AIP | Journal of Mathematical Physics

Note on "Large family of colliding waves in the Einstein–Maxwell theory"

M. Halilsoy

Citation: Journal of Mathematical Physics **34**, 3553 (1993); doi: 10.1063/1.530044
View online: http://dx.doi.org/10.1063/1.530044
View Table of Contents: http://scitation.aip.org/content/aip/journal/jmp/34/8?ver=pdfcov
Published by the AIP Publishing

Note on "Large family of colliding waves in the Einstein–Maxwell theory"

M. Halilsoy
Physics Department, EMU, G. Magosa (N. Cyprus), Mersin 10, Turkey

(Received 28 January 1993; accepted for publication 18 March 1993)

As a supplementary to the general class of solutions published in this journal [J. Math. Phys. **31**, 2694 (1990)] we remark that for a specific choice of our seed function we obtain an interesting solution which reduces to the well-known solutions of Khan–Penrose and Bell–Szekeres in particular limits.

Previously we had published a general family of colliding waves in the Einstein–Maxwell (EM) theory.[1] Our method was to combine a general class of similarity integrals to the Ernst equation[2] with a theorem published by Chandrasekhar and Xanthopoulos.[3] Our solution involved a free function X as seed which is obtained from the Euler–Darboux (ED) equation. Thus, given X we could easily construct a new cross polarized EM solution. After publishing Ref. 1 we have observed that by a particular choice of X it is possible to find a solution that in the limit of linear polarization reduces to the Khan–Penrose[4] solution in one limit and to the Bell–Szekeres[5] solution in another. Although there exist many solutions available in the literature published so far none of them possesses those simultaneous limits. By this note we aim at completing this missing link, simply because the famous solutions of Khan–Penrose and Bell–Szekeres have served as an undeniable source of reference to a large relativity literature.

Abiding by the notation of Ref. 1 entirely, we restate our space–time line element as

$$(ds)^2=e^{\nu+\mu_3}\sqrt{\Delta}\left(\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}\right)-\sqrt{\Delta\delta}\left(\chi\, dy^2+\frac{1}{\chi}(dx-q_2\, dy)^2\right). \tag{1}$$

As the particular integral of the ED equation

$$(\Delta X_\tau)_\tau-(\delta X_\sigma)_\sigma=0 \tag{2}$$

we choose the seed function X by

$$e^{2X}=\frac{1-\tau}{1+\tau}(\Delta\delta)^m, \tag{3}$$

where m is an arbitrary parameter. The most general separable solution of Eq. (2) was already given in Eq. (22) of Ref. 1. By making use of Eq. (3) and the method of Ref. 1 we obtain the following three parametric solution

$$e^{\nu+\mu_3}=\frac{1}{\Psi}\delta^{m^2}\Delta^{m^2+1/2}\left(\frac{1-\tau}{1+\tau}\right)^{2m},$$

$$\chi=\sqrt{\Delta\delta}/\Psi,$$

$$q_2=-q(2-a^2)\sigma(m\tau+1) \tag{4}$$

in which

$$2\Psi^{-1}=(2-a^2)\sqrt{1+q^2}\cosh 2X-2\sqrt{1-a^2}\sinh\, 2X+a^2.$$

In this solution q and $0\leqslant a\leqslant 1$ are parameters of rotation and electromagnetism, respectively, whereas m does not have an immediate interpretation. Let us also note that we are using shortly q instead of $\sin\alpha$ of Ref. 1. It is observed that there is nothing noticeable about this solution so far. However, if we set $q=0$ and $2m=1-a$ the line element reduces into

$$(ds)^2=\frac{\Delta}{\Psi}(\delta\Delta)^{[(1-a)/2]^2}\left(\frac{1-\tau}{1+\tau}\right)^{1-a}\left(\frac{d\tau^2}{\Delta}-\frac{d\sigma^2}{\delta}\right)-\Psi\, dx^2-\frac{\Delta\delta}{\Psi}\, dy^2, \tag{5}$$

where Ψ is given above with $q=0$. Checking the limits now we obtain the Khan–Penrose solution for $a=0$ and the Bell–Szekeres solution for $a=1$, therefore Eq. (5) combines those two solutions in a single metric.

[1] M. Halilsoy, J. Math. Phys. **31**, 2694 (1990).
[2] M. Halilsoy, Phys. Rev. D **38**, 2979 (1988).
[3] S. Chandrasekhar and B. C. Xanthopoulos, Proc. R. Soc. London, Ser. A **389**, 209 (1985).
[4] K. A. Khan and R. Penrose, Nature (London) **229**, 185 (1971).
[5] P. Bell and P. Szekeres, Gen. Relativ. Gravit. **5**, 275 (1974).