COMMENT ON COLLIDING PLANE GRAVITATIONAL WAVES

M. HALILSOY [1]

Department of Physics, Middle East Technical University, Ankara, Turkey

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It is shown that a theorem proved on colliding plane gravitational waves is not correct.

It is the purpose of the present note to show that the theorem stated some time ago in ref. [1] in connection with colliding plane gravitational waves is incorrect. If we quote the equations from this reference, the theorem states the following:

To any colliding gravitational plane-wave metric

$$\mathrm{d}s^2 = 2\mathrm{e}^{-M'}\,\mathrm{d}u\,\mathrm{d}v - \mathrm{e}^{-U}(\mathrm{e}^{V'}\,\mathrm{d}x^2 + \mathrm{e}^{-V'}\,\mathrm{d}y^2)\,, \quad (1)$$

one associates a new solution with $W \neq 0$,

$$\mathrm{d}s^2 = 2\mathrm{e}^{-M}\,\mathrm{d}u\,\mathrm{d}v - \mathrm{e}^{-U}(\mathrm{e}^{V}\cosh W\,\mathrm{d}x^2 + \mathrm{e}^{-V}\cosh W\,\mathrm{d}y^2 - 2\sinh W\,\mathrm{d}x\,\mathrm{d}y)\,, \quad (2)$$

where

$$\int \frac{\mathrm{d}W}{\cosh W\,(A^2\cosh^2 W - 1)^{1/2}} = \pm V\,, \quad (3a)$$

$$\int \frac{A\cosh W\,\mathrm{d}W}{(A^2\cosh^2 W - 1)^{1/2}} = \pm V'\,, \quad (3b)$$

and $M = M'A$, A = const. (The factor A in the integrand of (3b) is missing in ref. [1].) Stated in other words, in the newly generated solution W and V are assumed to be functionally related. We shall show that whenever W and V are functionally dependent, which is the basic assumption of the theorem stated above, it turns out that the metric becomes diagonalizable, hence the theorem fails.

Before we do this we would like to point out, for a better understanding, that the metric function W represents the polarization content of the colliding waves, which is however manifest prior to the collision. This is due to the fact that the plane wave nature of (2) is no more valid after the collision. The simplest case $W = 0$ corresponds to constant linear polarization [2,3]. In order to obtain solutions ($W \neq 0$) it is necessary that we must have $W \neq 0$ also in regions prior to collision [4,5]. Provided this requirement is satisfied then a consistent matching of solutions at the boundaries becomes possible. The vacuum Einstein equations must be satisfied everywhere including the boundaries and the resulting solution must be nondiagonalizable. We presented exact solutions to (2) before [4,5] which satisfied the properties that two single pulses may be diagonalized separately whereas the two pulses cannot be simultaneously diagonalized in the same coordinate patch.

Having this necessary information let us turn back to the above-state theorem: (3a) and (3b) are integrated to yield (the results of ref. [1] are incorrect)

$$\tanh V = \cos\alpha \tanh V'\,, \qquad \tanh W = \tan\alpha \sinh V\,, \quad (4)$$

where for convenience we introduced a new parameter by $\cos\alpha = A^{-1}$. Note also that the choice for M as $M = M'A$ is also not correct in the same reference, but should be $M = M'$

In conclusion, given a solution of (1) it seems that through (4) and ($U' = U$, $M' = M$) a new solution with $W \neq 0$ is generated. However, all this procedure does not give a solution other than (1): To see this, make a coordinate rotation,

$$x = \cos\tfrac{1}{2}\alpha\,\bar{x} + \sin\tfrac{1}{2}\alpha\,\bar{y}\,, \quad y = -\sin\tfrac{1}{2}\alpha\,\bar{x} + \cos\tfrac{1}{2}\alpha\,\bar{y}\,, \quad (5)$$

[1] Previous surname of the author was Halil, which is changed henceforth into Halilsoy.

and observe, after simple algebra, that (2) reduces to (1) in the rotated coordinates $(u, v, \bar{x}, \bar{y})$. In particular, if the incoming waves are impulsive waves the solution generated by the above theorem reads explicitly

$$ds^2 = 2e^{-M}\,du\,dv - \frac{e^{-U}}{1-|k|^2}\,[|1-k|^2\,dx^2 + |1+k|^2\,dy^2 + 2i\,dx\,dy\,(k-\bar{k})]\,, \tag{6}$$

where $k = e^{i\alpha}(pw + qr)$, with the usual notations

$$p = u\theta(u)\,, \quad q = v\theta(v)\,,$$

$$r^2 = 1 - p^2\,, \quad w^2 = 1 - q^2\,,$$

and U and M correspond to Khan–Penrose values. Solution (6) is readily identified as the solution of ref. [4] with the restriction $\alpha = \beta$. The same rotation (5) reduces (6) to the Khan–Penrose solution. Since $\alpha - \beta$ measures the incident polarization of the waves in collision we conclude that $\alpha - \beta \neq 0$ is the crucial quantity which generates a nontrivial solution to (2) with $W \neq 0$. Any solution of (2) which involves a single constant parameter (as the above theorem does) can be ruled out by a coordinate transformation. This completes the disproof.

References

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