Studies on Different Types of Facility Layout Problems

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ABSTRACT

Facility layout problems are applied to find the best arrangement of facilities in manufacturing and service environments. The main goal of these problems is to minimize the total weighted travelled distance of the facilities by the travelled frequency of them. The difficult part is how to measure these distances. A frequently used approximation is the Manhattan distance. However, it is significantly shorter than the real distance in many cases. This thesis suggests an exact mathematical model for closed loop layout that uses real distances instead of Manhattan distance. Many feasible solutions are generated for benchmark problems that are competitive with the solutions provided by metaheuristics. A generalization of multi-dimensional scaling (MDS) method is developed to reconstruct the layout problems from their distance matrix. MDS is a well-known method used in statistics to explore the hidden dependency among data. The reconstruction done by MDS is completely successful if the distance used in layout problems is of Euclidean type. Therefore the generalized MDS provides the opportunity to reconstruct the layout problems with any distance type. The results show that only the Quadratic Assignment Problems which are the models of real layout problems can be reconstructed successfully. The thesis also suggests a mathematical model based on Travelling Salesman Problem and its Dantzig-Fulkerson-Johnson formulation to rearrange the departments of a supermarket in order to increase the travelled path of customers and motivate them to buy more items. The study was done in one of the biggest supermarket chain of Hungary by considering the purchasing items of more than 13,000 customers. The

computational experiences show that the total travelled distance can be increased by approximately 4 percent.

Keywords: Facility layout problem, Quadratic assignment problem, Multi-dimensional scaling, Mixed integer linear model, Supermarket layout.

ÖZ

Tesis içi yerleşim, tesislerin üretim ve hizmet ortamlarında en iyi iç düzenlemesini bulmak için uygulanmaktadır. Bu problemlerin temel amacı seyahat sıklığına göre tesislerin toplam ağırlıklı seyahat mesafesini en aza indirmektir. Zor olan kısmı bu mesafelerin nasıl ölçüldüğü ile bağlantılıdır. Manhattan mesafesi sık kullanılan bir yaklaşıklamadır. Ancak birçok durumda gerçek mesafeden anlamlı derecede kısadır. Bu tez, kapalı döngü iç yerleşimi için Manhattan mesafesi yerine gerçek mesafe kullanmakta olan kesin sonuç veren bir matematiksel model önermektedir. Birçok uygulanabilir çözüm sezgi ötesi yöntemlerle sağlanan çözümlere rakip olabilecek denektaşı problemler için oluşturulmuştur. Genelleştirilmiş bir çok boyutlu ölçekleme metodu, tesis içi yerleşim problemlerini mesafe matrislerinden yeniden kurmak için geliştirilmiştir. Çok boyutlu ölçekleme, veriler arasındaki gizli bağlantıyı keşfetmek için istatistikte kullanılan bilinen bir yöntemdir. Çok boyutlu ölçekleme ile yeniden kurma, iç yerleşim probleminin öklit türü olması durumunda tamamen başarılıdır. Bu nedenle genelleştirilmiş çok boyutlu ölçekleme herhangi bir mesafe tipi olan iç yerleşim problemlerinin yerinden kurulması fırsatı yaratır. Sonuçlar göstermektedir ki sadece gerçek iç yerleşim problemlerinin modeli olan karesel atama problemleri başarılı bir şekilde yeniden kurulabilmektedir. Bu tez ayni zamanda müşterilerin süpermarkette daha fazla ürün almaları yönünde motive olmaları amacıyla katettikleri mesafeyi artırmak için gezgin satıcı problemi ve onun Dantzig-Fulkerson-Johnson biçimlendirmesini baz alarak süpermarket departmanlarının yeniden düzenlemesini sağlayan bir matematiksel model önermektedir. Bu çalışma Macaristanda bulunan en büyük süpermarket zincirlerinden birinde, 13000'den fazla müşterinin satın aldığı ürünler dikkate alınarak yapılmıştır. Hesaplamalı denemeler göstermektedir ki toplam seyahat edilen mesafe yaklaşık yüzde 4 oranında artırılabilmektedir.

Anahtar Kelimeler: Tesis içi yerleşim problemi, Karesel atma problemi, Çok boyutlu ölçekleme, Karma tamsayılı doğrusal model, Süpermarket iç yerleşimi.

To My

Dear ***Family***

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Chapter 1

INTRODUCTION

Facility planning consists of the location and the design of facilities which called facilities location and facilities layout respectively. Facility location problem analyzes and compares the alternative places for establishing the facility based on availability of some factors e.g. market, workforce, resources, etc. On the other hand, facility layout problem focuses on the optimal arrangement of departments of a facility after its location is found. This thesis focuses on facility layout problems in manufacturing and service sectors.

The main purpose in facility layout problems of manufacturing sector is minimization of material handling cost. Approximately 20-50 percent of operating cost in manufacturing is related to material handling and layout costs (Tompkins et al., 1996). In the last two decades, several mathematical models as facility layout problem were created to decrease the material handling cost in manufacturing systems. Facility layout problems generally can be classified to two types,

- Special facility layout problems such that mathematical model should be designed for them (positions are not determined)
- Facility layout problems which use Quadratic Assignment Problem as mathematical model (positions are determined).

First class of facility layout problems can be classified as either (i) a general Facility Layout Problem (FLP), which only considers each department's area and the determination of the shapes of the cells is the part of the problem, or (ii) a Machine Layout Problem (MLP), which considers each department's/machine's specific shape (Chae and Peters (2006)). In the manufacturing sector, this thesis addresses problem (ii), i.e., the MLP in a flexible manufacturing system environment.

There are four commonly used Flexible Manufacturing Systems (FMS) layout design shapes: the spine, circular (closed loop), ladder and open field layouts (Luggen, 1991) as follows (see Figure 1.1):

- The spine layout is a configuration in which cells are located on a single, direct line, which is the material handling path between cells. This configuration may be on one side of a line or the cells may be located on both sides of the line. All pick-up/drop-off points are also placed on the line,
- In a closed loop layout, the material handling path is a rectangle in which cells are either located inside or outside the rectangle, but all pick-up/drop-off points are on the edges of the rectangle. In this type of configuration, there may be shortcuts available to connect two opposite sides of the closed loop,
- The ladder layout includes several vertical and horizontal direct lines (formed like a ladder) that serve as material handling paths; one or more cells are placed in each rectangle formed by those lines. All pick-up/drop-off points are placed on these lines,
- In the open field layout, there is no restriction on the layout pattern. This means that there is no limitation for the material handling path also the pick-up/drop-off points may be placed everywhere on the plane.

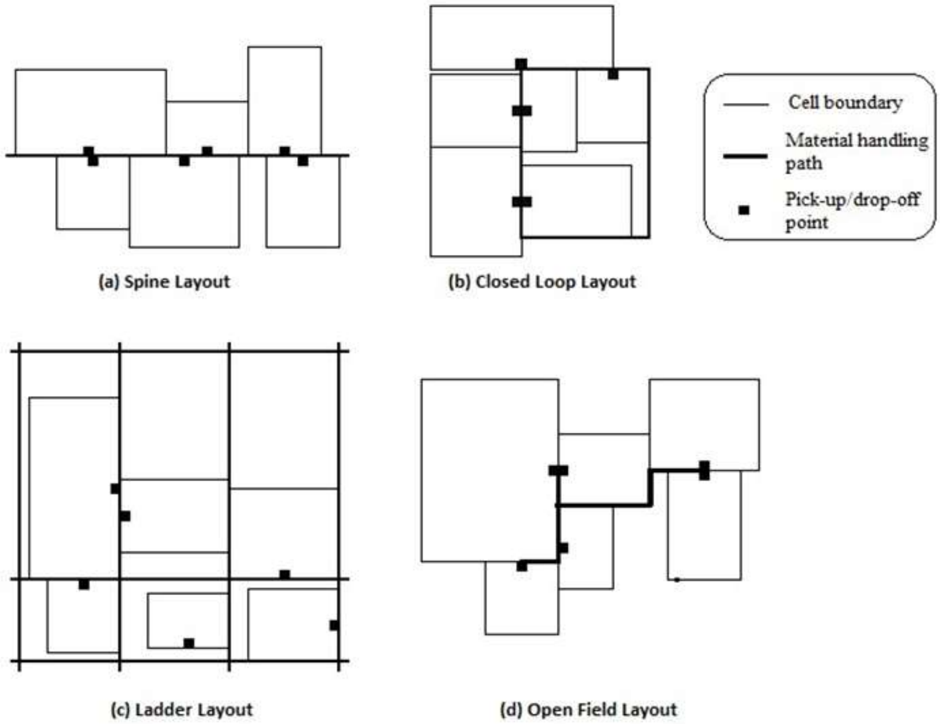

Figure 1.1. Different configurations commonly used in FMS layout design.

In all above mentioned layout configurations, there must not be overlapping between departments or machines which are placed on the plane. Mathematical models are used to prepare such configurations. The mathematical model of the above mentioned layout formations, consists of two parts. A mathematical model describes the layout problem by a set of constraints which forces the departments to be placed on the plane according to the specific layout shape and the non-overlapping constraints. The objective function minimizes the weighted distances of pick-up/drop-off points by amount of flow between them. Mostly, the distance used in objective function is Manhattan type.

The second class of facility problems uses Quadratic Assignment Problem (QAP) as mathematical model to minimize the material handling cost between the departments.

There are some predetermined positions for the departments on the plane or in the space, so the distances of the positions are input data of the problem. There is no need to non-overlapping constraints and only assignment constraints are used to assign each department to exactly one position by using a binary variable.

As mentioned before, the above-mentioned facility layout problems commonly are used to optimize the layout of manufacturing environments based on material handling cost. The same concept of facility layout problems is used in service sector. The objective of facility layout problems of service sector, is to optimize the travelled path of people between the departments. According to the nature of service centers both types of above-mentioned facility layout problems may be used for optimization. In some service centers like hospitals, banks, offices, etc. the goal is minimization of travelled path of customers or staffs, but in some other service centers like supermarket the goal may be maximization of travelled path of customers (which will be discussed in chapter 5).

Recently, the main concern of researchers is how to solve the layout problems. The complexity of most of the layout problems is NP-complete or NP-hard. Therefore, they are solved by enumerative methods exactly. The most famous exact method is the Branch & Bound method. Several softwares are available for applying Branch & Bound method e.g. Excel Solver, LINDO package, LINGO package, XPRESS, CPLEX, etc. These solvers solve NP problems optimally. The exact methods may not be able to solve NP-complete problems easily. Therefore, in the cases that exact methods are not efficient, based on the nature of the problems, metaheuristic algorithms, e.g. simulated annealing, genetic algorithm, ant colony, etc. are developed to obtain a good feasible solutions in a short time. NP-hard set consists of such problems that are at least as hard as the most difficult problem of NP set.

The aim of this thesis is to study the different types of facility layout problems in both manufacturing and service sectors. Chapter 2 contains a literature survey on facility layout problems. In Chapter 3 a complete study is done on closed loop layout problems. A reconstruction MILP model is introduced in Chapter 4. This MILP model reconstructs the distance matrix of QAPs in order to identify the QAPs related to layout problems from other QAPs. The experiments of this chapter is done by LINGO package. Chapter 5 focuses on facility layout problems of service sector. A big supermarket is considered as the case of study. The main goal is to rearrange the departments of the supermarket to increase the sales of the supermarket by forcing the customers to have longer travel in the store. The mathematical model uses QAP and travelling salesman problem to rearrange the departments. The XPRESS software is used to prove the optimality of solution.

Chapter 2

LITERATURE SURVEY

2.1 Introduction

Facility layout problem has a rich literature. The field of layout problem needs researchers who are experts in manufacturing systems, operations research, metahuristics, etc. This makes the topic a popular topic. The amount of research on facility layout problems is increased every year.

In continue the previous researches on manufacturing layout problems, reconstruction models and service facility layout problems are studied.

2.2 Manufacturing Facility Layout Problems

Literature of manufacturing layout problems can be studied from several point of views such as,

- Physical characteristics of the manufacturing system,
- Types of layout problems,
- Mathematical model of layout problems,
- Solution methodology.

2.2.1 Physical Characteristics of the Manufacturing System

Physical characteristics consist of variety of products, shape of facilities, material handling system, type of plane, etc. The previous layout studies based on physical characteristics are introduced in this part.

Based on variety and amount of product, there are four types of layout: fixed product layout, process layout, product layout and cellular layout which are mentioned by Dilworth (1996). Also as a part of cellular layout, Hamann and Vernadat (1992) considered a problem to find the best arrangement of machines in a cell which in named intra cell machine layout problem.

Shape of facilities also is one of the physical characteristics which is concerned in some studies. Irregular and regular types are considered for the shape of facilities. Regular shape mostly means rectangular facilities in the layout problems, e.g. Kim and Kim (2000), Das (1993), Chae and Peters (2006), etc., in these cases, fixed dimensions are defined for the facility (length and width). In irregular cases, area of the facility is important and any shape other than rectangle which satisfies the given area may be considered in the solution. This case was discussed by Chwif et al. (1998).

According to Tompkins et al. (1996) cost of material handling is about 20-50% of total manufacturing costs. They also mentioned that a suitable arrangement of manufacturing system reduces the material handling costs to 10-30%. Different types of tracks of the material handling system result different type of layout problems. For example there are, e.g. single row layout, open field layout, closed loop layout, ladder layout. Many researchers have obtained exact and heuristic approaches on single row layout e.g. Heragu and Kusiak (1988), Heragu and Alfa (1992), Suresh and Sahu (1993), Kumar et al. (1995), Braglia (1996), Solimanpur et al. (2005), etc. In the case of open field layout, the famous mathematical model of Das (1993), Rajasekharan et al. (1996) and Yang et al. (2005) can be mentioned. The closed loop layout consists of two types layout problems such that one sided closed loop layout and double sided closed loop layout, although there is no exact mathematical formulation for these type of layout problems, the studies of Tavakkoli-Moghaddam and Panahi (2007) and Chae and Peters (2006) can be mentioned for one sided and double sided closed loop layout, respectively. Tavakkoli-moghadam and Panahi (2007) introduced an approximate mixed integer linear programming model and Chae and Peters (2006) used an algorithm without any mathematical formulation to arrange the cells inside and outside of a loop.

From physical point of view, the number of floors which contain the facilities is another important factor in layout problems. When the number of floors exceeds one, the layout problem is named multi-floor facility layout problem. Some reasons like, lack of empty area, production process and etc., may force the facilities to be laid in several floors. Of course, an elevator should be considered to connect the floors. Johnson (1982) was the first one who introduced such a problem. Further studies are devoted to the multi-floor facility layout problem with elevator, e.g. Bozer et al. (1994), Meller and Bozer (1996), Lee et al. (2005), etc. In the problem, also the number and position of elevators may be fixed e.g. Lee et al. (2005) or can be determined as output of the model, e.g. Matsuzaki et al. (1999). The number of floor in this problem also could be known by Lee et al. (2005) or depending on available area by Patsiatzis and Papageorgiou (2002).

2.2.2 Types of Layout Problems

Currently, manufacturing companies need to be active in the global market in their life cycle. It is mentioned by Page (1991) that 40% of sales are related to the new products. Therefore, companies must be able to design new products. To make such design, the company needs a flexible arrangement of facilities. In most of the studies on layout problems, the demand is constant and the layout is designed once and assumed to be used forever. Such type of layout problems are called static layout problem. In static layout, no change is made in the system after arranging the facilities and establishing the system. However, the demand (flow amount between facilities) may be variable in different seasons, therefore, the best arrangement in a season may not be the best layout in other seasons. In these systems, the facilities' positions are changed based on the demand of the new season. This type of layout problems are called dynamic facility layout problem. The cost of change in the layout arrangement also is considered in the problem if any movement of facilities is done. Dynamic facility layout problem was studied by many researchers, e.g. Kouvelis et al. (1992), Baykasoglu and Gindy (2001), Balakrishnan et al. (2003), Barglia et al, (2003), Baykasoglu et al. (2006).

2.2.3 Mathematical Model of Layout Problems

Although some researchers like Porth (1992), Leung (1992), Kim and Kim (1995) used graph theory or Tsuchiya et al. (1996) applied neural network on layout problems, mostly mathematical formulation is used to optimize layout problems. The two common types of mathematical models used in layout problems are discrete and continuous formulations. Some papers, e.g. Evans et al. (1987), Grobelny (1987), Raoot and Rakshit (1991), Deb and Bhattacharyya (2005), etc., used fuzzy formulations for layout problems because they believed that the data are not absolutely known.

Discrete formulations are used in the layout problems that the potential positions of facilities are determined before optimizing. The most frequent mathematical formulation is Quadratic Assignment Problem (QAP). The QAP assigns each facility to only one location and also assigns only one facility to each location in order to obtain a layout with minimum material handling cost (usually between centers of the facilities). In many researches on layout problems, QAP was used to optimize the layout cost e.g. Balakrishnan et al. (2003), Wang et al. (2005), Fruggiero et al. (2006), etc. QAP also was applied as the mathematical formulation in dynamic layout problems of McKendall et al. (2006), Baykasoglu et al. (2006), etc.

In facility layout problems such that the positions are not determined a priori or transportation are done between pick-up/drop-off points (which are not the same as centers of the facilities), discrete formulation cannot be used. In these cases Mixed Integer Programming models are applied. Some sets of constraints are introduced to satisfy the restrictions in the layout problem, e.g. non-overlapping of facilities, determination of pick-up/drop-off points based on center of the facility, etc. The facilities can be placed anywhere on the plane. The objective function of the model minimizes the material handling cost of the layout (total distances weighted by flow values). Researchers like Das (1993), Chwif et al. (1998), Kim and Kim (1999), Meller et al. (1999), Dunker et al. (2005) used MILP in different types of open field layout problem.

2.2.4 Solution Methodology

As discussed above, the mathematical model of layout problems may be NP, NP-complete and NP-hard. Based on these difficulties several approaches were introduced to solve the layout problems which can be categorized as exact and approximate methods.

Mainly branch & bound algorithm was used to obtain exact optimal solution. Kim and Kim (1999) directly used branch & bound in layout problem. Meller et al. (1999) using acyclic sub-graph structure decreased the difficulty of a layout problem in order to apply branch & bound algorithm. A dynamic facility layout problem in small sizes (6 facilities and 5 periods) was optimized by Rosenblatt (1986) using branch & bound algorithm.

In layout problems with high level of difficulty (usually large amount of facilities causes the difficulty) exact methods cannot provide optimal solution. To solve these problems heuristic and metaheuristic algorithms are introduced. These methods are no used for optimality purposes. They are able to provide only a good feasible solution for the layout problem. Some classical heuristics were developed for layout problems, e.g. CRAFT by Armour and Buffa (1963), CORELAP by Lee and Moore (1967), ALDEP by Seehof and Evans (1967), COFAD by Tompkins and Reed (1976), etc.

Recently, metaheuristics, e.g. global search algorithms and evolutionary methods are use to solve difficult layout problems. Chiang and Kouvelis (1996) use tabu search in facility layout problems. Simulated annealing method was used by Chwif et al. (1998), McKendall et al. (2006), Chae and Peters (2006), etc, in layout problems. Genetic algorithm as more popular method was used in layout problems by many researchers, e.g. Banerjee and Zhou (1995), Azadivar and Wang (2000), Wu and Appleton (2002), Wang et al. (2005), etc.

2.3 Reconstruction Models and Multi-dimensional Scaling

The Multi-Dimensional Scaling (MDS) method is used in statistics to detect hidden interrelations among multi-dimensional data and it has a wide range of applications. The method's input is a matrix that describes the similarity/dissimilarity among objects of unknown dimension. The objects are generally reconstructed as points of a lower dimensional space to reveal the geometric configuration of the objects. In traditional MDS the sum of Euclidean distances (l_2) of distance matrix of reconstructed points (this matrix includes Euclidean distances of reconstructed points) and similarity matrix is minimized. On the other hand in some applications of MDS, the distances of objects may be of other types e.g. l_1 type (Manhattan distances). Facility layout problems can be considered as such applications of MDS where the distances of positions are l_1. In such cases, mathematical models can be applied to reconstruct the objects from similarities. This mathematical model is introduced in chapter 4.

Several different types of MDS procedure exists in the literature. These types can be classified based on input data used in the procedure. Based on another classification MDS is categorized as classical and replicated MDS. Classical MDS was studied by Kruskal (1964), Shepard (1962), Torgerson (1958) etc. This type of MDS uses a single similarity matrix containing either quantitative or qualitative data. Steyvers (2002) mentioned that replicated MDS (RMDS) deals with several matrices of dissimilarity data simultaneously but yields a single scaling solution, or one map. In MDS method, stress function is defined that measures the fit between input proximities and distances of similarity matrix. In the procedure, this function should be minimized. The most commonly used stress function was introduced by Kruskal (1964) and was applied by Giguere (2006), Steyvers (2002), Arce and Garling (1989), Davidson (1983), Kruskal and Wish (1978) etc. later.

MDS method has some applications in facility layout, geography, psychology, economy etc. In the case of facility layout problems, Niroomand et al. (2011) can be referred where the authors reconstructed the Kra30a problem (Hahn and Krarup (2001)) from the Quadratic Assignment Problem Library (QAPLIB), using the MDS method by introducing a mixed integer linear programming reconstruction model which used l_1 distances of facilities as the similarity matrix (this study is detailed in chapter 3). Some studies were done based on applications of MDS in geography e.g. Smallman-Raynor and Cliff (2001), Openshaw (1984), Massey (1999) etc. There exists a plenty of MDS studies on psychology. Ding (2006) applied MDS in personal profile construction. A comparison of cluster analysis and modal profile analysis using MDS, was studied by Kim et al. (2004). Sokolov (2000) used MDS method and designed an experiment aiming to create a spatial representation of emotion. In economics, MDS plays an important rule. Michael et al. (2008) applied MDS method to determine the monthly peak in economic analysis. Gabix et al. (2007) applied MDS to introduce a unified econophysics explanation for the power-law exponents of stock market activity. MDS method was applied in some other economic studies e.g. Michael et al. (2009), Knoop (2004) etc.

2.4 Service Facility Layout Problems

Although facility layout problems are mostly applied to find the best arrangement of facilities in manufacturing sector, the modification of these models and problems can be applied in service sector. Service sector consists of facilities like hospitals, supermarkets, fire stations, offices, schools etc.

There are some differences between facilities in manufacturing and service sectors. In service centers instead of total transported flow value between facilities, the travelled path of people is minimized, e.g. in hospital the travelled path of patients, nurses and doctors should be minimized. Also service centers are mostly established in multi-floor buildings but in manufacturing sector rarely multi-floor buildings are used.

In the literature of facility layout problems there are a few studies on service layout problems. the literature of hospital layout and supermarkets are focused here. One of the earliest studies was done by Krarup et al. (1972). The study was done to design a hospital in Germany. A QAP was used to assign 30 facilities to 30 positions in two floors but optimality could not be proved. Later Hahn and Krarup (2000) solved the problem optimally.

Supermarkets also are such type of service centers that need to be arranged optimally. A mathematical model is introduced and applied to a supermarket in Hungary in order to rearrange the departments optimally in chapter 4. Although the goal in service layout problems is to minimize the total travelled path of customers, in our model such path is maximized. In this way the customers will spend more time in the shop to buy more items in order to increase the sales of the supermarket.

The first systematic analysis of customer paths in supermarket was carried out by Farley and Ring (1966). The psychological customer research contributed considerably to the understanding of the effects of different factors on customer behaviour in shops by Mackay and Olshavsky (1975), Park *et al*. (1989). Harrell *et al*. (1980) analysed the path of more than 600 shoppers and explored sequential relationships among different variables pertinent to retail crowding. Underhill (1999) applied anthropological methods for better understanding of behaviour patterns in stores. Chandon (2002) and Sorensen (2003) mentioned that the shopping path determines the departments and products with which the customer comes into contact during the store visit. Hui *et al*. (1981) proved that if customers visit the store many times then they become more familiar with the layout and more purposeful, i.e., they are less likely to spend time on exploration and more likely to purchase according to their plan only. Hui *et al*. (2009) detected a positive relationship between the shopping path and the quantity of purchases. Kholod *et al*. (2010) proved a strong correlation between the length of the shopping path and the sales volume. They introduced a new concept: wandering degree which is calculated as the ratio of the distance walked by a costumer in a given area and the square root of the area size. The ratio between purchasing value and wandering degree is called purchasing sensitivity. Based on the wandering degree authors have classified the costumers (wandering, decisive and mixed) and categorized the different product groups according to purchasing intensity. Based on this information they were able to establish a relation between the customer types and the different product categories according to the purchasing sensitivity. Analysing the results it is obvious that the majority of the food, industrial and household products can be characterized as products with high or medium purchase sensitivity, that's why this research supports our hypothesis, that increasing the distance of the buyer's path increases the probability of increasing purchase value.

Chapter 3

AN EXACT MILP MODEL FOR CLOSED LOOP LAYOUT

3.1 Introduction

This chapter is developed to introduce a new mathematical model of the closed loop based layout problems.

Das (1993) introduced a mixed integer linear programming (MILP) model for the open field layout problem. This model considers Manhattan distance of cells in the objective function that is not the real distance in such cases when a cell is placed between a pair of cell. He also proposed a heuristic method that included four steps; the method was named the ’four-step open field layout’. In the first step, using the spine method, an upper bound to the FLP objective was obtained. A determination of the orientation and spatial sequencing of the cells was performed in the second step. In the third step, taking into account all of the other decisions made in step 2, the interference relationships of the cells were determined. In the last step, all decisions made in the second and third steps were considered as fixed variables and the locations of the pick-up/drop-off points and the spatial coordinates of the cells were determined. The spine method solution from step 1 was also taken into account in this final step.

Rajasekharan *et al.* (1998), applied a genetic algorithm to Das's MILP model as an alternative solution procedure. This algorithm improved the quality and the computational time of the solution. The solution methodology consists of two steps. The first step considers the open field floor area for each cell's location, and the second step applies the genetic algorithm to find a good solution in the restricted open field floor.

Chae and Peters (2006) continued Rajasekharan's and Das's studies. They restricted the material handling between cells to be located on a rectangular closed loop, and all pick-up/drop-off points had to be placed on this path. Although the problems are originally open field layout problems, the authors obtained acceptable solutions in some problems and ever better solutions in a few problems. They followed Das's MILP model and designed an algorithm to locate cells on a large-enough loop. In the next steps, they applied a simulated annealing method to improve the arrangement of cells in that fixed loop size. Then, the loop size becomes smaller, and the best arrangement in this situation will again be obtained by simulated annealing. The procedure is applied in an organized way for several loop shapes.

In all previous studies, the Manhattan distances between pick-up/drop-off points are considered to evaluate the material handling cost of the layout shape. Although this distance is correct in some arrangements, in other cases, it is not the real distance between two pick-up/drop-off points. For example, in the closed loop which was used in Chae and Peter (2006), when two cells are on opposite sides of the rectangle and material is forced to move on that rectangular closed loop, the real distance between these two cells is greater than the Manhattan distance between them. The same problem may also occur in an open field layout when there is one cell between a pair of cells. In this case, the real distance between that pair of cells is again greater than the Manhattan distance between them.

In this chapter, a new MILP model is introduced for the closed loop layout to eliminate such cases. The chapter is organized as follows. In Section 3.2, the basic model of Das is introduced. The new model is discussed in Section 4.3. Section 3.4 contains some remarks about the model. The computational experiments are described in the next section. The final section of the chapter contains the conclusions.

3.2 The Basic Model of Das

In all models, it is assumed that the rectangles of the cells have only horizontal and vertical edges and that the cells are not rotated in any other way.

An exact model of the layout of the rectangular cells must satisfy the following constraints:

- the cells must not overlap,
- the cells can be rotated by 90, 180 or 270 degrees,
- the method used to measure distances must be defined.

The model of Das (1993) gives a perfect solution for avoiding overlap and applying rotation but contains only an approximation for distances.

The notations used in the model are:

n: the number of cells (parameter)

i, j: indices of cells (index)

(x_i, y_i): coordinate of the center of cell i (variable)

z_i: binary variable; it is 1 (0) if cell i is in a vertical (horizontal) position (variable)

s_i: the length of the shorter edge of cell i (parameter)

t_i: the length of the longer edge of cell i (parameter)

ω_i: the distance of the pick-up point of cell i from the center of the cell (parameter)

(a_i, b_i): coordinate of the pick-up point of cell i (variable)

e_{ij}: $\max\{0, x_i - x_j\}$ (variable)

f_{ij}: $\max\{0, x_j - x_i\}$ (variable)

g_{ij}: $\max\{0, y_i - y_j\}$ (variable)

h_{ij}: $\max\{0, y_j - y_i\}$ (variable)

p_{ij}: $\max\{0, a_i - a_j\}$ (variable)

q_{ij}: $\max\{0, a_j - a_i\}$ (variable)

π_{ij}: $\max\{0, b_i - b_j\}$ (variable)

ρ_{ij}: $\max\{0, b_j - b_i\}$ (variable)

φ_{ij}: the flow value between cells i and j (parameter)

α_{ij}: a binary variable; it is 1 if $x_i \geq x_j$ (variable)

β_{ij}: a binary variable; it is 1 if $y_i \geq y_j$ (variable)

δ_{ij}: a binary variable; if it is 1, then cells i and j are not overlapping vertically, and if it is 0, then cells i and j are not overlapping horizontally (variable)

M: a large positive number

$\lambda_{1i}, \lambda_{2i}, \lambda_{3i}, \lambda_{4i}$: binary variables describing the position of the pick-up point of cell i according to its rotation (variable)

In the discussion that follows, the cells are represented by their center. The position of a cell is vertical (horizontal) if the position of its longer edge is vertical (horizontal). In the case of a square, the tie can be broken arbitrarily.

Two cells are overlapping if and only if their centers are too close to each other. The minimal required horizontal (vertical) distance such that two cells are not overlapping is half the sum of the length of their edges in the horizontal (vertical) position. The sum depends on the rotation of the cells. Notice that $e_{ij} + f_{ij}$ $(g_{ij} + h_{ij})$ is the horizontal (vertical) distance of the centers of the cells i and j. If there is no horizontal (vertical) overlap, then the distance must be at least as long as the sum of the two horizontal (vertical) half edges.

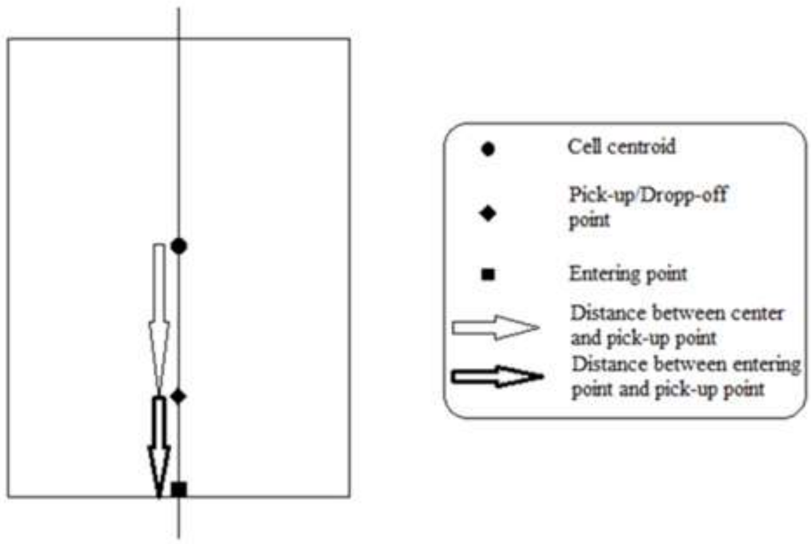

Figure 3.1. Cell with entering points and pick-up points.

This requirement is described by the following inequalities:

$$\forall\ i,j,i \neq j: e_{ij} + f_{ij} - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i - \frac{1-z_j}{2}t_j - \frac{z_j}{2}s_j \geq -M\delta_{ij} \tag{3.1}$$

and

$$\forall i,j,i \neq j:\ g_{ij} + h_{ij} - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i - \frac{1-z_j}{2}s_j - \frac{z_j}{2}t_j \geq -M(1-\delta_{ij}). \tag{3.2}$$

It is difficult to use the formulae of e_{ij}, f_{ij}, g_{ij} and h_{ij} explicitly in an optimization problem; therefore, they are described implicitly by the following constraints:

$$\forall i,j,i \neq j:\ x_i - x_j \ = \ e_{ij} - f_{ij} \tag{3.3}$$

$$\forall i,j,i \neq j:\ y_i - y_j \ = \ g_{ij} - h_{ij} \tag{3.4}$$

$$\forall i,j,i \neq j:\ e_{ij} \ \leq \ M\alpha_{ij} \tag{3.5}$$

$$\forall i,j,i \neq j:\ f_{ij} \ \leq \ M(1-\alpha_{ij}) \tag{3.6}$$

$$\forall i,j,i \neq j:\ g_{ij} \ \leq \ M\beta_{ij} \tag{3.7}$$

$$\forall i,j,i \neq j:\ h_{ij} \ \leq \ M(1-\beta_{ij}) \tag{3.8}$$

Notice that the inequalities (3.1) and (3.2) handle both overlapping and rotation. Constraints (3.5) and (3.6) with nonnegativity (see constraint (3.17) below) ensure that either e_{ij} or f_{ij} is equal to zero.

The next main step is the formulation of the objective function. It is the minimization of the sum of the flow between cells weighted by the distance of the pick-up points of the cells. The first step is to determine the coordinates of the pick-up points. The pick-up point is on one of the middle lines of the cell at distance ω_i (see Figure 3.1). If the pick-up point is on the shorter edge, then $\omega_i = t_i/2$, and if the pick-up point is on the longer edge, then $\omega_i = s_i/2$. However, the point can also be inside the cell. This means that if $\omega_i = 0$ then the two coordinates of the pick-up point are the same as those of the center point of the cell; otherwise, one coordinate is different, and the other one is equal to the same coordinate as the center point. In the latter case, the coordinates depend (i) on the definition of the position of the point, i.e., the point is on the middle line connecting the two shorter/longer edges, and (ii) on the rotation of the cell in the layout. The rotation of cell i is described by four binary variables, $\lambda_{1i}, \lambda_{2i}, \lambda_{3i}$, and λ_{4i} defined as follows:

$$\lambda_{1i} = \begin{cases} 1 & \textit{if the pick} - \textit{up point is on the right side of the center} \\ 0 & \textit{otherwise} \end{cases}$$

$$\lambda_{2i} = \begin{cases} 1 & \textit{if the pick-up point is below the center} \\ 0 & \textit{otherwise} \end{cases}$$

$$\lambda_{3i} = \begin{cases} 1 & \textit{if the pick-up point is on the left side of the center} \\ 0 & \textit{otherwise} \end{cases}$$

$$\lambda_{4i} = \begin{cases} 1 & \textit{if the pick-up point is above the center} \\ 0 & \textit{otherwise} \end{cases}$$

The λ and z variables are not independent. Two equations must hold between them. If the pick-up point is on the middle line connecting the two shorter edges, then the cell is in a vertical position if the pick-up point is below or above the center. Thus,

$$z_i \;=\; \lambda_{2i} + \lambda_{4i} \tag{3.9}$$

implying that

$$1 - z_i \;=\; \lambda_{1i} + \lambda_{3i}. \tag{3.10}$$

If the pick-up point is on the middle line connecting the two longer edges, then the form of the equations is as follows (using the same equation numbering):

$$z_i \;=\; \lambda_{1i} + \lambda_{3i} \tag{3.9}$$

and

$$1 - z_i \;=\; \lambda_{2i} + \lambda_{4i} \tag{3.10}$$

Finally, the two coordinates of the pick-up point of cell i are

$$\forall i: \;\; a_i \;=\; x_i \;+\; \omega_i(\lambda_{1i} - \lambda_{3i}) \tag{3.11}$$

and

$$\forall i: \;\; b_i \;=\; y_i \;+\; \omega_i(\lambda_{4i} - \lambda_{2i}). \tag{3.12}$$

The Manhattan distance of the cells i and j can be described by nonnegative variables p_{ij}, q_{ij}, π_{ij} and ρ_{ij} as follows.

$$p_{ij} \;-\; q_{ij} \;=\; a_i \;-\; a_j \tag{3.13}$$

$$\pi_{ij} - \rho_{ij} = b_i - b_j. \tag{3.14}$$

Then, the Manhattan distance of the two cells is

$p_{ij} + q_{ij} + \pi_{ij} + \rho_{ij}$.

In the model of [Das 1993], the total Manhattan distance weighted with the flow among the cells is minimized, i.e., the objective function is

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \varphi_{ij}(p_{ij} + q_{ij} + \pi_{ij} + \rho_{ij}) \tag{3.15}$$

To complete the model, the technical constraints defining the type of the variables must be mentioned. Without loss of generality, we may assume that the cells are in the nonnegative quarter of the plane:

$$\forall\, i\colon\ x_i, y_i, a_i, b_i \geq 0. \tag{3.16}$$

The distance variables are also nonnegative:

$$\forall\, i,j\colon\ e_{ij}, f_{ij}, g_{ij}, h_{ij}, p_{ij}, q_{ij}, \pi_{ij}, \rho_{ij}, \geq 0. \tag{3.17}$$

All other variables are binary:

$$\forall\ i,j\colon\ z_i, \alpha_{ij}, \beta_{ij}, \delta_{ij}, \lambda_{1i}, \lambda_{2i}, \lambda_{3i}, \lambda_{4i}, = 0 \quad or \quad 1. \tag{3.18}$$

The model (3.1)-(3.18) was developed by Das (1993). In the next section, the modification of the model for a closed loop layout with exact distances is elaborated.

3.3 Closed Loop Layout with Exact Distances

The vehicle transporting something to a cell enters the cell and moves to the pick-up point. The entering point is the middle point of the edge just in front of the pick-up point. The transportation performance within the cells is constant and is determined by the flow matrix and the position of the pick-up points within the cells. This quantity is introduced by Das (1993) and is denoted in that paper by $TVLP_{LB}$. *The model developed below does not contain the quantity* $TVLP_{LB}$. *The total transportation among the entering points of the cells is minimized*. Hence, a_i and b_i denotes the coordinates of the entering point.

If the distance of the cells is measured as the Manhattan distance between well-defined points of the cells, then this distance can be shorter than what the vehicle must pass. Figure 3.2 compares the Manhattan distance and the exact distance of two neighboring cells lying on a line. The Manhattan distance is shorter than the exact distance because the vehicle does not follow the whole path within the cell that it is required to follow. A 3-cell example is shown in Figure 3.3; as an objective function, the Manhattan distance gives an optimal solution that can be improved in an obvious way for the real distances by shifting cell B down. Further on, there are configurations for which the real path cannot use the logic of the Manhattan distance, e.g., see Figure 3.4.

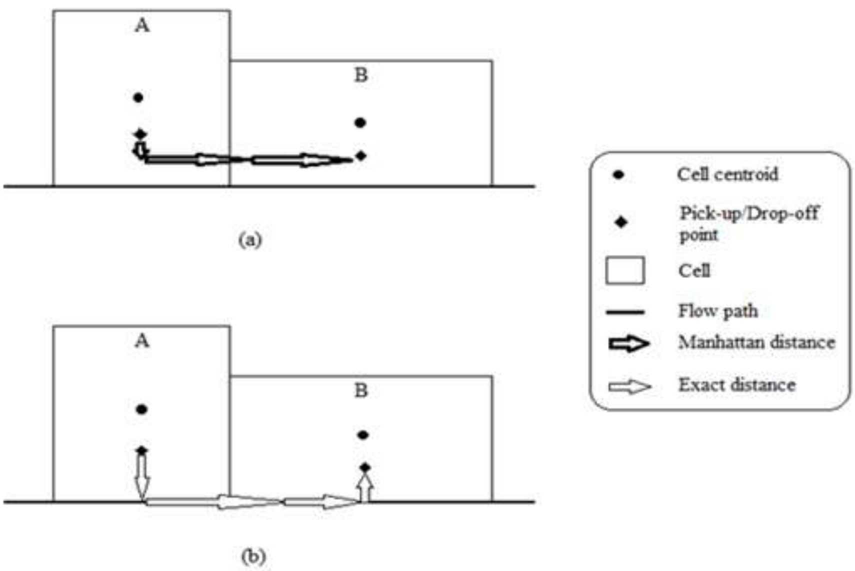

Figure 3.2. The real (exact) distance and the Manhattan distance.

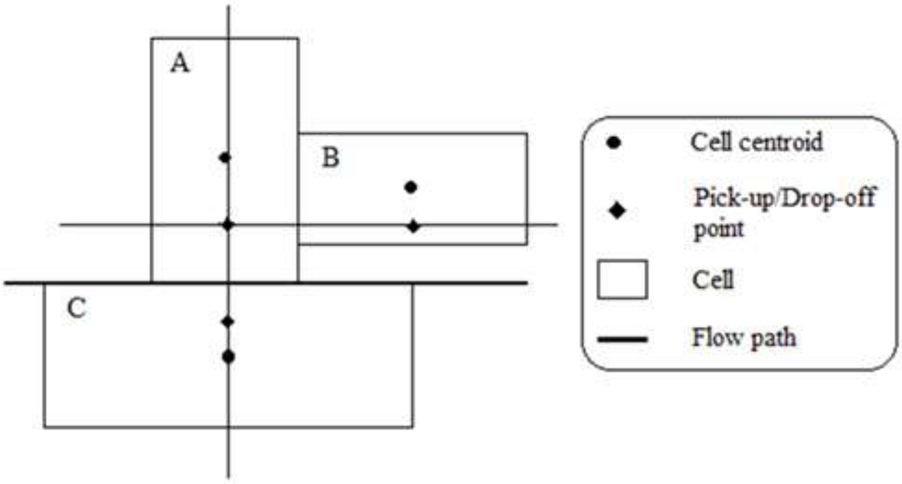

Figure 3.3. A solution that is optimal for the Manhattan distance, but is not optimal for the real distance.

In the case of closed loop layout, the shape of the track of the vehicle is a rectangle. *The entering points of the cells are on one edge of the rectangle*. The vehicle may use both directions. Between two entering points, the vehicle uses the direction that yields the shorter path.

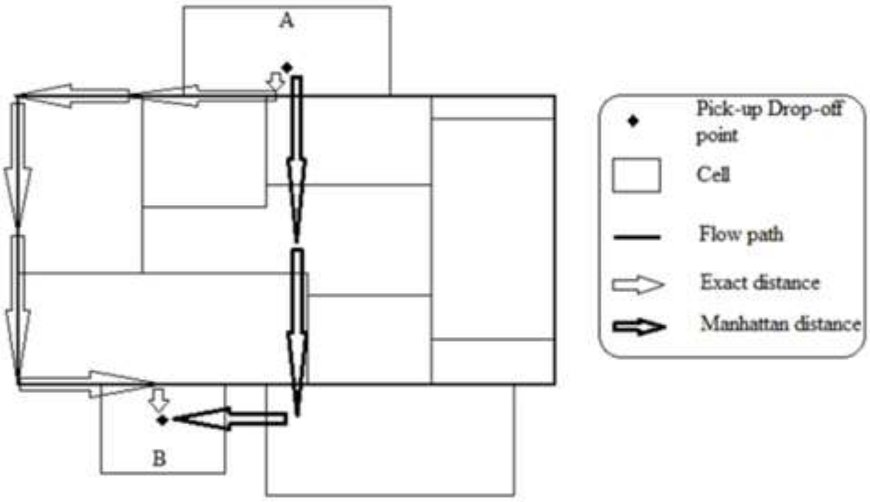

Figure 3.4. The route that is optimal for the Manhattan distance cannot be used.

Further notations related to distances are:

k, l: indices of the edges of the track (index)

w_{klij}: binary variable; it is 1 if cell i is on edge k and cell j is on edge l (variable)

v_1, v_2: the two vertical coordinates of the track ($v_2 \geq v_1$) (variable)

h_1, h_2: the two horizontal coordinates of the track ($h_2 \geq h_1$) (variable)

d_{ij}^{k}: the distance of cells i and j if both are on edge k; 0 otherwise (variable)

d_{ij}^{kl}: the distance of cells i and j if cell i is on edge k and cell j is on edge l ($k \neq l$); 0 otherwise (variable)

ψ_{ki}: a binary variable; it is 1 if cell i is on edge k (variable)

ϑ_i: a binary variable; it is 1 if cell i is outside the track (variable)

(a_i, b_i): the x (y) coordinate of the entering point of cell i (variable)

$u_{13ij}, u_{31ij}, u_{24ij}, u_{42ij}$: binary variables; they select the minimal path for the vehicle if cells i and j are on opposite edges (variable)

The edges of the track have the following indices: the upper horizontal edge is 1, the right vertical edge is 2, the lower horizontal edge is 3, and the left vertical edge is 4.

Notice that

$$w_{klij} \;=\; \psi_{ki}\psi_{lj}.$$

This relation can be described equivalently by two linear inequalities using an old integer programming technique:

$$2w_{klij} \;\leq\; \psi_{ki} + \psi_{lj}$$

$$\psi_{ki} + \psi_{lj} - 1 \leq w_{klij}.$$

The equivalence is based on the fact that all three variables are binary. Therefore, the first set of constraints of the model is

$$2w_{klij} \;\leq\; \psi_{ki} + \psi_{lj} \quad 1 \leq i < j \leq n,\; k,l = 1,2,3,4 \tag{3.19}$$

$$\psi_{ki} + \psi_{lj} - 1 \leq w_{klij} \quad 1 \leq i < j \leq n,\; k,l = 1,2,3,4. \tag{3.20}$$

The distances are restricted in the model only from below. In the optimal solution, the optimality condition forces them to be equal to their maximal lower bound.

There are several cases according to the (potential) position of the two cells.

Case 1: cells i and j are both on edge k. One of the coordinates of the two entering points is the same. If they are on edge 1 or 3, then the common coordinate is the y coordinate; otherwise, it is the x coordinate. Let d_{ij}^k be the distance of the entering points of the two cells. In any other case, d_{ij}^k is 0. Then, the distance must satisfy the following inequalities:

$$d_{ij}^k + M(1 - w_{kkij}) \;\geq\; a_i - a_j \quad 1 \leq i < j \leq n,\; k = 1,3 \tag{3.21}$$

$$d_{ij}^k + M(1 - w_{kkij}) \;\geq\; a_j - a_i \quad 1 \leq i < j \leq n,\; k = 1,3 \tag{3.22}$$

$$d_{ij}^k + M(1 - w_{kkij}) \;\geq\; b_i - b_j \quad 1 \leq i < j \leq n,\; k = 2,4 \tag{3.23}$$

$$d_{ij}^k + M(1 - w_{kkij}) \;\geq\; b_j - b_i \quad 1 \leq i < j \leq n,\; k = 2,4 \tag{3.24}$$

Notice that the constraints (3.21)-(3.24) are not restrictive if both of cells i and j are not on edge k; $w_{kkij} = 0$, and the constraint is satisfied automatically.

Case 2: cells i and j are on two adjacent edges. The vehicle must pass the intersection point of the two edges of the track. For example, if cell i is on edge 1 and cell j is on edge 2, then the vehicle must go through the upper left corner of the track. The coordinates of this point are (v_2, h_2). The pick-up point of cell i is to the left of this point, and the pick-up point of cell j is under this point. Hence, the distance d_{ij}^{12} must satisfy an inequality similar to the ones in (3.21)-(3.24):

$$d_{ij}^{12} + M(1 - w_{12ij}) \geq v_2 - a_i \ + \ h_2 - b_j. \tag{3.25}$$

Similarly, the distances of Case 2 must satisfy the following inequalities.

$$d_{ij}^{21} + M(1 - w_{21ij}) \geq v_2 - a_j \ + \ h_2 - b_i \tag{3.26}$$

$$d_{ij}^{23} + M(1 - w_{23ij}) \geq v_2 - a_j \ + \ b_i - h_1 \tag{3.27}$$

$$d_{ij}^{32} + M(1 - w_{32ij}) \geq v_2 - a_i \ + \ b_j - h_1 \tag{3.28}$$

$$d_{ij}^{34} + M(1 - w_{34ij}) \geq a_i - v_1 \ + \ b_j - h_1 \tag{3.29}$$

$$d_{ij}^{43} + M(1 - w_{43ij}) \geq a_j - v_1 \ + \ b_i - h_1 \tag{3.30}$$

$$d_{ij}^{14} + M(1 - w_{14ij}) \geq a_i - v_1 \ + \ h_2 - b_j \tag{3.31}$$

$$d_{ij}^{41} + M(1 - w_{41ij}) \geq a_j - v_1 \ + \ h_2 - b_i \tag{3.32}$$

Case 3: cells i and j are on two parallel edges. Assume that cell i is on edge 1 and that cell j is on edge 3. Any path between them must reach one of the vertical edges of the track first on a horizontal edge. After that, the path must pass the vertical distance $h_2 - h_1$. Finally, the path must reach the target cell on the other horizontal edge. If the vehicle starts to move to the right, then the two distances on the horizontal edges are $v_2 - a_i$ and $v_2 - a_j$. If the vehicle moves in the opposite direction, then the two distances are $a_i - v_1$ and $a_j - v_1$. The vehicle must choose the shorter of the two paths. Thus, in that case,

$$d_{ij}^{13} \ = \ \min\{h_2 - h_1 + 2v_2 - a_i - a_j, h_2 - h_1 - 2v_1 + a_i + a_j\}$$

It is not easy to use the minimum function in a model. Therefore, a new binary variable, u_{13ij}, is introduced, which will select the minimum from the two above-mentioned distances. Thus, d_{ij}^{13} must satisfy the following two inequalities:

$$d_{ij}^{13} + M(1 - w_{13ij}) + Mu_{13ij} \;\geq\; h_2 - h_1 + 2v_2 - a_i - a_j \tag{3.33}$$

and

$$d_{ij}^{13} + M(1 - w_{13ij}) + M(1 - u_{13ij}) \;\geq\; h_2 - h_1 - 2v_1 + a_i + a_j. \tag{3.34}$$

Notice that if $w_{13ij} = 0$, then neither (3.33) nor (3.34) is binding. In that case, d_{ij}^{13} can be on the lower bound, which is 0, as will be described later. As was mentioned above, the objective function will determine the value of u_{13ij} in such a way that d_{ij}^{13} is as small as possible. Formally, there are feasible solutions satisfying both (3.33) and (3.34) with a strict inequality, but they are not optimal. Based on a similar analysis, the following inequalities must be satisfied:

$$d_{ij}^{31} + M(1 - w_{31ij}) + Mu_{31ij} \;\geq\; h_2 - h_1 + 2v_2 - a_i - a_j \tag{3.35}$$

$$d_{ij}^{31} + M(1 - w_{31ij}) + M(1 - u_{31ij}) \;\geq\; h_2 - h_1 - 2v_1 + a_i + a_j \tag{3.36}$$

$$d_{ij}^{24} + M(1 - w_{24ij}) + Mu_{24ij} \;\geq\; v_2 - v_1 + 2h_2 - b_i - b_j \tag{3.37}$$

$$d_{ij}^{24} + M(1 - w_{24ij}) + M(1 - u_{24ij}) \;\geq\; v_2 - v_1 - 2h_1 + b_i + b_j \tag{3.38}$$

$$d_{ij}^{42} + M(1 - w_{42ij}) + Mu_{42ij} \;\geq\; v_2 - v_1 + 2h_2 - b_i - b_j \tag{3.39}$$

$$d_{ij}^{42} + M(1 - w_{42ij}) + M(1 - u_{42ij}) \;\geq\; v_2 - v_1 - 2h_1 + b_i + b_j. \tag{3.40}$$

The next set of constraints determines the positions of the cells from the closed loop point of view. Each cell must be either completely inside or completely outside the track. Furthermore, the edge of the cell where the vehicle may enter the cell must lie on one of the edges of the track.

The coordinates of the four corner points of cell i depend on the rotation of the cell described by the binary variable z_i. They are as follows:

$$(x_i - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i, y_i - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i),\quad (x_i - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i, y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i),$$

$$(x_i + \frac{1-z_i}{2}t_i + \frac{z_i}{2}s_i, y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i),\quad (x_i + \frac{1-z_i}{2}t_i + \frac{z_i}{2}s_i, y_i - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i).$$

A cell is inside the track if

$$h_1 \;\leq\; x_i - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i, \quad h_2 \;\geq\; x_i + \frac{1-z_i}{2}t_i + \frac{z_i}{2}s_i$$

and

$$v_1 \;\leq\; y_i - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i, \quad v_2 \;\geq\; y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i.$$

Furthermore one of these pairs of inequalities must be satisfied.

A binary variable ϑ_i is introduced to describe whether cell i is outside or inside the track. $\vartheta_i = 1$ if cell i is outside.

The cell must satisfy different conditions if it is inside the track than if it is outside.

Inside constraints. A pair of inequalities must be satisfied for each of the four edges of the track. The first inequality claims that the cell is inside the track, and the second one claims that its entering point is on the edge. Obviously, the first constraint must not be claimed if the cell is outside, and the cell can be on only one of the edges. This means that the constraints must be satisfied automatically in certain cases, and this situation is ensured with the binary variables ψ_{ki}'s and ϑ_i's.

Edge 1:

$$y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i - M\vartheta_i \leq h_2 \tag{3.41}$$

$$y_i + \frac{1-z_i}{2} s_i + \frac{z_i}{2} t_i + M\vartheta_i + M(1-\psi_{1i}) \geq h_2 \quad (3.42)$$

Edge 2:

$$x_i + \frac{1-z_i}{2} t_i + \frac{z_i}{2} s_i - M\vartheta_i \leq v_2 \quad (3.43)$$

$$x_i + \frac{1-z_i}{2} t_i + \frac{z_i}{2} s_i + M\vartheta_i + M(1-\psi_{2i}) \geq v_2 \quad (3.44)$$

Edge 3:

$$y_i - \frac{1-z_i}{2} s_i - \frac{z_i}{2} t_i - M\vartheta_i \geq h_1 \quad (3.45)$$

$$y_i - \frac{1-z_i}{2} s_i - \frac{z_i}{2} t_i - M\vartheta_i - M(1-\psi_{3i}) \leq h_1 \quad (3.46)$$

Edge 4:

$$x_i - \frac{1-z_i}{2} t_i - \frac{z_i}{2} s_i + M\vartheta_i \geq v_1 \quad (3.47)$$

$$x_i - \frac{1-z_i}{2} t_i - \frac{z_i}{2} s_i - M\vartheta_i - M(1-\psi_{4i}) \leq v_1 \quad (3.48)$$

$i = 1,2,\dots,n$

The fact that the entering point of cell i must be on exactly one edge is expressed by the equation

$$\psi_{1i} + \psi_{2i} + \psi_{3i} + \psi_{4i} = 1 \qquad i = 1,2,\dots,n. \quad (3.49)$$

Notice that the first constraints are automatically satisfied if cell i is outside as $\vartheta_i = 1$; thus, a ”large M” helps to make this possible. The two constraints of an edge together satisfy the equation if and only if $\vartheta_i = 0$, i.e., the cell is inside, and $\psi_{ki} = 1$, i.e., the indicator variable claims that the cell is on edge k.

Outside constraints. The lower edge of a cell cannot be higher than the upper edge of the track; otherwise, the cell is not on the track. Similarly, its left (upper, right) edge cannot be right (under, left) of the right (lower, left) edge of the track. However, the two edges must be on the same line if the indicator variable ψ_{ki} claims it. Moreover, if cell i is on a horizontal (vertical) edge of the track, then the horizontal (vertical) coordinate of its center point must be in the horizontal (vertical) range of the track. Hence, two pairs of inequalities must be satisfied for each edge of the track.

Edge 1:

$$y_i - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i \leq h_2 \quad (3.50)$$

$$y_i - \frac{1-z_i}{2}s_i - \frac{z_i}{2}t_i + M(1-\vartheta_i) + M(1-\psi_{1i}) \geq h_2 \quad (3.51)$$

$$v_2 + M(1-\psi_{1i}) \geq x_i \geq v_1 - M(1-\psi_{1i}) \quad (3.52)$$

Edge 2:

$$x_i - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i \leq v_2 \quad (3.53)$$

$$x_i - \frac{1-z_i}{2}t_i - \frac{z_i}{2}s_i + M(1-\vartheta_i) + M(1-\psi_{2i}) \geq v_2 \quad (3.54)$$

$$h_2 + M(1-\psi_{2i}) \geq y_i \geq h_1 - M(1-\psi_{2i}) \quad (3.55)$$

Edge 3:

$$y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i \geq h_1 \quad (3.56)$$

$$y_i + \frac{1-z_i}{2}s_i + \frac{z_i}{2}t_i - M(1-\vartheta_i) - M(1-\psi_{3i}) \leq h_1 \quad (3.57)$$

$$v_2 + M(1-\psi_{3i}) \geq x_i \geq v_1 - M(1-\psi_{3i}) \quad (3.58)$$

Edge 4:

$$x_i + \frac{1-z_i}{2}t_i + \frac{z_i}{2}s_i \geq v_1 \quad (3.59)$$

$$x_i + \frac{1-z_i}{2}t_i + \frac{z_i}{2}s_i - M(1-\vartheta_i) - M(1-\psi_{4i}) \leq v_1 \quad (3.60)$$

$$h_2 + M(1-\psi_{4i}) \geq y_i \geq h_1 - M(1-\psi_{4i}) \quad (3.61)$$

$i = 1,2,\ldots,n$

Notice that the constraints claiming that a cell must lay on a certain edge of the track are satisfied automatically again in all other cases.

For the sake of completeness, the nature of the new variables is claimed again:

$$\forall\, i, j, k, l\colon\ w_{klij}, \psi_{ki}, \vartheta_i \; = \; 0 \;\; or \;\; 1 \tag{3.62}$$

And

$$\forall\, i, j, k, l\colon\ d_{ij}^{k}, d_{ij}^{kl} \geq 0. \tag{3.63}$$

The objective function is the minimization of the total distance weighted by the flow values, i.e., it is

$$\min \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \varphi_{ij} \sum_{k=1}^{4} \left(d_{ij}^{k} + \sum_{l \neq k} d_{ij}^{kl} \right). \tag{3.64}$$

The model of the closed loop layout with exact distances is the optimization of (3.64) under the constraints (3.1)-(3.14) and (3.16)-(3.63).

3.4 Degenerated Solutions and the Multiplicity of the Solutions

The spine layout is a degenerated version of the closed loop layout. In the case of the spine solution, all cells lie on the same line. If the line is horizontal (vertical), then $h_1 = h_2$ ($v_1 = v_2$). This type of solution can also occur for the case in which one cell closes the track at the end of the track, i.e., the cell is rotated 90 degrees toward the track and its center line is the line of the track.

The problem has a high degree of symmetry. In the case of the non-degenerated solutions, the layout can be rotated by 90, 180, and 270 degrees such that the solution remains geometrically congruent to the original solution. Moreover, the solution can be mirrored to the horizontal and vertical middle lines of the closed loop. Obviously, the application of any sequence of these transformations results in a congruent layout.

The transformations generate the well-known dihedral group of the square. It consists of 8 elements: identity, the four reflections (to the horizontal and vertical axes and the two diagonals) and the three rotations (90, 180, and 270 degrees) (see Figure 3.5). It is a non-Abelian group, i.e., the operation is not commutative.

3.5 Computational Experiments

The high symmetry of the problem causes computational problems. This is true particularly in a branch and bound frame because there are eight equivalently good branches. The symmetry was broken by constraints similar to those used in [Sherali et al. 2003]. Any solution can be shifted on the plane without changing the transportation cost. It is equivalent to fixing the values h_1, and v_1. Both of them were fixed to 40. In this way there is enough space for all cells to be in the nonnegative quarter of the plane. Furthermore, the whole configuration can be put in a bounded area. For example, if the sums of the lengths, and widths of the cells are L, and W then all cells can be fitted into an area of size $L \times W$. The cell with the largest transportation flow was claimed to be in the lower left part of configuration. Finally, it was claimed that the layout has standing position, i.e., $v_2 \leq h_2$.

The computational experiments are carried out on the sequence of problems used by several authors. The sequence contains problems for $n = 4, 6, 8, 10, 12, 14, 16, 18$. The first five problems were proposed by Das (1993) and the last three were added to the sequence by Rajasekharan et al. (1998). The same problems are used in Chae and Peters (2006).

An optimal solution is found, and its optimality is proven only for $n = 4$ and 6. This solution is shown on Figure 3.6. In all other cases, only feasible solutions are

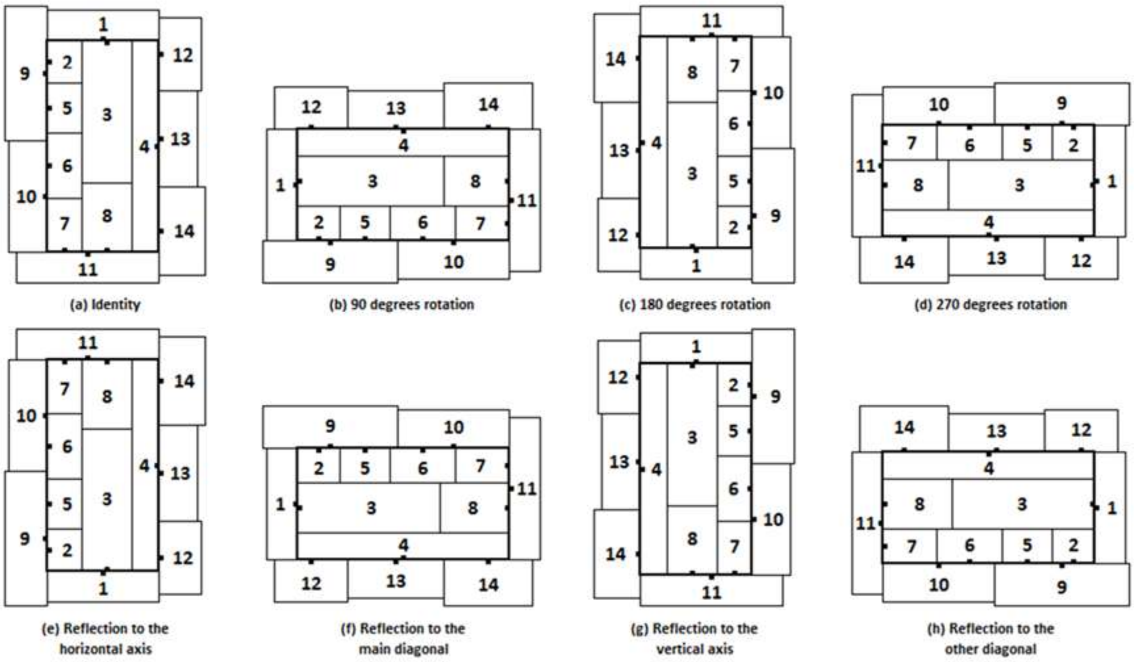

Figure 3.5. 8 equivalent solutions according to the 8 elements of the dihedral group.

obtained. The objective function values of the best-known feasible solutions are listed in Tables 3.1, and 3.2. It is important to emphasize that the previous values listed in the second column of Table 3.2 concern to the Manhattan distances; however, the values obtained from the model given in (3.1)-(3.14), (3.16)-(3.63), and (3.64) which are listed in Table 3.1, are exact. Thus, a higher exact value may represent a smaller real transportation performance than the transportation performance of a layout determined by using the Manhattan distance. Another difficulty of the comparison is that the previously generated layouts are unknown, with the exception that Das (1993) published three figures for the 6-cell problem. In all three cases, it is obvious that the Manhattan value is strictly less than the real objective function value. Moreover, the solutions called TAA-X, 4-STEP and the optimal solution obtained by optimizer can be improved for real distances. The reconstructions of the 4-STEP solution are not unique because cells 3 and 4 can be shifted horizontally. The optimal and TAA-X (reconstructed from Das (1993)) layouts are shown in Figure 3.7. Their real objective function values are 4034.8 and 4225.8 respectively, which are greater than the value of the best closed loop layout solution, which is 3255.8.

Table 3.1. The objective function values of the best-known feasible solutions for closed loop layout. The distances are exact. Solutions for 4, and 6 cells are optimal.

n	closed loop	inter-cell transportation cost	total transportation cost
4	547.5	1003.4	1550.9
6	1601.5	1654.3	3255.8
8	6522.5	4381.4	10906.6
10	13984.5	6627.4	20611.9
12	39765.0	14331.6	54096.6
14	45402.5	12980.0	58382.5
16	71744.2	15551.0	87295.2
18	96529.0	18525.0	115054.0

Table 3.2. The objective function values of the best-known feasible solutions for open field layout obtained from the literature and by optimizer. The distances are non-exact. Solutions for 4, and 6 cells are optimal.

n	total costs	
	literature	optimizer
4	1393.6	1393.6
6	2556.0	2556.0
8	8905.5	8789.3
10	15629.3	16245.1
12	36676.5	39940.6
14	41691.3	47661.5
16	55064.1	63506.4
18	66489.2	80090.4

The calculations were carried out by an Xpress-IVE system. The CPU times are long, i.e., greater than 10,000 seconds. Each problem was tried with different parameters of

Xpress. The following parameters have importance. The total number of physically existing and logical processors is XPRS THREADS. The B&B tree is different for different values of XPRS THREADS; thus, different sets of feasible solutions are generated. Xpress uses several heuristics, even during the B&B procedure. They can be applied in smaller or larger environments and with different frequencies. The parameter XPRS SEARCHEFFORT controls the number of calculations made by heuristics. The default value of the parameter is 1. The parameter is a multiplier, e.g., if XPRS SEARCHEFFORT=1, then the heuristics work double compared with the default case. The frequency of the application of heuristics is controlled by XPRS HEURFREQ. Heuristics are applied only at nodes such that their index is a positive integer multiple of XPRS HEURFREQ. The types of heuristics applied in the root and in the tree are selected by XPRS HEURSEARCHROOTSELECT and XPRS HEURSEARCHTREESELECT, respectively.

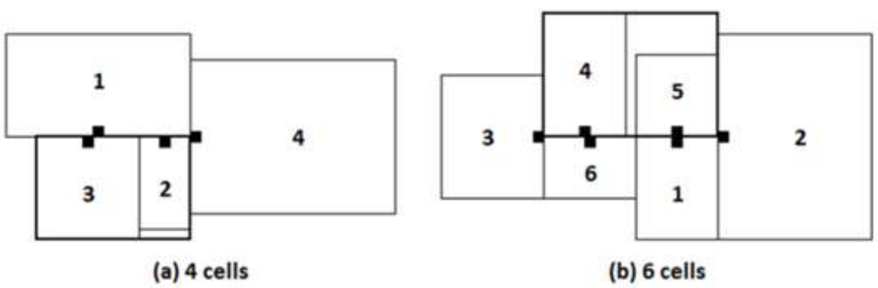

Figure 3.6. The optimal closed loop layout solution of the 4-cell, and 6-cell problem.

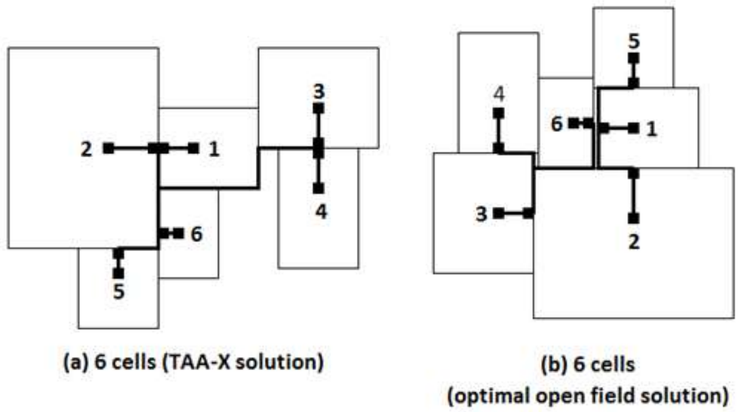

Figure 3.7. The TAA-X and optimal layouts of the 6-cell problem. The TAA-X layout is reconstructed from [Das 1993]. Notice that because the pick-up points are in the interiors of the cells, only the (1,2), (1,5), (2,6), (3,4), and (5,6) pairs in TAA-X layout, and only the (1,3), (1,6), (2,4), (2,6), (3, 5), and (5,6) pairs in optimal layout have a Manhattan distance.

The experiments show that the generation of feasible solutions is sensitive for the values of the parameters. Differences can exist even among similar problems. From the point of view of feasible solutions, the 14-cell problem was more difficult than any other problem. In the case of the closed loop layout problems, a good set of parameter values is as follows:

- XPRS HEURSEARCHEFFORT $= 50$,
- XPRS HEURFREQ $= 20$,
- XPRS HEURSEARCHROOTSELECT $= 7$,
- XPRS HEURSEARCHTREESELECT $= 7$.

The structure of the solution of the previous paper with the exception of the 4 and 6 cell problems of Das (1993) are unknown. The best solutions for all problems found during this research project are listed in Tables 3.3 to 3.5.

Table 3.3. Best-known closed loop solutions of problems with 4, 6 and 8 cells of Das (1993).

Problem	C4				C6				C8			
Cell	x_i	y_i	a_i	b_i	x_i	y_i	a_i	b_i	x_i	y_i	a_i	b_i
1	105	19	100	19	18	0	18	5	33	52.5	40	52.5
2	95.5	12.5	100	12.5	29.5	5	22	5	43.5	53	40	53
3	95	20	100	20	0	5	5	5	45	35	45	40
4	100	0	100	10	9	11	9	5	30	40	40	40
5					18	9	18	5	59	33.5	59	40
6					9.5	2	9.5	5	44	44	44	40
7									52.5	43	52.5	40
8									64.5	45.5	64.5	40
	h_1	h_2	v_1	v_2	h_1	h_2	v_1	v_2	h_1	h_2	v_1	v_2
	10	25	90	100	5	17	5	22	40	72	40	72

3.6 Conclusion

A new MILP model is presented for closed loop layout problems with exact distances. The model was solved by the optimizer Xpress. In two cases, an optimal solution was found, and its optimality is proven. In all other cases, the obtained best feasible solutions are competitive with those generated by different heuristics. There is only one opportunity to compare the best feasible solution obtained from the MILP model to the best one obtained from other methods. In that case, the former solution is better.

The design of a layout is not a problem that must be solved in real-time mode, i.e., long CPU times are still acceptable. The size of real-life layout problems in an industrial environment is limited. One can expect that both computers and optimizers will become faster in the future. Thus, exact models and feasible solutions generated during the not completed optimization procedures remain competitive methods of solving layout problems.

Table 3.4. Best-known closed loop solutions of problems with 10 and 12 cells of Das (1993) and 14 cells of Rajasekharan et al. (1998).

Problem	C10				C12				C14			
Cell	x_i	y_i	a_i	b_i	x_i	y_i	a_i	b_i	x_i	y_i	a_i	b_i
1	35	55.5	40	55.5	55.5	58.5	52	58.5	36.5	44.5	40	44.5
2	46	76	40	76	35	40	40	40	35	54.5	40	54.5
3	40	32.5	40	40	46	55	52	55	34	65	40	65
4	44.5	67.5	40	67.5	56	67	52	67	36	74	40	74
5	37	67.5	40	67.5	52	77	52	71	46	83	40	83
6	52.5	54	40	54	48	45	52	45	36	95	40	95
7	30	91	40	91	62	47	52	47	47.5	59	40	59
8	37.5	44	40	44	32.5	65.5	40	65.5	47.5	94.5	40	94.5
9	34.5	77.5	40	77.5	40	75.5	40	71	35.5	84	40	84
10	46.5	89.5	40	89.5	52	35	52	40	35	105	40	105
11					46.5	65.5	52	65.5	45.5	71.5	40	71.5
12					36	53	40	53	44	46	40	46
13									45	105	40	105
14									40	32.5	40	40
	h_1	h_2	v_1	v_2	h_1	h_2	v_1	v_2	h_1	h_2	v_1	v_2
	40	96	40	65	40	71	40	52	40	126	40	55

In the near future, a breakthrough can occur in the computer technology. It can be a cubic processor 1,000 times faster than the recent processor (White (2011)). IBM and Intel announced the design of such processors. Another option is the spreading of quantum computers. The first one has started to work (Quantum computer). Many problems which cannot be solved numerically for the time being, will become easy for the new computer technology. Non-real-time industrial design problems will belong to that category. This fact increases the importance of the exact models.

Table 3.5. Best-known closed loop solutions of problems with 16 and 18 cells of Rajasekharan et al. (1998).

Problem	C16				C18			
Cell	x_i	y_i	a_i	b_i	x_i	y_i	a_i	b_i
1	36.5	50.5	40	50.5	54.5	43.5	54.5	40
2	35	72.5	40	72.5	61.5	73	61.5	68
3	45	34	45	40	34	42	40	42
4	44	44	40	44	44	36	44	40
5	34	40	40	40	54	62	54	68
6	45	78.5	45	74.5	36	63	40	63
7	47.5	67	40	67	79.5	50	72	50
8	55.5	32.5	44.5	40	32.5	52.5	40	52.5
9	44.5	54	40	54	67.5	62	72	62
10	54	45	54	40	45	45	40	45
11	66.5	34.5	66.5	40	77.5	62.5	72	62.5
12	36	61	40	61	40	72	40	68
13	74	45	69	45	53	35	53	40
14	64	47.5	64	40	51	75.5	51	68
15	59	79.5	59	74.5	67	35	67	40
16	72.5	55	69	55	43.5	55	40	55
17					72	75	72	68
18					44	64	40	64
	h_1	h_2	v_1	v_2	h_1	h_2	v_1	v_2
	40	74.5	40	69	40	68	40	72

Chapter 4

ON THE GENERALIZATION OF MDS METHOD AND ITS APPLICATION IN FACILITY LAYOUT PROBLEMS

4.1 Introduction

The initial motivation of the research discussed in this chapter was as follows. The "quantity" of scientific research and its output has increased significantly over the last few decades. The number of SCI journals is far above 8,000. The *en masse* production of science has caused to some negative phenomena. It has been observed the phenomenon recently several times that a researcher carries out research in one field but finds that the results are not strong enough to publish in that field. The researcher then publishes them in a related but not identical field as an application. As a consequence of this practice, the authors do not know or even do not care of the results of the original field.

The Quadratic Assignment Problem (QAP) is known as one of the most difficult problems within combinatorial optimization. Therefore, it is a suitable experimental field for many algorithmic ideas, including artificial intelligence methods. However, these methods must compete with the special methods of QAP. The latter ones are far better in many cases. Moreover, it is easy to get information on the most recent developments in QAP from QAPLIB (2011) (online library of QAP), where the most important benchmark problems are also available.

The main reason why QAP specific methods are superior to AI methods is that they are based on the careful analysis of the structure of QAP, while AI methods are quite general and unable to exploit the special properties of QAP to the same extent. Thus, the experimental methods cannot be published in their own right. Their authors try to convert them to layout problems because QAP is well known to be a basic model in that application area.

However, it is easy to show by data analysis methods that the problems solved by some layout authors are not really layout problems. A special optimization model and a well-known statistical method called Multi-Dimensional Scaling (MDS) can be used for this purpose. The former can be used for exploring the geometric structure if the distances are l_1 distances (also called rectilinear or Manhattan distances). MDS can be applied for Euclidean distances.

The next section describes the problems in QAPLIB. A very short description of MDS can be found in Section 4.3. Section 4.4 discusses the reconstruction model in the case of l_1 , l_2 and l_∞ (infinite norm) distances. Section 4.5 covers the computational experiments, including both the exact solution of QAP problems and the reconstruction of layout configurations. Some recent papers are critisized in Sections 4.6 and 4.7. Section 4.6 gives a criterion for a QAP to be a layout problem. The contribution of AI methods to the solution of NP-complete problems is analyzed in Section 4.7. Section 4.8 concludes the chapter.

4.2 QAPLIB

The QAPLIB library was established in April, 1996 by R. Burkard, E. Çela, S.E. Karisch, F. Rendl in Graz, Austria (Burkard et al. 1997). Since August 2002, it has been updated by P. Hahn at Pennsylvania State University (QAPLIB 2011). The problems, it contains have very different origins. For example, the problems of Burkard, under the code names Bur26a through Bur26h, concern the speed of typing the 26 letters of the alphabet in different languages. The set of real layout problems constitutes only a minority of the problems in QAPLIB. They are summarized in Table 4.1. The name of a problem consists of two or three parts. The first indicates the author(s) of the problem. The second is the size of the problem. Finally, if the same author has several problems of the same size, then another letter is used to distinguish them. For example, Bur26a indicates Burkard's problem of size 26, as 26 is the number of letters in the alphabet, and the 'a' designates the first problem in this series.

QAPLIB contains many types of useful information besides numerical problems. The results of heuristics and lower bounds on the numerical problems are also reported with the best-known or optimal solution. Codes for computer programs as well as a long list of important papers are also available. The interested reader can find news on promising new results and ongoing research.

4.3 Multi-dimensional Scaling

Multi-dimensional scaling is a well-known method used in statistics to explore the hidden dependency among data. In that sense, it serves the same purpose as factor analysis. A short summary of the method can be found in MDS (2011) and STAT (2011). Assume that there are n comparable objects. The similarity of the objects is described by a nonnegative similarity matrix $(s_{ij})i, j = 1, \ldots, n$. The similarity value $s_{ij} = 0$ means that the two objects are identical, and the higher the value of s_{ij} is, the more dissimilar the objects are. The similarity matrix is supposed to be

Table 4.1. Layout problems in QAPLIB. In all cases where the distance type is not available, the data are integers; therefore, it can be supposed that they are not l_2 distances.

Author(s)	Problem name(s)	Type of distance	Optimal solution in QAPLIB
A.N. Elshafei	Els19	n.a.	YES
S.W. Hadley, F. Rendl, H. Wolkowicz	Had12, Had14, Had16 Had18, Had20	l_1	YES
J. Krarup, P.M. Pruzan	Kra30a, Kra30b, Kra32	weighted l_1	YES
C.E. Nugent, T.E. Vollmann, J. Ruml	Nug12, Nug14, Nug15, Nug16a, Nug16b, Nug17, Nug18, Nug20, Nug21, Nug22, Nug24, Nug25, Nug27, Nug28, Nug30	n.a.	YES
M. Scriabin, R.C. Vergin	Scr12, Scr15, Scr20	l_1	YES
J. Skorin-Kapov	Sko42, Sko49, Sko56, Sko64, Sko72, Sko81, Sko90, Sko100	l_1	NO
L. Steinberg	Ste36a	l_1	YES
L. Steinberg	Ste36b, Ste36c	l_2	YES
U.W. Thonemann, A. Bölte	Tho30	l_1	YES
U.W. Thonemann, A. Bölte	Tho40, Tho150	l_1	NO
M.R. Wilhelm, T.L. Ward	Wil50, Wil100	l_1	NO

symmetric, i.e. $\forall 1 \leq i,j \leq n : s_{ij} = s_{ji}$. If the objects are described by a sufficiently high number of parameters and the similarity is measured by the Euclidean distance, then it is possible to find parameter values such that each similarity value is equal to the appropriate Euclidean distance. Thus, the hidden structure of the objects is revealed only if they are described by a lower number of parameters. However, complete equality of similarity numbers and geometric distances cannot be expected in that case.

MDS works as follows. First, the number of parameters, say k, must be determined. The objects will be represented by k-dimensional vectors, say $X_1, X_2, \ldots, X_n$. Then, the parameter vectors are determined such that the total squared error is minimal, i.e., by the following unconstrained optimization problem:

$$\min_{X_1,X_2,\dots,X_n} \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} (\left\|X_i - X_j\right\|_2 - s_{ij})^2 \quad (4.1)$$

The value of k is either 2 or 3 in most applications. These low dimensions are selected so that the final results of MDS can be graphically represented and the hidden structure, if any, can be recognized by inspection. On the other hand, if the final result is a “random cloud” of points, then no hidden structure is detected.

If a similarity matrix contains Euclidean distances on a plane or in the 3-dimensional space, then MDS is able to reconstruct the relative positions of the points completely. Notice that distances are invariant under rotation and shifting of the whole set of points in any direction. Thus, if it is supposed that the distances of a QAP claimed to be a layout problem are Euclidean distances, then MDS is a perfect tool to use to see whether the problem is a layout problem.

If MDS is executed by an automatic system, then the system selects the lowest dimension d such that the loss of information compared to the case if the points are projected into the $d + 1$ dimensional space is not significant.

4.4 General Reconstruction Model

The reconstruction models can be used for different types of distances such as l_1, l_p and l_∞ distances. While l_1 is the Manhattan distance between points, l_2 is used to show the Euclidian distance between two points, which is a special case of l_p distance, and l_∞ is the maximum absolute difference between the coordinates of a pair of points.

To find the proper positions of the points, the types of distances between the reconstructed points should be determined in the reconstruction model. Usually this distance is of l_1, l_2 or l_∞ type. Additionally, the bias or tolerance of these distances from those of the similarity matrix should be calculated and minimized. This tolerance (bias) can be the same for all pairs of points. In this case, the bias is of l_∞ type. In other cases, the bias of each pair of points is different if type $l_p(1 \leq p < +\infty)$ distance is used in the reconstruction model.

The two main parts of the reconstruction model are the constraints, which are discussed first, and the objective function, which is the measure of error that must be minimized. The models are elaborated 2-dimensionally. The generalizations, however, are straightforward. The constraints and objective function can be introduced for l_1, $l_p(1 < p < +\infty)$ and l_∞ types of distances, separately. Therefore, there will be 3 types of constraints sets and 3 types of objective functions, which are introduced below.

4.4.1 l_1 Type Constraints

A mixed-integer linear programming model is discussed here for the case of 2 dimensions which includes the points on the plane.

The l_1 distance between points (x_j, y_j) and (x_k, y_k) is defined as

$$l_1\big((x_j, y_j), (x_k, y_k)\big) = |x_j - x_k| + |y_j - y_k| = \max\{x_j - x_k + y_j - y_k\,, x_j - x_k + y_k - y_j\,, x_k - x_j + y_j - y_k\,, x_k - x_j + y_k - y_j\} \tag{4.2}$$

If the reconstruction of n points on a plane is needed, let t_{jk} $(1 \le j, k \le n)$ be the l_1 distance between reconstructed j^{th} and k^{th} points in a square $((0,0),(0,h),(h,h),(h,0))$ where $h > 0$. This value must be at least the highest distance among the known distances of the similarity matrix.

The first set of constraints for each pair of cells will be used to force the four above-mentioned sums to be less than or equal to the reconstructed l_1 distance between the pair of points:

$$x_j - x_k + y_j - y_k \le t_{jk} \quad (4.3)$$

$$x_j - x_k + y_k - y_j \le t_{jk} \quad (4.4)$$

$$x_k - x_j + y_j - y_k \le t_{jk} \quad (4.5)$$

$$x_k - x_j + y_k - y_j \le t_{jk} \quad (4.6)$$

$$1 \le j < k \le n$$

In the second set of constraints, the opposite inequalities are claimed. At least one of the above-mentioned quantities on the left-hand sides must be greater than or equal to the reconstructed l_1 distance between two points. Let M be a large number, $M = 4h$ is then a proper choice. The constraints are

$$x_j - x_k + y_j - y_k + Mu_{jk1} \ge t_{jk} \quad (4.7)$$

$$x_j - x_k + y_k - y_j + Mu_{jk2} \ge t_{jk} \quad (4.8)$$

$$x_k - x_j + y_j - y_k + Mu_{jk3} \ge t_{jk} \quad (4.9)$$

$$x_k - x_j + y_k - y_j + Mu_{jk4} \ge t_{jk} \quad (4.10)$$

$$1 \le j < k \le n$$

with

$$u_{jk1}, u_{jk2}, u_{jk3}, u_{jk4} \in \{0,1\} \qquad 1 \le j < k \le n \quad (4.11)$$

Mu_{jki} is used as a correction value of i^{th} inequality for the above set of constraints. If $u_{jki} = 1$, then the i^{th} inequality automatically is satisfied. To obtain the l_1 distance, at least one of the constraints must be satisfied without using the correction term. Thus, the cut

$$u_{jk1} + u_{jk2} + u_{jk3} + u_{jk4} \leq 3 \tag{4.12}$$

must be applied.

The obvious set of constraints is to force the points to be in the square of h:

$$0 \leq x_j, y_j \leq h\,, \qquad j = 1, \dots, n \tag{4.13}$$

4.4.2 l_∞ Type Constraints

The l_∞ distance between points (x_j, y_j) and (x_k, y_k) is defined as

$$l_\infty\left((x_j, y_j), (x_k, y_k)\right) = \max\{|x_j - x_k|, |y_j - y_k|\} = \max\{x_j - x_k\,, x_j - x_k, y_j - y_k\,, y_k - y_j\} \tag{4.14}$$

Assume then that the problem is to reconstruct n points in the above-mentioned square by using l_∞ distance between reconstructed points. The constraint logic is similar to the l_1 case. For each pair of points, the first set of constraints claims that all four of the above terms are less than or equal to the reconstructed l_∞ distance:

$$x_j - x_k \leq t_{jk} \tag{4.15}$$

$$x_k - x_j \leq t_{jk} \tag{4.16}$$

$$y_j - y_k \leq t_{jk} \tag{4.17}$$

$$y_k - y_j \leq t_{jk} \tag{4.18}$$

$$1 \leq j < k \leq n$$

In the second set of constraints, with the help of binary variables, at least one of the above-mentioned quantities is greater than or equal to the reconstructed l_∞ distance. Using a large number with estimation of $M = 2h$, the constraints are:

$$x_j - x_k + Mu_{jk1} \geq t_{jk} \tag{4.19}$$

$$x_k - x_j + Mu_{jk2} \geq t_{jk} \tag{4.20}$$

$$y_j - y_k + Mu_{jk3} \geq t_{jk} \tag{4.21}$$

$$y_k - y_j + Mu_{jk4} \geq t_{jk} \tag{4.22}$$

$$1 \leq j < k \leq n$$

where

$$u_{jk1}, u_{jk2}, u_{jk3}, u_{jk4} \in \{0,1\} \quad 1 \leq j < k \leq n \tag{4.23}$$

If $u_{jki} = 1$, using Mu_{jki}, the i^{th} inequality automatically is satisfied. The j^{th} and k^{th} points are positioned properly, if at least one of the above-mentioned constraints is satisfied without using the correction term. Thus the cut

$$u_{jk1} + u_{jk2} + u_{jk3} + u_{jk4} \leq 3 \tag{4.24}$$

must be applied.

Additionally, the points are limited to fall in the square of h:

$$0 \leq x_j, y_j \leq h \ , \quad j = 1, \dots, n \tag{4.25}$$

4.4.3 l_p Type Constraints

The l_p distance between points (x_j, y_j) and (x_k, y_k) is defined as

$$l_p\big((x_j, y_j), (x_k, y_k)\big) = \sqrt[p]{|x_j - x_k|^p + |y_j - y_k|^p} \tag{4.26}$$

The nonnegative l_p $(1 < p < +\infty)$ distance can be expressed by a single equation:

$$|x_j - x_k|^p + |y_j - y_k|^p = t_{jk}{}^p \quad (4.27)$$

$$t_{jk} \geq 0 \quad (4.28)$$

The points also should be positioned in the square of h:

$$0 \leq x_j, y_j \leq h\,, \qquad j = 1, \dots, n \quad (4.29)$$

Of course, the well-known case of l_p distance is the Euclidean distance if $p = 2$.

4.4.4 l_1 Type of Objective Function

Before identification of the objective function, the bias between the reconstructed distances and the elements of the similarity matrix for each pair of points should be calculated, e.g., τ_{jk} for points j and k. Therefore, in l_1 type of objective function, this bias is separately defined for each pair of points and calculated by the following set of constraints:

$$d_{jk} - t_{jk} \leq \tau_{jk} \quad (4.30)$$

$$t_{jk} - d_{jk} \leq \tau_{jk} \quad (4.31)$$

Therefore the objective function will minimize the sum of all tolerances as follows:

$$\min \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \tau_{jk} \quad (4.32)$$

4.4.5 l_∞ Type of Objective Function

In this type of objective function, the same tolerance of τ is considered for the reconstructed distance and the related element of the similarity matrix for each pair of points. Thus, using the following set of constraints, the tolerance is calculated and subsequently minimized:

$$d_{jk} - t_{jk} \leq \tau \quad (4.33)$$

$t_{jk} - d_{jk} \leq \tau$ (4.34)

$\min \tau$ (4.35)

4.4.6 l_p Type of Objective Function

In l_p type of objective function, the different biases between the reconstructed distance and the distance from the similarity matrix for each pair of points are first calculated.

Next the sum of p^{th} power for all tolerances is minimized by use of the following set of constraints and the objective function:

$d_{jk} - t_{jk} \leq \tau_{jk}$ (4.36)

$t_{jk} - d_{jk} \leq \tau_{jk}$ (4.37)

$\min \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} {\tau_{jk}}^{p}$ (4.38)

4.4.7 Problem Types

Each type of objective function can be used with all types of constraints. This means that 9 possible reconstruction models may be considered.

The general notation of (a, b) is used to reference the utilized model. The first element of the notation signifies the type of constraints and the second element shows the type of objective function that is used in the reconstruction model. a and b can be selected from all above-mentioned distances, e.g., l_1, l_p and l_∞ distances. For example *the reconstruction model of* (l_1, l_∞) *distances*, refers to the mathematical model, which includes l_1 type constraints and l_∞ type objective functions.

4.5 Computational Results

The Had14 problem of QAPLIB has been solved by the program called qapbb.f (Burkard and Derigs 1980). It is also downloadable from QAPLIB. The program was running on a computer with an Intel Pentium Dual 2 GHz processor and 1024 Mb Ram. It solved the problem optimally in 5 seconds. Had14 was selected because it is the only problem that is experimentally discussed in Wong and See (2010).

To test the abilities of this approximately 30-year-old program, three further problems have been solved. Interestingly, in all three cases, an alternative optimal solution has been found that is not included in QAPLIB. They are contained in Table 4.2. In the case of the Els19 problem, the only difference is that the facilities 18 and 19 are interchanged. The CPU time was less than 1 second for both Els19 and Chr22a. It was 75 seconds for Chr25a. The optimal solutions found for the latter two problems are significantly different from the ones stored in QAPLIB. No further attempt to solve problems by qapbb.f was made, as the systematic reevaluation of earlier computer software is beyond the scope of the current research.

It is important to emphasize that the reconstruction methods determine only the relative positions of the points even in the case of perfect reconstruction. Then, to obtain the original structure, the reconstructed structure may need rotation and/or shifting.

The reconstruction is not successful in the case of the MDS method if the resulting set of points forms a random cloud. This means that either the underlying geometrical structure does not exist, or l_2 is not the proper distance. If we suppose that the distances are of type l_1, then the reconstruction is made by previously mentioned model. There is no reconstruction if there is no feasible solution. In such a case, if the problem is solved without the cut, the optimal solution may contain pairs

Table 4.2. The alternative optimal solutions found by qapbb.f

Problem	Location	1	2	3	4	5	6	7	8	9	10
Els19	Assigned Dept.	9	10	7	19	14	18	13	17	6	11
	Location	11	12	13	14	15	16	17	18	19	
	Assigned Dept.	4	5	12	8	15	16	1	2	3	
Chr22a	Location	1	2	3	4	5	6	7	8	9	10
	Assigned Dept.	6	2	15	16	11	13	7	4	19	21
	Location	11	12	13	14	15	16	17	18	19	20
	Assigned Dept.	14	22	10	9	1	5	12	8	18	17
	Location	21	22								
	Assigned Dept.	3	20								
Chr25a	Location	1	2	3	4	5	6	7	8	9	10
	Assigned Dept.	18	22	4	6	3	12	24	8	25	10
	Location	11	12	13	14	15	16	17	18	19	20
	Assigned Dept.	20	2	17	11	13	7	21	5	16	9
	Location	21	22	23	24	25					
	Assigned Dept.	19	23	14	15	1					

of points that coincide and other pairs with the wrong distance. It is also likely, according to our computational experiments, that many points are on the same vertical or horizontal line, as shown in Figure 4.6. The distances of Rou12 in QAPLIB do not satisfy the triangle inequality.

The results of the reconstruction are not necessarily congruent geometrically to the original structure. If a distance matrix has a clear underlying geometric structure, then the reconstructed problem will have such a structure as well, but the reconstructed structure may depend on the dimension of the space into which the points are projected and the type of the distance, which may be l_1 or l_2. What is

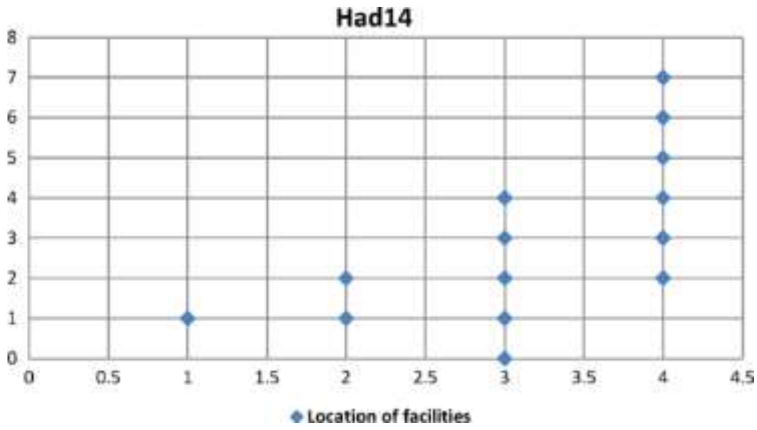

Figure 4.1. Reconstruction of the l_1 distances of the Had14 problem. The reconstruction is perfect in the sense that all l_1 distances are exactly the same as in the original problem.

important is that the generated figure shows a structure. A sequence of figures illustrates this principle. Figure 4.1 shows the Had14 problem. The reconstruction is perfect in the sense that the l_1 distances are exactly equal to the distances in the problem. Both methods give a good quality reconstruction for Kra30a in 3 dimensions. In the plane, the reconstructions are different but still have recognizable structure (see Figures 4.4 and 4.5). Kra30a has been selected because it is a benchmark problem and was not solved exactly for 27 years (Hahn and Krarup 2001).

4.6 To Lay Out or not to Lay Out

The Traveling Salesman Problem (TSP) is another important problem in combinatorial optimization. The literature on this problem is even richer than that of QAP. There are many computational studies on TSP. They can give hints as to what can be expected in the case of other problems, like QAP. Moreover, TSP has many applications in very different areas, including transportation, design and production of integrated circuits, scheduling in chemical industry, minimization of set-up times, and automatic movements of robot arms and machines, to name just a few examples. These problems pose very different difficulties from the algorithmic point of view

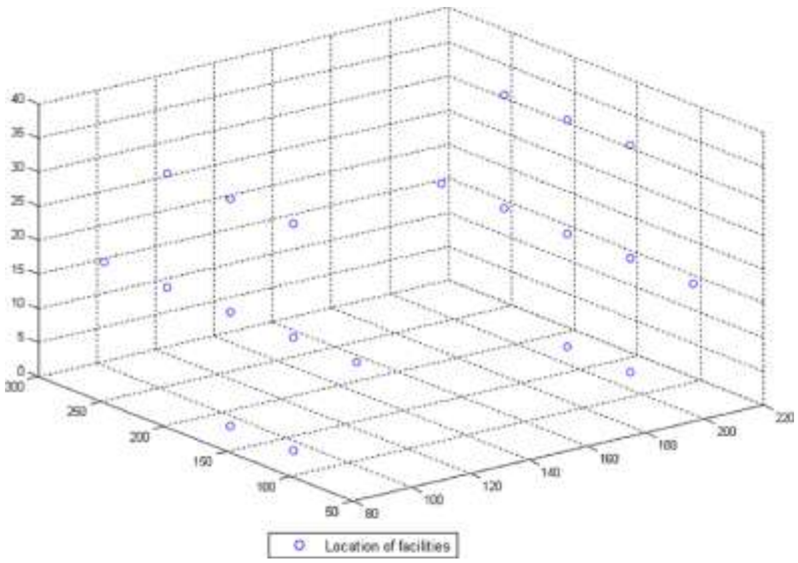

Figure 4.2. Reconstruction of the structure of the Kra30a problem by the introduced model in the 3-dimensional space. The reconstruction is not perfect, as the weights applied in the l_1 type distance are unknown. Note that the levels of the building are clearly recognizable.

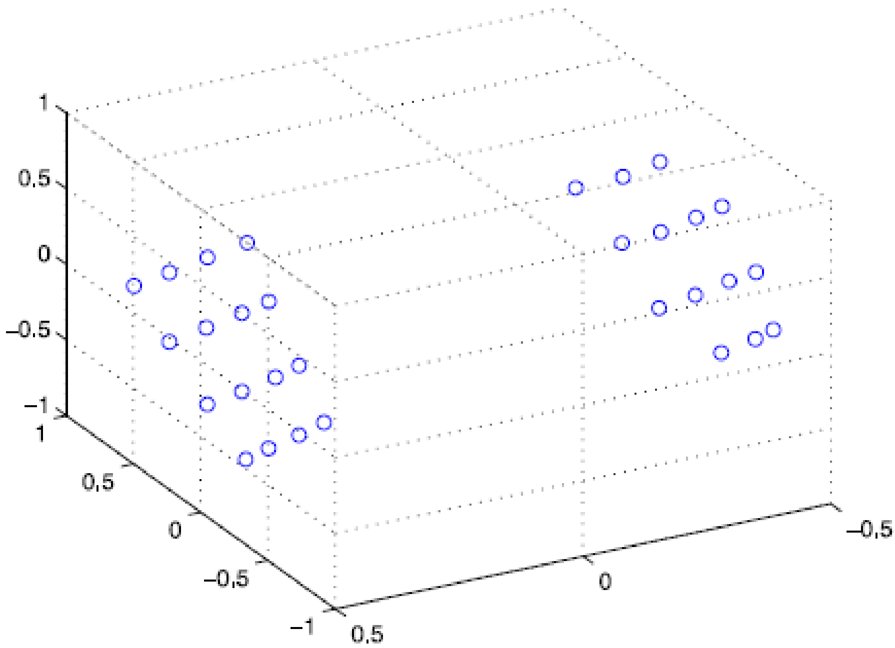

Figure 4.3. Reconstruction of the structure of Kra30a problem in 3-dimensional space by the MDS method. The configuration must be rotated to obtain the real positions.

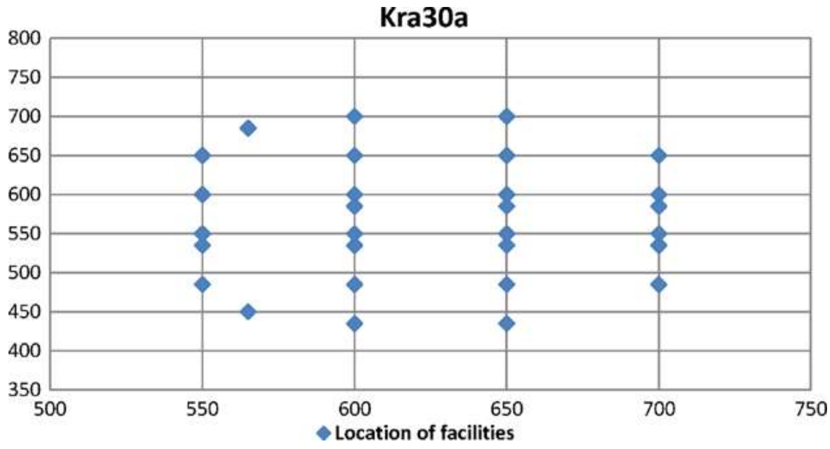

Figure 4.4. Reconstruction of the structure of the Kra30a problem in the plane by introduced model. The configuration has some symmetry and regularity properties.

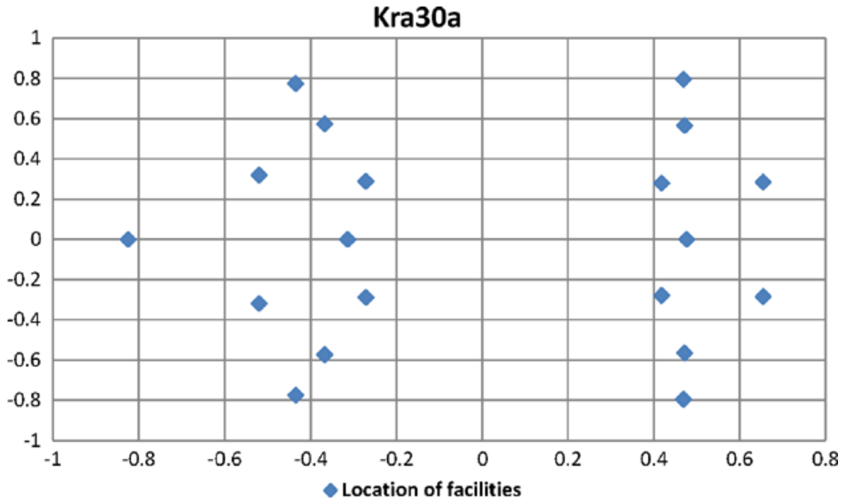

Figure 4.5. Reconstruction of the structure of Kra30a problem in the plane by the MDS method. This configuration also has some symmetry and regularity properties.

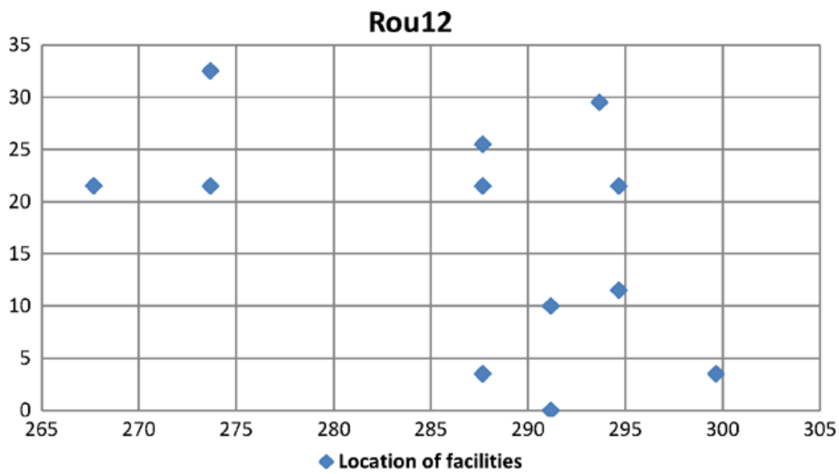

Figure 4.6. The problem Rou12 in QAPLIB. Its reconstruction is not possible. The attempt was made by the introduced model.

because the data sets of different practical problems have different structural properties. Not all of these properties are known and/or understood. Similarly, the instances of any other optimization problem may have different structural properties, depending on the origin of the practical problem to be modeled.

Any real layout problem can be reconstructed either in the plane, if the facilities must be positioned on a surface, or in 3-dimensional space, if the facilities must be assigned to positions in a building. In both cases, the reconstruction must show an easily recognizable geometric structure.

If a problem has no such reconstruction, then it is not a layout problem but another kind of problem that can also be modeled by QAP. There is no reason to suppose that their underlying QAPs can be solved by the same amount of computational efforts.

One can conclude that *if a paper deals with both reconstructable and non-reconstructable instances of QAP then that paper is a general purpose QAP paper regardless of whether it claims to be a layout paper.*

This is the case in the paper of Ramkumar et al. (2009). That paper provides computational results for almost all problem instances stored in QAPLIB. Only the three Thonemann-Bölte problems, the two Wilhelm-Ward problems and some very large-scale problems from the classes Li-Pardalos, Skorin-Kapov, and Taillard are missing. However, the origins of the majority of the problems are not layout problems, as in the case, for example, of the problems of the Burkard-Offermann class (Bru26x) model typewriting, which was mentioned above.

Ramkumar et al. (2009) does not give any information on the CPU times. Therefore, it is very difficult to evaluate its results in light of the results obtained by qapbb.f and mentioned in Section 4.5.

4.7 Further Remarks

Wong and See (2010) discusses a hybrid ant colony-genetic algorithm for QAP. The application of the ant colony method to QAP is not a new idea. For some early publications, see Colorni and Maniezzo (1999) and Taillard and Gambardella (1997). QAPLIB even contains a software system called FANT that was designed by Taillard and based on Taillard (1998). Wong and See (2010) solves a single problem, namely, the Had14 problem contained in QAPLIB. Furthermore, they even use the optimal value that is also reported in QAPLIB. As a matter of fact, they refer to QAPLIB on page 124 in the second-to-last paragraph. Therefore, they would have had to refer to both Taillard (1998) and the software FANT which are missing from their reference list.

At this point a very serious question arises:

> *What is the computational effort that the operations research community should expect from papers applying artificial intelligence methods to optimization problems?*

There are many questions connected to the main question. *Is the solution of a single problem enough?* Certainly not. The result of one measurement may reflect random effects. It is possible to draw conclusions on the properties of a method only if the method behaves in the same way for several problems. The more problems, the better. *Is it necessary that the new ideas must compete with all previous ideas?* Yes, it is. This principle concerns any kind of new results, not only AI methods. Wong and See (2010) finds the a *priori* known optimal solution of Had14 in 603 seconds but is unable to say anything about its optimality because of the nature of the method. Its optimality was recognized because it was already known. However, a branch and bound code, which is 30 years older, solved the problem in 5 seconds. If no further computational evidence is provided, then the only conclusion that can be drawn is that the new ant colony-genetic algorithm is useless. More generally, we can state the following principle: *a new method may be published only if the author is able to produce at least one case where the new method is superior to the known methods.*

When can the application of AI methods be justified? According to the present state of science, NP-hard problems can be solved exactly only by enumerative type methods, including dynamic programming. These methods have an exponential number of operations. Therefore, all of these methods/programs are subject to combinatorial explosion, so the problems larger than a certain threshold cannot be solved. Smarter methods and faster computers only shift the threshold. The use of AI methods is justified beyond the threshold, as they control the CPU time required. To predict what can be expected in that region is an important problem. The only basis for prediction is the behavior of the AI methods within the exactly solvable region. Hence, for that purpose, the computational experiments must be exhaustive in that region of the problems.

4.8 Conclusion

The initial objective of this chapter was to show that results of general quadratic assignment problems may appear under the name of layout problem. General quadratic assignment problems and layout problems can be distinguished from one another if a geometric structure of the position reconstructable. Two methods are used for reconstruction: generalization of MDS (the introduced optimization models) and the MDS method of statistics. An unexpected result is that a 30 years old public program behaves very well in contemporary computers. Thus new methods must compete with that program as well.

Chapter 5

ON THE LAYOUT PROBLEM OF EXISTING SUPERMARKETS

5.1 Introduction

The scientific analysis of shop layout has developed over recent decades parallel with customer research and marketing science and technologies. In the first phase of the development of supermarkets based on the referred studies in chapter 2, shop layout was based on some empirical experiences. In this phase the most important guidelines were the technological constraints of shops, e.g., minimization the distance of freezing equipment from the electrical power sockets. It can be stated that the current literature on shop layout (chapter 2) has focused more on description and analysis of shop layout/customer interaction and a lesser degree on optimization of layout with the purpose turnover maximization.

It is assumed that the storing system in the customer area, i.e., the system of shelves, and other logistic equipments, is given and cannot be changed, i.e., no new investment is possible. However, the *use* of the shelves can be changed in order to stimulate more purchases by customers. Thus, the main objective is to increase the walking distance the customers cover in the shop.

For practical purposes it is assumed that one shelf is filled by goods of a certain category, i.e., the number of shelves and categories are equal. Examples for categories are: bakery, fruit and vegetable, milk product, etc. Only complete categories can move from one place to another one.

To obtain an optimal layout the customers must be clustered according to the type and purpose of their purchases. The shoppings of a person may belong to different clusters if the purpose of the shopping is different. The clusters are discussed in section 4.3. The clusters represent the real customers in the mathematical model. The weight of a cluster is the frequency of the type of purchase.

In Section 4.2 the mathematical model is discussed. The classification of customers is described in Section 4.3. Computational results on real-life shops are presented in Section 4.4. Conclusions are drawn in Section 4.5.

5.2 Mathematical Model of Relocating Categories in a Supermarket

5.2.1 Basic Assumptions

Each category is in a certain place in the current layout of the supermarket. The following assumptions are applied when the categories are relocated.

1. Only a complete category is moved from one place to another.
2. The sets of categories and positions are not changed by the relocation.
3. The numbers of categories and positions are equal.
4. Each category can be moved only to one of a restricted set of positions, for example, goods requiring cooling can move to positions having refrigerators.
5. The distance of positions is measured according to the plan of the shop and not in a straight line.
6. There are finite types of customers. Each type is defined by the categories from which the customer buys something.

7. The customer enters at the entrance, visits all the categories he/she needs to and leaves at the nearest cashier.
8. The behaviour of the customer is supposed to be rational which means that his/her route from entrance through all necessary categories to cashier is optimized.
9. The customer may buy unplanned items what just observes. The quantity of these items is an increasing function of the length of the route covered by the customer.

It follows from the assumptions that a proper objective function is the maximization of the sum of the length of the minimal route length of the customers weighted by the frequencies of the customer types.

Each customer moves along a minimal length Hamiltonian path of the categories of his/her type according to Assumption 8. A Travelling Salesman Problem (TSP) occurs if the entrance and cashiers are considered to be the same point, so, what can be done as entrance and cashier are the two end points of the path? In general, the mathematical models of TSP are large and difficult to handle. There are two ways of incorporating TSP models into the layout model as discussed in the next section.

5.2.2 Remarks on TSP

Case 1: Few Visited Categories (FVC). In this case all possible orders of the categories can be considered explicitly. Assume that customers of type i visit k categories. Then there are $k!$ different orders. Each order determines a distance. The length of the path of the customer is equal to the minimal one of these distances, for example, assume that the customer visits categories a and b. The customer starts at the entrance denoted by E and finishes at the cashier denoted by C. As $2! = 2$, the two possible orders of his/her path are: $E - a - b - C$ and $E - b - a - C$. If d_{uw} denotes the distance between points (categories) u and w, and h_{ab} is the length of the path of the type visiting only a and b, then h_{ab} must satisfy the following inequalities:

$$h_{ab} \leq d_{Ea} + d_{ab} + d_{bC}$$

and

$$h_{ab} \leq d_{Eb} + d_{ba} + d_{aC}$$

It is enough to claim inequalities. The value of h_{ab} is maximized in the objective function. Thus, its value is automatically equal to the minimal right-hand side in the optimal solution. Notice however, that the distances on the right-hand sides, i.e., d_{Ea}, d_{Eb}, d_{ab}, d_{ba}, d_{aC}, and d_{bC}, are variables as they depend on the new positions of the categories.

Case 2: Large Number of Visited Categories (LNVC). If the number of visited categories is large then it is not practical to use $k!$ constraints for a single customer. In this case, the path of the customer must be described by a mathematical model of TSP. In this study, the Dantzig-Fulkerson-Johnson model (DFJ) is used among the possible options. Assume that the salesperson visits n cities. (In our case $n = k + 1$ if entrance and cashier are considered to be the same from modelling point of view). Let d_{ij} be the distance between cities i, and j. The set of cities is denoted by $N = \{1,2,\ldots,n\}$. One form of the DFJ model is the following

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \qquad (5.1)$$

$$\sum_{i=1}^{n} x_{ij} = 1 \qquad j = 1, 2, \ldots, n \qquad (5.2)$$

$$\sum_{j=1}^{n} x_{ij} = 1 \qquad i = 1, 2, \ldots, n \qquad (5.3)$$

$$\forall\, S \subset N, 2 \le |S| \le \tfrac{n}{2}\colon \textstyle\sum_{i\in S}\sum_{j\in N\setminus S} x_{ij} \ge 1\,, \quad \sum_{i\in N\setminus S}\sum_{j\in S} x_{ij} \ge 1 \tag{5.4}$$

$$\forall\, i,j \in N:\ x_{ij} = 0 \text{ or } 1. \tag{5.5}$$

The form (5.1)-(5.5) cannot be used directly in the model of the layout problem for two reasons. The direction of the optimization is minimization here and it is maximization in the layout problem. The distances $d_{ij}s$ are the function of other decision variables similarly to the case FVC. Thus, the objective function is at least quadratic and this fact may lead to computational difficulties. Therefore, the linear programming approximation of the DFJ model is used. It is obtained by substituting the integrality condition (5.5) by

$$\forall\, i,j \in N:\ 0 \le x_{ij} \le 1. \tag{5.5'}$$

Then the linear programming dual of the problem (5.1)-(5.4), (5.5') can be taken, which is a maximization problem. Let u_j, v_i, w_S, y_S and z_{ij} be the dual variables belonging to constraints (5.2), (5.3), the two inequalities of (5.4) and $x_{ij} \le 1$ inequalities of (5.5'), respectively. Then the dual problem is the following

$$\max \textstyle\sum_{j=1}^{n} u_j + \sum_{i=1}^{n} v_i + \sum_{S:2\le|S|\le\frac{n}{2}} w_S + \sum_{S:2\le|S|\le\frac{n}{2}} y_S - \sum_{i=1}^{n}\sum_{j=1}^{n} z_{ij} \tag{5.6}$$

$$\forall\, i,j \in N, i \ne j\colon u_j + v_i + \textstyle\sum_{i\in S, j\in N\setminus S} w_S + \sum_{i\in N\setminus S, j\in S} y_S - z_{ij} \le d_{ij} \tag{5.7}$$

$$\forall\, S \subset N, 2 \le |S| \le \tfrac{n}{2}:\ w_S, y_S \ge 0\,, \quad \forall\, i,j \in N\colon z_{ij} \ge 0. \tag{5.8}$$

Here, the two above-mentioned problems are avoided. The direction of the optimization is maximization and the distances stand alone on the right-hand sides and thus, there is no non-linearity in the problem.

5.2.3 Notations

When elaborating the notations, categories and their positions must be distinguished. Each category now occupies a position, which is not necessarily identical to its future position. The future position of a category is an element of a subset of positions consisting of positions such that the relocation of the category is technically possible and no other rule is violated. The distances between positions are known. The distances are measured from the middle points of the shelves and are equal to the distance that the customer must cover between the positions. The distances of the categories are functions of their positions.

The notations of the following mathematical model are mentioned here,

p : The number of positions and categories (constant),

q, r : Indices of positions (index),

E : The entrance (index),

C : The entrance (index),

i, j : Indices of categories (index),

P_i : The set of the potential new positions of category i (input data set),

δ_{qr} : The distance of positions q and r (constant),

d_{ij} : The distance of categories i and j (variable),

m : The number of types of customers (constant),

k : The index of customer type (constant),

B_k : The set of categories visited (input data),

$\prod(B)$: The set of all permutations of the set B,

h_k : The distance covered by customer type k(variable),

x_{iq} : Binary variable; it is 1 if category i is moved to position q and is 0 otherwise (variable),

t_{ijqr} : Binary variable; its value is $x_{iq}x_{jr}$ (variable),

C_{FVC} : The set of customer types visiting few categories (input data set),

C_{LNVC} : The set of customer types visiting a large number of categories (input data set),

$u_{kj}, v_{ki}, w_{kS}, y_{kS}, z_{kij}$: The variables in the dual of the TSP of customer type k.

5.2.4 Mathematical Formulation

The set of constraints form several groups according to the requirements that they express. The first group claims that categories must be assigned to positions and vice versa.

The first claim is that each category must be assigned to a position which is expressed by the following equations

$$\forall\, i : \sum_{q=1}^{p} x_{iq} = 1, \tag{5.9}$$

Similarly, each position must hold a category

$$\forall\, q : \sum_{i=1}^{p} x_{iq} = 1. \tag{5.10}$$

The distance of categories i and j is δ_{qr} if i is assigned to position q and j is assigned to position r, i.e., $x_{iq} = x_{jr} = 1$. In other words the equation $x_{iq}x_{jr} = 1$ must hold. Hence, the distance d_{ij} of the two categories can be expressed by the equation

$$d_{ij} = \sum_{q=1}^{p} \sum_{r=1}^{p} \delta_{qr} x_{iq} x_{jr}.$$

The product of the two binary variables is still a certain kind of non-linearity which can be eliminated by introducing the binary variables t_{ijpq} with

$t_{ijpq} = x_{iq}x_{jr}$.

Hence, the equations

$$\forall\, i,j:\ d_{ij} = \sum_{q=1}^{p}\sum_{r=1}^{p}\delta_{qr}t_{ijqr} \tag{5.11}$$

describe the distances of the categories. It is easy to see that if

$$\forall\, i,j,q,r:\ x_{iq} + x_{jr} \leq t_{ijqr} + 1 \tag{5.12}$$

and

$$\forall\, i,j,q,r:\ x_{iq} + x_{jr} \geq 2t_{ijqr} \tag{5.13}$$

then

$\forall i,j,q,r: x_{iq}x_{jr} = t_{ijqr}$.

All variables mentioned so far must be binary

$$\forall\, i,j,q,r:\ x_{iq}, t_{ijqr} = 0 \; or \; 1. \tag{5.14}$$

In the next two sets of constraints, the distance of the move of the customers is determined for the FVC and LNVC groups, respectively. In both cases, the variable h_k is restricted from above only. The value of h_k equals the minimal upper bound in the optimal solution as the sum of h_ks is maximized.

FVC. In this case, the length of the path is determined by the permutations of the categories visited by the customer. The same type of inequalities must be claimed for all customers belonging to the class C_{FVC}:

$$\forall\, k \in C_{FVC}, \forall\, \pi \in \prod(B_k): h_k \leq d_{E\pi(1)} + \sum_{i=2}^{|B_k|} d_{\pi(i-1)\pi(i)} + d_{\pi(|B_k|)C}. \tag{5.15}$$

LNVC. The dual of the DFJ model is applied as mentioned above. The constraints of the dual problem are the following according to (5.7) and (5.8)

$$\forall\, k \in C_{LNVC}, \forall\, i,j \in B_k (i \neq j): u_{jk} + v_{ik} + \sum_{\substack{i\in S \\ j\in N\setminus S \\ 2\le|S|\le|B_k|/2}} w_{Sk} + \sum_{\substack{j\in S \\ i\in N\setminus S \\ 2\le|S|\le|B_k|/2}} y_{Sk} - z_{ijk} \le d_{ij} \quad (5.16)$$

$$\forall\, k \in C_{LNVC}, \forall\, S \subset B_k \left(2 \le |S| \le \frac{|B_k|}{2}\right): w_{Sk}, y_{Sk} \ge 0 \quad (5.17)$$

$$\forall\, k, \forall\, i,j \in N: z_{ijk} \quad (5.18)$$

The objective function (5.6) of the dual problem is the upper bound of the related variable h_k

$$h_k \le \sum_{j\in B_k} u_{jk} + \sum_{i\in B_k} v_{ik} + \sum_{s:2\le|s|\le|B_k|} w_{sk} + \sum_{s:2\le|s|\le|B_k|} y_{sk} + \sum_{i\in B_k}\sum_{j\in B_k} z_{ijk} \quad (5.19)$$

The objective function of the whole model is the sum of the variables h_k, i.e.,

$$\max \sum_{k=1}^{p} h_k. \quad (5.20)$$

Thus, the model is formulated in (5.9)-(5.20).

5.3 Customers

The data of 13,300 purchases were collected from three shops. They were analysed by k-mean clustering. Some large clusters were divided by the same method into smaller clusters. The reason for the second level clustering is twofold: (i) to avoid cases when it is was doubtful that a category is purchased by a cluster; (ii) to ensure that all categories are purchased. Detergent is a typical example for the latter case; it is purchased rarely, however, when it is purchased then on average it is purchased in a large quantity. In this way, 27 clusters were generated. The k-mean tool of SPSS was used for carrying out the calculations.

The three shops are located in similar residential areas; the customers typically going to the supermarkets on foot. The clusters depend on income and from a more general perspective the social position of the regular customers. Therefore, the relevant clusters can be different for supermarkets located in different parts of a city. Similarly, super/hypermarkets serving motorized customers also have different clusters. It is also important to emphasize that different purchases by the same person can belong to different clusters, for example, the purchase for weekend cooking is quite different from buying some supplementary items.

The clusters (customer and the visited departments) are as follow:

Customer 1: Fruit and vegetable - Milk and milk products - Canned vegetables - Household paper - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Fresh meat - Coffee, tea and cocoa.

Customer 2: Milk and milk products - Sausage - Canned vegetables - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Praline, bonbon and biscuits - Cheese - Cakes.

Customer 3: Bread and bakery - Sweets and cakes - Chocolate, rice, salt and cornflakes - Cakes.

Customer 4: Milk and milk products - Bread and bakery - Sweets and cakes - Chips, flour and sugar - Cakes.

Customer 5: Bread and bakery - Wine, beer and alcohol - Pressing (fruit).

Customer 6: Milk and milk products - Bread and bakery.

Customer 7: Chips, flour and sugar.

Customer 8: Fruit and vegetable - Sausage - Canned vegetables - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Praline, bonbon and biscuits - Fresh meat - Cheese - Cakes.

Customer 9: Bread and bakery.

Customer 10: Milk and milk products.

Customer 11: Milk and milk products - Sweets and cakes - Chocolate, rice, salt and cornflakes - Chips, flour and sugar - Praline, bonbon and biscuits - Cakes.

Customer 12: Milk and milk products - Sausage - Sweets and cakes - Chocolate, rice, salt and cornflakes - Cakes.

Customer 13: Sausage - Bread and bakery - Chocolate, rice, salt and cornflakes.

Customer 14: Milk and milk products - Sausage - Bread and bakery.

Customer 15: Milk and milk products - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Cakes - Wine, beer and alcohol - Pressings (fruit).

Customer 16: Bread and bakery - Wine, beer and alcohol - Pressings (fruit).

Customer 17: Fruit and vegetable - Milk and milk products - Sausage - Other chemicals - Household paper - Detergents - Bread and bakery - Soup, spices, canned food - Chips, flour and sugar - Fresh meat.

Customer 18: Milk and milk products - Sausage - Other chemicals - Bread and bakery - Sweets and cakes - Chocolate, rice, salt and cornflakes - Praline, bonbon and biscuits - Cakes.

Customer 19: Milk and milk products - Sausage - Canned vegetables - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Frozen food - Coffee, tea and cocoa – Cakes.

Customer 20: Sausage - Bread and bakery - Sweets and cakes - Frozen food - Cakes.

Customer 21: Milk and milk products - Oil and vinegar - Bread and bakery.

Customer 22: Fruit and vegetable - Milk and milk products - Sausage - Canned vegetables - Oil and vinegar - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Coffee, tea and cocoa.

Customer 23: Cosmetics - Bread and bakery.

Customer 24: Milk and milk products - Sausage - Canned vegetables - Cosmetics - Household paper - Bread and bakery - Sweets and cakes - Soup, spices, canned food - Chips, flour and sugar - Coffee, tea and cocoa.

Customer 25: Milk and milk products - Household paper - Detergents - Bread and bakery.

Customer 26: Milk and milk products - Sausage - Mineral water - Canned vegetables - Bread and bakery - Sweets and cakes - Chips, flour and sugar – Cakes.

Customer 27: Mineral water - Bread and bakery.

5.4 Computational Results

Model (5.9)-(5.20) was solved by Xpress solver. The model has 44,278 continuous and 10,100 binary variables, 1,775 equations and 21,205 inequalities. A Toshiba laptop was the computer with an Intel i5 processor having 2x2.27 GHz frequency and 4GB memory. The time required to solve the model exactly was approximately 2 days.

The waited sum of the customers' walks in the case of the original layout is 2,363.75. The optimal value of the (5-9)-(5-20) model is 2,458. The current and optimal layouts can be seen on Figures 5.1 and 5.2. The optimal route of customer No. 3 is changed, but the distance is the same (Figure 5.3). The length of the route of customer No. 11 is increased in the optimal solution (Figure 5.4). The numbers on Figures 5.3 and 5.4 show the parts of the routes between categories.

5.5 Conclusion

A new problem is solved in this chapter. The categories of goods are rearranged in a supermarket such that the total distance covered by the customers is increased by 4 percent. The customers are classified based on a large number of real purchases. For different environments the clusters can be different as they depend on the social and cultural background of the customers. The moves of the customers are modelled by TSP. The DFJ model of TSP is used in the mathematical problem of the supermarket layout problem. It is a large scale mixed integer 0-1 programming problem. It can be solved in a long, but still reasonable, CPU time.

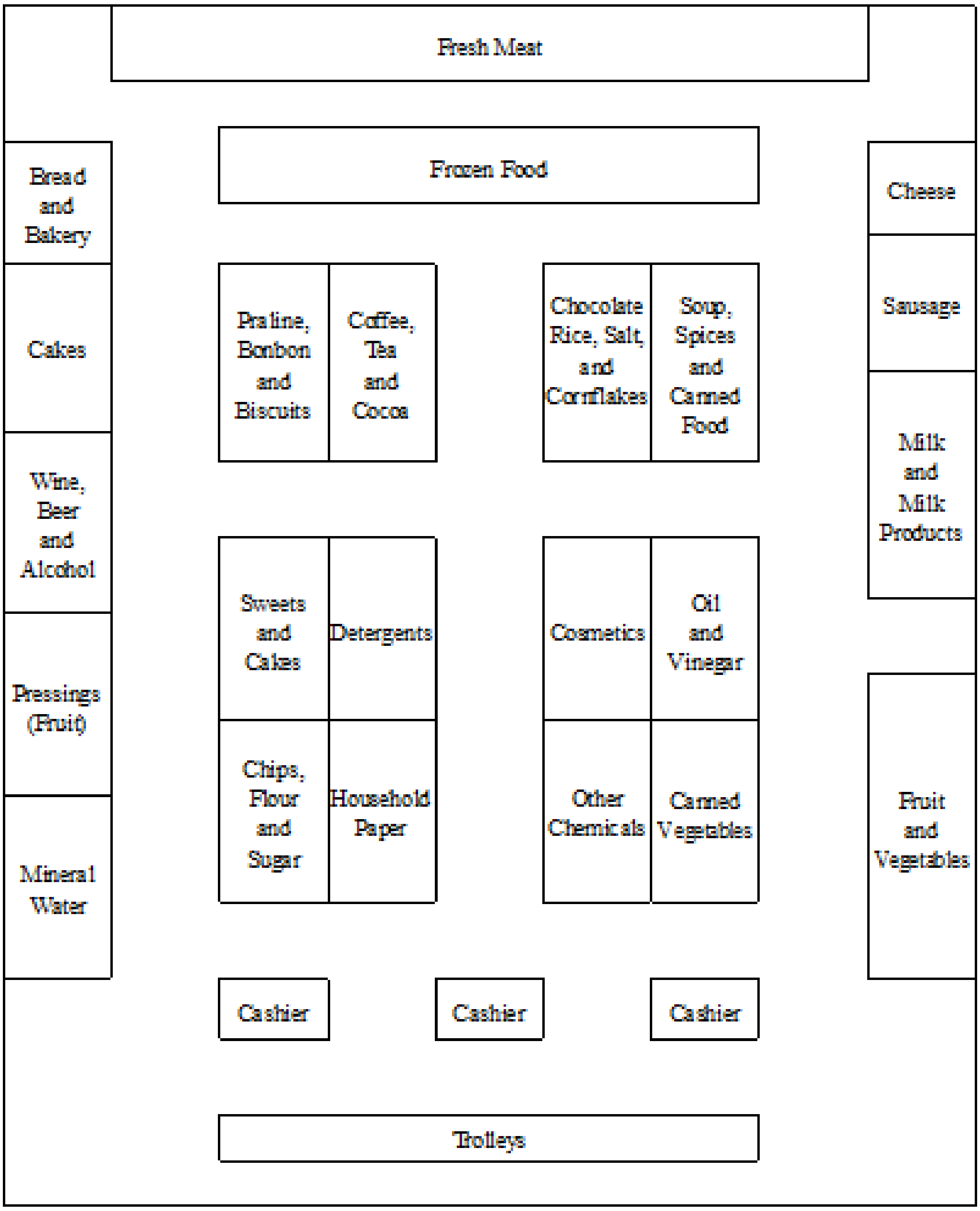

Figure 5.1. The original layout.

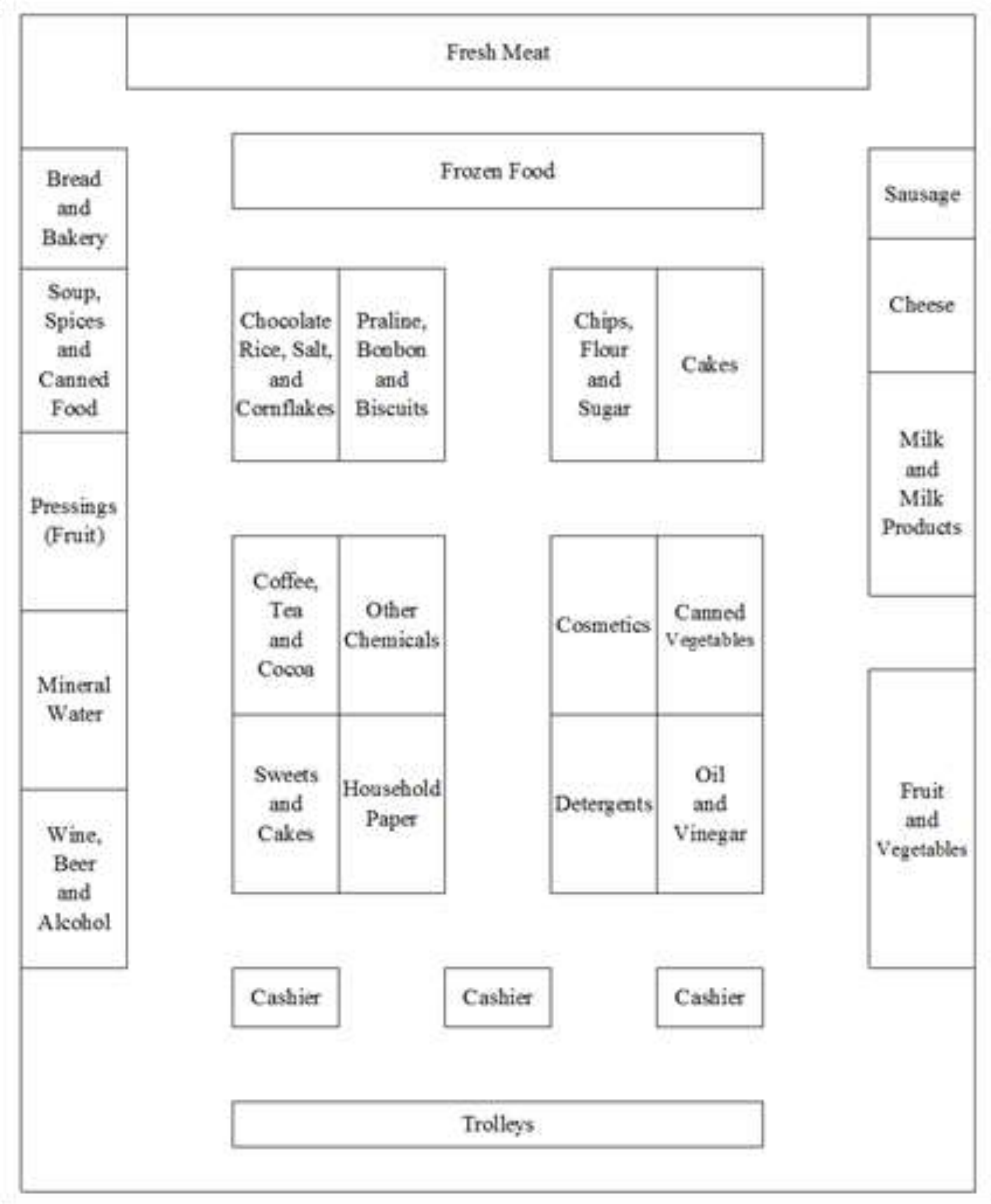

Figure 5.2. The optimal layout.

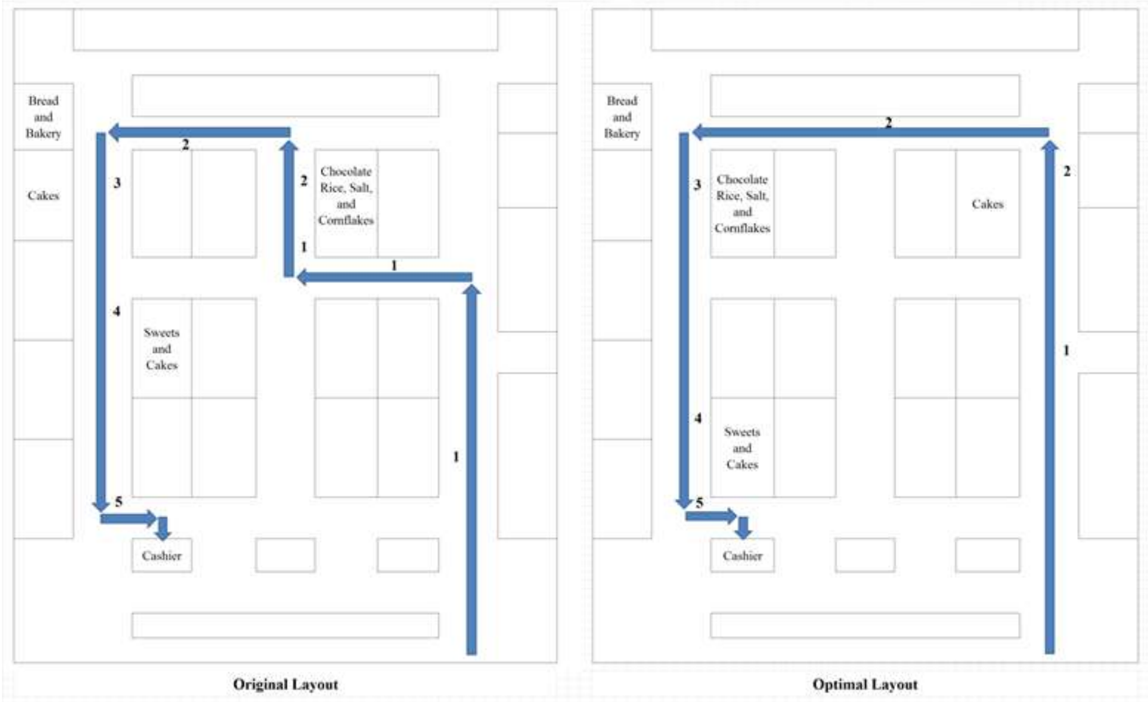

Figure 5.3. The shortest route of customer number 3 in original and optimal layout. The route has changed, but the distance is the same.

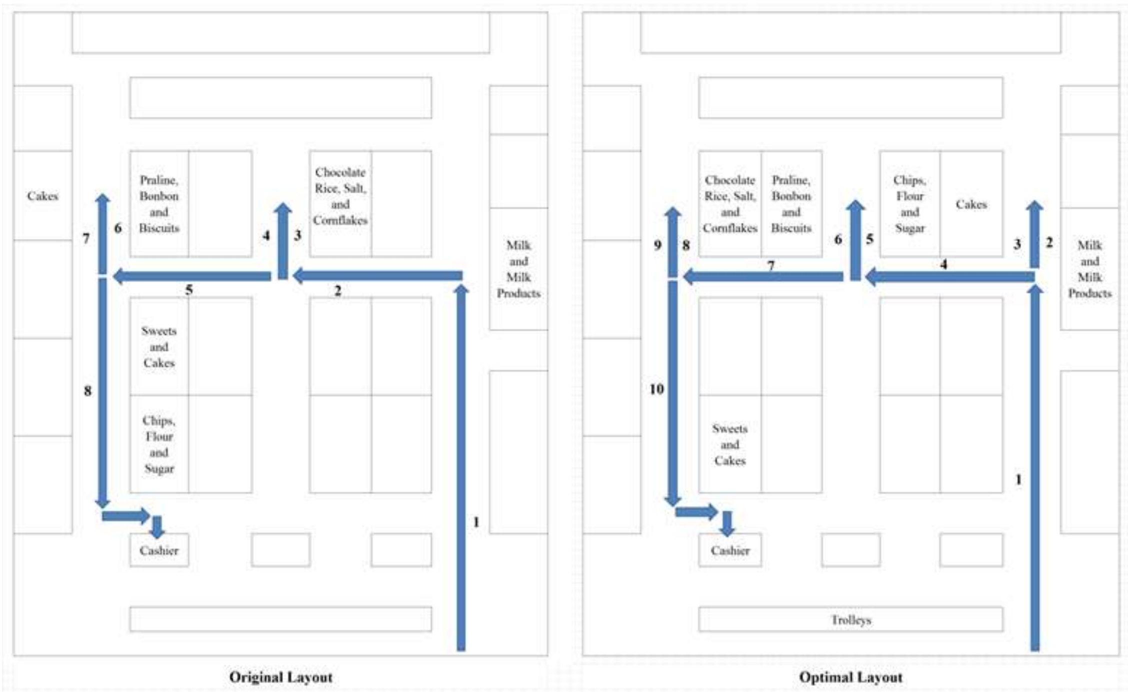

Figure 5.4. The shortest route of customer number 11 in original and optimal layout. The route has changed and the length of the route is increased.

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