Computation and Reasoning with Z-Numbers

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ABSTRACT

In most cases the information relevant to the real-world decision is partially reliable. This can be explained by the unreliability of the source of information, misinterpretation, inexperience etc. Z-numbers-based formalization of information (Z-information) represents a Natural Language (NL)-based value of a variable of interest in line with the related NL-based reliability. The necessary point that needs to be considered is that a Z-information generally represents the imperfect information specific to real-world, but at the same time a Z-information has a powerful description from perception point of view of human compared to fuzzy number.

In this thesis two conversion methods are given to convert the Z-number into the fuzzy number and also, to convert the fuzzy number into the crisp number.

An approach to decision making under Z-information based on direct computation over Z-numbers is presented. This approach utilizes the expected utility paradigm and is applied to a benchmark decision problem in the field of economics.

Keywords: Z-numbers, Fuzzy number, Discrete Z-numbers, Discrete fuzzy number, Utility value, Defuzzification, Decision making, Expected utility.

ÖZ

Bir çok durumda gerçek dünyadaki kararlarla ilgili bilgiler kısmen güvenilirdir. Bu durum, bilginin elde edildiği kaynağın kısmen güvenilir olması, yanlış yorumlama, deneyimsizlik vb. ile izah edilebilir. Bilginin Z-sayılara dayalı biçimselleştirilmesi (Z-bilgi) Doğal Dil tabanlı güvenilirliği doğrultusundaki bir değişkenle ilgili Doğal Dil tabanlı değerini temsil eder. Dikkate alınması gereken bir nokta ise Z-bilginin genellikle gerçek dünyaya özgü eksik bilgiyi temsil etmesidir. Aynı zamanda, Z-bilgi bulanık sayı ile kıyaslandığı zaman insan bakış algısı açısından güçlü bir açıklamaya sahip olmasıdır.

Bu tezde iki dönüşüm yöntemi verilmektedir: Z-sayıyı bulanık sayıya dönüştürmek ve bulanık sayıyı belirgin (crisp) sayıya dönüştürmek.

Z-sayılar üzerinde doğrudan hesaplamaya dayalı Z-bilgi altında bir karar verme yaklaşımı sunulmuştur. Bu yaklaşım beklenen fayda paradigmasını kullanır ve ekonomi alanında karşılaştırmalı değerlendirme (benchmark) karar problemine uygulanır.

Anahtar kelimeler: Z-sayılar, Bulanık sayı, Ayrık Z-sayılar, Ayrık bulanık sayı, Fayda değeri, Durulaştırma, Karar kabulü, Beklenen fayda

To my wife, sons and my daughter

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INTRODUCTION

1.1 Motivation

In respect of the theory of decisions, some popularizations of information linked to decisions are available. The first popularization targets crisp numbers, while the second is more interested with the use of intervals. In fact, theories of decisions are mainly based on these two popularizations. The principal available decision theories could be categorized as follows: expected utility and prospect theory which employ numeric information and the multiple priors-based theories like Maxmin expected utility that utilizes interval-valued information.

The third popularization is the fuzzy sets. In the scope of these sets, there seems to be a variety of works devoted by this domain to linguistic preferences, fuzzy utility function, fuzzy multi-criteria decision making and others. Unfortunately, all decision approaches based on these three popularizations apparently lack real reliability.

Uncertainty is quite evident in the real world where much of the decision-based information requires more carefulness due to the lack of the uncertainty. As a result, it is hard to formalize the ability to make rational decisions to be uncertain, inexact and/or incomplete information [1].

A decision theory proposed in [5] deals with imperfect information which is described in a natural language (NL).

Currently, one of the most effectual research areas is multi-criteria decision making, and fuzzy set [2] is considered as one of the key tasks that has been extensively used in the process of decision making since uncertainty and complexity is a constant phenomenon.

The computations with Z-numbers are the generalized form of computations with random numbers, crisp numbers, fuzzy numbers and intervals. As specified, the generality levels can be separated as follows: computation with numbers (ground level zero); computation with intervals (level one); computation with fuzzy numbers (level two) [2]; computation with random numbers (level two); and computation with Z-numbers (level three). The capability of building appropriate real-world models is increased by the increase of the generality level, in some areas like economics, risk assessment, decision analysis, planning, and causality analysis [1].

One of the current issues is that the reliability of information is not precisely taken into account. The reliability, among the most significant qualitative attributes of decision-relevant information, is not there in most cases. The decision making under Z-information as information represented by Z-numbers is considered in [3].

Z-numbers appear to be a sufficient formalization of real-world information which should approximately be considered in light of its reliability. The critical issue is that the reliability of information is not considered properly. Zadeh has proposed a new notion Z-number which is more appropriate to describe the uncertainty. Z-number takes both restraint and reliability. The ability of Z-number's description of the real information of human is higher than the classical fuzzy number [10].

The notion of Z-numbers is proposed to offer a basis for computation with numbers which are not totally reliable. The following examples of Z-numbers seem to be cogent [1]:

Population of Turkey (about 80 million, very sure)

Petrol price in the next two years (below 100 dollars, very probable)

In the suggested perspective, it is to explore an approach to decision making which generalizes the existing expected utility approach in case of Z-information. This approach, compared to the other works on decision making under Z-information, is based on direct computation over Z-numbers without converting them to fuzzy numbers. The direct computation over Z-numbers rules out the loss of information related to a conversion. In this study, an operational approach to solve the decision problems with Z-information by using expected utility is considered. This approach is based on computation over "original" Z-numbers (without conversion to fuzzy numbers) according to operations suggested by L.A. Zadeh. An example of the application of this suggested approach to solve a business decision making problem is provided.

1.2 Thesis Outline

The organization of the chapters of this thesis is as follows: Chapter one includes the introduction of the thesis. Chapter two considers the basic definitions and principles which are important for the description of the next chapters. Chapter three contains the description of the Z-numbers, conversion methods and application. Chapter four consists of computation with Z-numbers and application. Finally, the conclusion is provided in Chapter five.

Chapter 2

LITERATURE REVIEW AND PRELIMINARIES

2.1 General Review

Z-numbers were firstly presented by L.A. Zadeh in 2011 [1], and afterwards the studies initiated the discusion on the subject of Z-numbers in decision making under uncertainty, and in different related fields. One of the main goals of Z-numbers is to produce fuzzy numbers with degree of self-confidence in order to know the real information. By using the Z-numbers, humans' knowledge can be represented in a better way [8].

The concept of a Z-numbers which is introduced in [1] opens the door for a new research area, it should be viewed as a first step toward development of methods of computation with Z-numbers. Zadeh presents some operations for computation over Z-numbers based on his extension principle.

In [4], the authors suggest an approach to use Z-numbers for solving multi-criteria decision-making problems.

The author in [6] uses Z-numbers for the purpose of reasoning.

In [10] proposed approach is intended to use Z-numbers for the expected utility application to solve decision making problems

An approach to use Z-numbers for answering questions and decisions making is considered in [7]. Z-numbers converted into classical fuzzy numbers are suggested in [4] and [7].

In [8], Z-numbers are converted into classical fuzzy numbers and the fuzzy numbers are converted into crisp numbers.

A comprehensive and self-contained theory of Z-arithmetic and its applications is detailed with firstly appeared set of examples in literature [9].

In [11], the theoretical approach for computing arithmetic operations which are performed over discrete Z-numbers is proposed.

In [12], authors suggest the effective theoretic approach from general and computational points of view to be used in computations with discrete Z-numbers. The authors provide strong motivation and justification of the use of discrete Z-numbers as an alternative to the continuous counterparts. Particularly, the motivation is based on the fact that NL-based information has a discrete framework. The suggested arithmetic of Z-numbers includes basic arithmetic operations and important algebraic operations over Z-numbers. The proposed approach allows dealing with Z-information directly.

In [13] presented approach is used in decision making under Z-information based on direct computation over Z-numbers to utilize the expected utility paradigm and is applied to a benchmark decision problem in the field of economics.

2.2 Statement of Problem

2.2.1 Computation with Z-numbers

This section highlights the investigation of an approach for decision-making that generalizes the expected utility approach of Z-information. It is a direct-based computation over Z-numbers without converting them into fuzzy numbers. Furthermore, it differs from the existing works used for decision making problems. The direct computation with Z-numbers without conversion appears to eliminate the loss of information. In this research, an approach based on expected utility is strongly recommended to solve the decision making problems with Z-information. This approach is based on computation over Z-numbers accordingly to operations suggested in [1] and [11].

2.3 Preliminaries

Definition 2.3.1 A fuzzy set A which is defined on a universe set X is given as:

$$A = \left\{ (x, \mu_A(x) \mid x \in X \right\}$$

where $\mu_A : X \to [0,1]$ is the membership function of A, in which the membership value $\mu_A(x)$ is the degree of x for every x in A [14].

Definition 2.3.2 A trapezoidal fuzzy number $\tilde{A}$ can be written as $\tilde{A} = (a_1, a_2, a_3, a_4)$. The trapezoidal fuzzy number will be interpreted as follows [15]:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4} & a_3 \le x \le a_4 \\ 0 & otherwise \end{cases}$$

since $a_1 \le a_2 \le a_3 \le a_4$, such that $a_1, a_2, a_3,$ and a_4 are the real numbers. If $a_2 = a_3$, $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x) \mid x \in [0,1] \}$ becomes a triangular fuzzy number. Hence, the triangular fuzzy number can be considered as a special case of the trapezoidal fuzzy number, whereas the trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ is presented in Figure 1.

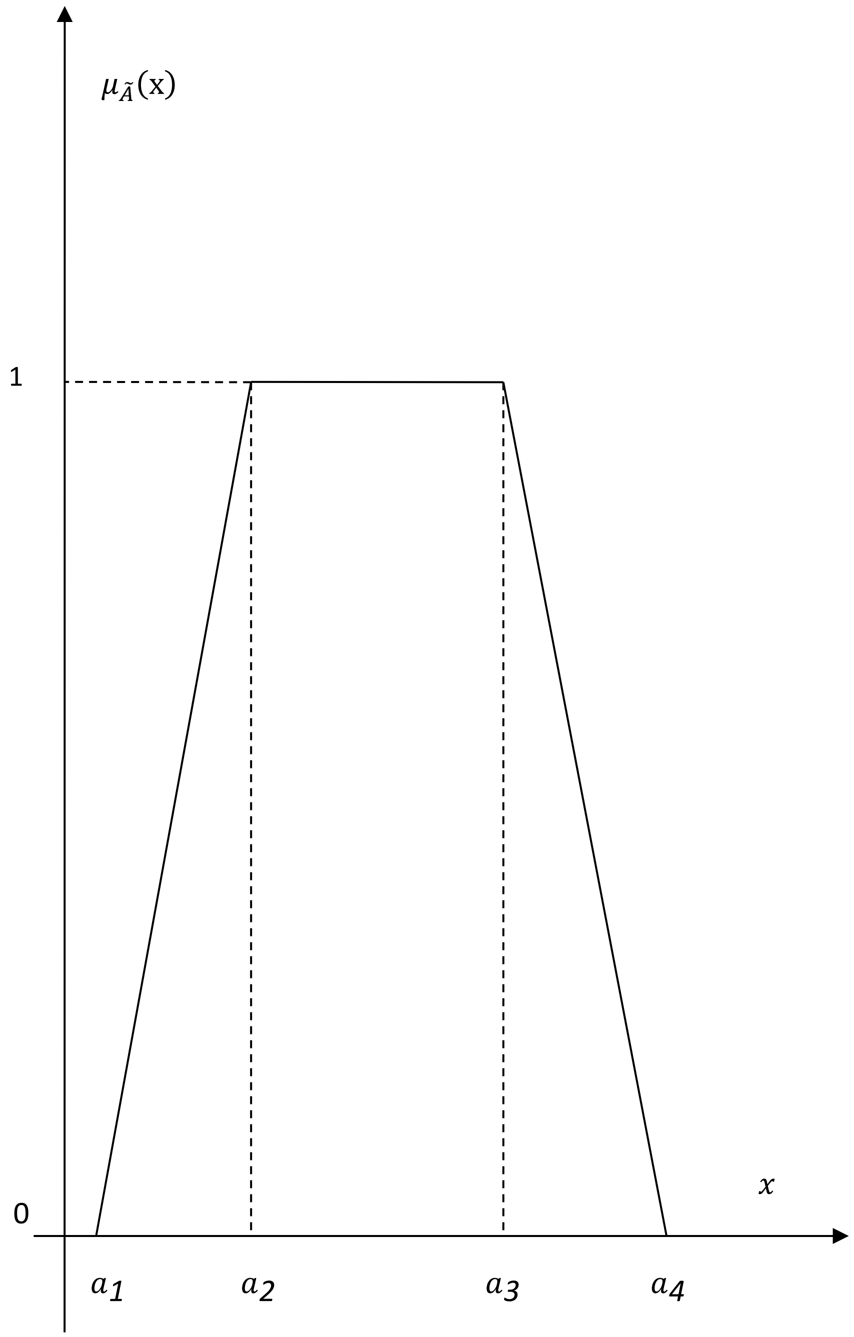

Figure 1: A trapezoidal fuzzy number

Let $\tilde{R} = \{ (x, \mu_{\tilde{R}}(x) \mid x \in [0,1] \}$ be a fuzzy set. Similarly, the membership function of the trapezoidal fuzzy number $\tilde{R} = (b_1, b_2, b_3, b_4)$ is defined by:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-b_1}{b_2-b_1} & b_1 \le x \le b_2\\ 1 & b_2 \le x \le b_3\\ \dfrac{x-b_4}{b_3-b_4} & b_3 \le x \le b_4\\ 0 & otherwise\end{cases}$$

since $b_1 \le b_2 \le b_3 \le b_4$, where b_1, b_2, b_3, and b_4 are the real numbers.

Definition 2.3.3 The regular fuzzy set is defined by $\tilde{Z}' = \{(x, \mu_{\tilde{Z}}'(x)) \mid \mu_{\tilde{Z}}'(x) = \mu_{\tilde{A}}\left(\frac{x}{\sqrt{\alpha}}\right), x \in [0,1]\}$ [7].

Definition 2.3.4 The concept of Z-numbers is highlighted in order to make a basis for computation available with numbers which are not totally reliable as explained in [1]. Z-number is an ordered pair of fuzzy number, $Z=(\tilde{A}, \tilde{R})$ for simplicity, $\tilde{A}$ and $\tilde{R}$ are assumed to be trapezoidal fuzzy numbers. The first component $\tilde{A}$ is a restriction on the values which a real-valued uncertain variable, X, is allowed to take. The second component $\tilde{R}$ is a measure of reliability of the first component. These components are described in Figure 2.

Let us give some examples of Z-numbers: Ali is young (Very High, Probable), Car is a comfortable (Very High, Very High).

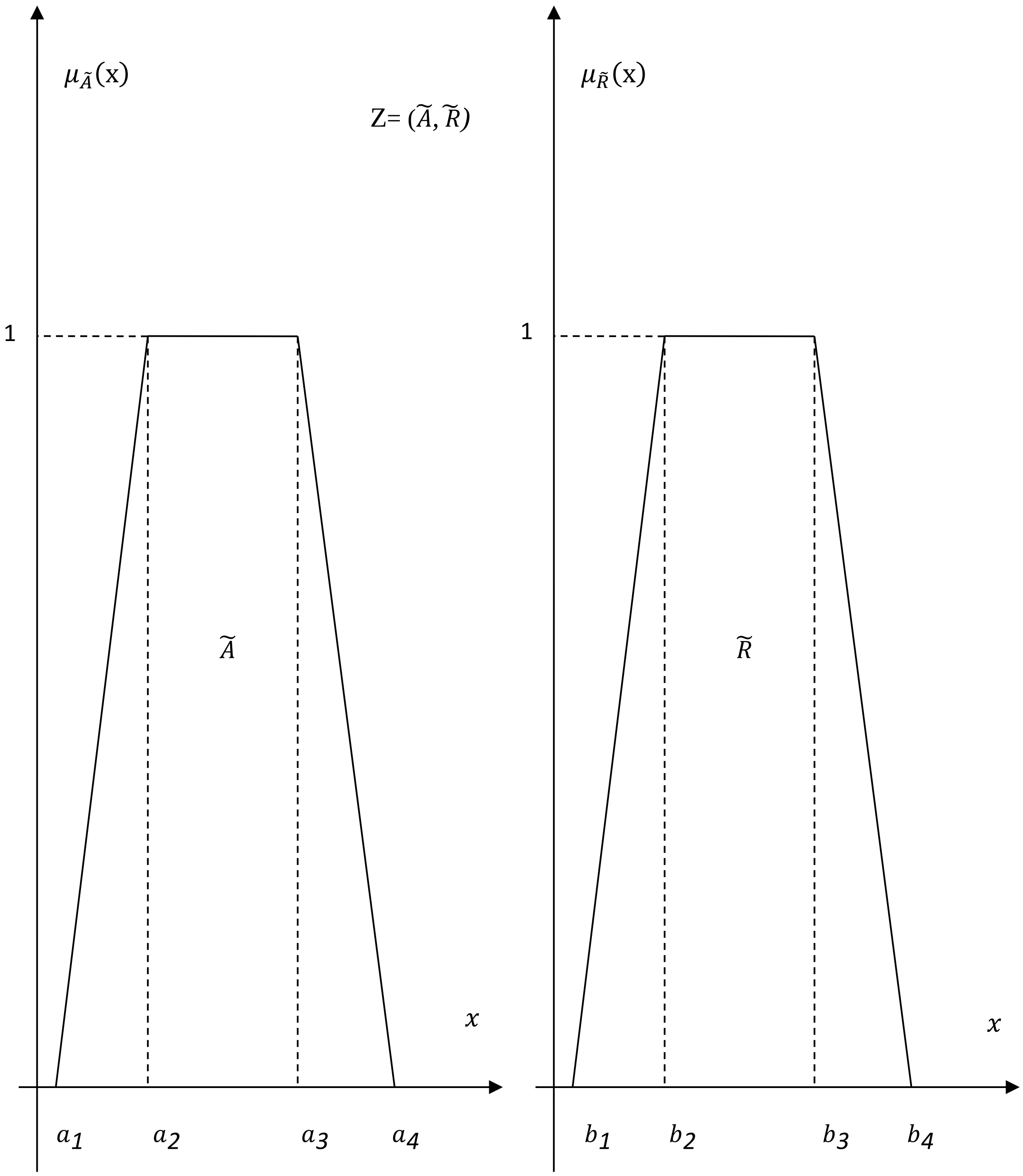

Figure 2: Z-number representation

Definition 2.3.5 (fuzzy number represented in discrete form [16-18]). A fuzzy subset A of $\mathcal{R}$ with membership function $\mu_A : \mathcal{R} \rightarrow [0,1]$ is a discrete fuzzy number in case if this number has a finite support, i.e. there exists $x_1, ..., x_n \in \mathcal{R}$ with $x_1 < x_2 < ... < x_n$, where $\text{supp}(A) = \{x_1, ..., x_n\}$ and there exists natural numbers s, t

with $1 \le s \le t \le n$ which must satisfy the following conditions:

1. $\mu_A(x_i) = 1$ for any natural number i with $s \le i \le t$;

2. $\mu_A(x_i) \le \mu_A(x_j)$ for any natural numbers i, j with $1 \le i \le j \le s$;

3. $\mu_A(x_i) \ge \mu_A(x_j)$ for any natural numbers i, j with $t \le i \le j \le n$.

Definition 2.3.6 (a discrete fuzzy number's probability measure [19]). Suppose A is a discrete fuzzy number, and $P(A)$ denotes A's probability measure. The probability measure $P(A)$ can be determined in the following form:

$$P(A) = \sum_{i=1}^{n} \mu_A(x_i) p(x_i) = \mu_A(x_{j1}) p_j(x_{j1}) + \mu_A(x_{j2}) p_j(x_{j2}) + ... + \mu_A(x_{jn_j}) p_j(x_{jn_j})$$

Below we present the definition of addition of discrete fuzzy numbers suggested in [16-18, 20], where the fuzzy numbers are considered in non-interactive form.

Definition 2.3.7 (addition of discrete fuzzy numbers [16-18, 20]). The addition of discrete fuzzy numbers $\tilde{A}_{12} = \tilde{A}_1 + \tilde{A}_2$ is a discrete fuzzy number with $\alpha - cut$ given as [16-18, 20]):

$$A_{12}^{\alpha} = \{x \in \{\text{supp}(\tilde{A}_1) + \text{supp}(\tilde{A}_2)\} \\ \left| \min\{A_1^{\alpha} + A_2^{\alpha}\} \le x \le \max\{A_1^{\alpha} + A_2^{\alpha}\}\right\}$$

where

$$\mathrm{supp}(\tilde{A}_1)+\mathrm{supp}(\tilde{A}_2)$$

$$=\{x_1+x_2 \mid x_j\in\mathrm{supp}(\tilde{A}_j), j=1,2\}$$

$$\min\{\tilde{A}_1^{\alpha}+\tilde{A}_2^{\alpha}\}=\min\{x_1+x_2 \mid x_j\in\tilde{A}_j^{\alpha}, j=1,2\}$$

$$\max\{\tilde{A}_1^{\alpha}+\tilde{A}_2^{\alpha}\}=\max\{x_1+x_2 \mid x_j\in\tilde{A}_j^{\alpha}, j=1,2\}$$

$$\mu_{\tilde{A}_1+\tilde{A}_2}(x)=\sup\{\alpha\in[0,1] \mid x\in\tilde{A}_1^{\alpha}+\tilde{A}_2^{\alpha}\}$$

Definition 2.3.8 (discrete fuzzy numbers' multiplication [11,12]). The multiplication of discrete fuzzy numbers $\tilde{A}_{12}=\tilde{A}_1\cdot\tilde{A}_2$ is a discrete fuzzy number with $\alpha-cut$ given as [11]:

$$A_{12}^{\alpha}=\{x\in\{\mathrm{supp}(\tilde{A}_1)\cdot\mathrm{supp}(\tilde{A}_2)\} \\ \mid \min\{A_1^{\alpha}\cdot A_2^{\alpha}\}\le x\le\max\{A_1^{\alpha}\cdot A_2^{\alpha}\}\}$$

where

$$\mathrm{supp}(\tilde{A}_1)\cdot\mathrm{supp}(\tilde{A}_2)=\{x_1\cdot x_2 \mid x_j\in\mathrm{supp}(\tilde{A}_j), j=1,2\}$$

$$\min\{\tilde{A}_1^{\alpha}\cdot\tilde{A}_2^{\alpha}\}=\min\{x_1\cdot x_2 \mid x_j\in\tilde{A}_j^{\alpha}, j=1,2\}$$

$$\max\{\tilde{A}_1^{\alpha}\cdot\tilde{A}_2^{\alpha}\}=\max\{x_1\cdot x_2 \mid x_j\in\tilde{A}_j^{\alpha}, j=1,2\}$$

$$\mu_{\tilde{A}_1\cdot\tilde{A}_2}(x)=\sup\{\alpha\in[0,1] \mid x\in\tilde{A}_1^{\alpha}\cdot\tilde{A}_2^{\alpha}\}$$

Definition 2.3.9 (discrete probability distribution). The discrete probability distribution is defined as a function p where if we suppose a discrete random variable X taking K different values with probability that $X = x_i$ defined to be $P(X = x_i) = p(x_i)$, the probability $p(x_i)$ must satisfy $0 \leq p(x_i) \leq 1$ for each i and $\sum_{i=1}^{k} p(x_i) = 1$ [21].

Definition 2.3.10 (discrete probability distributions's convolution). Suppose X_1 and X_2 are random variables in a discrete form with distribution functions p_1 and p_2. The distribution function $X_1 * X_2$ is given as [21]:

$$p_{12}(x) = \sum_{x = x_1 * x_2} p_1(x_1) p_2(x_2)$$

Definition 2.3.11 (a discrete Z-number [12]). A discrete Z-number is defined in the form of ordered pair $Z = (\tilde{A}, \tilde{B})$, where the numbers of the pair $\tilde{A}$ and $\tilde{B}$ are discrete fuzzy numbers, a random variable X may take the values, and $\tilde{A}$ is a fuzzy constraint on these values. $\tilde{B}$ is a fuzzy constraint in $\tilde{A}$:

$$P(\tilde{A}) \text{ is } \tilde{B}$$

The restriction concept has more generality than the constraint concept [22]. A restriction may be seen as a generalized constraint. A probability distribution is a restriction but clearly is not a constraint [23].

Z$^+$-number concept is related to discrete Z-number, i.e. Z$^+$-number is a pair of fuzzy number $\tilde{A}$ and random number R to be defined as:

$$Z^{+} = (\tilde{A}, R)$$

where $\tilde{A}$ plays the same role as in discrete Z-numbers $Z = (\tilde{A}, \tilde{B})$, and R plays the role of the probability distribution P in which [11]

$$P(A) = \sum_{i=1}^{n} \mu_A(x_i) p(x_i) = \mu_A(x_{j1}) p_j(x_{j1}) + \mu_A(x_{j2}) p_j(x_{j2}) + \ldots + \mu_A(x_{jn_j}) p_j(x_{jn_j})$$

Chapter 3

CONVERSIONS BETWEEN Z-NUMBER, FUZZY NUMBER AND CRISP NUMBER

3.1 Introduction

In this chapter, two methods are considered: converting Z-numbers into fuzzy number; and converting fuzzy number into crisp number. These methods are applied for the decision making problem in economics.

The corresponding theorems for the above conversions methods are proposed in [7]. Two methods for the conversions will be specified in this chapter.

3.2 Conversion Method from Z-number into Fuzzy Number

Assume $Z = (\tilde{A}, \tilde{R})$ is a Z-number such that the components $\tilde{A}$ and $\tilde{R}$ are the restriction and reliability, respectively. Let $\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x) \mid x \in [0,1] \right\}$ and $\tilde{R} = \left\{ (x, \mu_{\tilde{R}}(x) \mid x \in [0,1] \right\}$ where $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{R}}(x)$ are the trapezoidal membership functions.

Z-number can be converted into fuzzy number by using the method presented by Kang et.al [7]. First of all, the second component $\tilde{R}$ is converted into a crisp number. The value of α is calculated by the integral equation:

$$\alpha = \frac{\int x\mu_{\tilde{R}}(x)dx}{\int \mu_{\tilde{R}}(x)dx}$$

where symbol $\int$ represents an algebraic integration.

Secondly, the weight of the second component is added to the first component. The weighted Z-number is given by $\tilde{Z}\alpha = \{(x, \mu_{\tilde{A}_\alpha}(x)) \mid \mu_{\tilde{A}_\alpha}(x) = \alpha\mu_{\tilde{A}}(x),\ x \in [0,1]\}$ in the stated calculation.

Finally, the irregular fuzzy number (weighted restriction) is written as regular fuzzy number.

3.3 Conversion Method from Fuzzy Number into Crisp Number

The method which is used to convert fuzzy number to crisp number (defuzzification) by means of the centroid defuzzification technique is also identified as center of gravity or center of defuzzification. The center of gravity method first established by Sugeno in 1985, and it determines the center of the area of the combined membership functions as given in [24,25]. This is the most frequently used technique and is very precise for defuzzification. The centroid defuzzification technique can be written as

$$x^* = \frac{\int x\mu(x)dx}{\int \mu(x)dx}$$

Where x^* is the defuzzified output (crisp number), and the symbol $\int$ represents an algebraic integration.

3.4 An application of Z-number

In this application section, it seems to be relatively vital to explore a decision making problem in economics. The analyzed data obtained from SciTool Company in [26]. SciTools Company's expertise is in the center of scientific instruments, they, in the company, have a governmental contract to make a bid. According to the confidentiality agreement, they should submit a lot of instruments for the coming year and the lowest bid wins. SciTools Company estimates that the total cost will be \$5000 for each bid and the cost of producing one item will be \$95000 in order to win the contract.

From past experience, SciTools Company takes into its consideration that its contracts will depend mainly on the probabilities of the alternatives. The constraints of probabilities for any competition or low bids competition is conveyed as the following: if the low bid is less than \$115000, then the probability will be 0.2; if the low bid between \$115000 and \$120000, then the probability will be 0.4; if the low bid between \$120000 and \$125000, then the probability will be 0.3; if the low bid greater than \$125000, then the probability will be 0.1. The probability of not to bid is 30%. The goal is to find the best decision at the end of the calculation, in other words, it must be decided whether SciTools Company will bid or not.

In order to find out the best decision, there are two alternatives for the Company: to offer a bid, denoted by A_1, and not to offer a bid which is denoted by A_2. The state is denoted by $S_i\ ,i=1,...,5$: no bid, for the state $S_1 \Rightarrow P(S_1)=0.3$ (high); bid less than \$115000, for the state $S_2 \Rightarrow P(S_2)=0.2 \Rightarrow (1-0.3)\times 0.2=0.14$ (low); bid between \$115000 and \$120000, for the state

$S_3 \Rightarrow P(S_3) = 0.4 \Rightarrow (1-0.3) \times 0.4 = 0.28$ (more than medium); bid between \$120000 and \$125000, for the state $S_4 \Rightarrow P(S_4) = 0.3 \Rightarrow (1-0.3) \times 0.3 = 0.21$ (medium); bid greater than \$125000, and for the $S_5 \Rightarrow P(S_5) = 0.1 \Rightarrow (1-0.3) \times 0.1 = 0.07$ (low).

Z-information for the utilities of every accepting is used for different alternatives of the nature and probabilities on states of nature are obtained in Table 1 and Table 2, respectively.

Table 1: The utility value for different alternatives

	S_1	S_2	S_3	S_4	S_5
A_1	*(high; probable)*	*(low; probable)*	*(more than medium; probable)*	*(medium; probable)*	*(high; probable)*
A_2	*(low; probable)*	*(high; probable)*	*(low; probable)*	*(medium; probable)*	*(low; probable)*

Table 2: The values of probabilities of alternative

$P(S_1)$	$P(S_2)$	$P(S_3)$	$P(S_4)$	$P(S_5)$
(high; very sure)	(*low; very sure)*	(*more than medium; very sure)*	*(medium; very sure)*	(*low; very sure)*

The Z-numbers are represented for each alternative A and state S and then these representations are shown in Table 3.

Table 3: Representation of Z-numbers for each alternative and state

	S_1 Z_{31}	S_2 Z_{32}	S_3 Z_{33}	S_4 Z_{34}	S_5 Z_{35}
A_1	Z_{11}	Z_{12}	Z_{13}	Z_{14}	Z_{15}
A_2	Z_{21}	Z_{22}	Z_{23}	Z_{24}	Z_{25}

The data in terms of Z-numbers that are listed in Table 3 are given in the following:

$Z_{31} = (A_{31}, R_{31}) =$ (high; very sure) = [(0.0, 0.6, 0.6, 1.0), (0.0, 0.9, 0.9, 1.0)]

$Z_{32} = (A_{32}, R_{32}) =$ (low; very sure) = [(0.0, 0.3, 0.3, 1.0), (0.0, 0.9, 0.9, 1.0)]

$Z_{33} = \left(A_{33}, R_{33}\right) =$ (more than medium; very sure) = [(0.0, 0.5, 0.5, 1.0), (0.0, 0.9, 0.9, 1.0)]

$Z_{34} = \left(A_{34}, R_{34}\right) =$ (medium; very sure) = [(0.0, 0.4, 0.4, 1.0), (0.0, 0.9, 0.9, 1.0)]

$Z_{35} = \left(A_{35}, R_{35}\right) =$ (low; very sure) = [(0.0, 0.3, 0.3, 1.0), (0.0, 0.9, 0.9, 1.0)]

$Z_{11} = \left(A_{11}, R_{11}\right) =$ (high; probable) = [(0.0, 0.8, 0.9, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{12} = \left(A_{12}, R_{12}\right) =$ (low; probable) = [(0.0, 0.4, 0.5, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{13} = \left(A_{13}, R_{13}\right) =$ (more than medium; probable) = [(0.0, 0.7, 0.8, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{14} = \left(A_{14}, R_{14}\right) =$ (medium; probable) = [(0.0, 0.5, 0.6, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{15} = \left(A_{15}, R_{15}\right) =$ (high; probable) = [(0.0, 0.8, 0.9, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{21} = \left(A_{21}, R_{21}\right) =$ (low; probable) = [(0.0, 0.4, 0.5, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{22} = \left(A_{22}, R_{22}\right) =$ (high; probable) = [(0.0, 0.8, 0.9, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{23} = \left(A_{23}, R_{23}\right) =$ (low; probable) = [(0.0, 0.4, 0.5, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{24} = \left(A_{24}, R_{24}\right) =$ (medium; probable) = [(0.0, 0.5, 0.6, 1.0), (0.0, 0.7, 0.7, 1.0)]

$Z_{25} = (A_{25}, R_{25}) =$ (low; probable) = [(0.0, 0.4, 0.5, 1.0), (0.0, 0.7, 0.7, 1.0)]

Furthermore, Z-numbers are converted into fuzzy number using the method that is defined in Section 3.2. The decision matrix with the results of fuzzy numbers for the states is described in Table 4.

Table 4: Decision matrix with fuzzy number

	S_1 (0.0, 0.50, 0.50, 0.84)	S_2 (0.0, 0.25, 0.25, 0.84)	S_3 (0.0, 0.42, 0.42, 0.84)	S_4 (0.0, 0.33, 0.33, 0.84)	S_5 (0.0, 0.25, 0.25, 0.84)
A_1	(0.0, 0.62, 0.7, 0.77)	(0.0, 0.31, 0.39, 0.77)	(0.0, 0.54, 0.62, 0.77)	(0.0 ,0.39, 0.46, 0.77)	(0.0, 0.62, 0.7, 0.77)
A_2	(0.0, 0.31, 0.39, 0.77)	(0.0, 0.62, 0.7, 0.77)	(0.0, 0.31, 0.39, 0.77)	(0.0, 0.39, 0.46, 0.77)	(0.0, 0.31, 0.39, 0.77)

Finally, fuzzy numbers are converted into crisp numbers by means of the centroid defuzzification technique. By this conversion, the expected utilities of two alternatives, A_1 and A_2 are obtained and the expected utilities for A_1 and A_2 are given in Table 5.

Table 5: Decision matrix with crisp number

	S_1 0.24	S_2 0.17	S_2 0.22	S_2 0.20	S_2 0.17	**Expected Utility**
A_1	0.52	0.37	0.48	0.41	0.52	**0.46**
A_2	0.37	0.52	0.37	0.41	0.37	**0.40**

Consequently, the alternative A_1 is better than the alternative A_2, i.e. $A_1 > A_2$, according to the results which are shown in Table 5.

Z-numbers are ranked according to the degree of reliability and the experimental results show the effectiveness of a Z-number in decision making problems.

Chapter 4

COMPUTATION WITH DISCRETE Z-NUMBERS

4.1 Overview

A general approach for computations with Z-numbers according to Zadeh's extension principle is suggested in [1]. Many reseachers look into using Z-numbers, but the lack of a direct and easy way to compute Z-numbers forced them to start thinking about a way to convert them into fuzzy numbers.

An approach to convert a Z-numbers into classical fuzzy numbers is suggested in [7]. The second part is converted to crisp number, but this leads to loss of original information.

The studies [3,4,8] are used according to what has been put forward in study [7], but in fact the results of this method are not of high-reliable. As a result, the researchers looked for a new and simple way to calculate Z-numbers directly without conversion, based on what has been suggested in the study [1].

4.2 Addition and Multiplication of Discrete Z-numbers

Assume the discrete Z-numbers are $Z_1 = (\tilde{A}_1,\ \tilde{B}_1)$, and $Z_2 = (\tilde{A}_2,\ \tilde{B}_2)$ that describe the values of uncertain variables X_1 and X_2 with real values. The addition and multiplication of Z-numbers $Z_{12} = Z_1 * Z_2, * \in \{+, \times\}$ are determined as indicated below [12]. Let $Z_1^+ = (\tilde{A}_1,\ R_1)$ and $Z_2^+ = (\tilde{A}_2,\ R_2)$ be given. Then

$$Z_{12}^{+} = Z_{1}^{+} * Z_{2}^{+} = (\tilde{A}_1 * \tilde{A}_2, R_1 * R_2),$$

and the discrete probability distributions (Definition 2.3.9) represent R_1 and R_2 :

$$p_1 = p_1(x_{11}) \setminus x_{11} + p_1(x_{12}) \setminus x_{12} + ... + p_1(x_{1n}) \setminus x_{1n}$$

$$p_2 = p_2(x_{21}) \setminus x_{21} + p_2(x_{22}) \setminus x_{22} + ... + p_2(x_{2n}) \setminus x_{2n}$$

$\tilde{A}_{12} = \tilde{A}_1 * \tilde{A}_2$ is a sum (or multiplication) of fuzzy numbers which is determined according to the Definition 2.3.7 (Definition 2.3.8) and the convolution of probability distribution $R_1 * R_2$ is determined according the Definition 2.3.10.

Next, $\tilde{B}_{12}$ is constructed by solving the following problem:

$$\mu_{\tilde{B}_{12}}(b_{12s}) = \sup(\mu_{p_{12s}}(p_{12s}))$$

subject to

$$b_{12s} = \sum_{i} p_{12s}(x_i)\mu_{\tilde{A}_{12}}(x_i),$$

$$\mu_{p_{12}}(p_{12}) = \max_{\{p_1, p_2 | p_{12} = p_1 \circ p_2\}}[\mu_{p_1}(p_1) \wedge \mu_{p_2}(p_2)],$$

$$\mu_{p_j}(p_j) = \mu_{\tilde{B}_j}\left(\sum_{k=1}^{n}\mu_{\tilde{A}_j}(u_k)p_j(u_k)\right), \quad j = 1,2.$$

Thus, $Z_{12} = Z_1 * Z_2$ is obtained as $Z_{12} = (\tilde{A}_{12}, \tilde{B}_{12})$ [11,12].

4.3 Ranking of Discrete Z-numbers [12]

It is an important operation in the arithmetic of Z-numbers to rank Z-numbers which are in discrete form, and Zadeh has mentioned the importance of Z-numbers' ranking problem in [1]. Z-numbers are represented as ordered pairs in contradistinction to the real numbers, and a familiar method must be provided to rank these pairs. This thesis suggests considering Z-numbers' comparison which is based on the principle of fuzzy optimality (FO). Suppose Z-numbers $Z_1 = (\tilde{A}_1, \tilde{B}_1)$ and $Z_2 = (\tilde{A}_2, \tilde{B}_2)$ are given. First, it is required to calculate the functions n_b, n_e, n_w which are used to determine which one of the Z-numbers is better (worse) and how much better (worse) than another one or equivalent to another one by taking both first and second components into consideration [12]:

$$n_b(Z_i, Z_j) = Ps_b(\tilde{\delta}_{\tilde{A}}^{i,j}) + Ps_b(\tilde{\delta}_{\tilde{B}}^{i,j}),$$

$$n_e(Z_i, Z_j) = Ps_e(\tilde{\delta}_{\tilde{A}}^{i,j}) + Ps_e(\tilde{\delta}_{\tilde{B}}^{i,j}),$$

$$n_w(Z_i, Z_j) = Ps_w(\tilde{\delta}_{\tilde{A}}^{i,j}) + Ps_w(\tilde{\delta}_{\tilde{B}}^{i,j})$$

where $\tilde{\delta}_{\tilde{A}}^{i,j} = \tilde{A}_i - \tilde{A}_j$, $\tilde{\delta}_{\tilde{B}}^{i,j} = \tilde{B}_i - \tilde{B}_j$

$$Ps_l(\tilde{\delta}_{\tilde{A}}^{i,j}) = \frac{Poss\left(\tilde{\delta}_{\tilde{A}}^{i,j} \middle| n_l\right)}{\sum_{t\in\{b,e,w\}} Poss\left(\tilde{\delta}_{\tilde{A}}^{i,j} \middle| n_t\right)}, \quad Ps_l(\tilde{\delta}_{\tilde{B}}^{i,j}) = \frac{Poss\left(\tilde{\delta}_{\tilde{B}}^{i,j} \middle| n_l\right)}{\sum_{t\in\{b,e,w\}} Poss\left(\tilde{\delta}_{\tilde{B}}^{i,j} \middle| n_t\right)}, t \in \{b, e, w\},$$

$i, j = 1, 2,\ i \neq j$. As $\sum_{t \in \{b,e,w\}} Ps_l(\tilde{\delta}_k^{i,j}) = 1$ always available, and there is $n_b(Z_i, Z_j) + n_e(Z_i, Z_j) + n_w(Z_i, Z_j) = N$, where N defines components' number of a Z-number, i.e. $N = 2$. In figure 3 [12] the membership functions of $\tilde{n}_b, \tilde{n}_e, \tilde{n}_w$ are depicted.

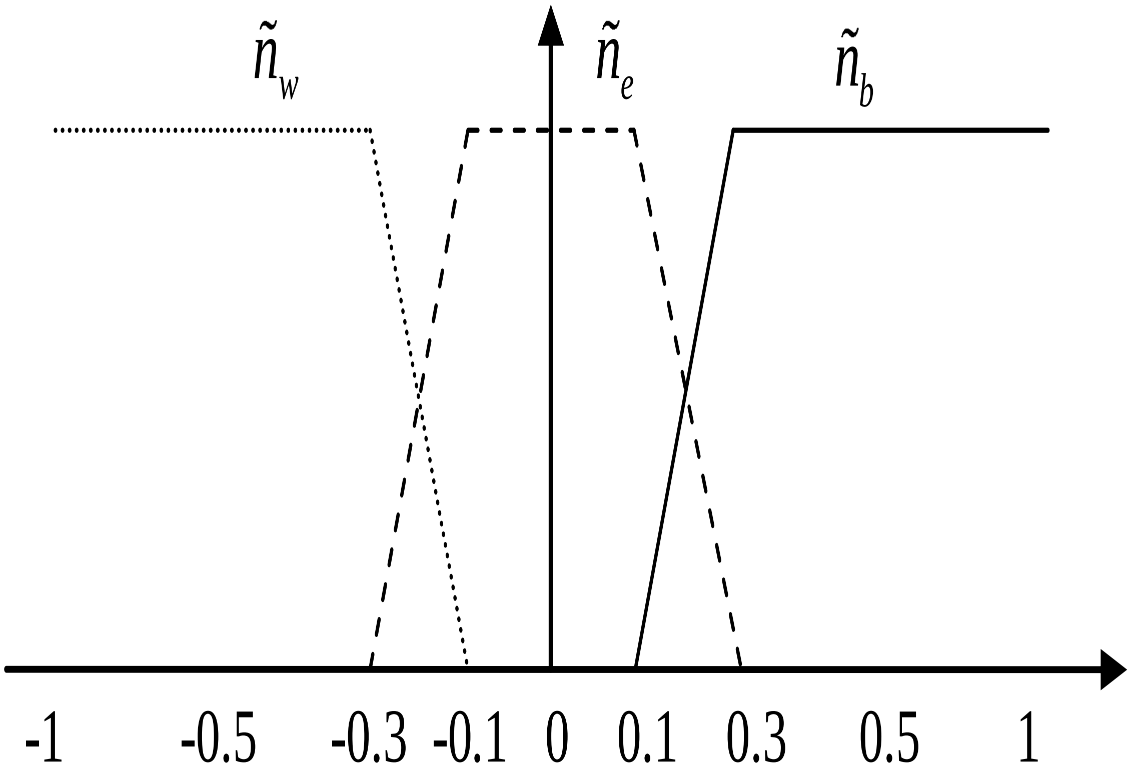

Figure 3: The membership functions of $\tilde{n}_b, \tilde{n}_e, \tilde{n}_w$

In the next stage the greatest k should be determined so that Z_i has a Pareto domination over Z_j to the $(1-k)$. To realize it, it is necessary to introduce a function d:

$$d(Z_i, Z_j) = \begin{cases} 0, \quad if \;\; n_b(Z_i, Z_j) \le \dfrac{2 - n_e(Z_i, Z_j)}{2} \\ \dfrac{2 \cdot n_b(Z_i, Z_j) + n_e(Z_i, Z_j) - 2}{n_b(Z_i, Z_j)}, otherwise \end{cases}$$

Taking into account the given d, we find the desired greatest k such as $k = 1 - d(Z_i, Z_j)$, and afterwards $(1-k) = d(Z_i, Z_j)$ is defined. The Pareto dominance of Z_i over Z_j is implied by $d(Z_i, Z_j) = 1$, and no Pareto dominance of Z_i over Z_j is implied by $d(Z_i, Z_j) = 0$. The optimality degree $do(Z_i)$ can be obtained in the following form:

$$do(Z_i) = 1 - d(Z_j, Z_i) .$$

Thus, in other words, $do(Z_i)$ describes the degree how one Z-number is greater than another one. Then [12]

$$Z_i > Z_j \text{ iff } do(Z_i) > do(Z_j) ,$$

$$Z_i < Z_j \text{ iff } do(Z_i) < do(Z_j)$$

and

$$do(Z_i) = do(Z_j) \text{, else.}$$

The degree-based comparison is very admirable for Z-numbers.

From the viewpoint of the considered approach, while ranking Z-numbers, it is suggested to take the degree of pessimism $\beta \in [0,1]$ into consideration. This degree is sent by a human whose desire is the comparison of the considered Z-numbers, but the fuzzy optimality approach mentioned above does not provide completely reliable results. So, given $do(Z_j) \le do(Z_i)$, two Z-numbers Z_1 and Z_2 are defined [12]:

$$r(Z_i, Z_j) = \beta do(Z_j) + (1-\beta) do(Z_i)$$

Then

$$Z_i > Z_j \text{ iff } r(Z_i, Z_j) > \frac{1}{2}(do(Z_i) + do(Z_j)),$$

$$Z_i < Z_j \text{ iff } r(Z_i, Z_j) = \frac{1}{2}(do(Z_i) + do(Z_j))$$

and $Z_i = Z_j$, otherwise [12].

The pessimism degree β is sent by a person and the ranking process of Z-numbers is adjusted to describe person's approach to do.

4.4 A method of decision making under Z-information utility function

Real-world decision relevant information is imprecise and uncertain, and this is a reason why results of decision analysis based on such information are partially reliable. This fact should prevent decision makers relying much on decision analysis results even when a very careful mathematical modeling was used.

A well-known approach to decision making under uncertainty is the use of expected utility function [27]. However, classical paradigm of expected utility function fails to express various adequate decisions due to incapability to handle imperfect decision-relevant information. The extension of this paradigm to the framework of Z-information may help achieve a more realistic decision analysis technique and, at the same time, to use a simple form of the utility function [13].

Let $\mathcal{S}=\{S_1,S_2,...,S_n\}$ be a set of states of nature and $\mathcal{X}=\{X_1,...,X_l\}$, $X_k=(\tilde{A}_k,\tilde{B}_k),\ k=1,...,l$, be a set of Z-valued outcomes. Denote by $\mathcal{F}_{\mathcal{S}}$ a σ - algebra of subsets of $\mathcal{S}$. So we consider $\mathcal{A}=\{f\in\mathcal{A}\,|\,f:\mathcal{S}\to\mathcal{X}\}$ the set of Z-valued actions as the set of all $\mathcal{F}_{\mathcal{S}}$-measurable Z-valued functions from $\mathcal{S}$ to $\mathcal{X}$ [35,32].

Linguistic information on likelihood Z_{P^l} of the states of nature is represented by Z-valued probabilities $Z_{P_i}=(\tilde{A}_{P_i},\tilde{B}_{P_i})$ of the states S_i:

$$Z_{P^l}=Z_{P_1}/S_1+Z_{P_2}/S_2+...+Z_{P_M}/S_M$$

In this framework, a classical neo-Bayesian nomenclature is extended as follows: elements of $\mathcal{X}$ are Z-valued outcomes; elements of $\mathcal{A}$ are Z-valued acts; elements of $\mathcal{S}$ are states of nature; elements of $\mathcal{F}_{\mathcal{S}}$ are events.

A framework of decision making with Z-information can be formalized as a 4-tuple $(\mathcal{S},Z_{P^l},\mathcal{X},\mathcal{A})$. The determination of an optimal act $f^*\in\mathcal{A}$ is the main issue of decision making problem with Z-valued information on the base of expected utility. It is necessary to find $f^*\in\mathcal{A}$ for which $Z_{U(f^*)}\ge Z_{U(f)},\ \forall f\in\mathcal{A}$.

Here $Z_{U(f)}$ is a Z-valued expected utility defined as

$$Z_{U(f)} = Z_{X_1} Z_{P_1} + ... + Z_{X_i} Z_{P_i} + ... + Z_{X_n} Z_{P_n} \ ,$$

where multiplication and addition are defined in Section 4.2. The comparison operation $\geq$ is as defined in Section 4.3.

The suggested approach is based on direct computations with Z-numbers, without converting them to fuzzy and/or crisp numbers. This allows preserving available imprecise and partially reliable information, and to use it in the final comparison of alternatives.

4.5 Practical Applications

In this part, economical decision making problem is considered. The data which are analized are taken from Techware Incorporated [26]. Techware Incorporated introduces to the market two different new software products. There are three alternatives the company has, and which are related to these two products. The products that are introduced are product one only, product two only, or both products. The expences spent for the research and development processes of above mentioned products are respectively \$180,000 and \$150,000. The national economical situation and the response of consumers to mentioned products are maily intented to impact what kind of the success these products will have for the next year. If the product one only is introduced by the company, then it is expected to have revenue of \$500,000, \$260,000, and \$120,000 in case of having, respectively, strong national economy, fair national economy, and weak national economy.

In the same manner, when it is to introduce the product two only, the expectation of a revenue to have is \$420,000, \$230,000, and 110,000 for the cases of having, respectively, strong national economy, fair national economy, and weak national economy.

Lastly, when both products one and two are introduced, the revenues to be expected are \$820,000, \$390,000, and \$200,000 for the cases of having, respectively, strong national economy, fair national economy, and weak national economy. The probabilities for strong national economy and fair national economy, estimated by the specialists of the Techware Incorporated Company as very sure, are expected to be about 0.30 and about 0.50, respectively. The problem consists in determination of the best decision.

The following proceeding is presented to formally describe the decision problem considered above. Z-numbers describe the information which is partially reliable. The alternatives are represented as a set in the following form:

$$\mathcal{A} = \{f_1, f_2, f_3\},$$

where the denotations f_1, f_2, and f_3 are for introducing product one only, product two only, and both products (one and two), respectively .

The states of nature are represented as a set given below:

$$\mathcal{S} = \{S_1, S_2, S_3\}$$

where S_1 denotes strong national economy, S_2 denotes fair national economy, S_3 denotes weak national economy. The states of nature are with the probabilities $Z_{P(S_1)} = (about\,0.3, very\,sure)$, $Z_{P(S_2)} = (about\,0.5, very\,sure)$.

The set of outcomes:

$$\mathcal{X} = \{(low, probable), (more\ than\ low, probable),$$

$$(medium, probable), (below\ than\ high, probable),$$

$$(high, probable)\}$$

The different alternatives with utility values and states of natures with probabilities are represented in Table 6.

In the Table 7 the corresponding matrix of decision with Z-number is represented.

Table 6: Different alternatives with utility values and states of natures with probabilities

	S_1	S_2	S_3
	(about 0.3, very sure)	(about 0.5, very sure)	(about 0.2, very sure)
f_1	$(high, probable)$	$(medium; probable)$	$(low; probable)$
f_2	$\begin{pmatrix} below\ than\ high; \\ probable \end{pmatrix}$	$(medium; probable)$	$(low; probable)$
f_3	$(high; probable)$	$\begin{pmatrix} more\ than\ low; \\ probable \end{pmatrix}$	$(low; probable)$

Table 7: Matrix of decision with Z-numbers

	S_1 Z_{41}	S_2 Z_{42}	S_3 Z_{43}
f_1	Z_{11}	Z_{12}	Z_{13}
f_2	Z_{21}	Z_{22}	Z_{23}
f_3	Z_{31}	Z_{32}	Z_{33}

The 1st and 2nd components of considered Z-numbers have membership functions which are used for probability and utility values given in the table represented in Figures 4-15.

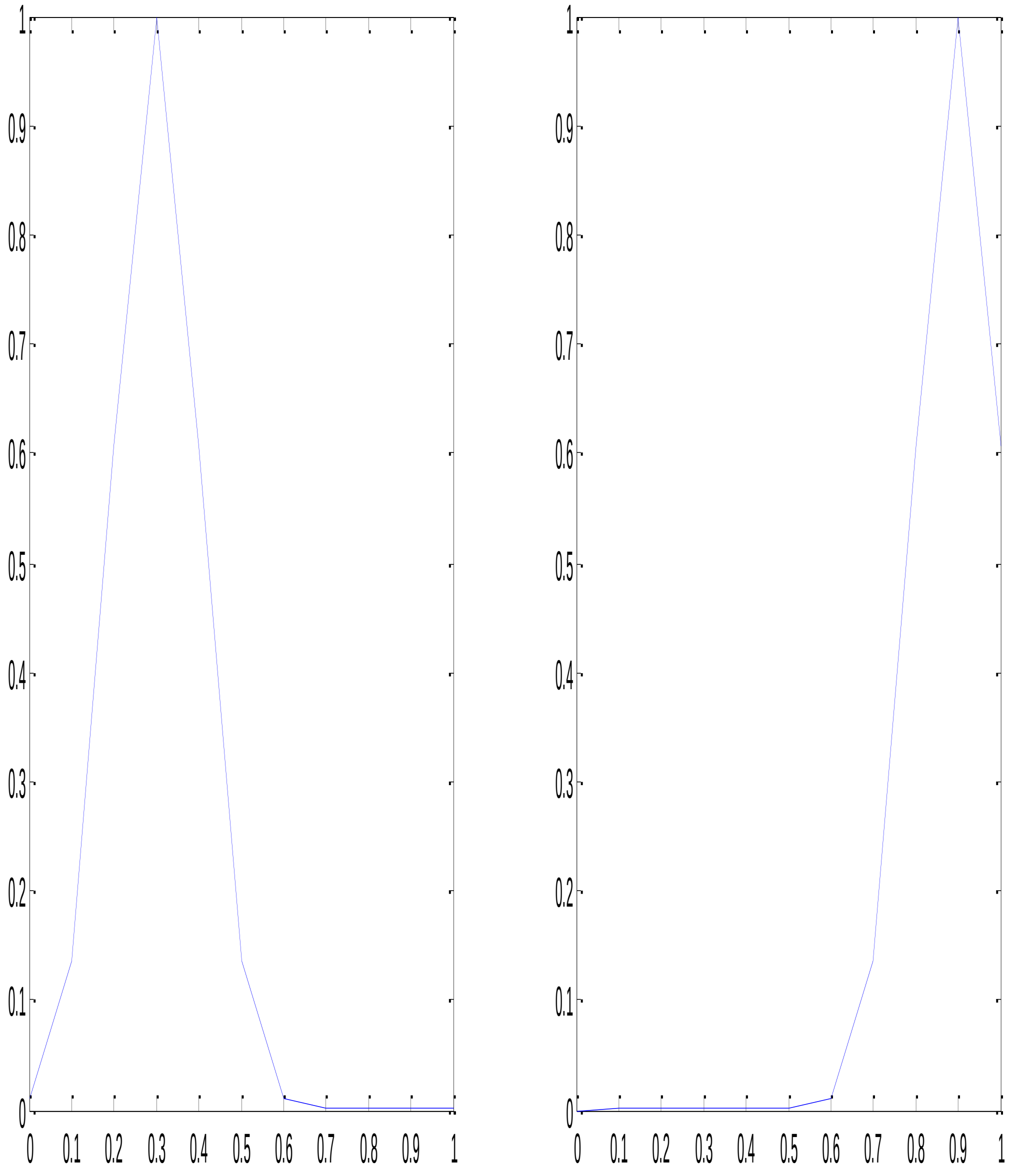

Figure 4: First state's (Z_{41}) representation in a Z-number form

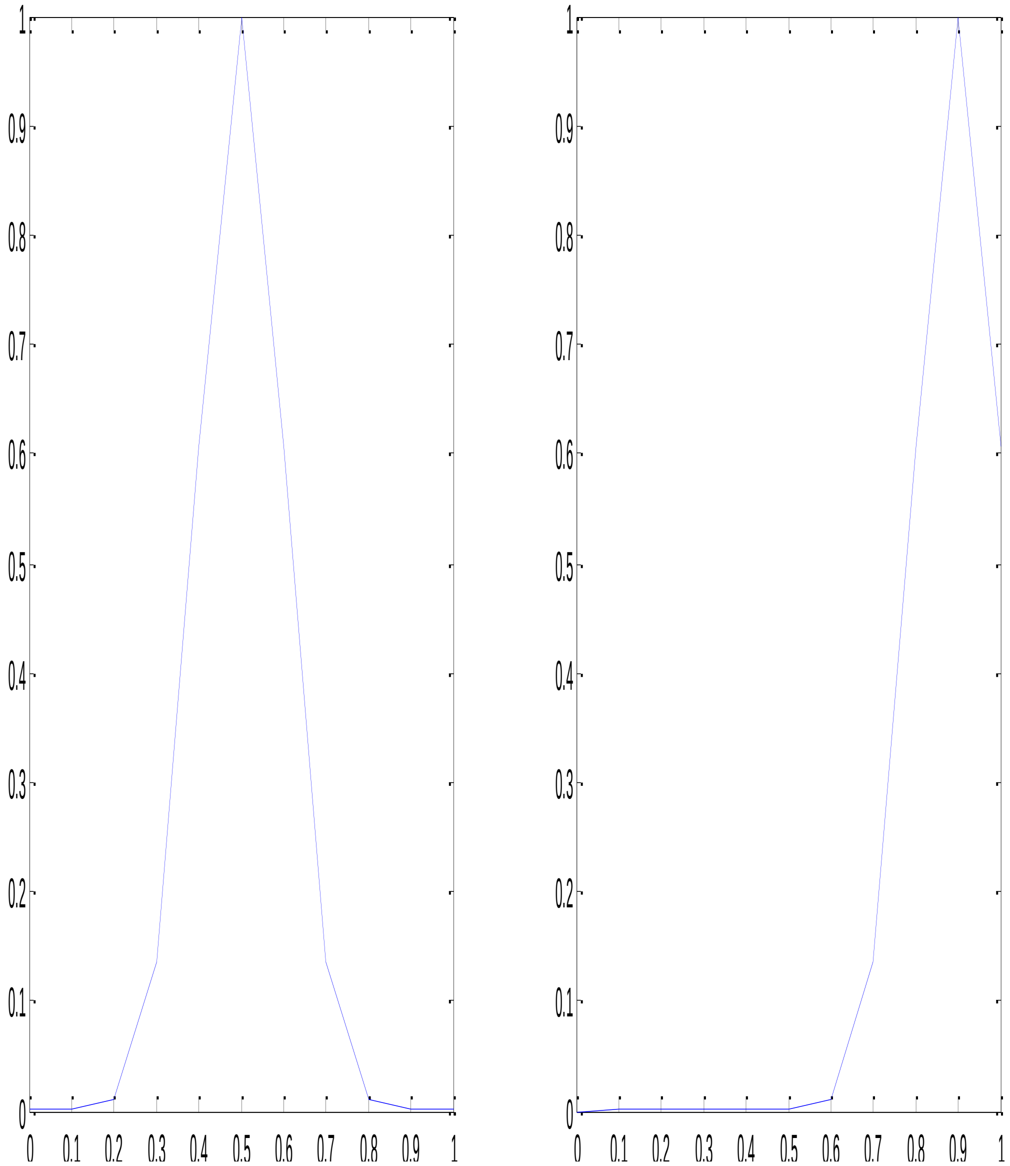

Figure 5: Second state's (Z_{42}) representation in a Z-number form

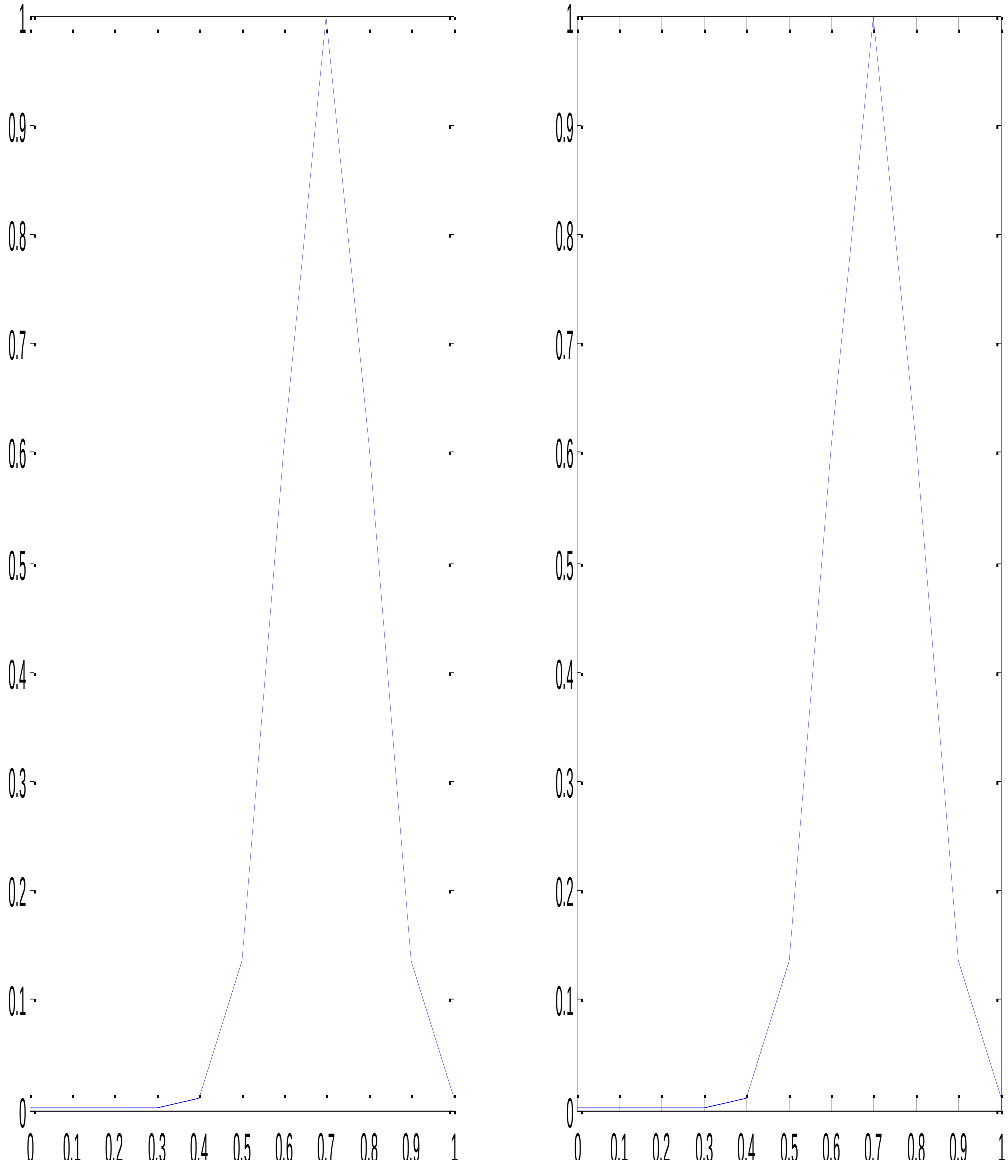

Figure 6: First alternative's representation in 1st state (Z_{11}) in a Z-number form

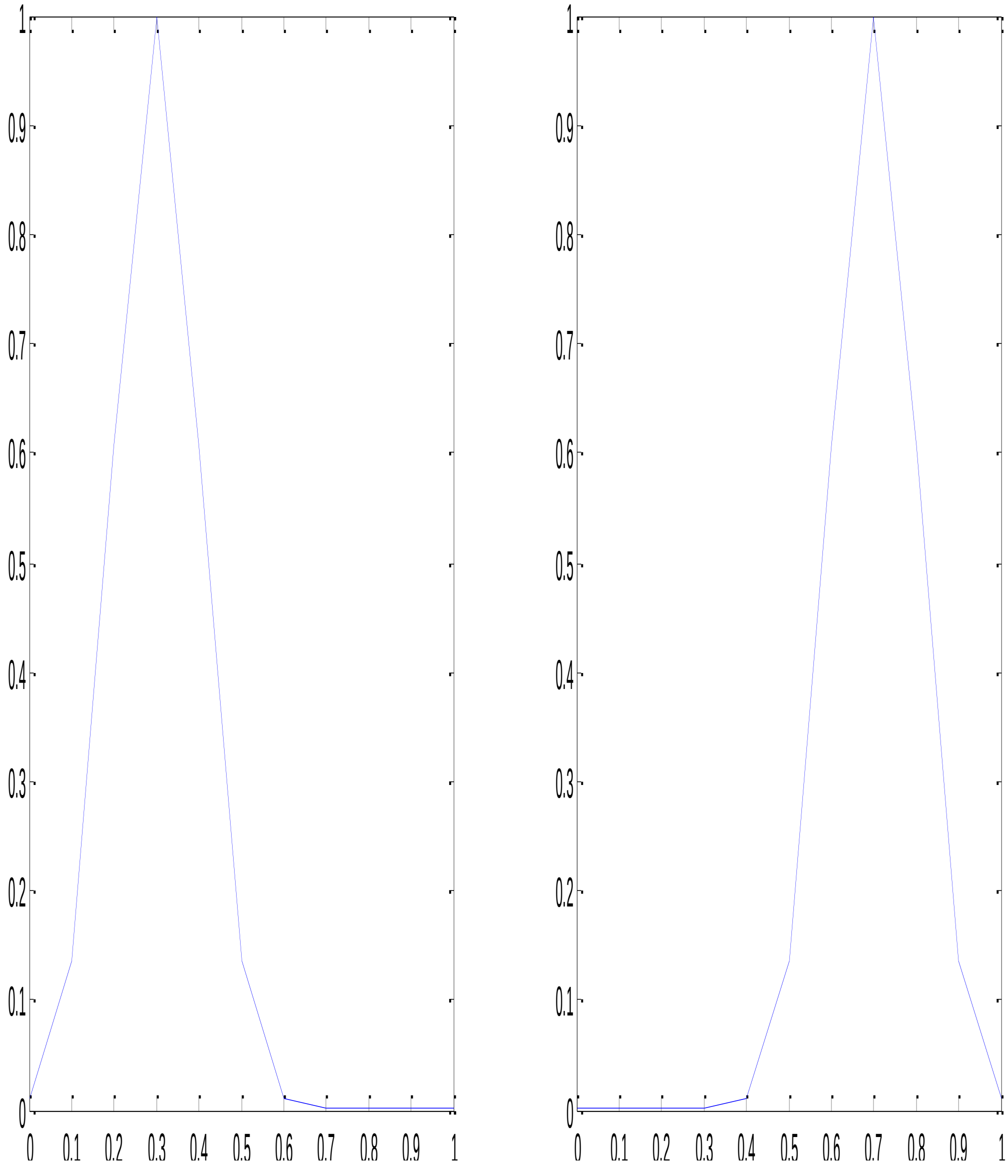

Figure 7: First alternative's representation in 2nd state (Z_{12}) in a Z-number form

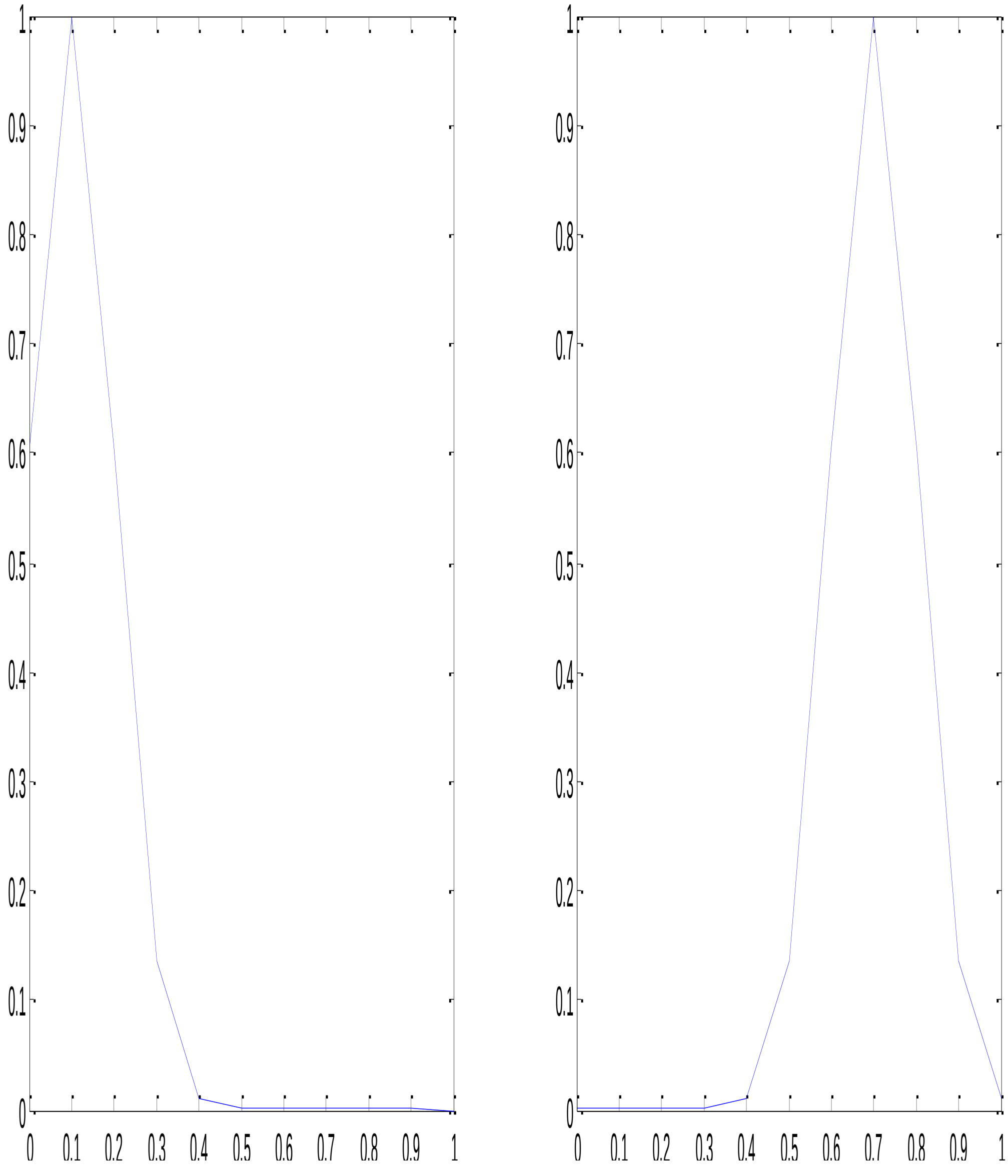

Figure 8: First alternative's representation in 3rd state (Z_{13}) in a Z-number form

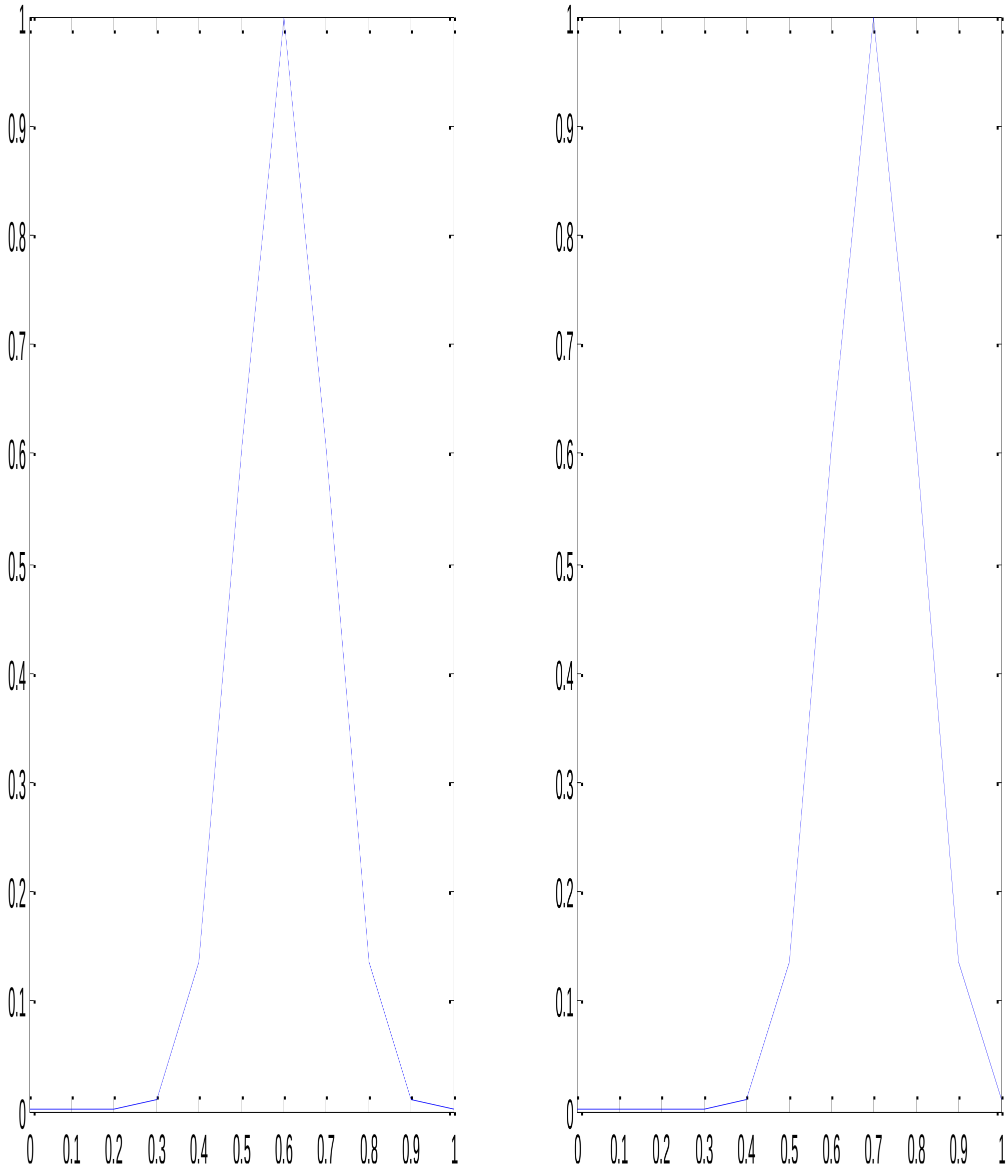

Figure 9: Second alternative's representation in 1st state (Z_{21}) in a Z-number form

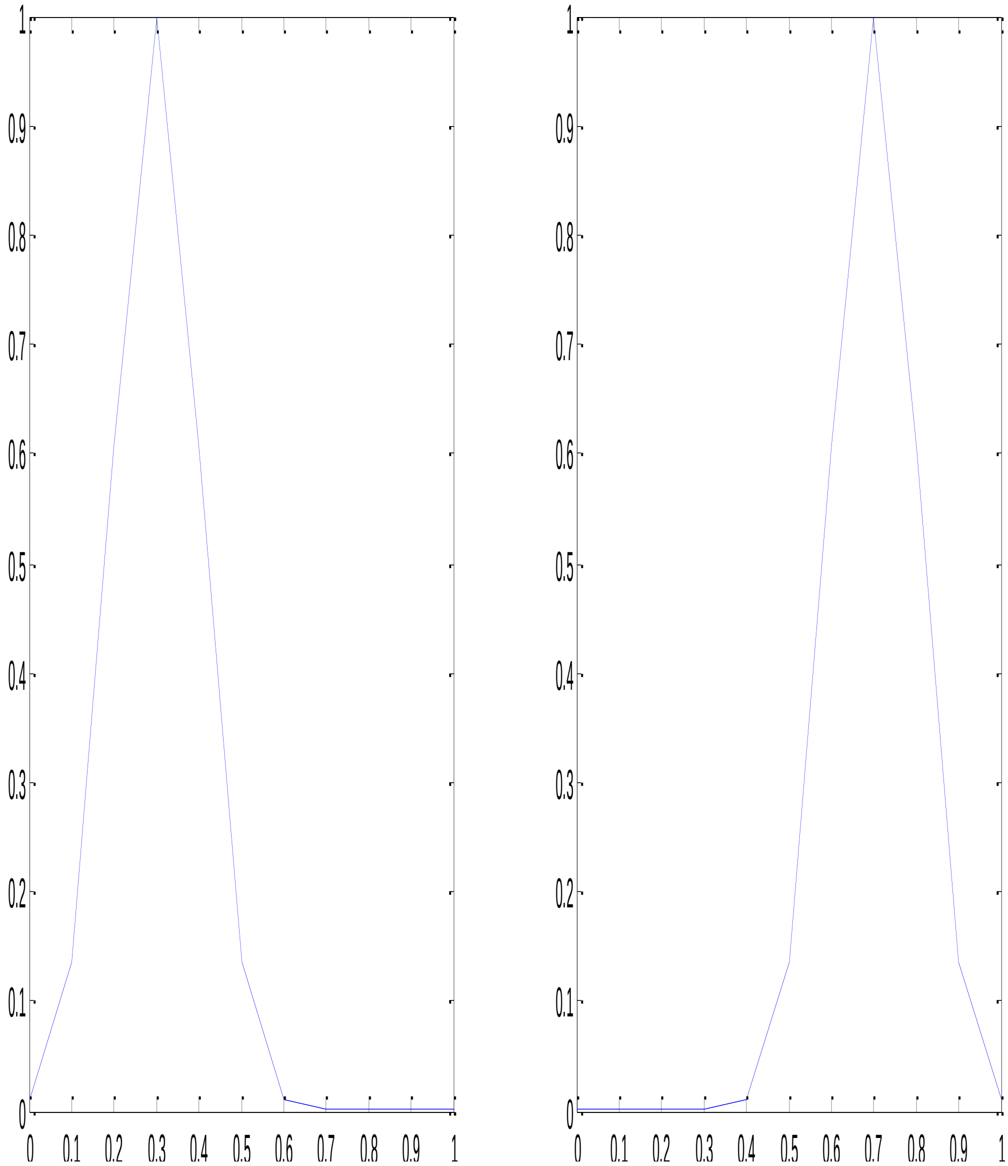

Figure 10: Second alternative's representation in 2nd state (Z_{22}) in a Z-number form

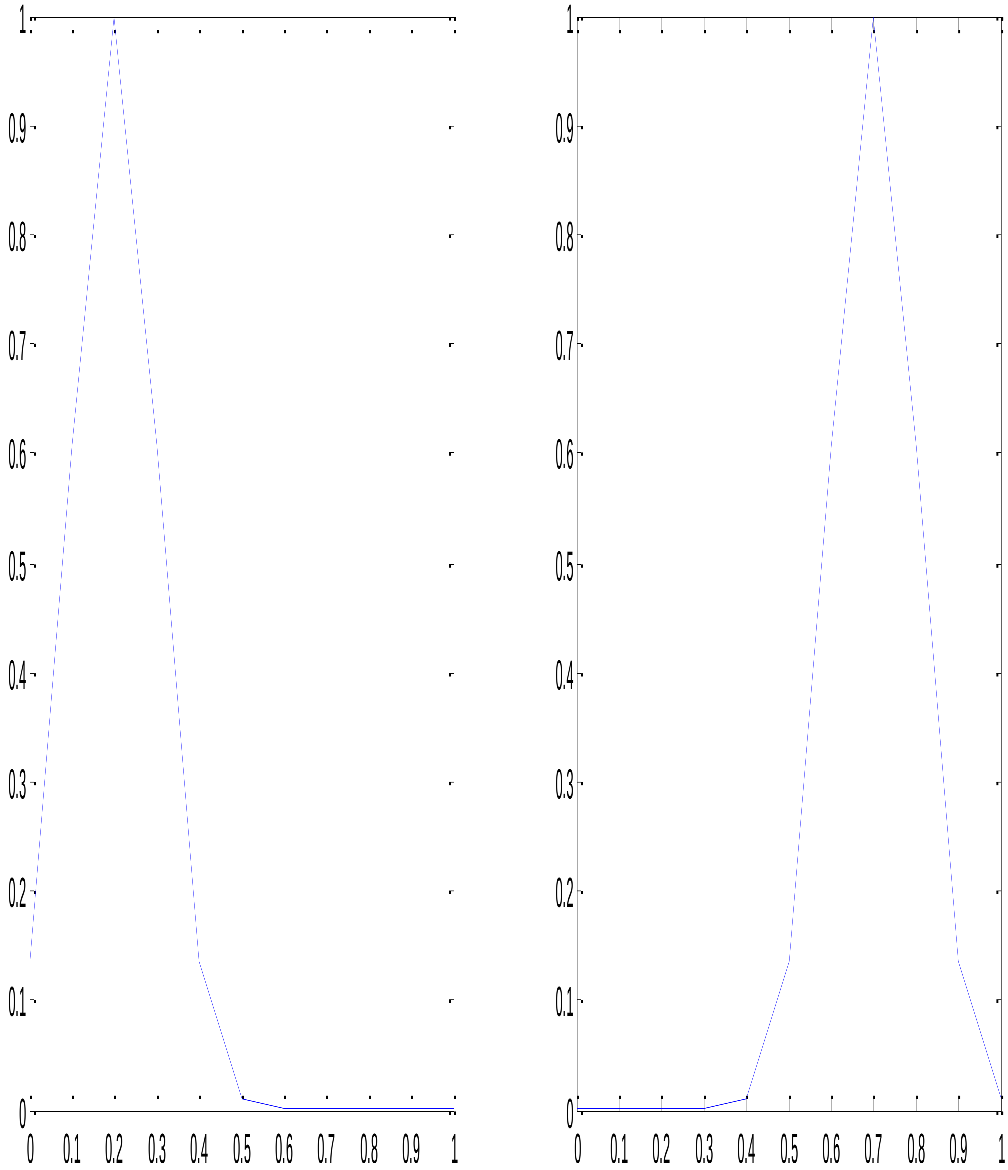

Figure 11: Second alternative's representation in 3rd state (Z_{23}) in a Z-number form

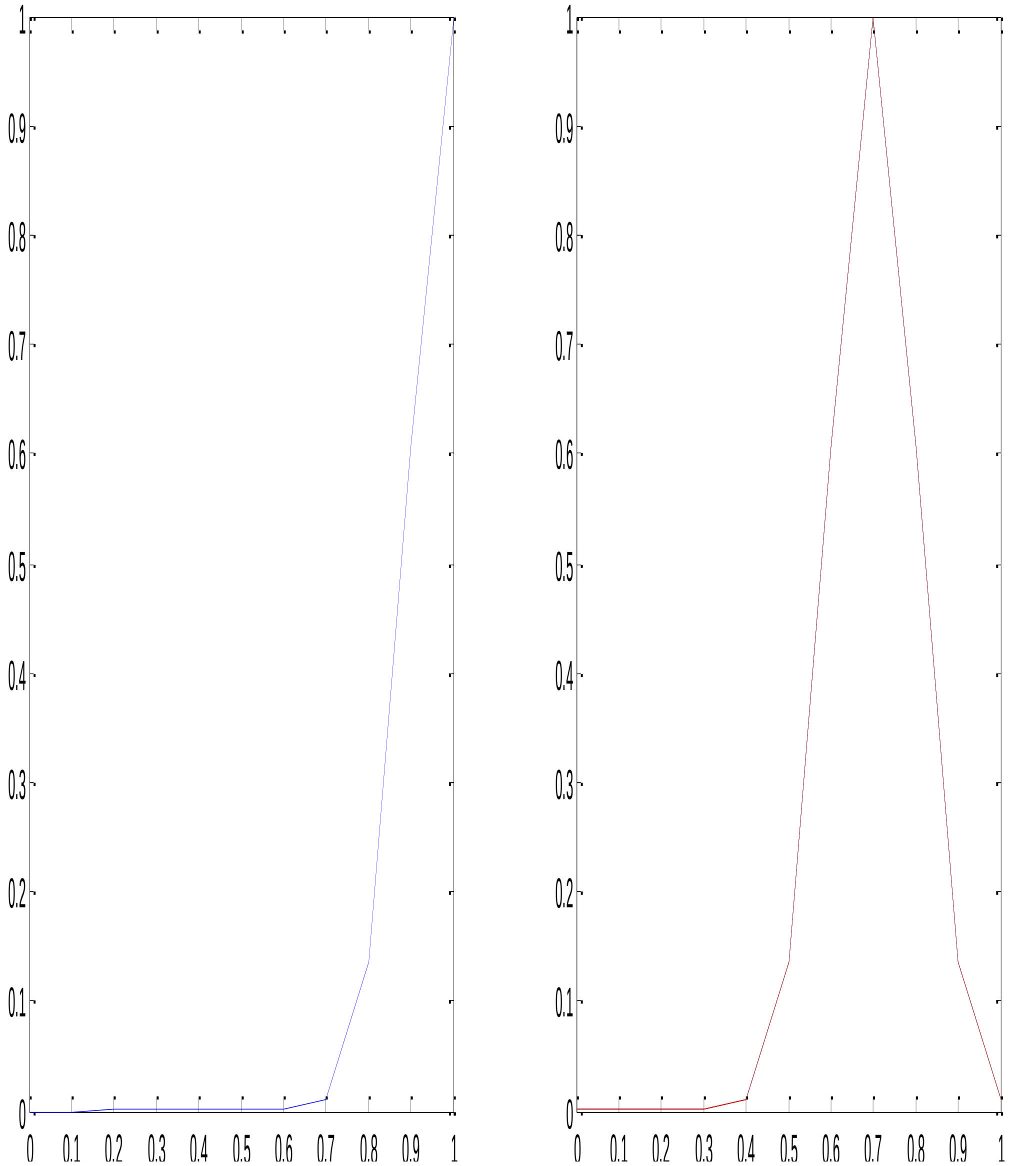

Figure 12: Third alternative's representation in 1st state (Z_{31}) in a Z-number form

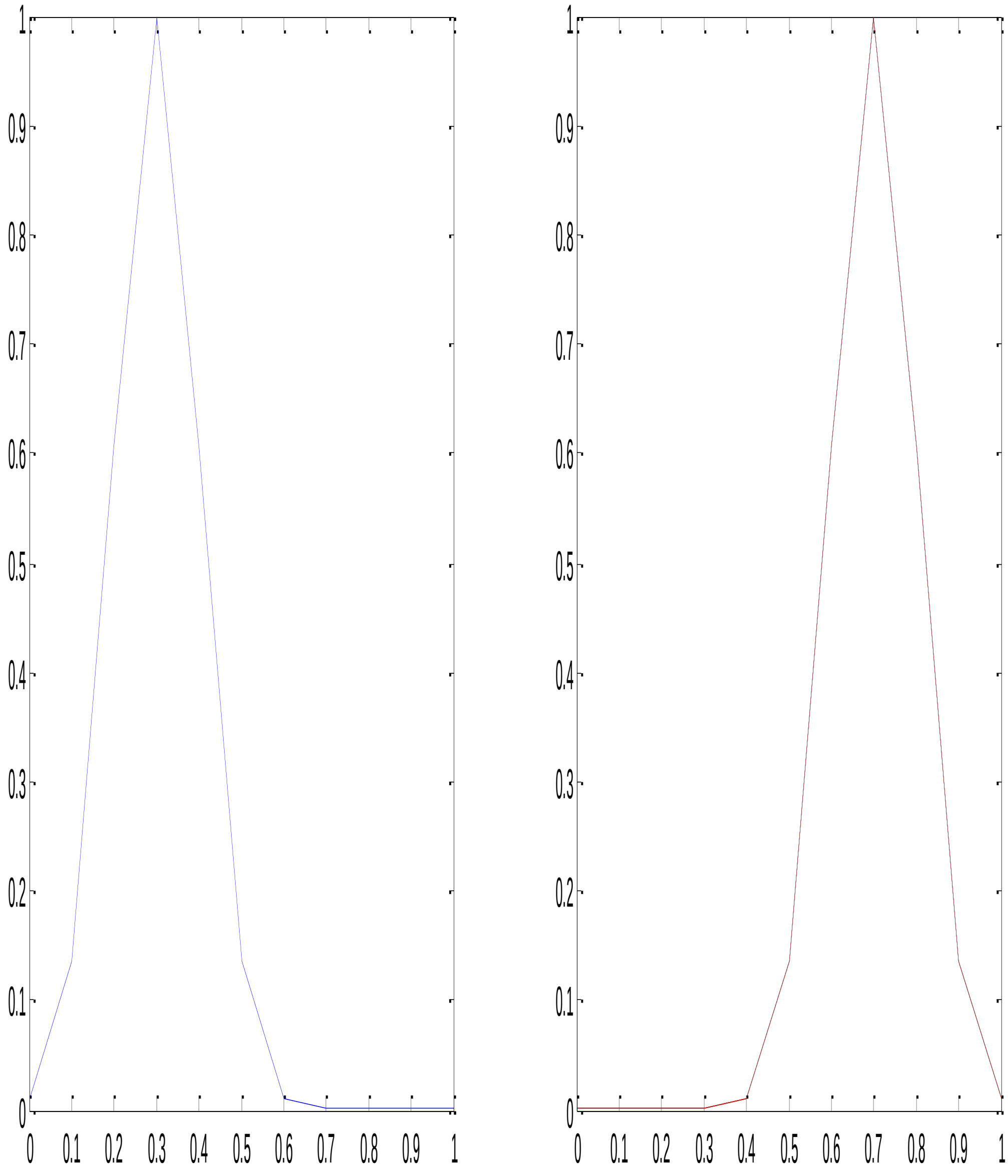

Figure 13: Third alternative's representation in 2nd state (Z_{32}) in a Z-number form

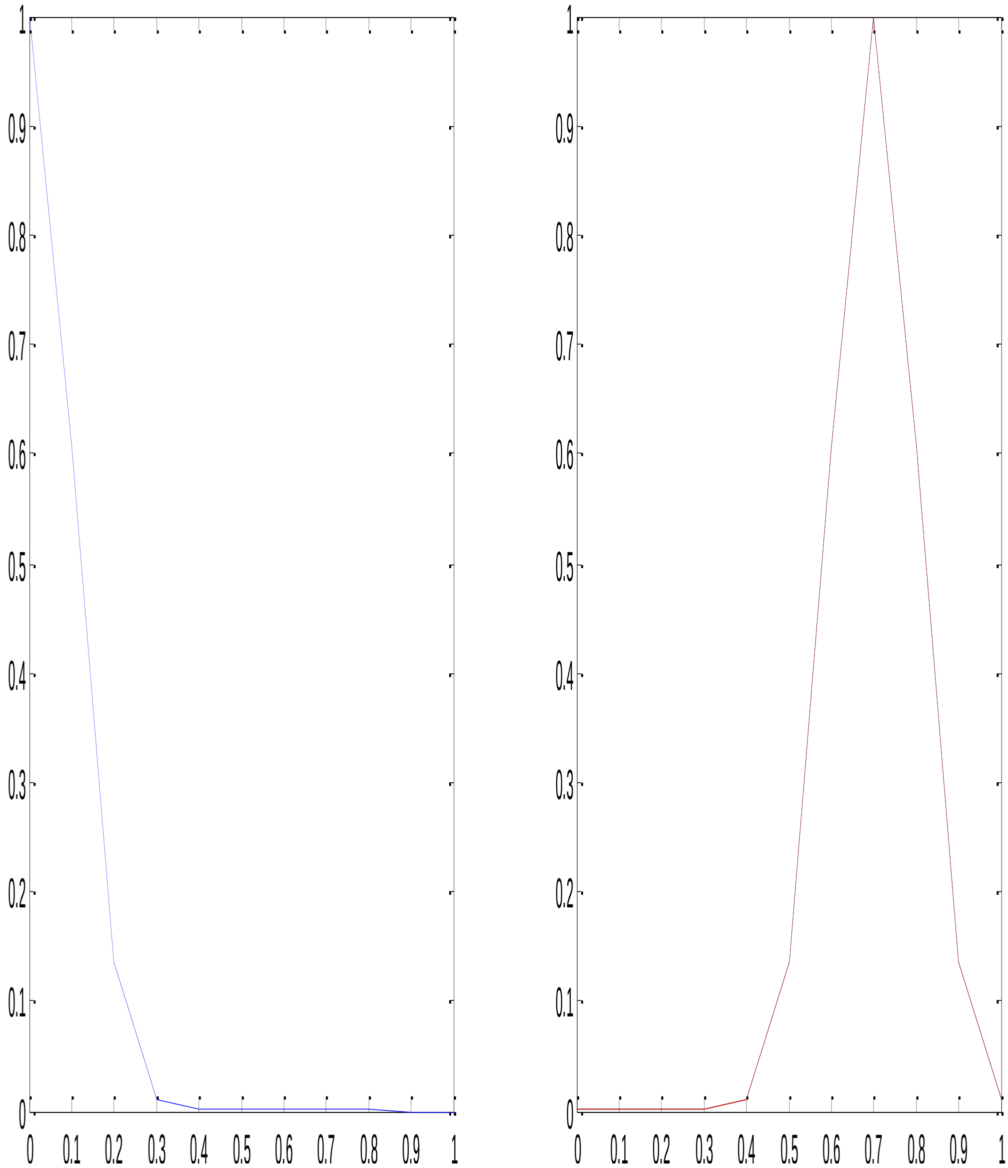

Figure 14: Third alternative's representation in 3rd state (Z_{33}) in a Z-number form

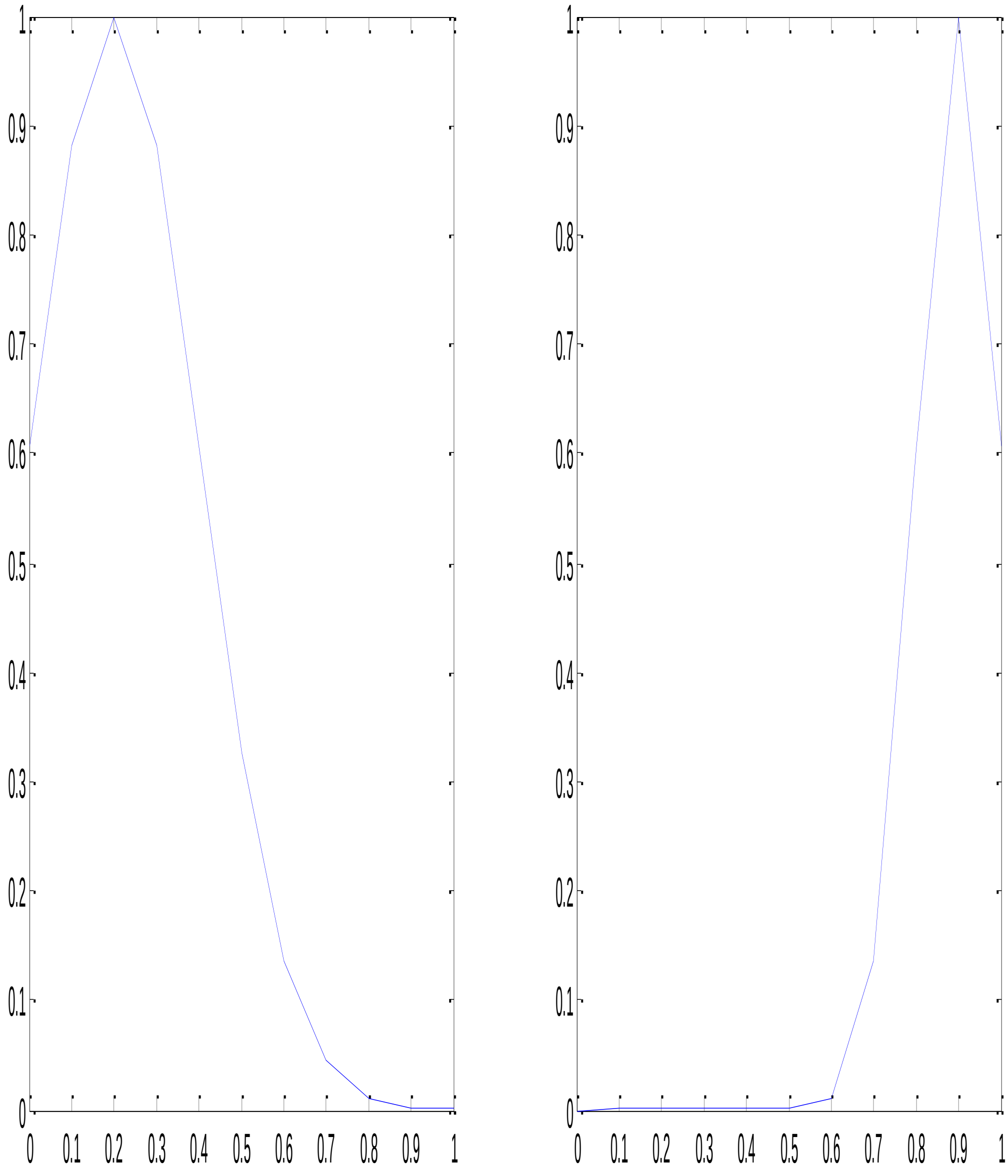

Figure 15: First state's (Z_{43}) representation in a Z-number form

Let's try to solve the problem. Initially, the probability for unknown Z-numbers must be evaluated as $Z_{P(S_3)} = Z_{43} = (A_{43}, B_{43})$, $Z_{P(S_1)} = Z_{41}$ and $Z_{P(S_2)} = Z_{42}$. $Z_{P(S_3)}$ is totally defined by $Z_{P(S_1)}$ and $Z_{P(S_2)}$, its reliability B_{43} will be the same as reliabilities B_{41} and B_{42}. Therefore, to complete the determination of $Z_{43} = (A_{43}, B_{43})$, A_{43} must be calculated according to A_{41} and A_{42}. In order to calculate A_{43}, the approach suggested in [23], is used. The calculated $Z_{43} = (A_{43}, B_{43})$ is depicted in Figure 15.

According to the previous data based on Z-numbers, the expected utility for each of the alternatives f_1, f_2, f_3 is computed in the following form:

$$Z_{U(f_1)} = Z_{11} \times Z_{41} + Z_{12} \times Z_{42} + Z_{13} \times Z_{43}$$

$$Z_{U(f_2)} = Z_{21} \times Z_{41} + Z_{22} \times Z_{42} + Z_{23} \times Z_{43}$$

$$Z_{U(f_3)} = Z_{31} \times Z_{41} + Z_{32} \times Z_{42} + Z_{33} \times Z_{43}$$

The expected utilities for all the alternatives are computed, and the results are represented in Figures 16, 17, and 18:

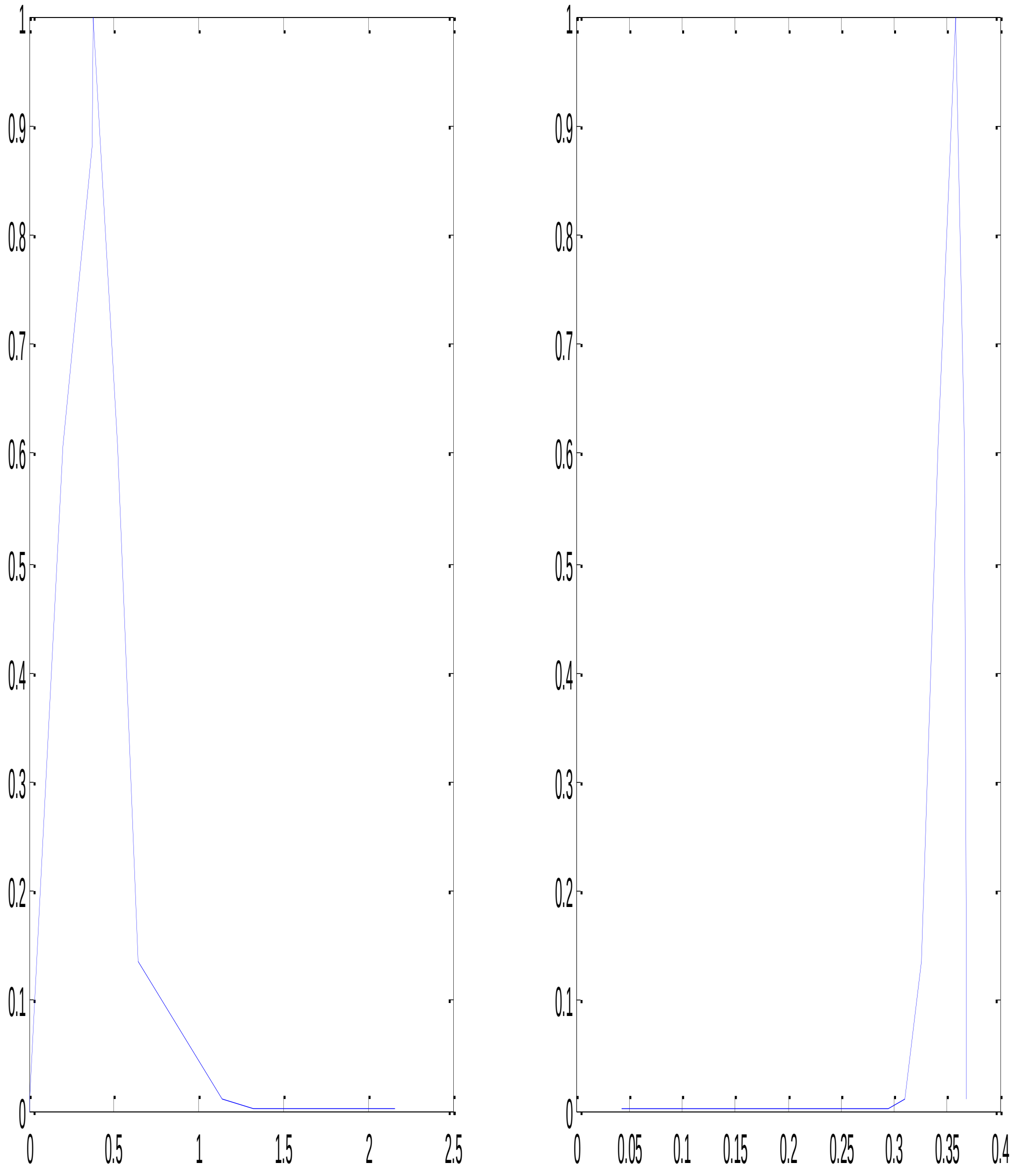

Figure 16: First alternative $Z_{U(f_1)}$ and the results of its expected utility

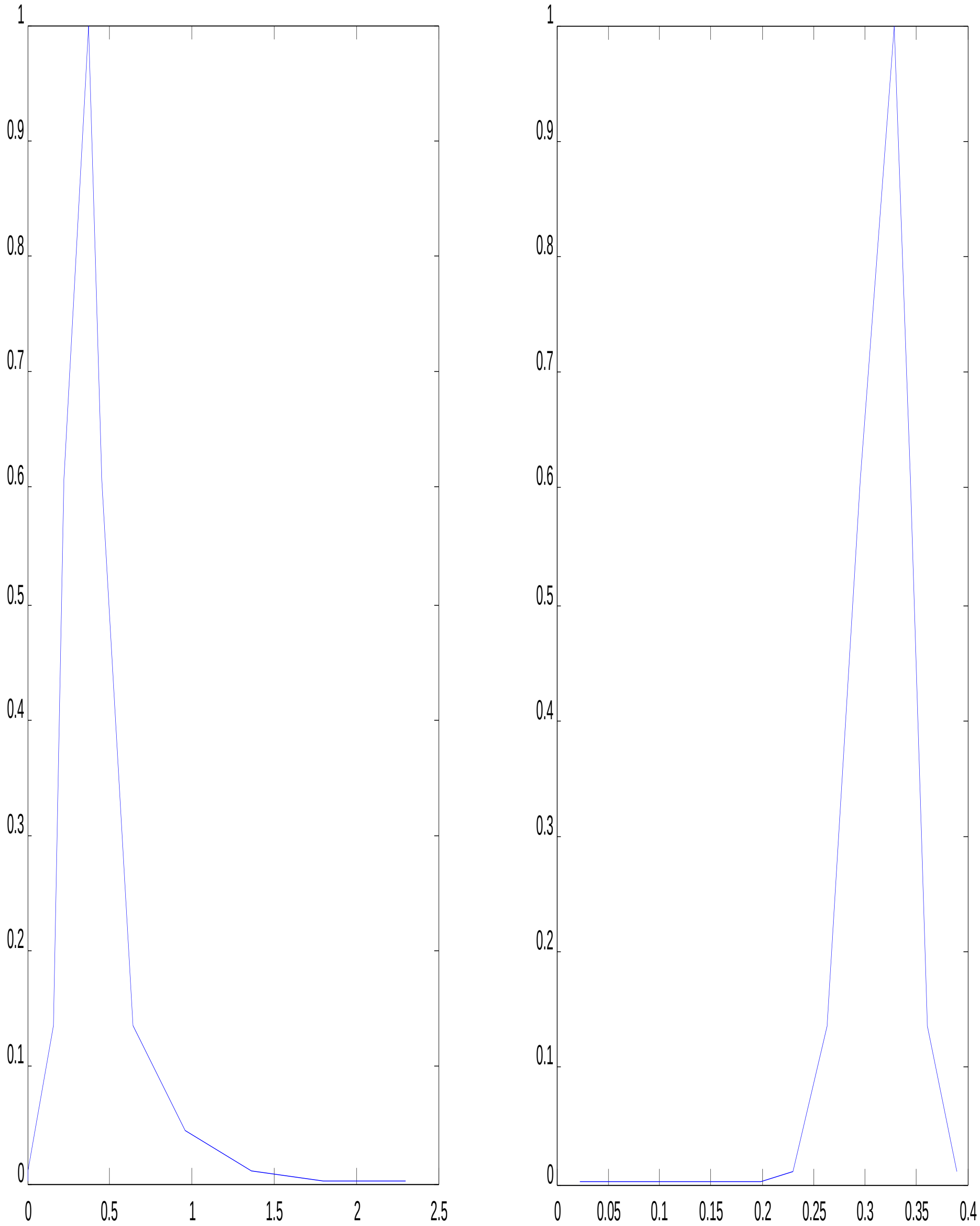

Figure 17: Second alternative $Z_{U(f_2)}$ and the results of its expected utility

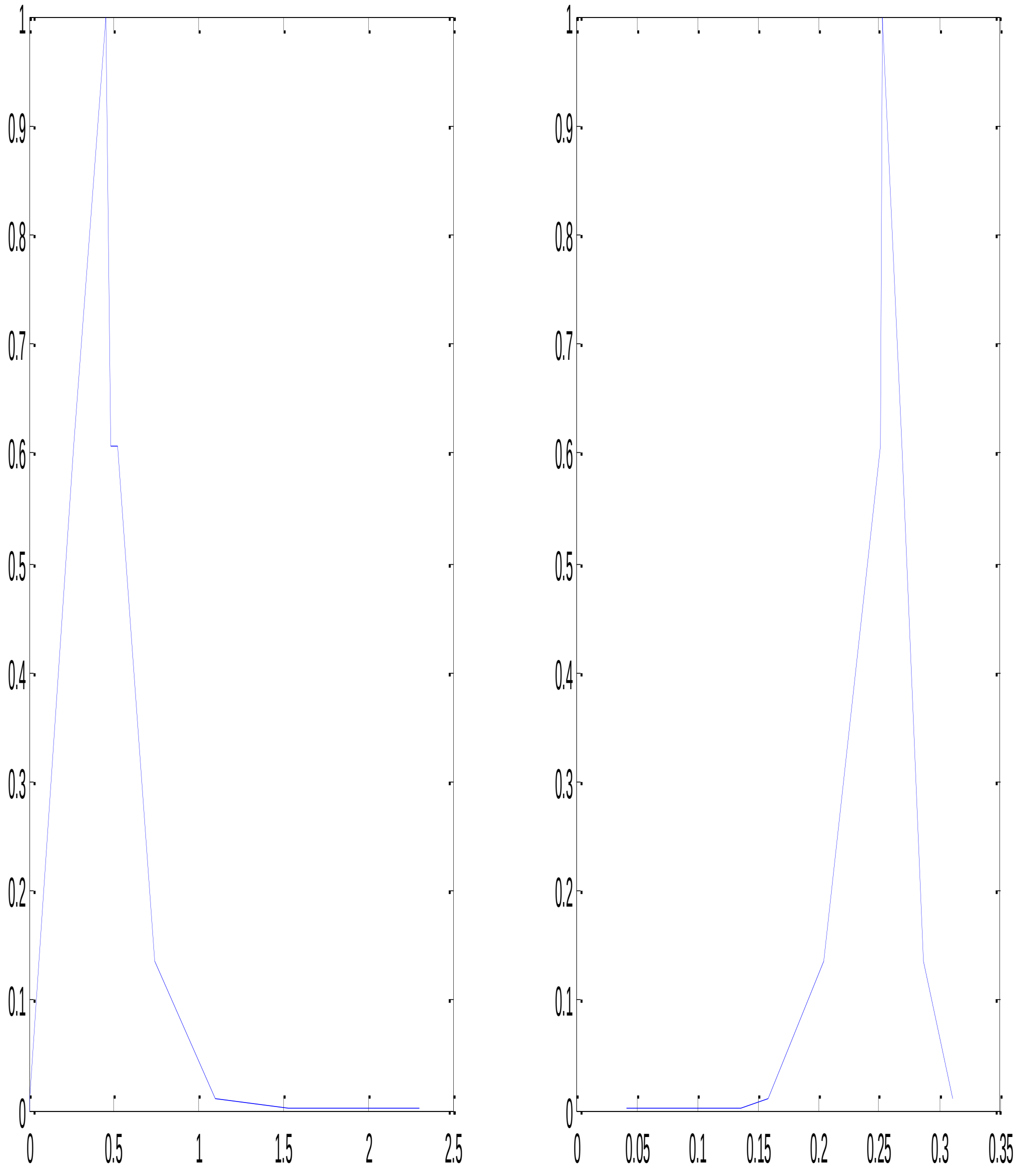

Figure 18: Third alternative $Z_{U(f_3)}$ and the results of its expected utility

In the last stage, the best alternative is determined by performing of comparison of the computed Z-number valued utilities. For this reason, it is important to use the method which is given in Section 4.2. This principle firstly provides the optimality degrees of the alternatives:

$$do(f_1)=1, do(f_2)=0,\ do(f_3)=0.92$$

It is deduced that the second alternative does not achieve Pareto optimality. The final comparison is carried out between first and third alternatives. Let us take the value of the pessimizm degree to compare these alternatives equal to $\beta = 0.3$.

We finally have:

$$r(Z_{U(f_1)}, Z_{U(f_3)}) = 0.976 > \frac{1}{2}(do(Z_{U(f_1)}) + do(Z_{U(f_3)})) = 0.96$$

From the results it can be concluded that the best action is f_1.

Chapter 5

CONCLUSION

Obviously, the issue of reliability of information is essential, and of key importance in planning, decision-making, management of information and formulation of algorithms. Recent studies have hugely targeted the multi-criteria decision making, and fuzzy set has been widely applied in the process of decision making since uncertainty and complexity is unescapably phenomenon in the real world. But the issue of the reliability of information is not taken into account seriously as well as efficiently. Decision-relevant information which is related to real-world is often partially reliable. The reasons for that is partial reliability of the source of information, psychological biases, misperceptions, incompetence etc.

Information-based decisions should be reliable. Zadeh proposed a new notion called Z-number in order to have accessibility to describe the knowledge which is uncertain. Essentially, a Z-number is related to the issue of reliability of information. There are two components in a Z-number, $Z= (\tilde{A},\tilde{R})$, where $\tilde{A}$ is a restriction on the values which a real-valued variable X can take. $\tilde{R}$ is a measure of reliability of the first component. It is to mention that a Z-number concept can be successfully applied in many fields of science, and mainly in such areas as economics, risk assessment, prediction decision analysis, rule-based characterization of imprecise functions and relations.

The concept of a Z-number only paves the way towards an unexplored field and should be viewed as a first step toward development of methods of computation with Z-numbers. The computation with Z-numbers may be seen as a generalization of computation with numbers, intervals, fuzzy numbers and random numbers.

Now the theories about Z-numbers are not mature, the conversion of Z-number to classical fuzzy number is rather significant for application.

In this thesis, a method of converting Z-number to classical fuzzy number is considered. The procedure of the proposed approach is illustrated by a practical example.

This dissertation aims at offering an analysis of the Z-numbers for decision making process in economic problem. For this purpose the data is achieved from SciTool Company, and the experimental calculations are carried out. The expected utilities of two alternatives are measured using the defuziffication process based on the center of gravity method to get an optimal decision. Z-numbers are categorized according to the degree of reliability and the experimental findings have proved the effectiveness of the Z-numbers in the decision making problem.

In addition, an approach is developed for decision making under Z-information described in NL. The suggested method utilizes the paradigm Expected Utility based on a direct computation with Z-numbers. The advantage of the approach is its ability to account for imprecision and partial reliability of information and relative simplicity of computations.

The approach is applied to solve a benchmark problem in the field of economics. The validity of the approach is confirmed by the obtained results.

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