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Decision-Feedback-Equalisation  
For Multipath Channels

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# A Sample Diversity Decision-Feedback-Equalisation For Multipath Channels

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**Abstract.** A new Decision-Feedback-Equalisation (DFE) technique is introduced, namely, the Sample-Diversity (SD) DFE. To realise a sample-diversity at the DFE input,  $N$  samples are taken from each symbol and these samples are used to drive  $N$  different DFE's. A simple selection mechanism is introduced to select the best sampling phase and DFE to be used in the tracking mode. Through simulations, for normalized-rms-delay-spread over the range from 0.01 to 1, the SDDFE is shown to have a much better performance than a conventional DFE. Being a combination of diversity and equalisation techniques, the SD approach is shown as a promising technique for symbol detection in multipath fading channels.

## 1 INTRODUCTION

The progress of mobile communication systems depends on the development of communication techniques for reliable operation in the hostile fading and interference environments. These techniques must also meet the reasonable complexity and performance trade-off made in the system design stage. The characteristic of the propagation medium has been shown to have a significant influence on the system design criteria.

The transmission rate in a multipath fading channel is limited by the normalised rms delay spread  $\tau/T$ , which introduces severe attenuation, Inter-Symbol-Interference (ISI) and Intra-Symbol-Interference (IaSI) in the received signal. Here,  $\tau$  is the rms delay spread and  $T$  is the bit duration. The ISI term appears when the excess delay in the channel is so large that, a symbol of duration  $T_s$  spills over several neighbouring symbols and contaminate them. As will be shown, the IaSI term is more pronounced at low delay spread values where the neighbouring samples within a symbol period are contaminating each other. The aim of this paper is to study the characteristics of IaSI and design a Decision-Feedback-Equaliser (DFE) to combat the resulting effects at  $\tau/T$  values below 1.0.

When the channel attenuation is so large that the received signal strength is below noise level, the channel is said to be in deep fade [1,2]. Typically, after such events, the equalisers can not recover until the next training symbol is received. As a result, the data in the period after the deep fade will be totally lost [3]. It is obvious that the probability of a deep fade is extremely small especially at high signal-to-noise ratio (SNR)

values. Therefore, ISI and IaSI remains to be the major impairment in such channels, which does not necessarily result in deep fade but may cause a burst of error. Among many methods suggested in order to reduce ISI, adaptive equalisation has attracted most of the attention throughout the years [3,4,6,7]. However, none of the equalisation techniques has succeeded to reduce the BER below  $10^{-3}$  in the range of normalised rms delay spread values  $\tau/T=0$  to 1 [3,4,5,8,9]. The IaSI is shown as an important impairment to be considered. Because, the IaSI can not be resolved by a symbol-spaced-equaliser, it is visualised as an additive coloured noise source by the DFE. Thus, in the conventional DFE with sampling point fixed in the middle of the symbol, the performance will be sub-optimum and the equaliser will show poor performance. In this study we show that the amount of interference changes at different sampling points. Therefore, an optimum sampling point will exist where the amount of interference will be minimum. This is the reason why we suggest the use of sample diversity together with equalisation.

The paper is organised as follows. In section 2 the communications system and characterisation of the multipath fading channel are presented. A brief review of the conventional detection and equalisation techniques is given in section 3. Section 4 introduces detection and equalisation techniques using the proposed sample diversity. Section 5 presents the results of simulations and conclusions are made in last section.

## 2 COMMUNICATIONS SYSTEMS CHARACTERISTICS

### 2.1 System Specifications

The following section provides description and design parameters of the mobile communications system. Mainly, the GSM system description and parameters are used with TDMA frames consisting of packets of 26 bits preamble and 148 bits data [4,10]. Our major interest is to improve the tolerance of communications systems to delay spread due to multipath propagation. Hence, we shall consider the normalised rms delay spread ( $\tau/T$ ) with respect to bit duration ( $T$ ) and energy-per-bit to noise-energy-spectral-density ( $E_b/N_0$ ) as the systems independent parameters. The modulation technique used is Quaternary-Phase-Shift-Keying (QPSK). The overall

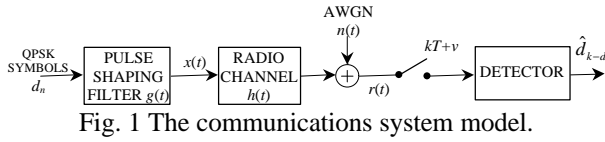


Fig. 1 The communications system model.

system block diagram is shown in Fig.1.

## 2.2 Mathematical Modelling

In the system block-diagram shown in Fig. 1, QPSK data symbols are fed to a rectangular shaping filter followed by a fourth order Butterworth transmit filter with a cut-off frequency of  $3/T_s$ . The radio channel is a typical  $L$ -ray multipath channel of the Digital European Cordless Telecommunications (DECT) type [11] where  $L=6$ . The  $L$  Rayleigh fading components have uniformly distributed phases and spacing such as to result in the target normalised rms delay spread  $\tau/T$ .

It has been shown that the statistics of the received signal at the output of the channel complies with a Rayleigh process with parameter  $\sigma=1.1$  [15]. Further investigations have shown that, at high  $E_b/N_0$ , the probability of a deep fade at which the received signal strength will be below noise level is extremely small. Specifically, at  $E_b/N_0=20\text{dB}$  and  $60\text{dB}$ , the probabilities of deep fades are  $10^{-2}$  and  $10^{-6}$ , respectively. Therefore, particularly at high  $E_b/N_0$ , it is not the deep fade that causes a burst of error but the IaSI and ISI. This can be shown mathematically as follows.

The impulse response of a frequency selective multipath fading channel is given by:

$$h(t) = \sum_{i=0}^{L-1} a_i \delta(t - t_i) \quad (1)$$

where  $a_i$  are the complex gains of the Rayleigh fading rays (with uniformly distributed phases  $\phi_i$  between 0 and  $2\pi$ ) and  $t_i$  are the delays of individual rays taken from the Power-Delay-Profile (PDP, which is defined as the average magnitude squared of the impulse response  $|h(t)|^2$ ) shown in Fig. 2. The shape of the average  $|h(t)|^2$

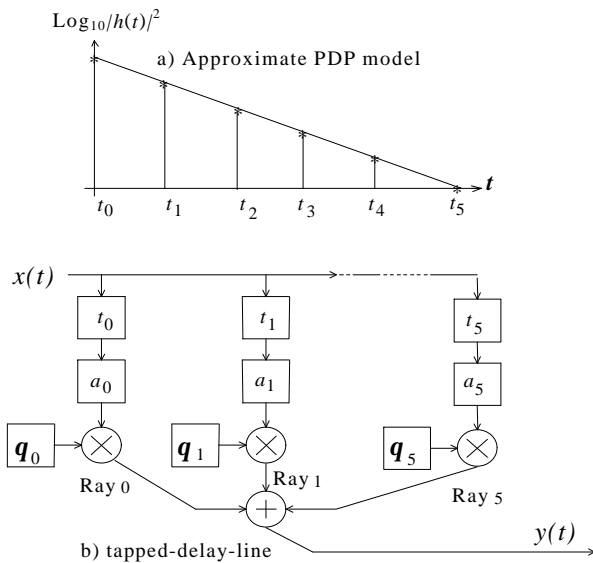


Fig. 2 Realisation of the PDP model

is exponential given by  $|h(t)|^2 = e^{-t/\tau}$  due to the measurement results presented in [12,13,14]. The signal  $x(t)$  at the output of the pulse shaping filter with impulse response  $g(t)$ , corresponding to an input sequence of  $d_n$  is given by:

$$x(t) = \sum_{n=-\infty}^{\infty} d_n g(t - nT_s) \quad (2)$$

Hence, the received signal in the absence of noise is written as:

$$r(t) = \sum_{n=-\infty}^{\infty} d_n \sum_{i=0}^{L-1} a_i g(t - t_i - nT_s) \quad (3)$$

and the sampler output corresponding to the  $k^{\text{th}}$  symbol with a delay of  $v$  is given by:

$$\begin{aligned} r(kT_s + v) &= \sum_{n=-\infty}^{\infty} d_n \sum_{i=0}^{L-1} a_i g(kT_s - t_i - nT_s + v) \\ &= \sum_{n=-\infty}^{\infty} d_n \sum_{i=0}^{L-1} a_i g((k-n)T_s - t_i + v) \end{aligned} \quad (4)$$

where the delay due to the channel spread and transmit filtering is,  $v = \mu T$  and  $0 \leq \mu < 1$ . Equation (4) can be rearranged as:

$$r(kT_s + v) = d_k \sum_{i=0}^{L-1} a_i g(v - t_i) + \sum_{\substack{n=-\infty \\ n \neq k}}^{\infty} d_n \sum_{i=0}^{L-1} a_i g((k-n)T_s - t_i + v) \quad (5)$$

Assuming that  $g(t)$  is of finite duration, we have:

$$r(kT_s + v) = d_k \sum_{i=0}^{L-1} a_i g(v - t_i) + \sum_{\substack{n=k-K_1 \\ n \neq k}}^{k+K_2} d_n \sum_{i=0}^{L-1} a_i g((k-n)T_s - t_i + v) \quad (6)$$

Defining  $j=k-n \Rightarrow j_1=K_1$  and  $j_2=-K_2$ . Hence,

$$\begin{aligned} r(kT_s + v) &= d_k \sum_{i=0}^{L-1} a_i g(v - t_i) + \sum_{\substack{j=-K_2 \\ j \neq 0}}^{K_1} d_{k-j} \sum_{i=0}^{L-1} a_i g(jT_s - t_i + v) \\ &= d_k a_0 g(v) + d_k \sum_{i=1}^{L-1} a_i g(v - t_i) + \sum_{\substack{j=-K_2 \\ j \neq 0}}^{K_1} d_{k-j} u_j(v) \end{aligned} \quad (7)$$

where  $u_j(v) = \sum_{i=0}^{L-1} a_i g(jT_s - t_i + v)$ . Eventually, the noisy received signal can be written in the form:

$$r(kT_s + v) = d_k a_0 g(v) + d_k \sum_{i=1}^{L-1} a_i g(v - t_i) + \sum_{\substack{j=-K_2 \\ j \neq 0}}^{K_1} d_{k-j} u_j(v) + n_k \quad (8)$$

The first term in (8) is the desired term, the second term is the IaSI, the third term is the ISI and the last term is the additive white Gaussian noise (AWGN) term. Equation (8) clearly shows the dependence of the received samples (to be used for detection) on delay  $v$ . Thus, a good choice of  $v$  should be made in order to maximise the ratio of the desired term to the undesired terms. For this purpose, we developed, what we call, the sample diversity approach in which the receiver is designed to operate on several values of  $v$ , i.e. using different samples taken within a symbol, and to select the best. Obviously, this is not an optimal approach because the number of samples that can be checked is finite.

In a specific case where  $t_i$  are selected uniformly, the separation between the rays will be given by  $\Delta t = t_{\max}/(L-1)$  and  $t_i = i\Delta t$ ; where  $t_{\max}$  is the maximum excess delay. As can be easily seen, the IaSI term disappears if  $v - t_i < 0 \forall i$ ; or equivalently,  $\Delta t > n$ . Numerically, for an exponential channel model shown in

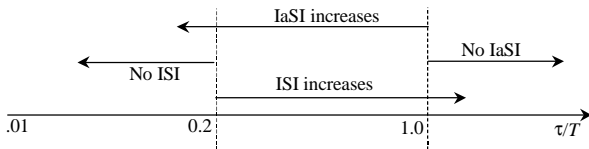


Fig. 3 Regions of IaSI and ISI with respect to  $\tau/T$ .

Fig. 2 with maximum excess delay  $t_{max} \hat{=} t_s = 10\tau$  [15], a normalised rms delay spread  $t/T=0.1$  and  $L=6$ , we have;  $\Delta t = 10 * 0.1/5 = 0.2T_s$ . That is, the IaSI term in (8) is absent for  $v < 0.2T_s$ . In general, for values of  $t_{max} > 5T_s$ , which corresponds to  $\Delta t > T_s$  or  $\tau/T > 1$ , the IaSI term is absent in (8) for all values of  $v$ . On the other hand, for  $t_{max} < T_s$ , which corresponds to  $\tau/T < 0.2$ , there is no ISI term in (8). The regions of IaSI and ISI are shown in Fig. 3 as a function of  $\tau/T$ . In conventional DFE, the IaSI term is not taken into account. However, we concentrated on the IaSI term and tried to minimise its impact on the system performance.

### 3 CONVENTIONAL DETECTION AND EQUALISATION

High bit rate digital transmission, through a mobile radio channel, suffer from severe IaSI and ISI due to multipath propagation. One way of reducing the effect of IaSI and ISI is to employ an adaptive equaliser at the receiver. Here, conventional detection and adaptive equalisation techniques for digital transmission through multipath fading channels are discussed and the design and implementation requirements with regards to their complexity and effectiveness against ISI and IaSI are investigated.

#### 3.1 The Coherent Detection

It has been reported in the literature that, in a multipath fading channel, coherent detection has a better performance than incoherent detection techniques [16,17]. Here, the receiver consists of two stages, namely a phase estimator and a timing recovery circuit as shown in Fig. 4.

In the phase estimation stage, the channel's phase rotation is estimated by multiplying the incoming sampled signal  $r_k$  by the conjugate of the preamble sequence  $I_k^*$  and averaging over the preamble data sequence. The delay estimator is a cross-correlator which estimates the time instance of maximum correlation between the two input sequences, which are the phase corrected version of the received data,  $r_k$  and the preamble  $I_k$ .

#### 3.2 The Decision-Feedback-Equalisation

The basic DFE consists of a feedforward transversal filter and a feedback transversal filter [18]. The input of the feedforward filter is the demodulated data sampled at 1 samples/symbol taken from the centre. The input of the feedback filter is a known sequence of bits corresponding to the preamble during the training mode and the detected data during the tracking mode. The aim of the feedforward section is to compensate for the ISI resulting from future symbols, which are not already detected. The feedback section is fed with detected symbols, thereby serves to remove the ISI arising due to

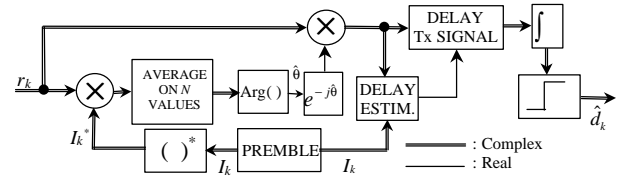


Fig. 4 A coherent detector for multipath signals

past symbols [18].

The equaliser makes the estimate:

$$\hat{y}_k = \sum_{j=0}^F c_j r_{k+j} - \sum_{l=1}^B b_l \hat{d}_{k-l} \quad (9)$$

and decide on the symbol  $d_k$  sent at  $t=kT$ , by passing  $\hat{y}_k$  through a threshold detector. In (9), F is the number of feedforward taps, B is the number of feedback taps,  $c_j$  and  $b_l$  are feedforward and feedback tap coefficients, respectively. The Recursive Least Squares (RLS) algorithm [18] is selected for coefficient updating. Even though the RLS algorithm is more complex than the Least Mean Squares (LMS) algorithm [18], it exhibits fast convergence and less sensitivity to the channel eigenvalue spread [9].

### 4 DETECTION AND EQUALISATION WITH SAMPLE DIVERSITY

#### 4.1 Coherent Detection with Sample Diversity

In the Sample Diversity Coherent Detector (SDCD), the signal at the detector input is over-sampled at a rate  $N$  times larger than the symbol rate of the system and the input is sequentially delayed by  $T_s/N$  as a means of sample diversity. Each Sample is then passed through a different CD, resulting in a total of  $N$  CDs. Finally, the sample that yields the best Bit-Error-Rate (BER) performance over the preamble is selected and kept fixed during data acquisition.

#### 4.3 DFE with Sample Diversity

The Sample Diversity DFE (SDDFE) uses a similar structure as the conventional DFE except that, the former is a combination of both sample diversity and equalisation techniques. Just as the bank of coherent detectors, the SDDFE has  $N$  parallel equalisers as shown in Fig. 5. Here, the received signal is sampled at a rate of  $T_s/N$  and each sample is fed to its own independent equaliser. The number of errors at the output of each equaliser is estimated during the training mode. The delays are set at  $nT/N$  for  $n=1, 2, 3, \dots, N$ . The equaliser with the minimum number of errors is selected and the time delay to input that particular equaliser is the estimated delay  $v$ . This is shown in Fig. 5. The equaliser complexity increases but on-line operation is possible without any delay during equalisation. The power consumption is also minimal since only one of the equalisers need to work in the tracking mode. The number of multiplications in the equaliser seems to increase by using  $N$  different equalisers. However, since only the sample at the optimum sampling point is selected, the net number of multiplications will be equal to that of a single DFE with  $N$  samples per symbol except for the delay

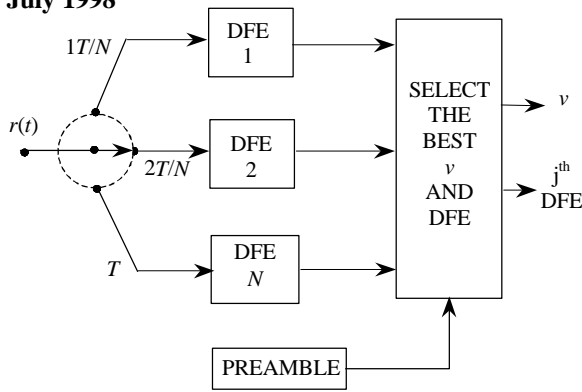


Fig. 5 Implementation of the SDDFE.

estimation process during training mode.

### 5 RESULTS OF SIMULATIONS

The performance of the proposed equalisation and detection techniques are assessed by computer simulations and the results are compared with those of the conventional synchronous DFE and coherent QPSK (CQPSK). Since our major interest is to find out the tolerance of these systems to rms delay spread, initially we considered a rather high  $E_b/N_0$  equal to 60dB. Results already published show that the conventional DFE works quite sufficiently with 3 forward (F) and 2 backward (B) taps [6,7]. Therefore in all the equalisers, unless otherwise specified, we used  $F=3$  and  $B=2$  taps, written as (3,2).

The simulations are performed mainly based on the GSM system parameters. Here, the number of preamble bits are taken as 26 and the number of samples for diversity  $N$  is taken as 10. Each packet consists of 148 data bits at a transmission rate of 10Mb/s.

Fig. 6 compares the BER versus  $\tau/T$  performance of SDDFE(3,2) and SDCD with the conventional DFE(3,2) and CQPSK. From these results, the obvious improvement in the performance obtained by the SD techniques can be clearly seen for  $\tau/T$  from 0.05 to 1.0. It can also be concluded that the improvement obtained by the SD techniques are more noticeable for values of  $\tau/T$  below 0.7. The SDDFE in particular has a quite

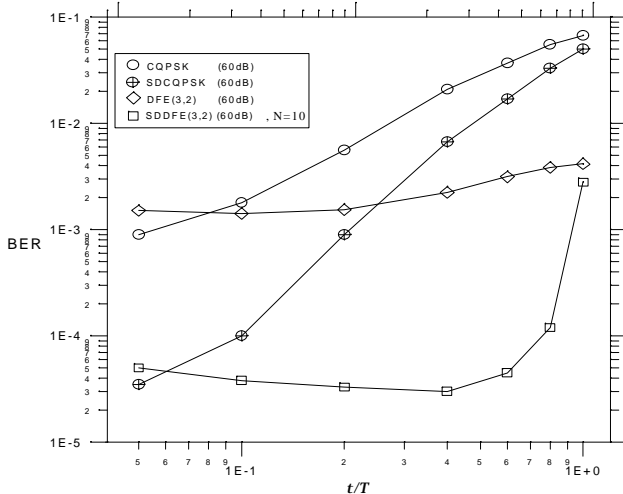


Fig. 6 The BER versus  $\tau/T$  plots for CQPSK, SDCQPSK, DFE(3,2) and SDDFE(3,2).

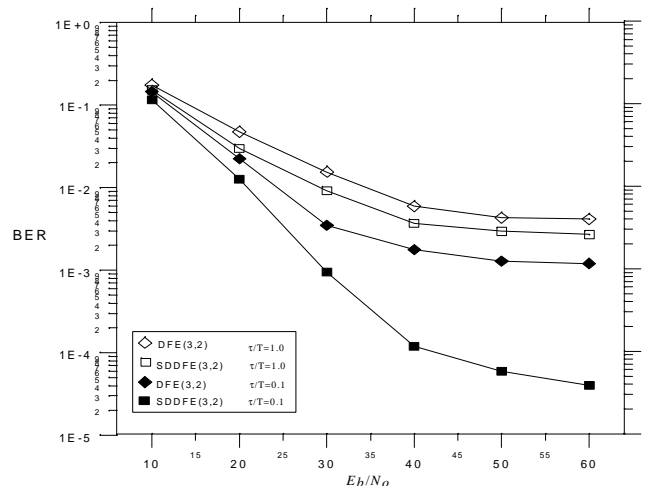


Fig. 7 The BER versus  $E_b/N_0$  plots for DFE and SDDFE at  $\tau/T=0.1$  and  $\tau/T=1.0$ .

satisfactory performance over the range up to  $\tau/T=0.8$  but the SDCD performance is unsatisfactory for  $\tau/T$  above 0.3. In both of the above cases, the equalisers are rendered useless for around  $\tau/T < 0.08$ .

The dependence of the DFE, SDCD and SDDFE on  $E_b/N_0$  are also investigated for two different  $\tau/T$  and the results are presented in Fig. 7. It can be seen that when  $\tau/T=0.1$ , the SDDFE reaches an irreducible BER of  $1 \times 10^{-4}$  and the DFE reach an irreducible BER of  $3 \times 10^{-3}$  both at around  $E_b/N_0=40$ dB. When, however,  $\tau/T=1.0$ , the SDDFE reaches an irreducible BER of  $4 \times 10^{-3}$  and the DFE reach an irreducible BER of  $6 \times 10^{-3}$  both at around  $E_b/N_0=40$ dB. It is also depicted in Fig. 7 that the improvement by SDDFE is more pronounced at smaller  $\tau/T$  values.

Simulations are performed for number of samples used for diversity,  $N=2, 5$  and 10 and the results are presented in Fig. 8. The results clearly show that there is a significant improvement when  $N$  is increased up to 5. However, for larger values of  $N$ , the amount of improvement is insignificant.

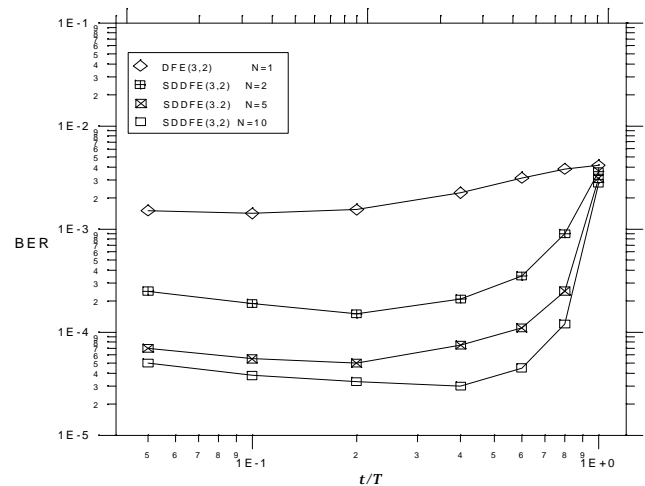


Fig. 8 Comparison of the BER versus  $\tau/T$  plots for SDDFE with number of equaliser equal to 1, 2, 5 and 10.

## 6 CONCLUSION

A novel approach called the sample diversity has been developed and its effectiveness both in coherent detection and in DFE has been illustrated. Among the detection techniques simulated, the SDCD outperformed the conventional DFE and CQPSK, but the SDDFE has shown an outstanding performance against both. This is mainly due to the fact that the SDDFE is not only an equalisation technique but also a diversity technique.

Our major goal was to reduce average BER of the mobile radio system below  $10^{-3}$  in the rms delay spread,  $\tau/T$ , range from 0.01 up to 1.0. The SDDFE has shown to achieve this goal up to  $\tau/T \approx 0.9$  at  $E_b/N_0 = 60$  dB. At  $E_b/N_0 = 22$  dB with a two-branch diversity, the target BER is satisfied for  $\tau/T \leq 0.8$ .

It has been shown that, the performance improvement obtained by sample diversity start to saturate for  $N$  greater than 5 and  $N=2$  is a good compromise between receiver complexity and performance. More optimal procedures for sampling point selection are currently under investigation.

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