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Simple model for vector bosons

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Using the analogy between the SL(2,*C*) gauge theory of gravitation and the Yang-Mills theory, we propose a model for massive vector bosons. The model is based on the Geroch-Held-Penrose treatment of gravitation in which a reduction from SL(2,*C*) to an Abelian subgroup of it is made. It is shown that the proposed model is unitary at the two-loop level.

I. INTRODUCTION

Spontaneous symmetry breaking was considered to be the only possible method for introducing mass terms for the vector bosons.[1] Recently Hsu and Mac[2] proposed a new SU(2) model where intrinsic[3] rather than spontaneous breaking of gauge symmetry is used. Their Lagrangian is not invariant under the usual local SU(2) transformations but is invariant under a local Abelian gauge transformation. In this paper we propose still another SU(2) model for massive vector bosons where the masses are introduced without spontaneously breaking the gauge symmetry. Our Lagrangian also is not invariant under the local SU(2) transformation but is invariant under a local Abelian subgroup C^0 of SU(2). Our global C^0-invariant Lagrangian contains charged scalar fields $\phi^\pm$ and a pair of Maxwellian fields as the carrier fields of the formalism. These fields can be replaced by spinor and Proca fields, respectively. Then extension to local C^0 invariance requires the introduction of a massless vector boson (photon) in accordance with Utiyama's[4] theory of compensating fields. The photon introduced by this method together with the initial pair of Maxwell fields constitutes the local SU(2) Yang-Mills[5] (YM) triplet. However, instead of Maxwellian fields we shall choose the initial carrier fields to be Proca fields and demand local C^0 invariance rather than local SU(2) invariance. The basic idea in our theory therefore is to reduce from a non-Abelian group invariance to an Abelian subgroup invariance and exploit the local gauge freedom in the manner of Utiyama. This choice provides us with the decomposition of the three SU(2) YM fields into a photon and two massive vector bosons. The same procedure can be generalized to gauge groups of arbitrary rank which admit an Abelian subgroup. For the SU(N) case there are as many photons as the rank, namely $N-1$, and one finds $N(N-1)$ massive vector bosons. Similarly for the group SU(2) × U(1) there are two photons (the first is the usual photon of electrodynamics and the second comes from the local C^0 invariance of the theory) and two charged massive vector bosons.

The idea of formulating a non-Abelian gauge theory within the context of one of its Abelian subgroups seems an interesting concept although it is not a completely new one. We can refer to a previous example of such an idea in the SL(2, C) gauge theory of gravitation. It is well known that the general theory of relatively, in the null-tetrad version of Newman and Penrose[6] (NP), can be cast as a gauge theory of gravitation with structure group SL(2, C).[7] In a particular version of the null-tetrad method, Geroch, Held, and Penrose[8] (GHP) have formulated a reduction[9] from SL(2, C) to an Abelian subgroup of it. The resulting theory is a bona fide theory of gravity which, in a class of space-times, completely reproduces the results of the SL(2, C) formalism. The procedure for such a reduction amounts to identification of two of the four principal directions of the Riemann tensor as the direction of propagation of gravitational fields, and the gauge freedom left in the problem turns out to be tetrad rotations for the principal vectors. Our procedure for YM theories amounts to the same procedure, namely to single out $N-1$ directions of SU(N) in the internal space and study the theory within the reduced gauge freedom.

II. THE FORMALISM

Consider charged massive scalar fields $\phi^\pm$ together with given massive vector fields $a_\mu^\pm$ all of which transform according to the adjoint representation of an Abelian subgroup C^0 of SU(2), namely

$$\begin{pmatrix} e^{i\lambda} & 0 \\ 0 & e^{-i\lambda} \end{pmatrix}, \tag{1}$$

where λ denotes half of the angle of rotation around the third internal direction. Therefore by this choice we geometrically single out the third internal axis as the invariant direction. The Lagrangian of the uncoupled system is given by

$$L_0 = |\partial_\mu \phi^+|^2 - \tfrac{1}{4}|F^+_{\mu\nu}|^2 - m^2|\phi^+|^2 + \tfrac{1}{2}M^2|a^+_\mu|^2 , \quad (2)$$

where $F^+_{\mu\nu} = \partial_\mu a^+_\nu - \partial_\nu a^+_\mu$ is a Maxwell tensor. L_0 is invariant under the constant-phase transformations

$$\phi^- \to e^{i\lambda}\phi^- , \qquad a^-_\mu \to e^{i\lambda} a^-_\mu . \quad (3)$$

After making λ an arbitrary function of space-time, we introduce the compensating field A_μ in order to preserve the local gauge invariance. In contrast to the case of electrodynamics we introduce Pauli-moment-like terms,[10] so that we can make correspondence with the usual YM theory. The Lagrangian of the model becomes

$$L = |\partial_\mu \phi^+ - ieA_\mu \phi^+|^2 - m^2|\phi^+|^2 + \tfrac{1}{2}M^2|a^+_\mu|^2 - \tfrac{1}{4}|f^+_{\mu\nu}|^2 - \tfrac{1}{4}F_{\mu\nu}{}^2 , \quad (4)$$

where

$$f^+_{\mu\nu} = \partial_\mu a^+_\nu - \partial_\nu a^+_\mu - ie(A_\mu a^+_\nu - A_\nu a^+_\mu) , \quad (5)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \tfrac{1}{2} ie(a^+_\mu a^-_\nu - a^+_\nu a^-_\mu) , \quad (6)$$

and the local gauge transformations under which L is invariant are

$$\phi^- \to e^{i\lambda(x)}\phi^- , \qquad a^-_\mu \to e^{i\lambda(x)} a^-_\mu , \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \lambda . \quad (7)$$

The correspondence of the YM part of the Lagrangian (4) with the usual YM theory is provided by the identifications

$$A_\mu = A^3_\mu , \qquad A^+_\mu = A^1_\mu - iA^2_\mu . \quad (8)$$

In order to define the scalar parts $a^\pm_S$ of the vector bosons $a^\pm_\mu$, we introduce a subsidiary Lagrangian L_ξ due to Lee and Yang[11]:

$$L_\xi = -\frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - \frac{\xi}{2}|(\partial_\mu - ieA_\mu)a^{+\mu}|^2 , \quad (9)$$

where the masses of $a^\pm_S$ are $M_S{}^2 = M^2/\xi$ and α is a constant. We introduce further the gauge-fixing Lagrangian L_G of three Lagrange multipliers,[12] $\chi^\pm$ and χ^0,

$$L_G = M\chi(\partial_\mu A^\mu) + \tfrac{1}{2}\alpha M^2 \chi^2 + \tfrac{1}{2}M\chi^+(\partial_\mu + ieA_\mu)a^{-\mu} + \tfrac{1}{2}M\chi^-(\partial_\mu - ieA_\mu)a^{+\mu} + \frac{M^2}{2\xi}\chi^+\chi^- . \quad (10)$$

The field equations derived from $L + L_G$ are

$$(\partial^\mu - ieA^\mu)(\partial^\mu - ieA_\mu)\phi^+ + m^2\phi^+ = 0 , \quad (11)$$

$$(\partial_\mu + ieA_\mu) f^{-\mu\nu} - iea^-_\mu F^{\mu\nu} + M^2 a^{-\nu} - M(\partial^\mu + ieA^\mu)\chi^- = 0 , \quad (12)$$

$$\partial_\mu F^{\mu\nu} + \tfrac{1}{2}ie(a^-_\mu f^{+\mu\nu} - a^+_\mu f^{-\mu\nu}) - ieJ^\mu - M\left(\partial^\nu \chi^0 - \frac{ie}{2}\chi^+ a^{-\nu} + \frac{ie}{2}\chi^- a^{+\nu}\right) = 0 , \quad (13)$$

together with the complex conjugate of Eq. (12). Here J_μ is the conserved current

$$J_\mu = \phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+ + 2ieA_\mu\phi^+\phi^- . \quad (14)$$

The remaining equations are the three constraint equations for the Lagrange multipliers

$$\partial_\mu A^\mu + \alpha M \chi^0 = 0 , \quad (15)$$

$$(\partial_\mu - ieA_\mu)a^{+\mu} + \frac{M}{\xi}\chi^+ = 0 , \quad (16)$$

together with the complex conjugate of Eq. (16). Taking the divergence of (12) and (13) and using the constraint conditions, we get

$$\left(\Box + \frac{M^2}{\xi}\right)\chi^- + 2ieA_\mu\partial^\mu\chi^- - e^2A_\mu A^\mu\chi^- + \frac{e^2}{2}a^-_\mu(\chi^- a^{+\mu} - a^{-\mu}\chi^+) = -\frac{e^2}{M}a^-_\mu J^\mu , \quad (17)$$

$$\Box\chi^0 = 0 . \quad (18)$$

In the light of Eq. (18) we set $\chi^0 = 0$, which by the constraint equation implies

$$\partial_\mu A^\mu = 0 . \quad (19)$$

We next introduce a fictitious Lagrangian L_f which contains a pair of fictitious particles $D^\pm$ whose statistics we do not specify at the moment but consider them of parastatistical[13] nature. We shall exploit the behavior of these nonphysical particles to cancel the contributions coming from the indefinite-metric, spin-zero part of the vector bosons at the two-loop level. Such a fictitious Lagrangian can be constructed with the help of Eq. (17),

$$L_f = |\partial_\mu D^+|^2 - M_S{}^2|D^+|^2 - 2ieA_\mu(\partial^\mu D^-)D^+ + e^2A_\mu A^\mu|D^+|^2 + \frac{e^2}{4}|D^+a^-_\mu - D^-a^+_\mu|^2 . \quad (20)$$

The Feynman rules are derived from the effective Lagrangian

$$L_{\rm eff} = L + L_\xi + L_f . \quad (21)$$

It should be noted that the structure of Eq. (17) does not provide us with a compact unitarized Feynman amplitude for $L_{\rm eff}$. Those terms which are not suitable for the functional integration must be treated by perturbation expansion.

III. UNITARITY

The physical fields of the formalism are $\phi^{\pm}$, transverse photon, and spin-1 part of $a_\mu^{\pm}$, while nonphysical fields are $a_S^{\pm}$ and $D^{\pm}$. In order to verify unitarity we examine the imaginary parts of the self-energy diagrams for the physical vector bosons $a_\mu^{\pm}$. For the one-loop case the nonphysical contribution comes from the process

$$a_\mu^- \to A_\mu a_S^- \to a_\mu^- ,$$

which vanishes identically. For the two-loop case there are three types of diagrams. Denoting their amplitudes by b_1, b_2, and b_3, respectively, we have the following:

First diagram: $a_\mu^- \to a_S^- a_S^+ a_S^- \to a_\mu^-$,

$$a_\mu^- \to A_\mu a_S^- \to a_S^- a_S^+ a_S^- \to a_\mu^- ,$$

$$\operatorname{Im} b_1 = -\frac{1}{2}\frac{e^4}{M^4}(P_3\cdot\epsilon)^2 ,$$

where ϵ_μ is the polarization and P_3 the momentum vector of a_μ^-;

Second diagram: $a_\mu^- \to a_S^- D^+ D^- \to a_\mu^-$,

$$a_\mu^- \to A_\mu a_S^- \to a_S^- D^+ D^- \to a_\mu^- ,$$

$$\operatorname{Im} b_2 = \frac{e^4}{M^2}(P_3\cdot\epsilon)^2 ;$$

Third diagram: $a_\mu^- \to a_S^+ D^- D^- \to a_\mu^-$,

$$\operatorname{Im} b_3 = -\frac{e^4}{2M^2}(P_2\cdot\epsilon)^2 .$$

The total contribution to the imaginary part coming from nonphysical processes is

$$\operatorname{Im} B = \sum_{i=1}^{3}\int \operatorname{Im} b_i \,\delta(P_2^{\,2}-M_S^{\,2})\delta(P_3^{\,2}-M_S^{\,2})\delta(P_4^{\,2}-M_S^{\,2}) \times \delta(P_1-P_2-P_3-P_4) \times \theta(P_{20})\theta(P_{30})\theta(P_{40})d^4P_2 d^4P_3 d^4P_4 , \tag{22}$$

and one can easily show that it vanishes identically. We have therefore verified the unitarity at the two-loop level. Let us note that the assigning bosonic or fermionic statistics for the fictitious particles does not provide the unitarity, and we should be forced to introduce excess fields in order to cancel the nonphysical contributions. Finally, the formalism is renormalizable by means of standard power counting. Extension of this model to SU(N) will be discussed elsewhere.

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