma}C^{(a)}_{(b)(c)}A^{(b)}_\mu A^{(c)}_\nu, \tag{3}$$

where $C^{(a)}_{(b)(c)}$ are the structure constants of $\frac{(N-1)(N-2)}{2}$-parameter Lie group G, σ is a coupling constant, $A^{(a)}_\mu$ are the gauge potentials, and α_2 and α_3 are GB and third order Lovelock coefficients. Variation of the action with respect to the spacetime metric $g_{\mu\nu}$ yields the Einstein–Yang–Mills–Gauss–Bonnet–Loelock (EYMGBL) equations

$$G^E_{\mu\nu} + \alpha_2 G^{\text{GB}}_{\mu\nu} + \alpha_3 G^{(3)}_{\mu\nu} = T_{\mu\nu}, \tag{4}$$

where the stress–energy tensor is

$$T_{\mu\nu} = \operatorname{tr}\left[2F^{(a)\lambda}_{\mu}F^{(a)}_{\nu\lambda} - \frac{1}{2}F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma}g_{\mu\nu}\right], \tag{5}$$

$G^E_{\mu\nu}$ is the Einstein tensor, while $G^{\text{GB}}_{\mu\nu}$ and $G^{(3)}_{\mu\nu}$ are given explicitly as [2]

$$\begin{aligned}G^{\text{GB}}_{\mu\nu} = {} & 2\left(-R_{\mu\sigma\kappa\tau}R^{\kappa\tau\sigma}{}_{\nu} - 2R_{\mu\rho\nu\sigma}R^{\rho\sigma} - 2R_{\mu\sigma}R^{\sigma}{}_{\nu} + RR_{\mu\nu}\right) \\ & - \frac{1}{2}\mathcal{L}_{\text{GB}}g_{\mu\nu},\end{aligned} \tag{6}$$

$$\begin{aligned}G^{(3)}_{\mu\nu} = {} & -3\left(4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda}_{\nu\tau\mu} - 8R^{\tau\rho}{}_{\lambda\sigma}R^{\sigma\kappa}{}_{\tau\mu}R^{\lambda}{}_{\nu\rho\kappa}\right. \\ & + 2R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\lambda\rho}R^{\lambda\rho}{}_{\tau\mu} - R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\rho}R_{\nu\mu} \\ & + 8R^{\tau}{}_{\nu\sigma\rho}R^{\sigma\kappa}{}_{\tau\mu}R^{\rho}{}_{\kappa} + 8R^{\sigma}{}_{\nu\tau\kappa}R^{\tau\rho}{}_{\sigma\mu}R^{\kappa}{}_{\rho} \\ & + 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\mu\rho}R^{\rho}{}_{\tau} - 4R_{\nu}{}^{\tau\sigma\kappa}R_{\sigma\kappa\tau\rho}R^{\rho}{}_{\mu} \\ & + 4R^{\tau\rho\sigma\kappa}R_{\sigma\kappa\tau\mu}R_{\nu\rho} + 2RR_{\nu}{}^{\kappa\tau\rho}R_{\tau\rho\kappa\mu} + 8R^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\sigma}R^{\rho}{}_{\tau} \\ & - 8R^{\sigma}{}_{\nu\tau\rho}R^{\tau}{}_{\sigma}R^{\rho}{}_{\mu} - 8R^{\tau\rho}{}_{\sigma\mu}R^{\sigma}{}_{\tau}R_{\nu\rho} - 4RR^{\tau}{}_{\nu\mu\rho}R^{\rho}{}_{\tau} \\ & \left. + 4R^{\tau\rho}R_{\rho\tau}R_{\nu\mu} - 8R^{\tau}{}_{\nu}R_{\tau\rho}R^{\rho}{}_{\mu} + 4RR_{\nu\rho}R^{\rho}{}_{\mu} - R^2R_{\nu\mu}\right) \\ & - \frac{1}{2}\mathcal{L}_{(3)}g_{\mu\nu}.\end{aligned} \tag{7}$$

Variation of the action with respect to the gauge potentials $A^{(a)}_\mu$ yields the Yang–Mills equations

$$F^{(a)\mu\nu}_{;\mu} + \frac{1}{\sigma}C^{(a)}_{(b)(c)}A^{(b)}_\mu F^{(c)\mu\nu} = 0, \tag{8}$$

while the integrability conditions are

$$*F^{(a)\mu\nu}_{;\mu} + \frac{1}{\sigma}C^{(a)}_{(b)(c)}A^{(b)}_\mu * F^{(c)\mu\nu} = 0, \tag{9}$$

in which * means duality [6].

3. Wu–Yang Ansatz in $N > 5$ dimensions

The N-dimensional line element is chosen as

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2_{N-2}, \tag{10}$$

in which the S^{N-2} line element will be expressed in the standard spherical form

$$d\Omega^2_{N-2} = d\theta_1^2 + \sum_{i=2}^{N-3}\prod_{j=1}^{i-1}\sin^2\theta_j\,d\theta_i^2,$$
$$0 \leqslant \theta_1 \leqslant \pi,\ 0 \leqslant \theta_i \leqslant 2\pi. \tag{11}$$

We use the Wu–Yang ansatz [7] in N-dimensional case as

$$A^{(a)} = \frac{Q}{r^2}(x_i dx_j - x_j dx_i), \quad Q = \text{charge},\ r^2 = \sum_{i=1}^{N-1}x_i^2,$$
$$2 \leqslant j+1 \leqslant i \leqslant N-1, \quad \text{and} \quad 1 \leqslant a \leqslant (N-1)(N-2)/2, \tag{12}$$

where we imply (to have a systematic process) that the super indices a is chosen according to the values of i and j in order.

The YM field 2-forms are defined as follow

$$F^{(a)} = dA^{(a)} + \frac{1}{2Q}C^{(a)}_{(b)(c)}A^{(b)} \wedge A^{(c)}. \tag{13}$$

We note that our notation follows the standard exterior differential forms, namely d stands for the exterior derivative while $\wedge$ stands for the wedge product [7]. The integrability conditions

$$dF^{(a)} + \frac{1}{Q}C^{(a)}_{(b)(c)}A^{(b)} \wedge F^{(c)} = 0, \tag{14}$$

are easily satisfied by using (12). The YM equations

$$d * F^{(a)} + \frac{1}{Q}C^{(a)}_{(b)(c)}A^{(b)} \wedge *F^{(c)} = 0, \tag{15}$$

are also satisfied. The energy–momentum tensor (5), becomes after

$$\operatorname{tr}\left[F^{(a)}_{\lambda\sigma}F^{(a)\lambda\sigma}\right] = \frac{(N-3)(N-2)Q^2}{r^4}, \tag{16}$$

with the non-zero components

$$T^a{}_b = \frac{(N-3)(N-2)Q^2}{2r^4}\operatorname{diag}[-1,-1,\kappa,\kappa,\ldots,\kappa],$$
$$\text{and} \quad \kappa = -\frac{N-6}{N-2}. \tag{17}$$

The EYMGBL equations (4) reduce to the general equation

$$\begin{aligned}& \left(r^5 - 2\tilde\alpha_2 r^3\left(f(r)-1\right) + 3\tilde\alpha_3 r\left(f(r)-1\right)^2\right)f'(r) \\ & \quad + (n-1)r^4\left(f(r)-1\right) - (n-3)\tilde\alpha_2 r^2\left(f(r)-1\right)^2 \\ & \quad + (n-5)\tilde\alpha_3\left(f(r)-1\right)^3 + (n-1)r^2Q^2 = 0,\end{aligned} \tag{18}$$

in which a prime denotes derivative with respect to r, $n = N-2$, $\tilde\alpha_2 = (n-1)(n-2)\alpha_2$ and $\tilde\alpha_3 = (n-1)(n-2)(n-3)(n-4)\alpha_3$. This equation is valid for $N \geqslant 4$ (i.e., $n \geqslant 2$), but for $N = 4$ (i.e., $n = 2$) we get

$$r^3 f'(r) + r^2\left(f(r)-1\right) + Q^2 = 0, \tag{19}$$

which clearly is $\alpha_{2,3}$ independent and therefore it will be the Einstein–Yang–Mills equation admitting the well-known Reissner–Nordström form

$$f(r) = 1 - \frac{2m}{r} + \frac{Q^2}{r^2}. \tag{20}$$

For $N = 5$ (i.e. $n = 3$) Eq. (18) has already been considered in [5]. We note that in Eq. (18), r, $\tilde\alpha_2$ and $\tilde\alpha_3$ can all be scaled by Q (i.e. $r \to |Q|r$, $\tilde\alpha_2 \to Q^2\tilde\alpha_2$ and $\tilde\alpha_3 \to Q^4\tilde\alpha_3$, as long as $Q \neq 0$) so that we set in the sequel, without loss of generality, $Q = 1$.

4. The EYMGB case, $\alpha_2 \neq 0$, $\alpha_3 = 0$

Eq. (18) with $\alpha_3 = 0$ takes the form

$$\left(r^3 - 2\tilde{\alpha}_2 r\left(f(r) - 1\right)\right) f'(r) + (n-1)r^2\left(f(r) - 1\right) - (n-3)\tilde{\alpha}_2\left(f(r) - 1\right)^2 + (n-1) = 0, \quad (21)$$

which may be called as Einstein–Gauss–Bonnet–Yang–Mills (EGBYM) equation. This equation admits a general solution in any arbitrary dimensions N as follows

$$f(r) = 1 + \frac{r^2}{2\tilde{\alpha}_2}\left(1 \pm \sqrt{1 + \frac{4\tilde{\alpha}_2 m}{r^{n+1}} + \frac{4\tilde{\alpha}_2(n-1)}{(n-3)r^4}}\right), \quad n > 3, \quad (22)$$

where m is the usual integration constant to be identified as mass.

4.1. The EYMGB solution in 6-dimensions

In this section we shall explore some physical aspects of the solution (22) in 6-dimensions. This is interesting for the reason that, $N = 5$ and $N = 6$ are the only dimensions which will not be effected by the non-zero third order Lovelock gravity. For $N = 6$ ($n = 4$), the metric function $f(r)$ in Eq. (22) takes the form

$$f_{\pm}(r) = 1 + \frac{r^2}{2\tilde{\alpha}_2}\left(1 \pm \sqrt{1 + \frac{4\tilde{\alpha}_2 m}{r^5} + \frac{12\tilde{\alpha}_2}{r^4}}\right), \quad (23)$$

in which $\tilde{\alpha}_2 (= 6\alpha_2)$ and $\pm$ refer to the two different branches of the solution. Asymptotic behaviors of $f_{\pm}(r)$ can be shown to be as

$$\lim_{r\to\infty} f_+(r) \to 1 + \frac{r^2}{\tilde{\alpha}_2}, \quad \text{and} \quad \lim_{r\to\infty} f_-(r) \to 1,$$

which imply that, the positive branch is Asymptotically-de-Sitter (A-dS) with positive α_2 and Asymptotically-Anti-de-Sitter (A-AdS) with negative α_2. It is seen obviously that the negative branch is an Asymptotically Flat (A-F) space. One can also show that

$$\lim_{r\to 0^+} f_+(r) \to +\infty, \quad \text{and} \quad \lim_{r\to 0^+} f_-(r) \to -\infty,$$

which clearly, shows that, $f_+(r)$ is an A-dS solution while $f_-(r)$ represents an A-F black hole solution. In the sequel we shall consider $\alpha_2 > 0$ with the negative branch of the solution (i.e., the A-F black hole solution). One can easily show that, this solution admits a single horizon (i.e., event horizon) given by

$$r_+ = \left(\frac{m}{2} + \sqrt{\left(\frac{m}{2}\right)^2 - (1-2\alpha_2)^3}\right)^{1/3} + \left(\frac{m}{2} - \sqrt{\left(\frac{m}{2}\right)^2 - (1-2\alpha_2)^3}\right)^{1/3}, \quad (24)$$

which is real and positive for any values of m and $\tilde{\alpha}_2$. In Fig. 1, (i.e., the dashed curves) we plot the radius of event horizon r_+, in terms of α_2 (i.e., Gauss–Bonnet parameter), for some fixed values for m. This figure displays the contribution of the Gauss–Bonnet parameter to the possible radius of the event horizon of the black hole. By looking at Fig. 1, one may comment that for any value of m, $\lim_{\alpha_2\to\infty} r_+ \to 0$. We notice further that, with $\alpha_2 = 0$, we get the radius of event horizon for the six-dimensional EYM black hole solution which was given in Ref. [5]. That is, for very large α_2, the event horizon coincides with the central singularity. As a particular choice, we consider $\alpha_2 = \frac{1}{2}$ which implies that

$$r_+ = m^{1/3}. \quad (25)$$

The surface gravity, κ defined by [8]

$$\kappa^2 = -\frac{1}{4} g^{tt} g^{ij} g_{tt,i} g_{tt,j}, \quad (26)$$

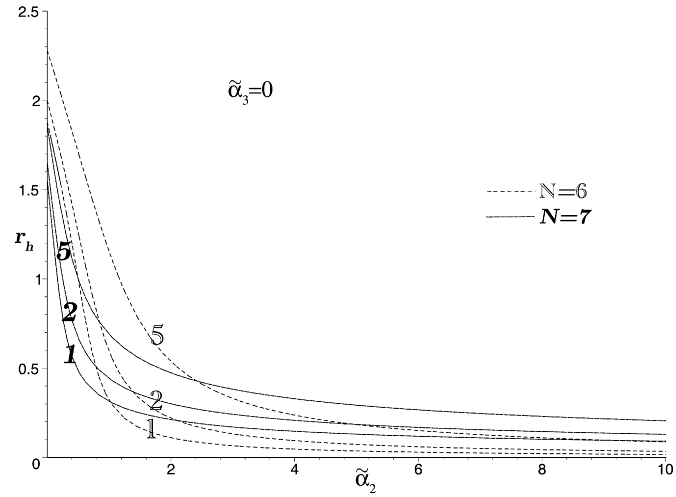

Fig. 1. The horizon radius r_h versus $\tilde{\alpha}_2$ for $\tilde{\alpha}_3 = 0$, and different values for mass (each mass is written on the correspondence curve) in $N = 6$ and $N = 7$.

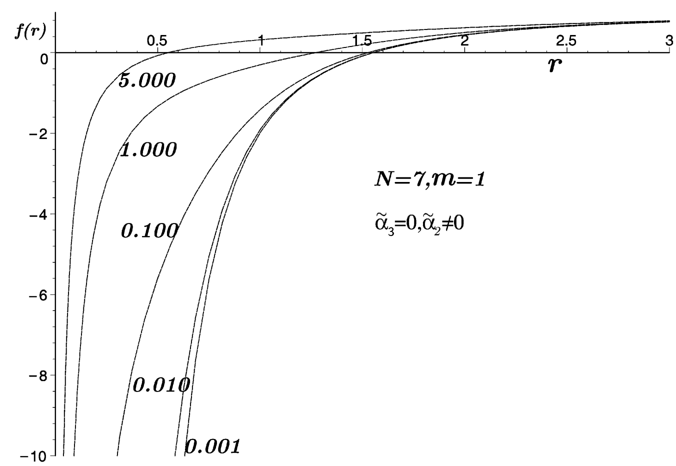

Fig. 2. Plot of the metric function $f(r)$ versus r for $N = 7$, $m = 1$, $\tilde{\alpha}_3 = 0$, $\tilde{\alpha}_2 = 5$, 1, 0.1, 0.01 and 0.001.

takes the value

$$\kappa = \left|\frac{1}{2} f'(r_+)\right| = \frac{3}{2}\frac{m^{1/3}}{m^{2/3} + 6}. \quad (27)$$

The associated Hawking temperature depending on mass m and $\alpha_2 = \frac{1}{2}$ becomes

$$T_H = \frac{\kappa}{2\pi} = \frac{3}{4\pi}\frac{m^{1/3}}{m^{2/3} + 6}, \quad (28)$$

in the choice of units $c = G = \hbar = k = 1$.

4.2. The EYMGB solution in 7-dimensions

In this section we represent some physical aspects of the solution (22) in 7-dimensions. In 7-dimensions, both second and third order Lovelock terms contribute but still we set $\alpha_3 = 0$ in order to identity the contribution of α_2. For $N = 7$ ($n = 5$), the metric function $f(r)$ in Eq. (22) takes the form

$$f_{\pm}(r) = 1 + \frac{r^2}{2\tilde{\alpha}_2}\left(1 \pm \sqrt{1 + \frac{4\tilde{\alpha}_2 m}{r^6} + \frac{8\tilde{\alpha}_2}{r^4}}\right), \quad (29)$$

in which $\tilde{\alpha}_2 (= 12\alpha_2)$ and $\pm$ refers to the two individual branches of the solutions. In Fig. 2 we plot $f_-(r)$ which goes to asymptot-

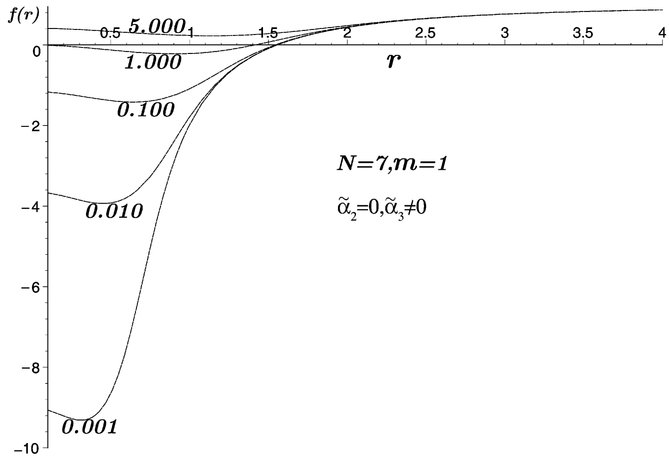

Fig. 3. Plot of the metric function $f(r)$ versus r for $N = 7$, $m = 1$, $\tilde{\alpha}_2 = 0$, $\tilde{\alpha}_3 = 5$, 1, 0.5, 0.1, 0.01 and 0.001.

ically flat value for $r \to \infty$. Asymptotic behaviors of $f_{\pm}(r)$ can by written as

$$\lim_{r\to\infty} f_+(r) \to 1 + \frac{r^2}{\tilde{\alpha}_2}, \quad \text{and} \quad \lim_{r\to\infty} f_-(r) \to 1,$$

which imply that, the positive branch is Asymptotically-de-Sitter (A-dS) for positive α_2 and Asymptotically-Anti de Sitter (A-AdS) for negative α_2, while the negative branch leads to an Asymptotic Flat (A-F) space. Also one can show that

$$\lim_{r\to 0^+} f_+(r) \to +\infty, \quad \text{and} \quad \lim_{r\to 0^+} f_-(r) \to -\infty,$$

which clearly, manifests that, $f_+(r)$ is an A-dS solution while $f_-(r)$ represents an A-F black hole solution. Hence in the sequel we just consider $\alpha_2 > 0$ and the negative branch of the solution (i.e., the A-F black hole solution). One can easily show that, this solution admits only an (event horizon) which can be written as

$$r_+ = \sqrt{1 - 6\alpha_2 + \sqrt{(1 - 6\alpha_2)^2 + 4m}}, \tag{30}$$

which implies that r_+ is real and positive for any values of m and $\tilde{\alpha}_2$. In Fig. 1 we plot the radius of event horizon r_+, in terms of α_2 (i.e., the solid curves), with some fixed values of m. This figure displays the contribution of the Gauss–Bonnet parameter in place of possible radius of the event horizon of the EYM black hole. We notice that, with $\alpha_2 = 0$, we recover the radius of event horizon for the 7-dimensional EYM black hole solution which is given in Ref. [7].

As a particular choice, we consider $\alpha_2 = 1/6$ which implies that

$$r_+ = (4m)^{1/4}, \tag{31}$$

and the surface gravity, (26) in this case has the value

$$\kappa = \left|\frac{1}{2} f'(r_+)\right| = \frac{2m^{1/4}}{m^{1/4} + 4}. \tag{32}$$

With the associated Hawking temperature given by

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{\pi}\frac{m^{1/4}}{m^{1/4} + 4}, \tag{33}$$

which is comparable with the 6-dimensional case (28).

5. The EYML case with, $\alpha_2 = 0, \alpha_3 \neq 0$

In this section we just consider the effect of α_3 on the solution of the field equation. Eq. (18) with $\alpha_2 = 0$ takes the form

$$\begin{aligned}&\left(r^5 + 3\tilde{\alpha}_3 r\left(f(r) - 1\right)^2\right) f'(r) + \nu r^4\left(f(r) - 1\right) \\ &\quad + (\nu - 4)\tilde{\alpha}_3\left(f(r) - 1\right)^3 + \nu r^2 = 0,\end{aligned} \tag{34}$$

in which $\nu = N - 3$ ($= n - 1$ therefore). The proper general solution of this equation is given by

$$f(r) = 1 + \frac{r^4 \Omega^{\frac{1}{3}}}{6\tilde{\alpha}_3(\nu - 2) r^{\frac{(8+\nu)}{3}}} - \frac{2(\nu - 2) r^{\frac{(8+\nu)}{3}}}{\Omega^{\frac{1}{3}}}, \tag{35}$$

in which

$$\Omega = -108A + 12\sqrt{\frac{3}{\tilde{\alpha}_3}\left[27\tilde{\alpha}_3 A^2 + 4(\nu - 2)^2 r^{2\nu+4}\right]},$$

and

$$A = \nu r^{\nu-2} + (\nu - 2)m.$$

One can easily show that in the limit $\tilde{\alpha}_3 \to 0$,

$$\lim_{\tilde{\alpha}_3\to 0} f(r) = 1 - \frac{m}{r^\nu} - \frac{\nu}{(\nu - 2)r^2}, \tag{36}$$

which is the EYM solution.

By setting $\nu = 4$ in Eq. (35) for $N = 7$ one obtains

$$f(r) = 1 + \frac{\Omega^{\frac{1}{3}}}{12\tilde{\alpha}_3} - \frac{4r^4}{\Omega^{\frac{1}{3}}}, \tag{37}$$

in which

$$\Omega = -108\left(4r^2 + 2m\right) + 12\sqrt{\frac{3}{\tilde{\alpha}_3}\left[27\tilde{\alpha}_3\left(4r^2 + 2m\right)^2 + 16r^{12}\right]}. \tag{38}$$

In Fig. 3 we plot the $f(r)$ function versus r depending on different $\tilde{\alpha}_3 \neq 0 = \tilde{\alpha}_2$.

6. The general case, $\alpha_3 \neq 0 \neq \alpha_2$

For $N \geqslant 7$ ($n \geqslant 5$), with $\alpha_3 \neq 0 \neq \alpha_2$ we shall see the role of the third order Lovelock parameters as well as the second order. This leads us to a tedious set of differential equations which fortunately reduces to Eq. (18) and can be integrated exactly. Indeed, the general solution of Eq. (18) with arbitrary values of α_2 and α_3, in any arbitrary dimension $N \geqslant 7$ can be expressed by the following expression

$$\begin{aligned}f(r) = 1 &+ \frac{(4\Omega^2/\Delta)^{\frac{1}{3}}}{6(n+1)(n-3)\tilde{\alpha}_3 r^n} + \frac{\tilde{\alpha}_2 r^2}{3\tilde{\alpha}_3} \\ &+ \frac{r^{n+4}(n-3)(n+1)(\tilde{\alpha}_2^2 - 3\tilde{\alpha}_3)}{6\tilde{\alpha}_3}\left(\frac{16}{\Delta}\right)^{\frac{1}{3}},\end{aligned} \tag{39}$$

in which we use the abbreviations

$$\begin{aligned}\Omega = &-r^{2n+2}(n-3)^2(1+n)^2 \\ &\times \left\{-3\sqrt{\delta}\tilde{\alpha}_3 + \left[-\tilde{\alpha}_2\left(2\tilde{\alpha}_2^2 - 9\tilde{\alpha}_3\right)(1+n)\right.\right. \\ &\times (n-3)r^4 + 27\left(n^2 - 1\right)\tilde{\alpha}_3^2\left.\right]r^n \\ &+ 27\tilde{\alpha}_3^2 m r^3(1+n)(n-3)\left.\right\},\end{aligned} \tag{40}$$

$$\begin{aligned}\delta = 54(1+n)^2&\left\{\frac{3}{2}\tilde{\alpha}_3^2(n-1)^2 r^{2n}\right. \\ &+ \left[(n-1)(n-3)\tilde{\alpha}_2\left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right)r^{2n+4}\right. \\ &+ \left.3\tilde{\alpha}_3^2 m(n-1)(n-3)r^{n+3}\right] + \frac{2}{9}\left(\tilde{\alpha}_3 - \frac{1}{4}\tilde{\alpha}_2^2\right)(n-3)^2 r^{8+2n} \\ &+ \left(\tilde{\alpha}_3 - \frac{2}{9}\tilde{\alpha}_2^2\right)(n-3)^2\tilde{\alpha}_2 m r^{7+n} \\ &+ \left.\frac{3}{2}\tilde{\alpha}_3^2 m r^6(n-3)^2\right\},\end{aligned} \tag{41}$$

$$\Delta = 2(n-3)^2\left\{\frac{3}{2}\sqrt{\delta}\tilde{\alpha}_3 + (1+n) \times\left[\left(-\frac{27}{2}(n-1)\tilde{\alpha}_3^2 - \frac{9}{2}\tilde{\alpha}_2\tilde{\alpha}_3 r^4(n-3) + \tilde{\alpha}_2^3 r^4(n-3)\right)r^n - \frac{27}{2}\tilde{\alpha}_2^3 m r^3(n-3)\right]\right\}r^{2n+2}(1+n)^2. \quad (42)$$

To proceed further with these expressions does not seem feasible from technical points, therefore in the sequel we shall adopt a special simplifying relation between α_2 and α_3.

6.1. The EYML solution in 7-dimensions with $\tilde{\alpha}_3 = \tilde{\alpha}_2^2/3$

In 7-dimensional spacetime we can see the roles of both second and third order Lovelock parameters simultaneously. One of the simplifying, yet interesting case in the solution (39) can be obtained if one sets $\tilde{\alpha}_3 = \frac{\tilde{\alpha}_2^2}{3}$. The metric function $f(r)$ reads rather simple

$$f(r) = 1 + \frac{r^2}{\tilde{\alpha}_2}\left\{1 - \left[1 + \frac{3\tilde{\alpha}_2 m}{r^{n+1}} + \frac{3(n-1)\tilde{\alpha}_2}{(n-3)r^4}\right]^{\frac{1}{3}}\right\}, \quad (43)$$

which yields an asymptotically flat metric. For $r \to \infty$ in order to have black hole solutions one should investigate the existence of roots of the metric function (i.e. $f(r) = 0$). To this end, one finds the solutions of the following equation

$$r^{n+1}\left[r^4 + \left(\tilde{\alpha}_2 - \frac{n-1}{n-3}\right)r^2 + \frac{1}{3}\tilde{\alpha}_2^2\right] - mr^6 = 0, \quad (44)$$

which generally seems a difficult task. But in seven dimensions we get the following roots

$$r_\pm = \left[\left(1 - \frac{\tilde{\alpha}_2}{2}\right) \pm \sqrt{\left(1 - \frac{\tilde{\alpha}_2}{2}\right)^2 + \left(m - \frac{\tilde{\alpha}_2^2}{3}\right)}\right]^{\frac{1}{2}}, \quad (45)$$

in which for $1 > \sqrt{\frac{\tilde{\alpha}_2}{2}}$ and $[\frac{\tilde{\alpha}_2^2}{3} - (1 - \frac{\tilde{\alpha}_2}{2})^2] < m < \frac{\tilde{\alpha}_2^2}{3}$, so that we have both inner and outer radii of black hole. In the upper and lower limits of m, we will get just the radius of the event horizon of the black hole solutions, i.e. when $m = \frac{\tilde{\alpha}_2^2}{3}$, then $r_+ = \sqrt{2 - \tilde{\alpha}_2}$ while $r_- = 0$. On the other hand when $m = \frac{\tilde{\alpha}_2^2}{3} - (1 - \frac{\tilde{\alpha}_2}{2})^2$, $r_+ = \sqrt{1 - \frac{\tilde{\alpha}_2}{2}}$ while r_- does not exist. For the choice $\tilde{\alpha}_2 = 2$, we can define a critical mass, $m_{\text{crit}} = 4/3$, so that for $m > m_{\text{crit}}$ we can have a black hole solution.

The surface gravity in $N = 7$ has the form

$$\kappa = \left|\frac{1}{2}f'(r_+)\right| = \left|\frac{r_+}{\tilde{\alpha}_2}\left(1 - \frac{1}{\Delta^{(2/3)}}\right) - \frac{2}{r_+^3\Delta^{(2/3)}}\right|, \quad (46)$$

in which

$$\Delta = 1 + \frac{3\tilde{\alpha}_2 m}{r_+^6} + \frac{6\tilde{\alpha}_2}{r_+^4},$$

where r_+ is the radius of event horizon of the possible black hole. For instance when $\tilde{\alpha}_2 = 2$, (i.e. $r_+ = (m - \frac{4}{3})^{1/4}$) one gets

$$\kappa = \left|9\sqrt{3}\frac{(3m-4)^2[(3m-4)\Xi^{2/3} - (3m+4)]}{2(9m-12)^{11/4}\Xi^{2/3}}\right|, \quad (47)$$

in which

$$\Xi = 1 + \frac{162m}{(9m-12)^{3/2}} + \frac{36}{(3m-4)}.$$

The associated Hawking temperature $T_H = \frac{\kappa}{2\pi}$ can be found by using the above result, on which one may expect that, T_H is strongly $\tilde{\alpha}_2$ (and $\tilde{\alpha}_3$) dependent.

6.2. Asymptotically dS (AdS) property

The general solution (39) as $r \to \infty$ reads

$$f_\infty(r) \cong 1 + \left[\left(\frac{\tilde{\alpha}_2^2}{3\tilde{\alpha}_3} - 1\right)\left(\frac{2}{\Sigma}\right)^{1/3} + \frac{1}{3\tilde{\alpha}_3}\left(\frac{\Sigma}{2}\right)^{1/3} + \frac{\tilde{\alpha}_2}{3\tilde{\alpha}_3}\right]r^2, \quad (48)$$

where

$$\Sigma = \left[3\tilde{\alpha}_3\sqrt{3\left(4\tilde{\alpha}_3 - \tilde{\alpha}_2^2\right)} + 2\tilde{\alpha}_2^3 - 9\tilde{\alpha}_2\tilde{\alpha}_3\right].$$

It is observed that the metric function (39), can be rewritten as a N $(n = N - 2)$ dimensional dS spacetime in which $f_\infty(r) \cong 1 - \frac{2\tilde{\Lambda}}{(n-3)(n-4)}r^2$ one defines a cosmological constant without cosmological constant $\tilde{\Lambda}$ as an effective cosmological constant given by

$$\tilde{\Lambda} = -\frac{(n-3)(n-4)}{2}\left[\left(\frac{\tilde{\alpha}_2^2}{3\tilde{\alpha}_3} - 1\right)\left(\frac{2}{\Sigma}\right)^{1/3} + \frac{1}{3\tilde{\alpha}_3}\left(\frac{\Sigma}{2}\right)^{1/3} + \frac{\tilde{\alpha}_2}{3\tilde{\alpha}_3}\right]. \quad (49)$$

It is seen that in this effective cosmological constant both α_2, and α_3 play role. One can easily show that, depending on $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$, $\tilde{\Lambda}$ can take zero, positive or negative values, and consequently the general solution becomes asymptotically flat, dS or AdS, respectively. For instance, from Eq. (43), one can show that the choice $\tilde{\alpha}_3 = \tilde{\alpha}_2^2/3$ is asymptotically flat.

These results verify that, the Lovelock parameters $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ significantly modify the properties of EYM black holes as well as their asymptotic behaviors.

7. Conclusion

We introduced YM fields through the Wu–Yang ansatz into the third order Lovelock gravity with spherical symmetry. The ansatz and symmetry aided in overcoming the technical difficulty and obtaining exact solutions in higher dimensions. In this sense our work is partly an extension of our previous work which included only the GB parameter and for $N = 5$ [5]. Our solutions include black hole possessing parameters of mass, magnetic charge (which is scaled to $Q = 1$), α_2 (for $N \geqslant 5$) and α_3 (for $N \geqslant 7$). Our analysis indicates that higher order Lovelock parameters have less significant contributions compared with the lower order terms superposed to the EH Lagrangian. At least this is the picture that we expect in case that α_k $(k \geqslant 4)$ terms are taken into account. Depending on the choice/relative magnitudes of the parameters, formation of the black holes with inner/outer horizons is conditional. Asymptotic solutions give rise to dS (AdS) or flat spacetimes which are topologically trivial but from the field theoretic point of view they are rather important.

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