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A New Solution of the Deep Bed Filter Equations

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ABSTRACT

We present a more general solution of the standard deep bed filtration equations which explains the initial improvement of effluent quality as a suspension passes through a porous medium. Our solution gives a good fit to experimental data and predicts the experimental observations.

1. INTRODUCTION

Filtration of suspensions through granular beds is a mass transport problem with important practical applications. Considerable theoretical and experimental research has been done to investigate the physics of mass deposition in porous media [1 - 3]. However most of the theoretical research was aimed at explaining the final stages of the degradation of the effluent quality, termed "filter breakthrough". Although it has been well documented experimentally, the initial improvement of the effluent quality, "filter ripening" has not been studied in detail analytically [4, 5].

The aim of this paper is to present a new general solution of the standard deep bed filter equations. Unlike the previous models, the solution accounts for both the ripening and the breakthrough periods of deep bed filters. It fits the experimental data well and gives a theoretical explanation for the behaviour of the ripening period as observed by other investigators. A particular case of the solution gives the well-known "Logit" equation which has been suggested as a design equation for deep bed filters and adsorption beds.

2. THEORY

Figure 1 shows a schematic flow diagram of a deep bed filter. C_0 is the concentration of the inlet suspension, C is the effluent concentration, v is the approach velocity and x is the granular filter bed depth.

Mass accumulation in the pores of the granular media is described by two basic equations, the mass balance equation

$$\frac{\partial(q + pC)}{\partial t} + v\frac{\partial C}{\partial x} + D\frac{\partial^2 C}{\partial x^2} = 0 \tag{1}$$

and the kinetic equation

$$\frac{\partial q}{\partial t} = \mathrm{F}(q)C \tag{2}$$

Here D is the diffusion coefficient, p is the bed porosity, q is the mass of deposited

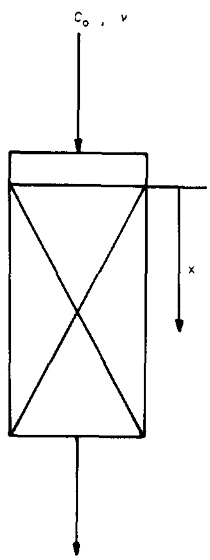

Fig. 1. Schematic diagram of a deep bed filter.

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particles per unit volume, $F(q)$ is a function of q which describes the accumulation kinetics and t represents the time variable.

The diffusion term and the term pC representing the moving particles with respect to the deposited particles are usually neglected as in ref. 1, and eqn. (1) simplifies to

$$\frac{\partial q}{\partial t} + v\frac{\partial C}{\partial x} = 0 \tag{3}$$

Using eqn. (2) and the same simplifications made to obtain eqn. (3), the mass balance equation can be written in a different way as

$$\frac{\partial q}{\partial x} + F(q)\frac{q}{v} = 0 \tag{4}$$

Finding $\partial C/\partial x$ from eqns. (2) and (3) and dividing with eqn. (4) gives

$$\frac{C}{C_0} = \frac{q}{q_i} \tag{5}$$

where q_i is the accumulation in the inlet layer.

From eqn. (4)

$$v\int\frac{\partial q}{qF(q)} = -x + f(t) \tag{6}$$

where $f(t)$ is a function of t only.

Different equations have been suggested for $F(q)$ to describe the kinetics of accumulation in porous media. If the linear kinetics model [1, 3] is chosen

$$F(q) = k(N-q) \tag{7}$$

eqn. (6) can be solved analytically to give

$$q = \frac{N}{1+\exp\{Ax - Af(t)\}} \tag{8}$$

and

$$q_i = \frac{vC_0}{df(t)/dt} \tag{9}$$

Here k is the attachment coefficient in the bed, N is the filter capacity coefficient and $A = kN/v$.

Using eqn. (5),

$$y = \frac{C}{C_0} = \frac{N/q_i}{1+\exp\{Ax - kC_0N\int(1/q_i)\,dt\}} \tag{10}$$

To solve eqn. (10), an equation which describes the kinetics of accumulation in the inlet layer is required. Although the removal of solids is primarily within the filter bed in deep bed filters, removal at the bed surface is also significant and the dominant mass transport and attachment mechanisms at the inlet layer may be different from those within the depths of the medium. The dominant mechanisms will depend on the physical and chemical characteristics of the suspension and the medium, the rate of filtration, and the chemical characteristics of the water [5].

In this approach, the linear kinetic model described by eqn. (7) will be assumed to hold for the inlet layer with a different attachment coefficient k_i, which is specific for the inlet layer. Any model chosen for this purpose must satisfy both the mass balance and the kinetic equations and should show the saturation character of the surface layer at high t values.

Thus

$$\frac{dq_i}{dt} = k_i(N-q_i)C_0 = K(N-q_i) \tag{11}$$

where $K = k_iC_0$. From eqn. (11)

$$q_i = N\{1-\exp(-k_iC_0t)\} \tag{12}$$

Substituting eqn. (12) into eqn. (10) gives

$$\begin{aligned} y = {} & [1-\exp\{-k_iC_0(t+m)\}]^{k/k_i-1} \\ & \times\left\{[1-\exp\{-k_iC_0(t+m)\}]^{k/k_i} + \exp\{kNx/v - kC_0(t+s)\}\right\}^{-1} \end{aligned} \tag{13}$$

where m is the time scale shift parameter and s is an integration constant or ordinate shift parameter.

Equation (13) is scale invariant; therefore a scale invariance factor can be introduced. Also for the same cases as observed within the practical limits of field and laboratory filter runs

$$y \longrightarrow y_{\text{limit}} < 1$$

If needed, eqn. (13) can be modified to incorporate this experimental observation. Then

$$y = \frac{u^{B/K-1}}{u^{B/K} + \exp\{Ax - B(t+s)\}} \tag{14}$$

where

$$u = E - \exp\{-K(t+m)\}$$

$$B = \frac{kC_0}{E} \text{ and } E = \frac{1}{y_{\text{limit}}}$$

3. APPLICATIONS

Equation (14) shows a minimum (the ripening effect) if $k < k_i$. For $k = k_i$ eqn. (14) reduces to the "logistic" equation which was applied as a design method for adsorption and deep bed filtration [3]. The logistic equation has also been used by biologists in converting data to a linear form and for estimating future populations.

Taking the derivative of eqn. (14) with respect to t gives

$$\frac{\partial y}{\partial t} = \frac{(K-B)y}{1-\exp(Kt)} + By(1-y) \tag{15}$$

The first term on the right-hand side of eqn. (15) is the slope of the initial improvement phase and the second term is the slope of the breakthrough part. At the minimum

$$y_m = \frac{1 - K/B\exp(-Kt_m)}{1-\exp(-Kt_m)} \tag{16}$$

and

$$\frac{\partial t_m}{\partial y_m} = \frac{B-K}{K(1-y_m)(K-By_m)} > 0 \tag{17}$$

where y_m and t_m are the C/C_0 and t values at the minimum respectively.

Equation (16) gives the relationship between B and K (or k and k_i) for experimental t and y values. For values of $t > t_m$, eqn. (13) approaches the "logistic" equation

$$y = \frac{1}{1+\exp\{Ax - B(t+s)\}} \tag{18}$$

which can be transformed to the linear form

$$\ln\left(\frac{1}{y} - 1\right) = Ax - B(t+s) \tag{19}$$

Linear regression of the logit term ($\ln(1y - 1)$) and t values gives the B value as the slope and the $(Ax - Bs)$ value as the intercept. Knowing the B value, K can be calculated from eqn. (16).

In the following examples some applications of our solution will be presented.

Figures 2 and 3 show the application of eqn. (14) to the experimental data of other investigators [5, 6].

The solution also predicts certain important characteristics of filter effluent quality which have been observed experimentally [5].

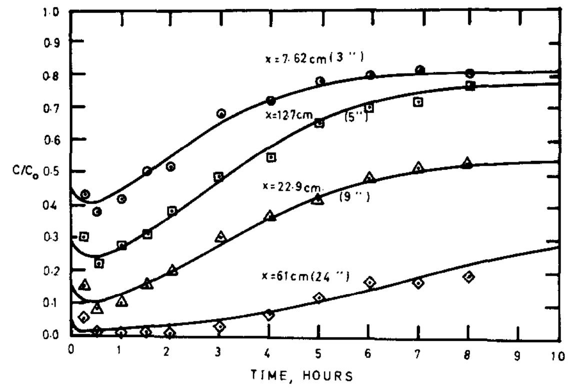

Fig. 2. Comparison of the model with Cleasby's data for filtration of kaolinite clay after alum was added to mixture [5]. Constants for 7.62 cm (3"): C_0 = 20 ppm; v = 24.45 cm min^{-1}; K = 1 h^{-1}; k = 42 $\text{cm}^3\ \text{mg}^{-1}\ \text{h}^{-1}$; N = 6.6 mg cm^{-3}; m = 0.1; s = 0.5; E = 1.22.

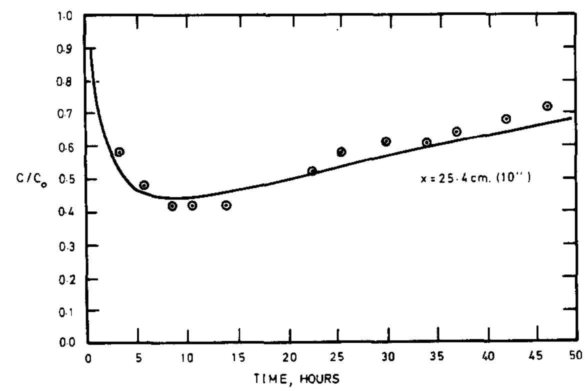

Fig. 3. Comparison of the model with Ling's run 28 [6]. Constants: x = 25.4 cm (10"); C_0 = 26.4 ppm; v = 8.16 cm min^{-1}; K = 0.3 h^{-1}; k = 1 $\text{cm}^3\ \text{mg}^{-1}\ \text{h}^{-1}$; N = 10.8 mg cm^{-3}; m = 1; s = 0; E = 1.

(1) Equation (15) predicts that if k_i is small, the slope of the initial improvement part of the concentration curve will be small and the initial improvement will last for a long time. Consequently, since $k < k_i$, the breakthrough curve will be flat which is a desirable factor. This behaviour fits the experimental observations [5].

(2) If k is large and close to the k_i value, no initial improvement will be observed and a steep breakthrough is expected. It has been experimentally observed that rapid degradation or breakthrough occurs when there is no initial improvement [5].

(3) Addition of filter aids and chemical pretreatment of the filter effluent will promote attachment and will increase the filter

capacity coefficient N. As k and the N values increase, the second exponential term in the denominator of eqn. (13) will be dominant, decreasing the effect of the initial improvement terms. It has been observed that when clay suspensions are pretreated, the initial improvement disappears or becomes much briefer [5].

(4) It can be seen from eqns. (13) and (17) that as the depth of the filter increases, the y_m value decreases; and as the y_m value decreases, t_m decreases. Thus, the experimentally observed fact that initial improvement is more evident and lasts longer in deeper filters than in shallow filters [5, 6], is also predicted by the theory.

4. CONCLUSIONS

A more general solution of the deep bed filter equations has been presented. The solution offers an explanation for the occurrence of the ripening period in deep bed filters. Ripening occurs if the inlet layer attachment coefficient is greater than the attachment coefficient within the filter depth. Any previously suggested deep bed logistic design equation is a particular case of the solution presented here. The solution predicts certain experimentally observed characteristics of filter ripening.

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APPENDIX A: NOMENCLATURE

A	kN/v (cm^{-1})
B	kC_0/E (h^{-1})
C	effluent concentration (mg cm^{-3})
D	diffusion coefficient (cm^2 min^{-1})
E	$1/y_{\text{limit}}$
$\mathrm{F}(q)$	a function of which describes the accumulation kinetics
$\mathrm{f}(t)$	a function of time
K	$k_i C_0$ (h^{-1})
k	attachment coefficient (cm^3 mg^{-1} h^{-1})
m	time scale shift parameter
N	filter capacity (mg cm^{-3})
p	bed porosity
q	mass of deposited particles per unit filter volume (mg cm^{-3})
s	integration constant or ordinate shift parameter
t	time (min)
v	approach velocity (cm min^{-1})
x	filter bed depth (cm)
y	C/C_0, ratio of outlet concentration to the inlet concentration

Subscripts

i	inlet layer
m	minimum
0	initial