Signature Waveform Estimation By Using Short Training Sequences

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Abstract

Multiuser detection techniques are known to be effective strategies to counter the presence of multiuser interference in code division multiple access channels. Generally, multiuser detectors can provide excellent performance only when the signature waveforms[1] of all users are precisely known. Hence, the estimation of signature waveforms is a challenging issue in mobile communication systems. In this paper, we compare the performance of two training based estimators of using short training sequences. One is maximum likelihood type signature waveform estimator that requires the knowledge of spreading sequences and training sequences. The other estimator is based on subspace method and requires the knowledge of training sequences only. Through the simulations, we show the signature waveform estimation performance of both systems and the effect of the estimation error on the performance of a multiuser detector. The complexity comparisons of both systems are also given.

1. Introduction

Direct Sequence Code Division Multiple Access (DS-CDMA) has become one of the favorite candidates for future mobile radio systems. Recently, there is a growing interest in multiuser detectors that provide excellent detection performance. A significant performance improvement of multiuser detectors is due to the capability to suppress Multiple Access Interference (MAI). Although, CDMA based systems provide high power efficiency and moderate error rates, coherent modulation does not provide reliable communication on fading channels if the impulse responses of these channels or the signature waveforms are not known. Traditionally, channel estimation is achieved by sending training sequences or using pilot channel. These approaches rely on periodic transmission of long training sequences [1], making the identification of signature waveforms feasible since both input and output signals are known during the transmission of these sequences. Generally, for better estimation accuracy, more training symbols or higher power for pilot channel shall be required. Consequently, one must pay the price of using long training sequences with a significant reduction of channel efficiency. Subspace based methods were also proposed for signature waveform estimation and blind multiuser detection [2, 3].

Two training methods were recently proposed for synchronous CDMA systems [4, 5]. A subspace signature waveform estimating method using short training sequences is proposed in [5] and in [4] a Maximum Likelihood (ML) channel estimation method, which uses the known spreading sequences together with short training sequences is presented. The signature waveform estimation methods considered in this paper uses short training sequences for estimation, which will not result in significant reduction in net data rate. The ML channel estimation method requires the knowledge of spreading codes and training sequences where the subspace based method requires only the training sequences for signature waveform estimation. If the spreading sequences are known by the receiver, the use of ML estimation method provides significant performance improvement over the subspace based method.

The rest of the paper is organized as follows. In the next section, the assumed communication system model is presented. Section 3 describes the signature waveform estimation methods with short training sequences and compares their complexities. Finally, the simulation

[1] We use the term “signature waveform” to refer to the convolution of the channel and the spreading code throughout the paper.

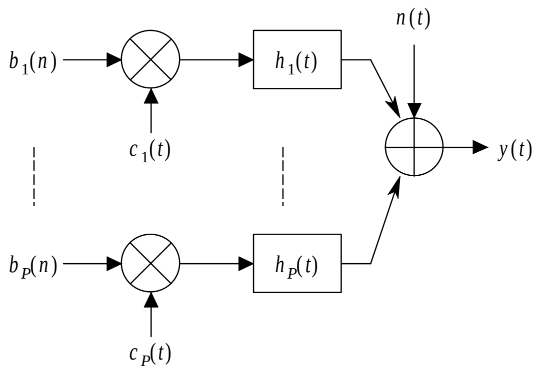

Figure 1. Synchronous CDMA System

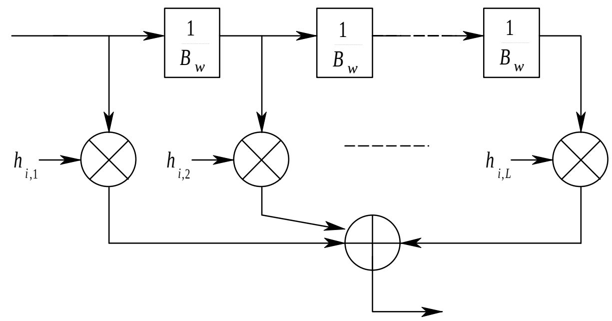

Figure 2. Tapped-delay line model of channel (*$h_i(t)$*)

results, illustrating the performance of the estimation methods are presented together with some conclusions.

2. System Model

In a CDMA system, several users transmit simultaneously over a common channel. Figure 1 shows the equivalent baseband system model used in this paper. The received baseband signal with a single receiver from P users can be represented as the superposition of the active users with additive channel noise

$$y(t) = \sum_{i=1}^{P} r_i(t) + n(t) \tag{1}$$

where subscript i denotes the user index, and $n(t)$ is assumed to be white Gaussian noise with zero mean and a two-sided power spectral density of $N_o/2$.

The multipath-fading channel, which can be represented by the tapped delay line model as shown in Figure 2, can be implemented by a series of Dirac delta functions [6]:

$$h_i(t) = \sum_{l=1}^{L} h_{i,l}\delta(t - \tau_l) \tag{2}$$

Where L is the total number of propogation paths of the channel, $h_{i,l}$ is the complex gain of the l-th propagation path, and τ_l is the delay of the l-th propagation path (l/B_w). The channel coefficients, $h_{i,l}$, are zero-mean complex Gaussian variables and are not changing within a symbol duration.

It is assumed that $h_i(t)$ has a finite support [0 LT_c] where T_c is the chip duration. In addition, we may assume that the channel order is much less than the code length ($L<<L_c$) since the maximum delay spread of channel is usually insignificant in relative to the symbol period [7]. The received signal $r_i(t)$ for each user can be represented as:

$$r_i(t) = \sum_{n=1}^{N} A_i b_i(n) \sum_{l=1}^{L} h_{i,l} c_i(t - \tfrac{l-1}{B_w} - (n-1)T_s) \tag{3}$$

where
A_i transmitted amplitude,
N length of the packet containing N_e preamble bit,
T_s symbol duration,
$b_i(n)$ transmitted sequence containing N_e preamble bit,
$c_i(t)$ pre-assigned spreading code waveform, and
B_w is the bandwidth of spreading code waveform.

The signature waveform of the i-th user can be denoted as:

$$w_i(t) = \sum_{l=1}^{L} h_{i,l} c_i(t - \tfrac{l-1}{B_w}) \tag{4}$$

It is seen that Intersymbol Interference (ISI) exists because the duration of the signature waveform exceeds T_s. However, since $L<<L_c$, this ISI is negligible in general [6].

The discrete counterpart of the signature waveform in (4) for one symbol period is given by:

$$w_i(n) = \sum_{l=1}^{L} h_{i,l} c_i(n - l), \quad n = 1, \cdots, L_c \tag{5}$$

The general problem addressed by the signature waveform estimators considered in this paper is the estimation of $\mathbf{w}_i=[\, w_i(1) \;\; w_i(2) \;\; \ldots \;\; w_i(L_c)]^T$, where $(\cdot)^T$ denotes the transpose operation.

3. Signature Waveform Estimation

We assume a training sequence of N_e symbols. By sampling the received signal $y(t)$ at chip rate (T_c), over the training period, the received signal vector $\mathbf{Y}$ of length N_eL_c can be obtained as follows:

$$\mathbf{Y} = \mathbf{CH} + \mathbf{N} \tag{6}$$

where, $\mathbf{C} = [\mathbf{C}_1 \quad \mathbf{C}_2 \quad \cdots \quad \mathbf{C}_P]$ is the spreading sequences matrix, $\mathbf{H} = [\mathbf{h}_1 \quad \mathbf{h}_2 \quad \cdots \quad \mathbf{h}_P]^T$, in which the amplitudes A_i are accounted for as $\mathbf{h}_i = A_i[h_{i,1} \quad h_{i,2} \quad \cdots \quad h_{i,L}]$. The spreading sequence matrix of each user can be represented as,

$$\mathbf{C}_i = [\mathbf{g}_i(0) \quad \mathbf{g}_i(1) \quad \cdots \quad \mathbf{g}_i(L-1)] \tag{7}$$

where,

$$\mathbf{g}_i(n) \overset{\text{def}}{=} [\underbrace{0 \;\; \cdots \;\; 0}_{n} \;\; \mathbf{f}_{i,1}(L_c) \quad \mathbf{f}_{i,2}(L_c) \quad \cdots \quad \mathbf{f}_{i,N_e}(L_c - n)]^T$$

and

$$\mathbf{f}_{i,n}(l) \overset{\text{def}}{=} b_i(n)[c_i(1) \quad c_i(2) \quad \cdots \quad c_i(x)] \quad x \le L_c \,.$$

3.1 Maximum Likelihood Signature Waveform Estimation

The maximum likelihood estimation of the i-th user channel vector $\mathbf{h}_i$, which amounts to the multiplication of $\mathbf{Y}$ by $\tilde{\mathbf{C}}_i$, is given by [4]:

$$\hat{\mathbf{h}}_i = \tilde{\mathbf{C}}_i \mathbf{Y} \tag{8}$$

where $\tilde{\mathbf{C}}_i$ is an $L\times(L_cN_e)$ matrix of the i-th user, which contains the corresponding L columns of the pseudoinverse [8] of $\mathbf{C}$.

The signature waveform of the i-th user ($\hat{\mathbf{w}}_i$) can be obtained from the multiplication of the estimated channel by the $L\times L_c$ Toeplitz matrix $\mathbf{G}_i$ having $\mathbf{c}_i=[c_i(1) \quad c_i(2) \ldots c_i(L_c)]$ as its first column and $[c_i(1) \;\; 0 \ldots 0]$ as its first row.

$$\hat{\mathbf{w}}_i = \mathbf{G}_i \tilde{\mathbf{C}}_i \mathbf{Y} \tag{9}$$

After the estimation of the signature waveforms, we set up the Minimum Mean Square Error (MMSE) detector as [9]:

$$\hat{b}_i(n) = sign(\mathbf{z}_i^H \mathbf{Y}(n)) \tag{10}$$

where, $\hat{b}_i(n)$ is the i-th user's n-th bit detected, $\mathbf{Y}(n)$ is the discrete vector of the received samples at the n-th symbol interval, $\mathbf{z}_i$ is the i-th column of $\mathbf{Z} = \mathbf{W}(\mathbf{W}^H\mathbf{W} + N_o\mathbf{I})^{-1}$ and $\mathbf{W} = [\hat{\mathbf{w}}_1 \quad \hat{\mathbf{w}}_2 \quad \cdots \quad \hat{\mathbf{w}}_P]$. The superscript $(\cdot)^H$ is used to denote the conjugate transpose operation.

3.2 Subspace Based Signature Waveform Estimation

A subspace-training algorithm is proposed in [5] for signature waveform estimation in a synchronous DS-CDMA system. In this algorithm, the received signal matrix $\mathbf{X}$ having N symbols is constructed as:

$$\mathbf{X} = [\mathbf{Y}(1) \quad \mathbf{Y}(2) \quad \cdots \quad \mathbf{Y}(N)] \tag{11}$$

The eigen decomposition of the matrix $\mathbf{XX}^H$ yields:

$$\mathbf{XX}^H = \mathbf{U\Lambda U}^H \tag{12}$$

where $\mathbf{U}=[\mathbf{U}_s \;\; \mathbf{U}_n]$, $\mathbf{\Lambda}=\text{diag}(\mathbf{\Lambda}_s, \mathbf{\Lambda}_n)$ and $\mathbf{\Lambda}_s=\text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_P)$ contains the P largest eigenvalues of $\mathbf{XX}^H$ in descending order and $\mathbf{U}_s=[\mu_1 \;\; \mu_2 \ldots \mu_P]$ contains the corresponding eigenvectors. The signature waveform estimate of the i-th user can be estimated as [5]:

$$\tilde{\mathbf{w}}_i = \mathbf{U}_s \mathbf{\Lambda}_s (\mathbf{U}_s^H \mathbf{\Gamma\Gamma}^H \mathbf{U}_s)^{-1} \mathbf{U}_s^H \mathbf{\Gamma b}_i \tag{13}$$

where $\mathbf{b}_i=[b_i(1) \quad b_i(2) \quad \ldots \; b_i(N_e)]^T$ is the vector of the training symbols of the i-th user and $\mathbf{\Gamma}=[\mathbf{Y}(1) \quad \mathbf{Y}(2) \ldots \mathbf{Y}(N_e)]$ is the matrix of N_e corresponding data vectors. The linear MMSE receiver of the i-th user, which minimizes the MSE, is then given by [5, 9]:

$$\mathbf{z}_i = \mathbf{U}_s (\mathbf{U}_s^H \mathbf{\Gamma\Gamma}^H \mathbf{U}_s)^{-1} \mathbf{U}_s^H \mathbf{\Gamma b}_i \tag{14}$$

3.3 System Complexities

In the maximum likelihood signature waveform estimation method, the spreading sequences matrix $\mathbf{C}$ associated with preamble sequence of dimension

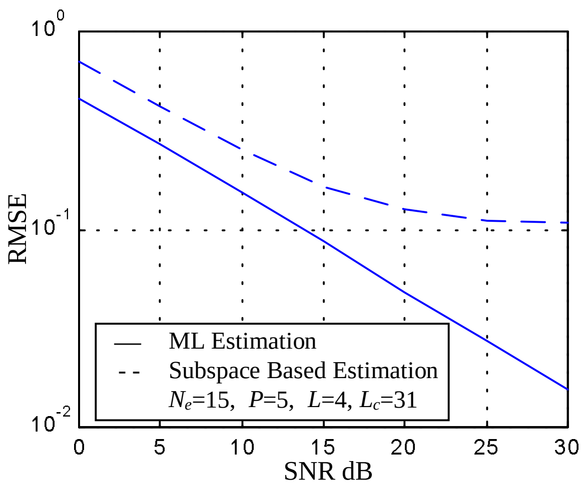

Figure 3. RMSE versus SNR for user 1

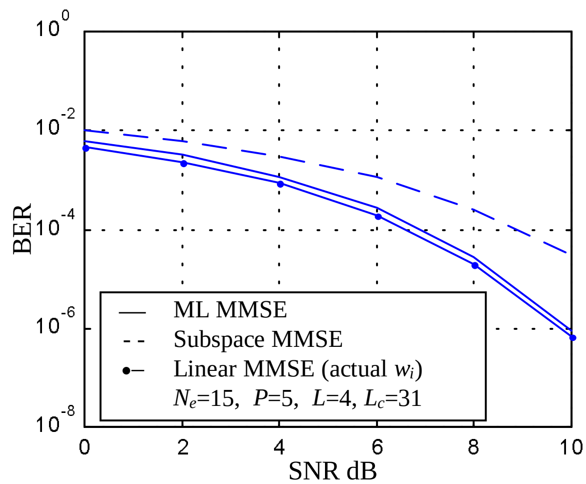

Figure 4. BER versus SNR for user 1

$LP\times(L_cN_e)$ should be inverted. Each user requires the $L\times(L_cN_e)$ part of the inverted matrix ($\tilde{\mathbf{C}}_i$) for signature waveform estimation. Since the spreading and preamble sequences are same for each packet transmission, the necessary matrices are calculated once and stored in memory. Hence, for ML channel estimation each user should store $\tilde{\mathbf{C}}_i$ and the spreading sequence associated with that user ($\mathbf{c}_i$).

In the subspace based estimation method, data matrix $\mathbf{X}$, which is defined in (11) and having dimension $L_c\times N$, should be constructed in each packet transmission. Then, eigen value decomposition should be applied to $L_c\times L_c$ matrix $\mathbf{XX}^H$ to obtain $\mathbf{U}_s$ having the dimension of $L_c\times P$. As defined in (13) $\mathbf{\Gamma}$ of dimension $L_c\times N_e$ and the vector $\mathbf{b}_i$ containing N_e preamble symbols of the corresponding user is necessary for the estimation. The only information required for signature waveform estimation with subspace method is the preamble sequences. Although, it needs less information than ML estimation method, because of the necessary calculations in each packet transmission, it is more complex and slower than the system having ML estimation.

4. Simulation Results

In this section, the Root Mean Square Error (RMSE) and Bit Error Rate (BER) performances of the training based estimation methods are illustrated and compared by extensive computer simulations. In all of the following examples, a synchronous CDMA system with P=5 users was simulated. The number of symbols in each packet is N=400 with N_e=15 training symbol. The channel length (L) was preselected to be four and the channel response of each user was generated Gaussian randomly based on (2). The desired and interfering users employed Gold sequences [10] of length L_c=31 as spreading codes. The preamble bits and data bits were generated randomly for each user as antipodal signaling and perfect power control (i.e. all users have equal powers, A_i=1) was assumed.

The RMSE performances of signature waveform estimating methods are presented Figure 3 with Signal to Noise Ratio (SNR) varying from 0 to 30. The RMSE of the estimation is defined by,

$$\text{RMSE} = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left\|w_i - \hat{w}_i\right\|^2} \qquad (15)$$

where, M is the number of iteration in each Monte Carlo trials, and $\hat{w}_i$ is the estimation of the signature waveform from the i-th iteration. It is seen that the ML estimator has significant performance improvement over the subspace based estimator, especially at high SNR. The subspace based estimator tends to exhibit error floor at high SNR values due to the finite length of the packet size N. This phenomenon has also been observed in [2].

In the next case, the effect of signature waveform estimation by the ML estimator and subspace based estimator on bit error probabilities are investigated. The performances of MMSE detectors defined in (14) and (10) are examined with respect to SNR. To be able to demonstrate how close the signature waveform estimates to the actual waveforms are, the performance of linear MMSE detector with perfect knowledge of signature waveforms are also shown. The results are

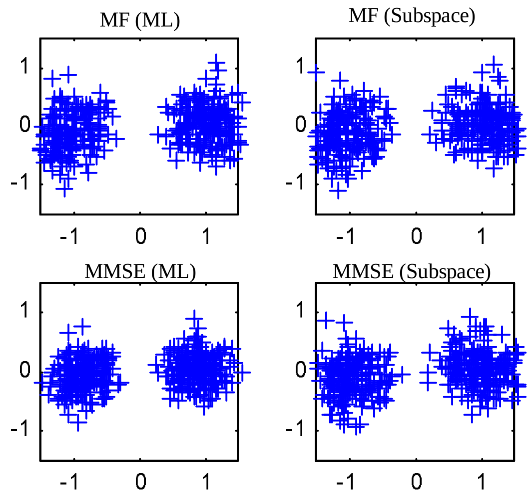

Figure 5. Signal constellation plots for user 1 (N_e=15, P=5, L=4, L_c=31, SNR=8dB)

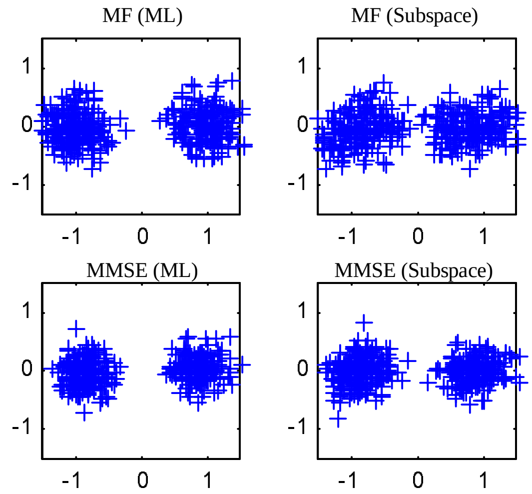

Figure 6. Signal constellation plots for user 1 (N_e=15, P=5, L=4, L_c=31, SNR=12dB)

plotted in Figure 4. It is evident from the figure that the MMSE detection with the signature waveform information obtained by ML estimator offers substantial performance gain over the subspace based method, especially at high SNR. The performance obtained by using ML estimates is very close to the one obtained with perfect signature waveform knowledge.

The signal constellation obtained by the Match Filter (MF) and MMSE detector using the signature waveform estimates of ML and subspace based methods are presented in Figure 5 and Figure 6 respectively. In these simulations, after the signature waveforms were obtained by the training based estimators, the rest of the symbols were applied to the MF and MMSE detector and the signal constellations of two systems are compared at 8dB and 12dB. This is another way of visualizing the difference between two approaches, as can be easily seen the cluster in ML approaches are separated much better than those in the subspace approached.

5. Conclusion

In this paper, two signature waveform estimation techniques using short training sequences were considered. It is observed that, the maximum likelihood signature waveform estimator offers better performance gain over the subspace based method especially at high SNR values, thanks to the knowledge of spreading sequences. The bit error performance of the MMSE receiver using ML waveform estimates is very close to the results obtained with the perfect signature waveform knowledge. Although the ML based estimation system is computationally simple, there is a need to store some matrices for the estimation. But the memory requirement to store these matrices is not important if the length of preamble sequences is kept moderately short. The signature waveforms can be estimated without any prior knowledge of the users' spreading sequences by the subspace based estimator. But the computational complexity of such estimation is prohibitive because of the necessary decompositions and required matrices to be constructed in each packet transmission. However, if the spreading sequences of the users are not known by the receiver, the subspace based estimation should be used.

6. References

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