Iterative Least Squares Channel Estimation in Frequency Selective CDMA Systems

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Abstract

Multiple access interference (MAI) is the main factor affecting the performance of channel estimation techniques for code division multiple access (CDMA) systems. Although, several multi-user channel estimation algorithms have been proposed to mitigate MAI, these algorithms require high computational complexities. In this paper, we address the problem of least squares (LS) mobile channel estimation at high channel efficiency that requires a short training sequence along with the spreading sequences. Then, we employ an efficient iterative method based on conjugate gradient (CG) algorithm to reduce the computational complexity of the estimation method. Computer simulations illustrate that the proposed method performs almost identical to the exact LS estimate for reasonable training lengths.

1. Introduction

Multiple access interference and near-far effect can severely degrade the performance of CDMA systems. Recently, there is a growing interest in accurate multi-user channel estimation techniques to obtain performance improvement [1]-[3]. A significant performance improvement of multi-user estimators is due to the capability to suppress MAI. This paper proposes an iterative multi-user least squares (LS) channel estimation method that is suitable for use in, for instance, slotted systems where each user transmits a data burst with a short training sequence. A similar problem for multi-user channel estimation in long-code systems is considered in [1] and a gradient descent (GD) algorithm is suggested for iterative channel estimation. However, it requires long training sequences for successful estimation of channel parameters. In this paper, an iterative method, based on the conjugate gradient (CG) algorithm [4], is presented to improve the convergence rate or, equivalently, reduce the length of the training sequence required. The iterative algorithm is modified to update the channel estimates at each training symbol instant and spreads the computation over the whole training length to provide a reasonable complexity. The proposed scheme is not a direct application of CG algorithm that solves a deterministic linear equation, but a modified online CG-type algorithm that updates both the solution and the parameters of the linear equation as training symbols are received. Simulation results illustrate that the iterative estimator performs almost identical to the exact LS estimate for reasonable training lengths. The multi-user interference is also well taken care of by the proposed channel estimator.

2. System Model

We considered a P-user synchronous CDMA system where direct paths from all users are perfectly synchronized. The proposed technique can easily be extended to the asynchronous case. The baseband signal plus additive white Gaussian noise $n(t)$ at the receiver is given by,

$$r(t) = \sum_{i=1}^{P} A_i \sum_{n=1}^{N} b_i(n) \sum_{l=1}^{L} h_{i,l} c_i(t-(l-1)T_c-(n-1)T_s) + n(t) \qquad (1)$$

where A_i is the transmitted amplitude of the ith user, $\{b_i(n) \in \pm 1\}$ is the information-bearing symbol, $\{c_i(j) \in \pm 1, j = 1 \cdots L_c\}$ is the pre-assigned spreading code of the ith user, T_c is the chip period of spreading

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code waveforms, T_s is the symbol duration, L is the total number of propagation paths, each spaced at T_c time intervals, of the channel, $h_{i,l}$ is the complex attenuation of the lth propagation path and N is the length of the frame containing N_p training symbols. The channel is assumed to vary slowly such that the channel parameters are constant over the training period and changed independently from frame to frame.

By sampling the received signal $y(t)$ at chip rate over the training period, the received signal vector $\mathbf{y}$ of length N_pL_c can be obtained as follows:

$$\mathbf{y} = \mathbf{Ch} + \mathbf{n} \tag{2}$$

where $\mathbf{n}$ is the noise vector, $\mathbf{C} = [\mathbf{C}(0) \quad \mathbf{C}(1) \quad \cdots \quad \mathbf{C}(N_p - 1)]^T$ is spread training sequences matrix, $\mathbf{h} = [\mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_P]^T$ is the channel coefficients vector in which the amplitudes, A_i, are accounted for as $\mathbf{h}_i = A_i[h_{i,1} \quad h_{i,2} \quad \cdots \quad h_{i,L}]$, and $(\cdot)^T$ denotes the transpose operation. The $L_c \times L$ Toeplitz matrix of spreading sequence of the ith user at the nth bit interval can be represented as, $\mathbf{C}_i = \mathbf{C}_i^R + \mathbf{C}_i^L$, where, $\mathbf{C}_i^R$ is constructed with the right part of the spreading code of the ith user corresponding to the current symbol and $\mathbf{C}_i^L$ is constructed using the left part of the spreading code of the ith user corresponding to the previous symbol. These matrices can be defined as,

$$\mathbf{C}_i^R = \begin{bmatrix} c_i(1) & 0 & \cdots & 0 \\ c_i(2) & c_i(1) & \ddots & \ddots \\ \vdots & \vdots & \ddots & \ddots \\ c_i(L_c) & c_i(L_c - 1) & \cdots & c_i(L_c - L + 1) \end{bmatrix}$$

and,

$$\mathbf{C}_i^L = \begin{bmatrix} 0 & c_i(L_c) & \cdots & c_i(L_c - L + 2) \\ 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & c_i(L_c) \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

Hence, the $L_c \times PL$ spread training sequences matrix at the nth symbol interval can be constructed as,

$$\mathbf{C}(n) = [b_1(n-1)\mathbf{C}_1^L + b_1(n)\mathbf{C}_1^R \quad \cdots \quad b_p(n-1)\mathbf{C}_p^L + b_p(n)\mathbf{C}_p^R]$$

3. Channel Estimation

3.1 Conventional Channel Estimation

The conventional correlative single-user (SU) channel estimation can be obtained by a simple averaging technique as,

$$\hat{\mathbf{h}}_i^{su} = \frac{1}{N_p} \sum_{n=1}^{N_p} b_i(n) \mathbf{C}_i^H \mathbf{y}(n) \tag{3}$$

where $\mathbf{y}(n)$ is the discrete vector of the received samples at the nth symbol interval, and $(\cdot)^H$ denotes the conjugate transpose.

3.2 Least Squares Channel Estimation

The LS channel estimate $\hat{\mathbf{h}}_{LS}$ satisfies the following equation,

$$\hat{\mathbf{h}}_{LS} = \arg \min_{\mathrm{h}} E\left[(\mathbf{y} - \mathbf{Ch})^H (\mathbf{y} - \mathbf{Ch}) \right] \tag{4}$$

Denote $\mathbf{C}^\dagger$ as the pseudoinverse [4] of $\mathbf{C}$, under assumption that $\mathbf{C}$ has full column rank and $\mathbf{C}^\dagger \mathbf{C} = \mathbf{I}$. The multi-user LS channel estimation of all the users, which amounts to the multiplication of $\mathbf{y}$ by the pseudoinverse of $\mathbf{C}$, is given by [2], [4]:

$$\hat{\mathbf{h}}_{LS} = \mathbf{C}^\dagger \mathbf{y} . \tag{5}$$

The condition for $\mathbf{C}$ matrix to be full column rank is provided if $N_p > \lceil LP / L_c \rceil$. Current standards provide enough preambles to satisfy this condition.

3.3 Iterative Channel Estimation

A direct computation of the exact LS channel estimate $\hat{\mathbf{h}}_{LS}$ involves the matrix pseudoinverse at the end of the training period that is computationally expensive. Moreover, $\mathbf{C}^\dagger$ should be calculated periodically for each frame at the end of the preamble sequence, which makes the problem even more severe. In order to solve (5) and, yet avoid matrix pseudoinverse, iterative methods like GD or CG algorithms should be utilized to solve the linear system $\mathbf{Ch}$=$\mathbf{y}$. However, these algorithms in their basic forms only solve the equation systems with a symmetric positive definite (SPD) coefficient matrix. To solve the non-square system $\mathbf{Ch}$=$\mathbf{y}$, the normal equation $\mathbf{C}^T\mathbf{Ch} = \mathbf{C}^T\mathbf{y}$ can be used. Since $\mathbf{C}$ has full column rank then $\mathbf{C}^T\mathbf{C}$ is SPD. Defining $\mathbf{Y}_C = \mathbf{Cy}$ and $\mathbf{R}_C = \mathbf{C}^H\mathbf{C}$, the GD algorithm can be applied for the nth training symbol as:

$$\hat{\mathbf{h}}_{GD}(n) = \hat{\mathbf{h}}_{GD}(n-1) + \mu(\mathbf{Y}_C(n) - \mathbf{R}_C(n)\hat{\mathbf{h}}_{GD}(n-1)) \quad (6)$$

where μ is the step-size parameter that should be chosen between 0 and $2/\lambda$ for convergence, and λ is the maximum eigenvalue of $\mathbf{C}^T\mathbf{C}$ [5]. $\mathbf{R}_C(n)$ and $\mathbf{Y}_C(n)$ are computed in each iteration as shown in Table 1. In order to improve the convergence rate of GD algorithm, conjugate gradient (CG) algorithm is used as presented in Table 1. In this algorithm $\alpha(n)$ is the step size that minimizes the cost function along the conjugate gradient vector and $\beta(n)$ is chosen to make $\mathbf{g}(n)$, $\mathbf{R}_C$-conjugate or $\mathbf{R}_C$-orthogonal to $\mathbf{g}(n-1)$, such that $\mathbf{g}(n)^H\mathbf{R}_C\mathbf{g}(n-1) = 0$. The algorithm starts with an initial guess $\hat{\mathbf{h}}_{CG}(0)$ as $\mathbf{C}(1)^H\mathbf{y}(1)$ to improve the convergence rate. In each iteration, the estimate of the channel vector and parameters of the linear equation are updated by taking a step along the search direction $\mathbf{p}(n)$. In other words, the new channel vector $\hat{\mathbf{h}}_{CG}(n)$ is computed as a linear combination of the previous weight vector and the search direction.

4. Simulation Results and Computational Complexity

The performance of a synchronous CDMA system with 7 users is analyzed using N=400 data symbols in each frame having N_p training symbols. A four-path slowly varying propagation channel is simulated for each user. Channels are randomly realized with exponential power delay profile with a dynamic range of 20 dB. Each user employed Gold sequence of length 31 as spreading codes. The preamble bits and data bits are generated randomly for each user as antipodal signaling.

We evaluate the computational complexities of the estimation algorithms by counting the number of floating-point operations (flops) [4]. The results for a single frame transmission are shown in Table 2. The actual LS method is the most complex algorithm because of the required matrix inverse. In the iterative algorithms, the most complex part is the calculation of $\mathbf{R}_C$. The CG algorithm requires additional $(4PL+9)PLN_P + 4N_P$ flops than GD algorithm for the calculation of the search direction. To be able to understand the meaning of these complexities, the numerical results with a representative system setup is also presented in Table 2. As seen, the proposed iterative CG based channel estimation method requires

TABLE 1

PROPOSED CONJUGATE GRADIENT ALGORITHM

Set initial conditions

$$\mathbf{R}_C(0) = \mathbf{C}(1)^H\mathbf{C}(1)$$
$$\mathbf{Y}_C(0) = \hat{\mathbf{h}}_{CG}(0) = \mathbf{C}(1)^H\mathbf{y}(1)$$
$$\mathbf{g}(0) = \mathbf{p}(0) = \mathbf{Y}_C(0) - \mathbf{R}_C(0)\hat{\mathbf{h}}_{CG}(0)$$

for $n = 1$ to N_P

$$\mathbf{R}_C(n) = \left(\frac{n-1}{n}\right)\mathbf{R}_C(n-1) + \frac{1}{n}\mathbf{C}(n)^H\mathbf{C}(n)$$
$$\mathbf{Y}_C(n) = \left(\frac{n-1}{n}\right)\mathbf{Y}_C(n-1) + \frac{1}{n}\mathbf{C}(n)^H\mathbf{y}(n)$$
$$\alpha(n) = \frac{\mathbf{g}(n-1)^H\mathbf{g}(n-1)}{\mathbf{p}(n-1)^H\mathbf{R}_C(n)\mathbf{p}(n-1)}$$
$$\hat{\mathbf{h}}_{CG}(n) = \hat{\mathbf{h}}_{CG}(n-1) + \alpha(n)\mathbf{p}(n-1)$$
$$\mathbf{g}(n) = (\mathbf{Y}_C(n) - \mathbf{R}_C(n)\hat{\mathbf{h}}_{CG}(n-1)) - \alpha(n)\mathbf{R}_C(n)^H\mathbf{p}(n-1)$$
$$\beta(n) = \frac{(\mathbf{g}(n) - \mathbf{g}(n-1))^H\mathbf{g}(n)}{\mathbf{g}(n-1)^H\mathbf{g}(n-1)}$$
$$\mathbf{p}(n) = \mathbf{g}(n) + \beta(n)\mathbf{p}(n-1)$$

TABLE 2

COMPUTATIONAL COMPLEXITIES OF THE CHANNEL ESTIMATORS

Alg.	Complexity (Flops)	Rep. Sys. (K flops)
SU	$2N_pL_cPL$	27.8
LS	$2N_p^3L_c^3 + 4N_p^2L_c^2LP + 2N_PL_CLP$	271629.4
GD	$(PL+1)(2L_c+3)PLN_P + 5N_P$	844.6
CG	$P^2L^2(2L_c+7)N_P + 2PL(L_c+6)N_P + 9N_P$	898.8

approximately 320 times less flops than exact LS method and has almost the same complexity as the GD method.

The effect of training length on the average mean squared error (MSE) performance of the estimation methods are presented in Fig. 1 by averaging 200 Monte-Carlo runs. In each run, different channel parameters are used. The iterative algorithms shows only slight overshoot at the early stages and then converge to a stable level. It is evident from the figure that the iterative estimators offer

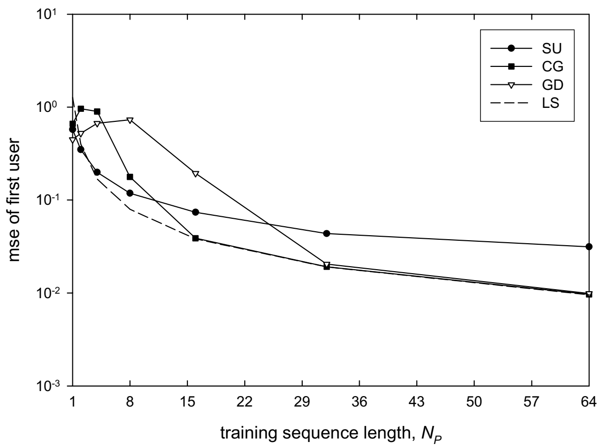

Fig. 1 MSE versus N_p for user 1 (SNR=8 dB, P=7, L=4, L_c=31)

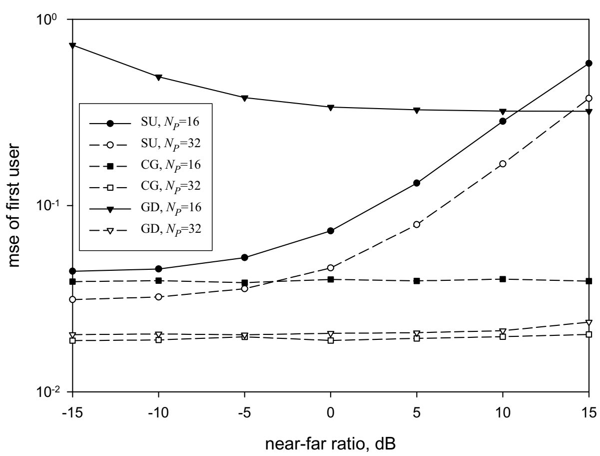

Fig. 2 MSE versus NFR for user 1 (SNR=8 dB, P=7, L=4, L_c=31)

substantial performance gain over the conventional SU estimation method and their performance are the same as the exact LS estimation provided that enough training symbols are present. The

CG reaches the exact LS performance more than twice as fast as the GD algorithm (μ=0.01). That is, the proposed method (CG) requires less training symbols than the GD algorithm for successful training.

Fig. 2 depicts MSE performance of the proposed method under near-far conditions for several preamble sizes at 8 dB SNR. We define the near-far ratio (NFR) as the ratio of the average received power of each interferer

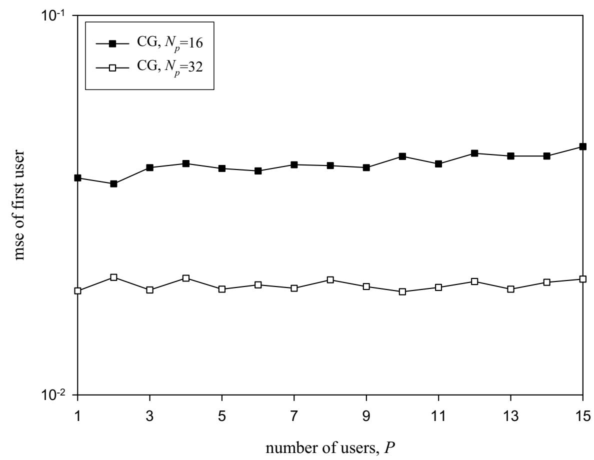

Fig. 3 MSE versus number of users in the system for user 1 (SNR=8 dB, NFR=0 dB, L=4, L_c=31)

to that of the desired user. As evident from the simulation results, the iterative CG-based algorithm is near-far resistant. The performance of GD method is also independent of the near-far effect provided that longer training sequences are used. The conventional SU estimator is not near-far resistant and for strong MAI it fails to show satisfactory performance. Thus, power control is necessary with SU estimator to improve channel estimation performance of all the users.

In Fig. 3, the effect of the MAI on the CG-based estimator is investigated by increasing the system load (number of users). Curves are obtained by using 16 and 32 training symbols in each slot. Simulation result shows that, with increasing the number of users in the system, the performance is not degraded. This is because the multi-user interference is well mitigated by the proposed iterative channel estimator.

5. Conclusion

In this paper, we consider an iterative LS channel estimation method based on CG algorithm to reduce the computational complexity of exact LS channel estimation technique. It is observed that, the CG-based iterative channel estimator offers significant performance gain over the conventional SU estimator. The iterative method performs as well as the exact LS estimation method with reasonable computational complexity by spreading the complexity over the length of the preamble. The estimation performance of the proposed iterative method is independent from the powers of the interfering users. The system can be heavily loaded since the multi-user interference can well be taken care of by the iterative estimator.

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