Capacity Enhancement of a MIMO Cognitive Radio System Using Water-Filling Strategy

Umar Ali

Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Science
in
Electrical and Electronics Engineering

Eastern Mediterranean University
February 2016
Famagusta, North Cyprus

Approval of the Institute of Graduate Studies and Research

Prof. Dr. Cem Tanova
Acting Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Prof. Dr. Hasan Demirel
Chair, Department of Electrical and Electronic Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Electrical and Electronic Engineering.

Assoc. Prof. Dr. Erhan İnce
Co-Supervisor

Prof. Dr. Şener Uysal
Supervisor

Examining Committee

1. Prof. Dr. Hasan Demirel
2. Prof. Dr. Hüseyin Özkaramanli
3. Prof. Dr. Şener Uysal
4. Assoc. Prof. Dr. Erhan İnce
5. Asst. Prof. Dr. Rasime Uyguroğlu

ABSTRACT

The mobile communication systems will be differentiated by higher throughput, flexibility, and large combination of services in the future. Thus, the structure and configuration of wireless networks have begun to change rapidly. In the present situation, the aim is not only to provide data communication to mobile users, but also provide significant high capacity and data rate within the same bandwidths. There are optional schemes to meet these requirements like space time coding (STC), advance antenna system (AAS), multiple inputs and multiple outputs (MIMO) systems. The MIMO systems can be used for interference reduction, spatial multiplexing and diversity. There are many algorithms for MIMO system to increase the data rate and capacity of the channels.

In this thesis, we evaluate the capacity of the MIMO system using water-filling solution as well as compare it with other systems like SISO, MISO and SIMO. The strategic choice of power allocation is based on Lagrange multiplier which provides the best power allocation under the fix value of water level to the sub channels. We noted that the proper power allocation to the sub channels can cause significant improvement in capacity for high data transmission. Our simulation results, when we applied water filling strategy over (2×2) MIMO system which shows the improvement by 1.3017 bps/Hz at SNR 0 dB as well as 1.74323 bps/Hz at 15 dB with 2 kW power budget. When we apply the water filling strategy over (4×4) MIMO system with the same 2 kW power budget, it provides a significant performance in capacity by 3.8321 bps/Hz enhancement at SNR 0 dB as well as the capacity improve 5.837 bps/Hz at SNR of 15 dB.

Keywords: Multiple Input Multi Output (MIMO), Water Filling, Channel Capacity, Spectral efficiency, Signal to noise ratio (SNR).

ÖZ

Mobil haberleşme sistemleri gelecekte daha yüksek çıktı, esneklik ve geniş hizmet birleşimleri ile farklılaştırılacaktır. Dolaysıyla kablosuz ağ yapıları ve yapılandırmaları hızlı bir şekilde değişmeye başlamış bulunmaktadır. Mevcut durumdaki amaç mobil kullanıcılara data haberleşme olanağını sağlamanın yanı sıra aynı bant genişliğinde önem arz eden yüksek kapasite ve veri hızı sağlamaktır. Alan Zaman Kodlama (AZK), Gelişmiş Anten Sistemi (GAS), Çoklu Giriş ve Çoklu Çıkış Sistemler (ÇGÇÇ) olmak üzere bu gereksinimleri karşılamak için alternatif planlar bulunmaktadır. ÇGÇÇ sistemleri parazitleri azaltmak, uzaysal çoğullama ve çeşitlilik için kullanılabilmektedir. Veri hızı ve kanal kapasitesini yükseltmek amacıyla ÇGÇÇ sistemleri için birçok algoritma bulunmaktadır.

Bu tez çalışmasında su-doldurma çözümü kullanılarak ÇGÇÇ sisteminin kapasitesi değerlendirilmiş olup aynı zamanda Tekli Giriş Tekli Çıkış (TGTÇ), Çoklu Giriş Tekli Çıkış (ÇGTÇ) ve Tekli Giriş Çoklu Çıkış (TGÇÇ) gibi diğer sistemler ile karşılaştırılmıştır. Stratejik güç dağıtım seçimi sabit su düzey değeri altında alt kanallara en iyi güç dağıtımını gerçekleştiren Lagrange çarpanına dayanmaktadır. Alt kanallara uygun güç dağıtımının yüksek data iletimi açısından önemli ölçüde iyileştirmeye neden olabileceği saptanmıştır. 2 kW güç bütçesi ile (2x2) ÇGÇÇ sistemi üzerinde su doldurma işlemi gerçekleştirilerek elde edilen benzetim sonuçları, SNR 0dB’de 1.3017 bps/Hz ve aynı zamanda 15 dB’de 1.74323 bps/Hz düzeyinde iyileşme göstermektedir. Su doldurma stratejisi 2kW güç bütçesine sahip (4x4) ÇGÇÇ sistemi üzerinde uygulandığında kapasite açısından SNR 0dB’de

3.8321 oranında önemli ölçüde artış sağlınmış olup aynı zamanda SNR 15 dB'de 5.837 bps/Hz oranında kapasite gelişimi elde edilmiştir.

Anahtar Kelimeler: Çoklu Giriş Çoklu Çıkış (ÇGÇÇ), Su Doldurma, Kanal kapasitesi, Spektral Verim, Sinyal Parazit Oranı (SPO)

I am dedicating this to my lovely mother, father and to all my family & friends.

ACKNOWLEDGMENT

I am very thankful to my supervisors Prof. Dr. Şener Uysal and Assoc. Prof. Dr. Erhan İnce for their guidance, support and motivation throughout the duration of my master study. This thesis would not have happened without their guidance and endless patience.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AMC	Adaptive Modulation and Coding
AWGN	Additive White Gaussian Noise
bps	Bits Per Second
BER	Bit Error Rate
BPSK	Binary phase-shift keying
BS	Base Station
FEC	Forward Error Correction
ITU	International Telecommunication Union
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
OFDM	Orthogonal Frequency Division Multiplexing
PB	Power Budget
QAM	Quadrature Amplitude Modulation
QoS	Quality of Service
QPSK	Quadrature Phase Shift Keying
RB	Radio Broadcasting
R_X	Receiver
SISO	Single Input Single Output
SM	Spatial Multiplexing
SNR	Signal to Noise Ratio
STBC	Space–Time Block Code
STC	Space–Time Code

Tx	Transmitter
WF	Water-Filling
ZF	Zero Forcing

Chapter 1

INTRODUCTION

In wireless system radio frequency spectrum increasing for higher data rate has led to be use of multiple input multiple output (MIMO), which give us higher throughput in term of transmitter power or bandwidth without any overhead as compared to single input single output (SISO) in wireless communication system.

By using spatial diversity, the capacity of the wireless communication system can be increased [1]. In radio link, the multiple antenna can be employed to overcome the problems. The capacity of the MIMO system is much better than SIMO/MISO system [2] [3].

The diversity gain of SISO, transmit or receive proven by multiplexing of MIMO and the number of parallel channels show the multiple input single output (MISO) system [4]. It can also give us high capacity but with high expense of hardware and computational complexity.

The capacity of the MIMO system can be further increased by performing optimal power allocation over the transmit antennas which is known as water-filling (WF) solution. In this scenario, the channel state information (CSI) must be known for the parallel Gaussian channels to find out the mean capacity of the channel.

1.1 Thesis Aims

Our main aim is to study and analyze the channel capacity by using water filling algorithm in a MIMO system and try to enhance the capacity. With today's technology, our main focus is on data rate which provide us high quality of voice and video communication. However, QoS depends on number of factors but we will try to analyze channel capacity and improve the capacity by using optimal power allocation over the transmit antennas.

1.2 Thesis Overview

The thesis will be organized by following sections:

The first chapter describes the thesis introduction, aims, and outline of the research. The 2nd chapter describes the overview of the channel capacity for SISO, SIMO, MISO and MIMO systems. The 3rd chapter explores the water-filling algorithm for power allocation. The fourth chapter is related to performance analysis which is based on Gaussian parallel channels. In fifth chapter, Conclusion and future work have been presented.

Chapter 2

CHANNEL CAPACITY FOR SISO, SIMO, MISO AND MIMO SYSTEMS

In wireless, the use of diversity is known as to improvement in results. By repeating the transmitted symbols in a frequency, in time and implement the multiple antennas at receiver provide the good result to achieve the diversity.

Generally, the diversity gain is increased the SNR by coherent combination of received signals at receiver side and even in the absence of fading, It reduces the average noise power. The state is more complicated when a larger system require a great deal in potential advantages at price and flexibility in design. The extensive value of spatial multiplexing can achieve by MIMO techniques which have significant impact on introduction of MIMO technology in wireless systems. The channel capacity for different system like SISO, SIMO, MISO and MIMO have described in this chapter as well as the differences between them.

Consider, there are N number of transmitter and receiver antennas as well as the transmitter and receiver symbols denoted by $s(t)$ and $y(t)$. The 'h' is denoted by fading gain from transmit antenna to receive antenna.

$$y(t) = hs(t) + n(t) \tag{2.1}$$

where $n(t)$ denotes the noise of the channel.

2.1 SISO System

Single input single output (SISO) is the simplest form of the communication system in which single transmitter and single receiver antenna become uses at source and destination side respectively [5]. The SISO systems are mostly using for simplex channels like TV, radio broadcasting, bluetooth etc. The capacity for SISO system derived from output SNR is given as

$$SNR_{output} = \frac{|h|^2 P}{\sigma^2} \tag{2.2}$$

$$C_{SISO} = log_2(1 + \frac{|h|^2 P}{\sigma^2}) \tag{2.3}$$

here σ^2 is the Noise variance of the system and P is the signal power as well as h denotes the channel gain. The capacity of the channel depend on signal to noise ratio and bandwidth. Figure 2.1 shows the SISO system.

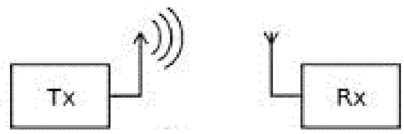

Figure 2.1: SISO system

where *Tx* and *Rx* shows the transmitter and receiver in the SISO system. The capacity of the SISO system is very low as compared to the (2×2) MIMO system.

2.2 MISO System

Multiple input single output (MISO) is that system which consist of multiple transmitter at source side and single receiver at destination side as shown in Figure 2.2 [6].

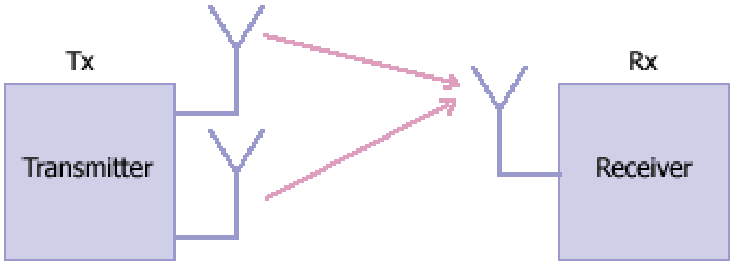

Figure 2.2: MISO system

The mean capacity of the MISO system is higher than SISO system. When we use multiple receive at destination side then the effect of packet loss, delay or multipath wave propagation etc. can be reduce. So MISO system is not much efficient than SISO system. The capacity for SIMO system derived from output of the SNR expressed as

$$SNR_{output} = \sum_{i=1}^{M_t} \frac{|h_i|^2 P}{\sigma^2}) \tag{2.4}$$

$$C_{MISO} = log_2(1 + \sum_{i=1}^{M_t} \frac{|h_i|^2 P}{\sigma^2}) \tag{2.5}$$

where M_t denotes the number of transmitter antennas.

2.3 SIMO System

Single input multiple output (SIMO) is that system which consist of single transmitter at source side and multiple transmitter at receiver side as shown in Figure 2.3.

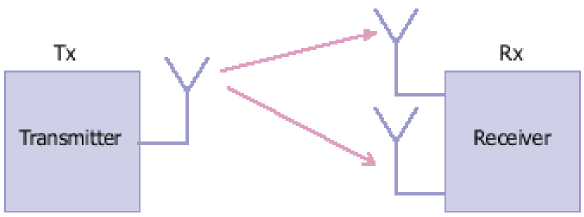

Figure 2.3: SIMO system

The mean capacity of the SIMO system is higher than SISO or MISO system. The diversity scheme can be deploy at receiver side in order to reduce the bit error rate of the channel. The capacity of the SIMO system derived from output of the SNR can be expressed as

$$SNR_{output} = \sum_{i=1}^{M_r} \frac{|h_i|^2 P}{\sigma^2}) \tag{2.6}$$

$$C_{SIMO} = log_2(1 + \sum_{i=1}^{M_r} \frac{|h_i|^2 P}{\sigma^2}) \tag{2.7}$$

where M_r denotes the number of receiving antennas.

2.4 MIMO System

Multiple input multiple output (MIMO) is that system which consist of multiple transmitter at source side and multiple receiver at destination side [7]. The main advantage of the MIMO technology is that, there are different paths available which can be cause of delay and packet loss reduction. The mean capacity of the MIMO system is higher than SISO, SIMO and MISO systems. The Figure 2.4 shows the MIMO system.

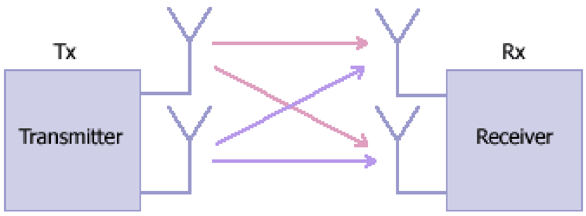

Figure 2.4: MIMO system

The MIMO system is becoming Popular in every wireless technology for high data rate and parallel communication in wireless system. The MIMO technique is the hot topic now a days for researcher and they are trying to add in other network to increase the capacity of the system. The capacity of the MIMO system derived from output of the SNR can be expressed as

$$SNR_{output} = \frac{\lambda P}{\sigma^2} \tag{2.8}$$

$$\lambda = \max_{u,v} |u^H H v|^2 \tag{2.9}$$

where λ are singular values of H matrix.

$$C_{MIMO} = log_2[\det er\,(I + \frac{\rho}{N_t} HH^H)] \tag{2.10}$$

where ρ is the signal to noise ratio and N_t is number of transmitter antennas. The $(\,.\,)^H$ denotes the Hermitian transpose of channel gain. The mean capacity comparison between SISO, MISO, SIMO and MIMO systems can be shown in Table 2.1.

Table 2.1: Mean capacity comparison between different systems

Mean capacity in bps/Hz				
SNR(dB)	(1×1) SISO	(2×1) MISO	(1×2) SIMO	(2×2) MIMO
-10	0.131260324	0.132062691	0.251496606	0.25969396
-9	0.163206764	0.169704044	0.320749872	0.330419874
-8	0.197391659	0.202424287	0.369034589	0.407331425
-7	0.238291015	0.249477617	0.458091453	0.497064027
-6	0.290923146	0.306567382	0.571786514	0.585910476
-5	0.341307611	0.374446045	0.647435868	0.747756664
-4	0.419490528	0.452756678	0.768823935	0.863053881
-3	0.543014746	0.553619409	0.909953028	1.032454689
-2	0.607058638	0.666241527	1.101295181	1.233577108
-1	0.74259595	0.791790594	1.262257579	1.430551043
0	0.888956735	0.914386434	1.415477182	1.653210816
1	0.999826732	1.073826789	1.640728748	1.961853764
2	1.139986886	1.257971035	1.891189575	2.245499751
3	1.308993705	1.445496159	2.077677183	2.592138898
4	1.520878228	1.684522337	2.350972263	2.921384434
5	1.704165708	1.870520742	2.562106166	3.322518206
6	1.906978406	2.126963115	2.884287978	3.7173753
7	2.063688693	2.368200664	3.178143324	4.159439951
8	2.357993563	2.635835728	3.457290226	4.529695701
9	2.700721136	2.880196725	3.768931424	5.077959161
10	2.893317816	3.122855571	4.07865709	5.606611587
11	3.202971472	3.402504307	4.404916721	6.035432417
12	3.486641891	3.792067144	4.64350606	6.585233655
13	3.780908829	4.044871611	4.990003647	7.09333477
14	3.997743697	4.359371687	5.321805453	7.609940085
15	4.308937394	4.680180381	5.632816507	8.291888718
16	4.715988519	4.995712638	5.917444929	8.891179429
17	4.923333597	5.307713677	6.277158738	9.410861753
18	5.267279728	5.65071441	6.568655886	10.00318688
19	5.474010796	5.928944952	6.874569287	10.60775688
20	5.8097992	6.263273225	7.286180301	11.3604139
21	6.310466884	6.633662109	7.609173229	11.96769355
22	6.480255531	6.917880775	7.967603924	12.56213112
23	6.856815607	7.211047552	8.290569285	13.14131288
24	7.129766277	7.628204358	8.586091171	13.85402582
25	7.523570113	7.853223804	9.002079536	14.47838235
26	7.765014369	8.233187864	9.226921886	15.04831553
27	8.099522954	8.598830526	9.581024926	15.7468847
28	8.340228361	8.908664057	9.917263363	16.36196942
29	8.703214089	9.288981913	10.23655489	17.14156364
30	9.20576988	9.613281057	10.57886229	17.72776084

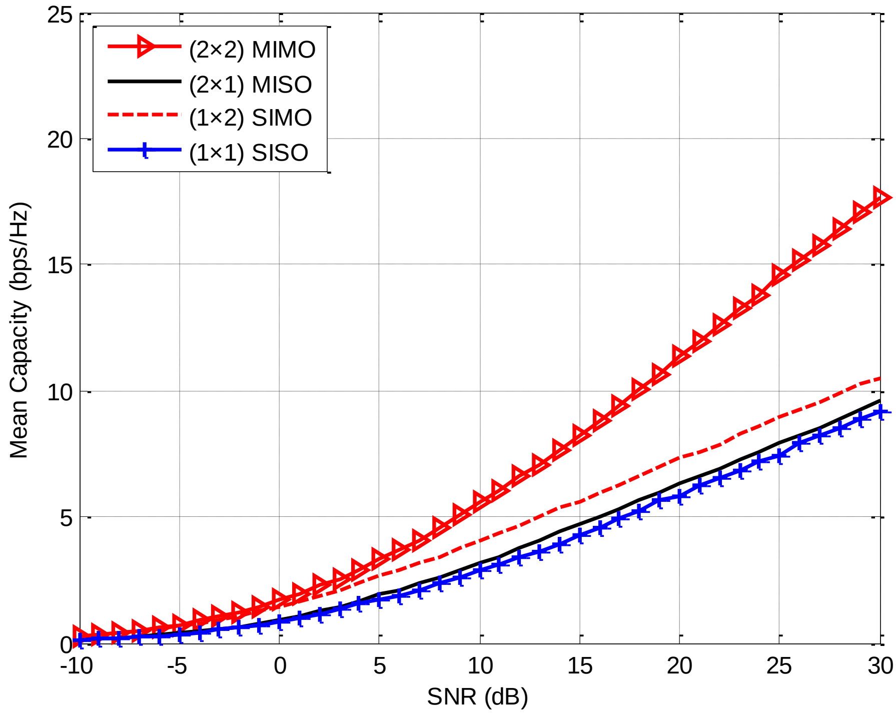

Figure 2.5: Mean capacity comparisons

In Figure 2.5, the mean capacity of the MIMO system gradually increases with respect to other SISO, MISO and SIMO systems. As discussed before, the SIMO system is more efficient and reliable than MISO system because of diversity gain at receiver side.

2.5 Antenna Array Techniques

As we know, multiple antenna at transmitter side and at receiver side provide an enormous value of diversity and high data rate via space-time signal processing as well as in sectorization.

The directional antennas in MIMO techniques can also provide a significant range of cell, reduce flat-fading and channel delay spread. Due to directional antenna, we can make differences to null, which causes of increase in system capacity. So by using multiplexing, it is premier to exploit the more antennas to enhance the data rates.

The switched-beam or phased (directional) antennas are most common directive arrays, as shown in Figure 2.6. The arrays make by multiple fixed antenna beams in this system.

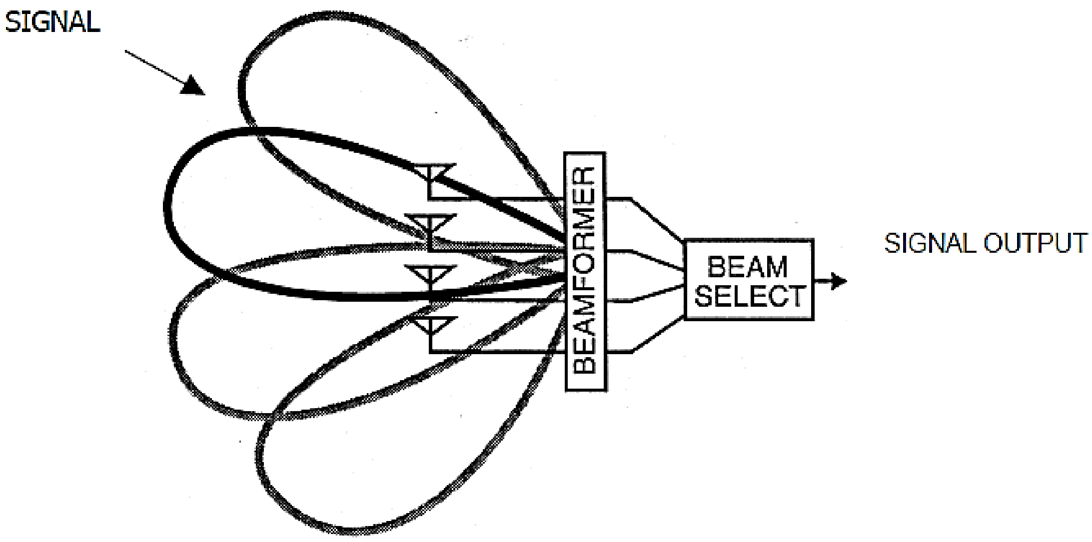

Figure 2.6: Sectorized switched-beam array

Basically, sectorization can install only on base station to cut down on interference between users. In this technique, users can be disturbed by interference only on within the sectors, if and only when sectors will assigned by different frequencies and time slots. Its mean, increasing the sectors is inversely proportional to the interference factors between users.

2.5.1 MIMO Schemes

We will highlight two fundamental tradeoffs in this section: The first tradeoff is between multiplexing gain and diversity gain [8]-[9] as well as the second one is between complexity and performance.

Consider, there are (2×2) MIMO systems. One transmit the symbols by two transmit antennas. In this scenario, the signal crosses four propagation paths. It means signal will travel and effected by four paths (independent fading), so the achievable diversity will be four.

On the other hand, if two independent signal transmitting at same time, then one of them crosses two individualistic paths and will give two diversity. But in MIMO every channel transmits two signals at a time which result in two fold multiplexing gain. The best way to achieve diversity gain and multiplexing gain is to transmit signals over MIMO channels, which is attained by coding across time and space [10].

The second tradeoff is between complexity and performance. It is very complex to design optimum receiver. Consider in MIMO system, there are N number of transmitter and receiver antennas as well as the transmitter symbols denoted by $T_1 \ldots\ldots T_N$. The *hij* is denoted by fading gain from transmit antenna *i* to receive antenna *j*,

$$r = HT + N \tag{2.11}$$

where, square matrix *H* denoted by fading gain and *T* shows the transmitted symbols as well as *r* is the received vector.

2.5.2 Diversity of Transmission

Space–time block code scheme was proposed by Alamouti in [11] for downlink transmits diversity. Generally, Alamouti's system was proposed to keep the users stations simple and reduce the bit error rate.

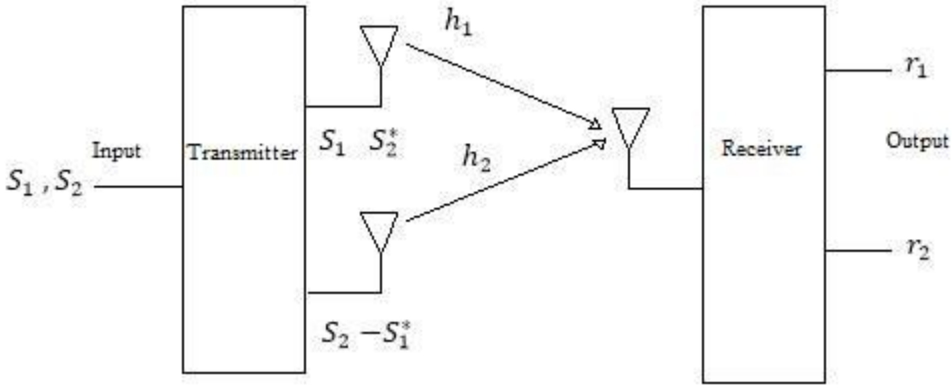

Figure 2.7: Alamouti's transmit diversity Schematic block diagram

In Figure 2.7, we assume that (S_1, S_2) show the symbols of a group in input data stream and going to be transmit. The first transmitter antenna (*Tx*) transmits the first symbol S_1 during first symbols period as well as the 2nd transmitter antenna transmits S_2 symbol during the same period. Now during the 2nd symbols period, the 1st transmitter antenna transmits S_2^* and 2nd transmitter antenna transmits S_1^*. The h_1 denoted by channel response from 1st transmitter antenna to receiver and h_2 shows the channel response from 2nd transmitter antenna to same receiver. The received signal samples from 1st and 2nd transmitter antenna can be written as

$$r_1 = h_1 * S_1 + h_2 * S_2 + n_1 \tag{2.12}$$

$$r_2 = h_1 * S_2^* - h_2 * S_1^* + n_2 \tag{2.13}$$

2.5.3 Spatial Multiplexing (SM)

The second multiple antenna profile is (2×2) MIMO technique showed by Matrix form. From transmitter side the Spatial Multiplexing (SM) doesn't give any diversity gain at receiver end.

In (2×2) spatial multiplexing, we eliminate frequency and time dimensions, but work with only space dimensions. S_1 And S_2 are denoted by transmitted symbols and they are transmitting from Tx_1 and Tx_2 respectively. The h_{ij} is denoted by the channel response. The signals received at receiver antennas are given below.

$$r_1 = h_{11}s_1 + h_{12}s_2 + n_1 \tag{2.14}$$

$$r_2 = h_{21}s_1 + h_{22}s_2 + n_2 \tag{2.15}$$

We can write in matrix form as well:

$$\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

2.6 Comparison of Alamouti and Spatial Multiplexing

The (2×2) spatial multiplexing schemes have diversity order 2 while the Alamouti schemes have 4 diversity orders, so when the same coding and modulation patterns will used in these systems the Alamouti scheme will have better BER performance rather than spatial multiplexing scheme. Consequently, when both schemes are required to provide same BER then Alamouti scheme can be used for higher modulation.

We noted that Alamouti technique provide same spectral efficiency by transmitting 2*m* bits per symbols as comparison with MIMO spatial multiplexing technique that transmits *m* bits per symbols. We made a performance when spatial multiplexing technique uses Quadrature Phase Shift Keying (QPSK) and Alamouti technique uses 16- quadrature amplitude modulation (16-QAM).

It can be noted that the slope of BER curve is approximately half that of ML receiver as compare to ZF receiver. So according to this, the diversity of spatial multiplexing will not be exploiting. It can be observed the Alamouti scheme provides better BER rather than spatial multiplexing Maximum likelihood ML receiver. These results are reported in [12 - 13] as shown in Figure 2.8.

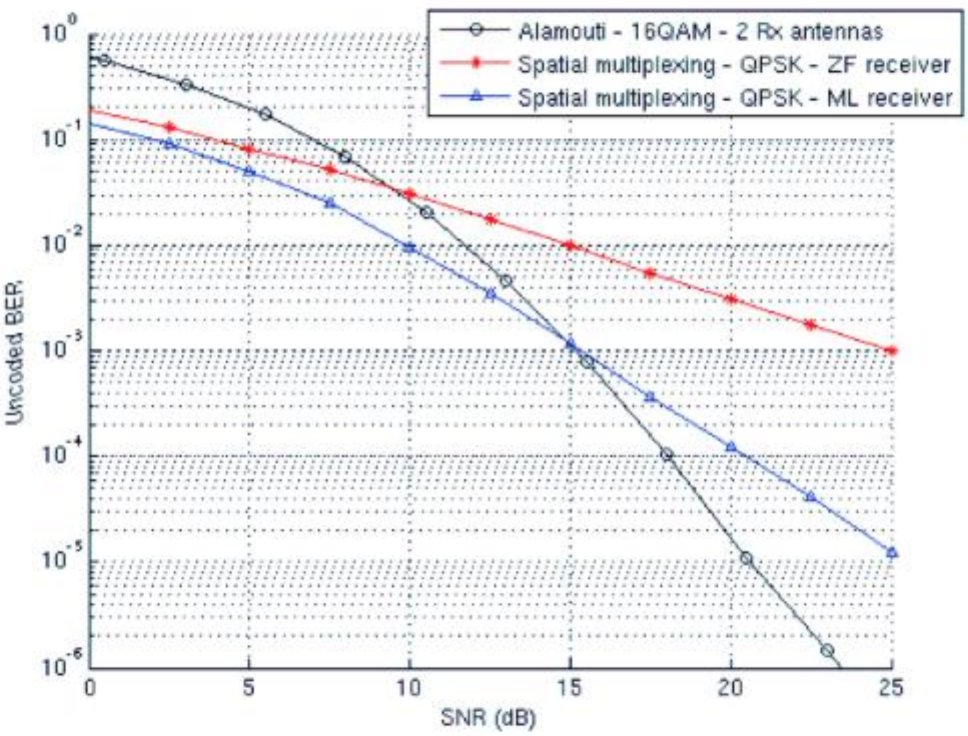

Figure 2.8: Comparison of (2×2) spatial multiplexing with Alamouti [12]-[13]

Figure 2.8 shows that the Alamouti technique with 16-QAM provide a better results rather than (2×2) SM-MIMO scheme with ML observation. The best MIMO technique is depend on required throughput and channel SNR as well as other parameter like interference cancellation capability.

2.7 AMC in MIMO System

The throughput is optimized by using single antenna through link adaptation. The technique in which code rate and constellation is select as a purpose of the channel, then the idea is known as Adaptive Modulation and Coding (AMC) scheme. This technique provides a optimize result by using single antenna. The main aim is to determine the channel excellence by estimation the received SNR and received power at mobile station. The Base Station (BS) adopt the modulation and coding scheme according to the channel measurements and some parameter which related to delay and estimation error.

The AMC concept shown in Figure 2.8, the forward error correction (FEC) must be less than10^{-3}. At transmitter end, the SNR thresholds are evaluated for the system using MIMO matrix and the pedestrian channel "*A*" equating to a speed of 3 kilometer per hour.

Matrix A: Represent the multiple antennas at transmitter and receiver side that are using maximum combing ratio technique at receiver side.

Matrix B: Shows the spatial multiplexing and send the data independently over each antenna that provides a good spectral efficiency.

Furthermore the modulation 16QAM (matrix *A*) with code rate 3/ 4 which is exceeding to 9 dB and showing the spectral efficiency near to 3 bit/symbols but 16QAM (matrix *B*) provide a good spectral efficiency than 16QAM (matrix *A*) as shown in Figure 2.9.

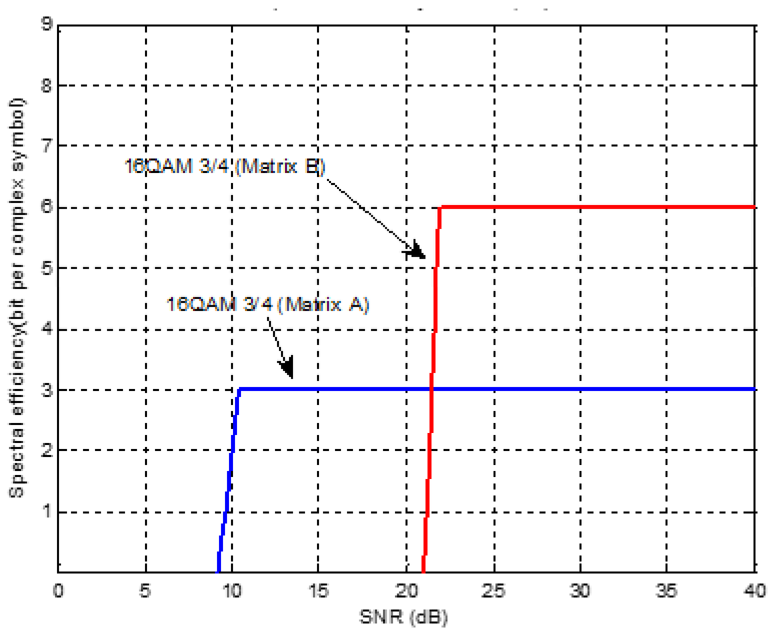

Figure 2.9: Comparison of 16QAM 3/4 with matrix *A* and matrix *B* [13]

So it is the best to use MIMO techniques and select the best MIMO scheme, best modulation and coding via adaptation link.

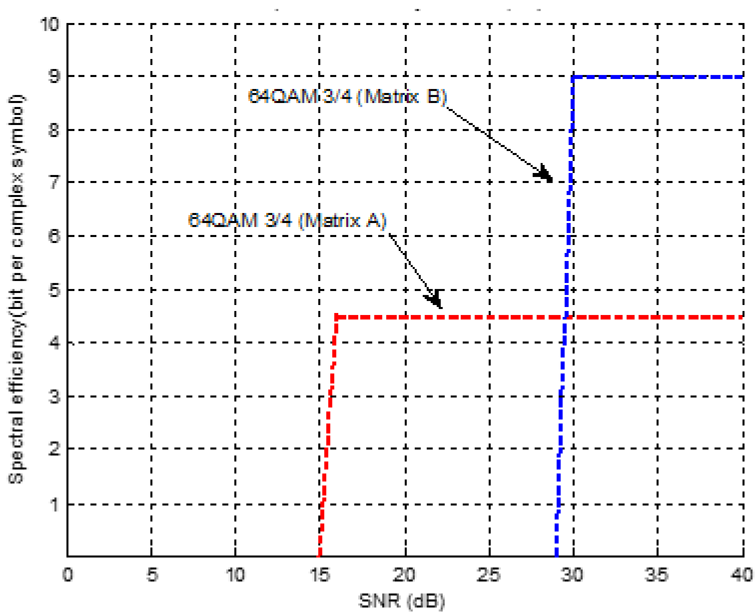

Figure 2.10: Comparison of 64QAM 3/4 with matrix *A* and matrix *B* [13]

Furthermore in Figure 2.10, the 64QAM with code rate 3/4 provides a 9 bit/symbol spectral efficiency, because the SNR value upper than 29 dB, this means the 64QAM with 3/4 code rate is more efficient than 64QAM with 3/4 code rate of matrix *A* which is using MRC at receiver side. But matrix *A* provide a best performance in bit error rate than matrix *B* because of adaptive modulation technique. So we can say that AMC provide a good result in spectral efficiency but it shows the high bit error rate as illustrated in Figure 2.8. That’s way, the high code rate modulation provides to those users who are near to BTS.

Chapter 3

WATER-FILLING ALGORITHM FOR POWER ALLOCATION

3.1 Gaussian Channel

The Gaussian channel is the most important continuous alphabet channel as illustrated in Figure 3.1. So we can express in form of equation.

$$Y_i = X_i + Z_i \qquad i = 1, 2, \ldots, m \tag{3.1}$$

where $Z_i \sim N(0, \sigma^2)$.

The noise Z_i assumed to be independent of the input signal. If the noise variance is zero then the data can be recoverable perfectly without any error rate at destination. At the input side, the most common thing is power limitation. We assume that the input signals are $(x_1, x_2, x_3, \ldots, x_m)$ transmitted over the channel then we must require

$$\frac{1}{n} \sum_{i=1}^{m} x_i^2 \le P \tag{3.2}$$

This communication model is using in radio and satellite links in practically. With power constraint the information capacity is

$$C = \max_{E[x^2] \le P} I(X; Y) \tag{3.3}$$

Now according to equation (3.3) the information capacity of Gaussian channel becomes

$$C = \max_{E[x^2] \le P} I(X;Y) = \frac{1}{2}\log(1 + P/N) \tag{3.4}$$

3.2 Water-Filling on Parallel Gaussian Channels

We consider, there are *m* parallel Gaussian channels with common power constraint. The objective is to maximize the capacity of the channel by distributing the total power among the channels. We assume that there are set of parallel channels as illustrated in Figure 3.1.

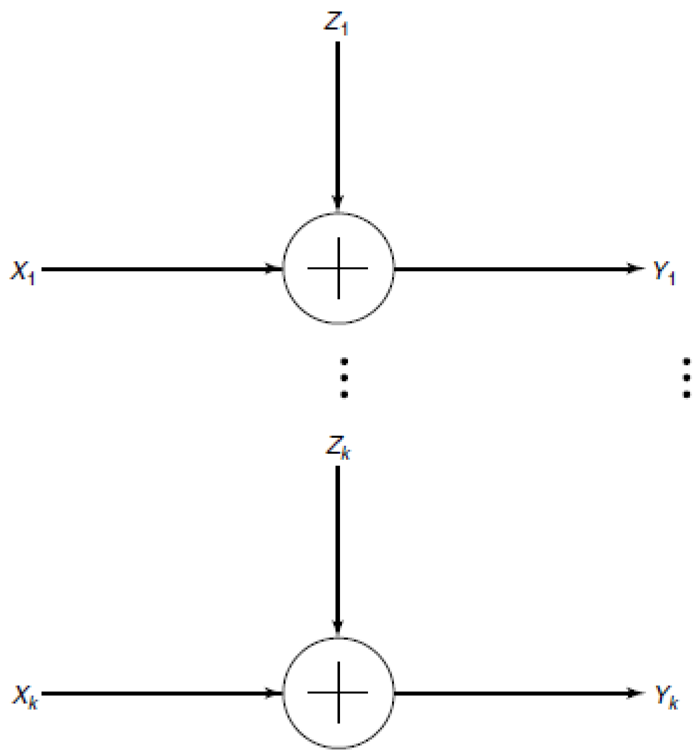

Figure 3.1: Parallel Gaussian channels

where *x*, *y* denote the input and output signals of the system as well as *z* shows the noise of the channel. The output of the each channel is the sum of noise and input signals. If noise is zero the receiver can receive the transmitted data perfectly.

The problem is reduced to finding the allotment of power which can maximize the capacity of the channel. The expression $\sum P_i = P$ shows that the sum of different channel powers is equal to the total power of the system. So by using Lagrange multiplier, we can find the best power allocation to each channel.

$$\mathcal{L}(P_1, P_2, P_3, \dots, P_m) = \sum_{i=1}^{m} \frac{1}{2} \log\left(1 + \frac{P_i}{N_i}\right) + \lambda(\sum_{i=1}^{m} P_i) \tag{3.5}$$

Now after differentiating the equation (3.5) with respect to P_i, we have

$$\frac{1}{2} \frac{1}{P_i + N_i} + \lambda = 0 \tag{3.6}$$

$$P_i = v - N_i \tag{3.7}$$

where "v" denotes the water level. According to the power allocation P_i must be nonnegative, so

$$P_i = (v - N_i)^+$$

where ,
$$P_i = (v - N_i)^+ = \begin{cases} v - N_i & \text{iff} \quad v \geq N_i \\ 0 & \phantom{\text{iff}} \quad v < N_i \end{cases}$$

This equation shows the allotment of power to each channel after observe the noise level with respect to the water level as shown in Figure 3.2. If the noise level is over the water level then there is no need to allocate power to the channel according to this theorem (i.e. N_3 is over v for channel 3 in Figure 3.2). This process is known as water filling algorithm.

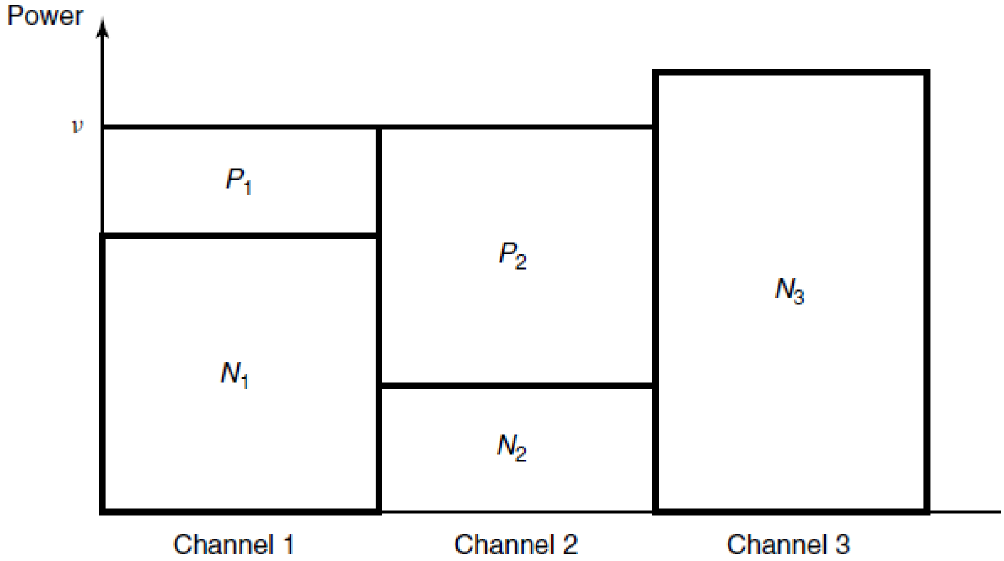

Figure 3.2: Power allocation to parallel channels

3.3 How Water-Filling Algorithm Works

Ones the water level known then solution is readily obtain. The problem reduce to finding the water level to satisfy the constraint. So by fixing the water level "v" according to the requirements then the iterative algorithm can be obtain.

N_c ……… Number of sub channels

U………… Total number of users

P_t………… Total Power (Power Budget)

(Iteration Water Filling Algorithm) Power Allocation

Fix: u= P_{Al}^{u} and λ = H_n^u

Loop:

Number of Iterations with Fix step (0.001)

Loop 2: Power Allocation for all users

P_{Al}^{u} (i) = P_{Al}^{u} (i) + step size

Check constraint

$\sum_{u=1}^{U} \sum_{K=1}^{Nc} P_k^u \leq P_{Total}$

If satisfied

P_k^u (i) = P_k^u (i) + stepsize

Else

P_k^u (i) = P_k^u (i)

End Loop 2

End Loop 1

End

3.4 Example of WF Strategy for (2×2) MIMO System

In a (2×2) MIMO system the received signals y_1 and y_2 at the two receiver side antennas can be represented as:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$$

where,

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \sim N \left(0, \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \right)$$

We assume power budget (PB) = $P_t = 4$ kW.

Where the transmitting and receiving antennas are $m = 2 = N_t = N_r$ as well as the noise consider to be $N_1 = 3$ & $N_2 = 5$. Now we will check the capacity for (2×2) MIMO System with equal power distribution.

So, $P_i = \frac{P_t}{m} = \frac{4\text{ kW}}{2} = 2$ kW, i = 1, 2.

As we know

$$C = \max_{E[x^2] \leq P} I(X; Y)$$

The total capacity of parallel Gaussian channel without (WF) strategy is

$$C = \frac{1}{2}\sum_{i=1}^{m} log_2\left(1 + \frac{P_i}{N_i}\right) = \frac{1}{2} log_2 \prod_{i=1}^{m}\left(1 + \frac{P_i}{N_i}\right)$$

$$= \frac{1}{2} log_2\left(1 + \frac{2}{3}\right)\left(1 + \frac{2}{5}\right) = 0.6112\ bps/Hz$$

Now by using optimal power allocation, we will find out total capacity.

We assume water level *v* is 6 kW. So we can obtain optimal power by using

$$P_i = (v - N_i)^+ \qquad i = 1, 2.$$

for $$P_i = (v - N_i)^+ = \begin{cases} v - N_i & \text{iff} \quad v \geq N_i \\ 0 & \phantom{\text{iff}} \quad v < N_i \end{cases}$$

So, we find out $P_1 = 3$ kW & $P_2 = 1$ kW.

We obtained total capacity by using water filling strategy that is

$$C_{wf} = \frac{1}{2}\sum_{i=1}^{m} log_2\left(1 + \frac{P_i}{N_i}\right) = \frac{1}{2} log_2 \prod_{i=1}^{m}\left(1 + \frac{P_i}{N_i}\right)$$

$$= \frac{1}{2} log_2\left(1 + \frac{3}{3}\right)\left(1 + \frac{1}{5}\right) = 0.6315\ bps/Hz$$

Hence,

Capacity gain = $C_{wf} - C = 0.6315 - 0.6112 = 0.0203\ bps/Hz$

Chapter 4

PERFORMANCE ANALYSIS

4.1 Improvement in MIMO using water filling Algorithm

As we know MIMO is more efficient and provide higher date rate as compare to SISO system with same SNR and power budget. In order to achieve the same capacity, SISO system need the higher transmit power than the MIMO system. As our requirement, we need to consume minimum power at input and need maximum capacity response from output that is possible only when we will use MIMO system. We will assume 2 receiver antennas and 2 transmitter antennas as well as 4 receiver antennas and 4 transmitter antennas. According to the result, if we increase transmit and receive antennas then we can get higher data rate and capacity of the channels.

We can increase the data rate and capacity, if we know the parameters of the channels at receiver and at transmitter side as well as allocating optimal power at transmitter by using water filling algorithms to each channel with respect to the power budget. In MIMO system we will use water filling algorithm that can give better results

4.2 Simulation of Water Filling Algorithm

As we know the mean capacity of (2×2) MIMO system with WF strategy is higher than the (2×2) MIMO system without WF. We noted that the difference between graph lines of (2×2) MIMO system with WF and (2×2) MIMO system without WF gradually decrease as SNR increases as shown in Figure 4.4. There are two parallel transmitters and two receivers that are denote by N_t and N_r respectively. These parallel channels have different noise level denote by N_1 and N_2.

For equal distribution,

$$P_i = P_t/m \tag{4.1}$$

where P_t is the total power and m is the number of channels. So we can find the distributed power by calculating P_i. The total capacity of the parallel channels using same distributed power with different noise level N_i is

$$C = \frac{1}{2}\sum_{i=1}^{m} log_2\left(1 + \frac{P_i}{N_i}\right) \tag{4.2}$$

Now according to the water level denoted by "v" we will adjust the power on every channel with respect to the noise level.

$$P_i = (v - N_i)^+ \tag{4.3}$$

where, $$P_i = (v - N_i\,)^+ = \begin{cases} v - N_i & \text{iff} \quad v \geq N_i \\ 0 & \quad\;\; v < N_i \end{cases}$$

We can get the water filling channel capacity to put the value of P_i in equation 4.2.

$$C_{wf} = \frac{1}{2}\sum_{i=1}^{m} log_2\left(1 + \frac{P_i}{N_i}\right) \tag{4.4}$$

The power allocation to any channel depends on the noise level of the channel. According to the power budget, if the noise level is low then we have to allocate high power than other channels. But we cannot allocate more power than power budget which already calculated by water level "v".

4.2.1 Results of WF Over (2×2) MIMO System

According to our simulation results, the noise of the first and second channel illustrated in Figure 4.1.

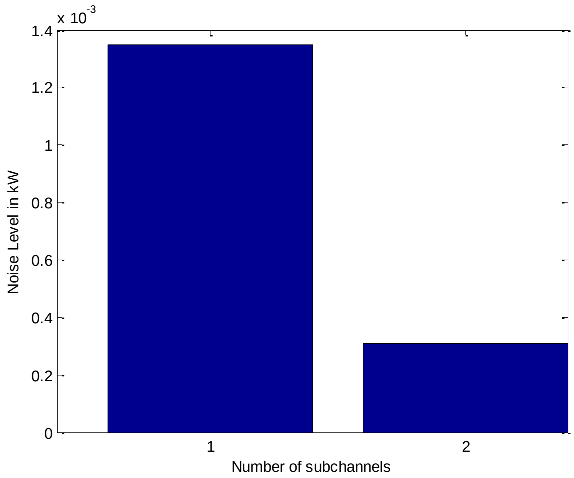

Figure 4.1: Noise in each channel

As we noted, the noise of the first and second channel is 1.35 W and 0.34 W respectively. Now according to the water-filling algorithm, the power will distribute to both of channels under the condition of water level "*v*'' as illustrated in Figure 4.2.

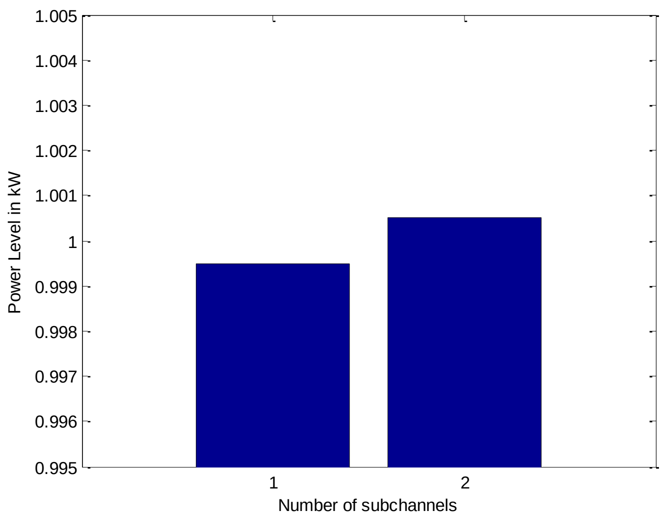

Figure 4.2: Power distribute to each channel

Figure 4.2 shows the optimal power allocation to each channel that can be helpful for enhancement of the channel capacity. The allotment of power to the first and second channel is 0.997 kW and 1.003 kW respectively.

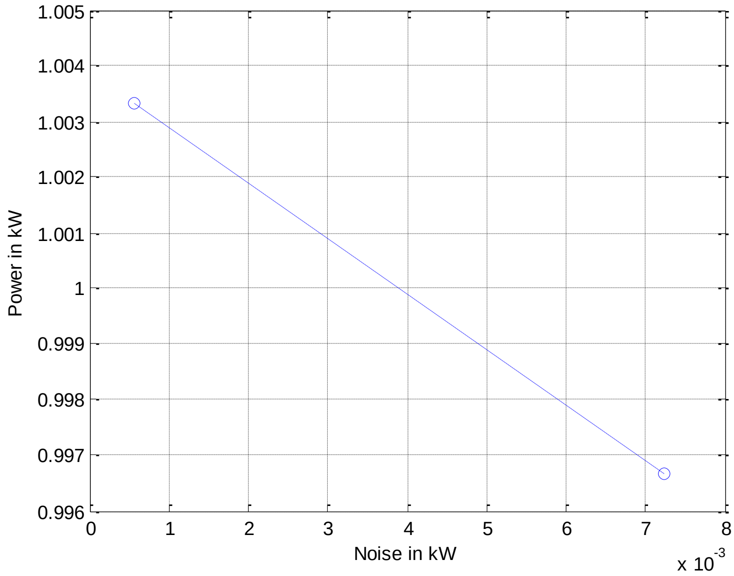

Figure 4.3: Power distribute over noisy channels

Figure 4.3 shows the amount of power distributed over the noisy channels. The circles in the graph represent the channels. Now we can observe the mean capacity of the (2×2) MIMO channel by using optimal power allocation to the channels with power budget 2 kW as shown in the Table 4.1.

Table 4.1: Mean capacity comparison with (2×2) MIMO WF

Mean capacity in bps/Hz					
SNR(dB)	**(1×1) SISO**	**(2×1) MISO**	**(1×2) SIMO**	**(2×2) MIMO**	**(2×2) MIMO with WF**
-10	0.131260324	0.132062691	0.251496606	0.25969396	0.735811501
-9	0.163206764	0.169704044	0.320749872	0.330419874	0.868671508
-8	0.197391659	0.202424287	0.369034589	0.407331425	1.016323829
-7	0.238291015	0.249477617	0.458091453	0.497064027	1.195841717
-6	0.290923146	0.306567382	0.571786514	0.585910476	1.37140651
-5	0.341307611	0.374446045	0.647435868	0.747756664	1.590259686
-4	0.419490528	0.452756678	0.768823935	0.863053881	1.830738692
-3	0.543014746	0.553619409	0.909953028	1.032454689	2.091477912
-2	0.607058638	0.666241527	1.101295181	1.233577108	2.374361989
-1	0.74259595	0.791790594	1.262257579	1.430551043	2.639504035
0	0.888956735	0.914386434	1.415477182	1.653210816	2.95489981
1	0.999826732	1.073826789	1.640728748	1.961853764	3.300639168
2	1.139986886	1.257971035	1.891189575	2.245499751	3.622819189
3	1.308993705	1.445496159	2.077677183	2.592138898	3.9913372
4	1.520878228	1.684522337	2.350972263	2.921384434	4.367618455
5	1.704165708	1.870520742	2.562106166	3.322518206	4.862653439
6	1.906978406	2.126963115	2.884287978	3.7173753	5.167114291
7	2.063688693	2.368200664	3.178143324	4.159439951	5.692766823
8	2.357993563	2.635835728	3.457290226	4.529695701	6.155535224
9	2.700721136	2.880196725	3.768931424	5.077959161	6.705737492
10	2.893317816	3.122855571	4.07865709	5.606611587	7.237822839
11	3.202971472	3.402504307	4.404916721	6.035432417	7.808767225
12	3.486641891	3.792067144	4.64350606	6.585233655	8.364446411
13	3.780908829	4.044871611	4.990003647	7.09333477	8.927027576
14	3.997743697	4.359371687	5.321805453	7.609940085	9.470045724
15	4.308937394	4.680180381	5.632816507	8.291888718	10.03522985
16	4.715988519	4.995712638	5.917444929	8.891179429	10.66072451
17	4.923333597	5.307713677	6.277158738	9.410861753	11.29366757
18	5.267279728	5.65071441	6.568655886	10.00318688	11.98975428
19	5.474010796	5.928944952	6.874569287	10.60775688	12.65816372
20	5.8097992	6.263273225	7.286180301	11.3604139	13.25271893
21	6.310466884	6.633662109	7.609173229	11.96769355	13.77849463
22	6.480255531	6.917880775	7.967603924	12.56213112	14.38167354
23	6.856815607	7.211047552	8.290569285	13.14131288	15.12058847
24	7.129766277	7.628204358	8.586091171	13.85402582	15.75015412
25	7.523570113	7.853223804	9.002079536	14.47838235	16.43949877
26	7.765014369	8.233187864	9.226921886	15.04831553	17.12682256
27	8.099522954	8.598830526	9.581024926	15.7468847	17.6567495
28	8.340228361	8.908664057	9.917263363	16.36196942	18.38744467
29	8.703214089	9.288981913	10.23655489	17.14156364	19.0729329
30	9.20576988	9.613281057	10.57886229	17.72776084	19.74872841

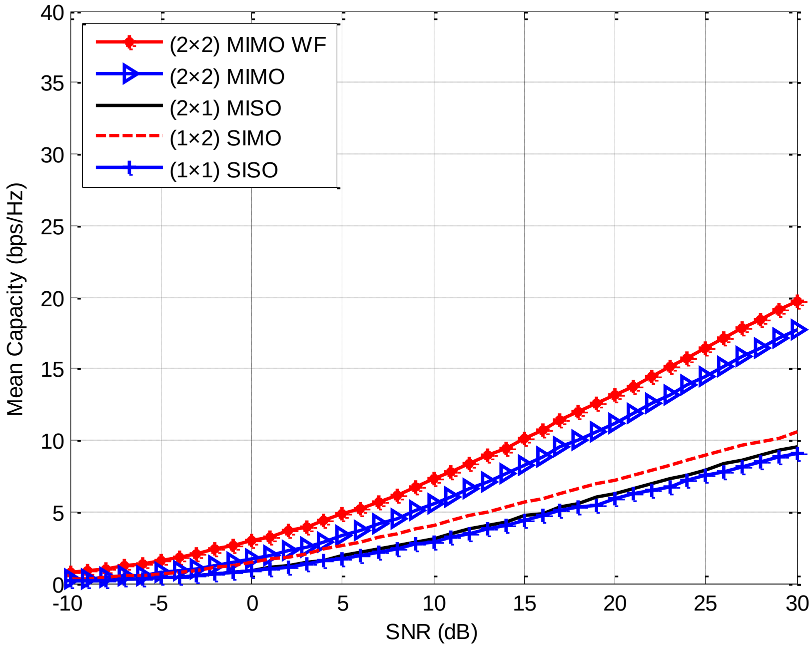

Figure 4.4: Mean capacity vs SNR (dB)

As we noted, the (2×2) MIMO system with water-filling strategy is more efficient than other systems and provide us higher capacity for the users as illustrated in Figure 4.4. The capacity line of (2×2) MIMO system with WF is increasing gradually as compare to the other system like SISO, SIMO and MISO system. We can also evaluate the water-filling capacity gain with respect to other systems that can be expressed as shown below.

WF capacity gain w.r.t SISO system = $C_{wf} - C_{1x1}$ (4.4)

WF capacity gain w.r.t SIMO system = $C_{wf} - C_{1x2}$ (4.5)

WF capacity gain w.r.t MISO system = $C_{wf} - C_{2x1}$ (4.6)

WF capacity gain w.r.t MIMO system = $C_{wf} - C_{2x2}$ (4.7)

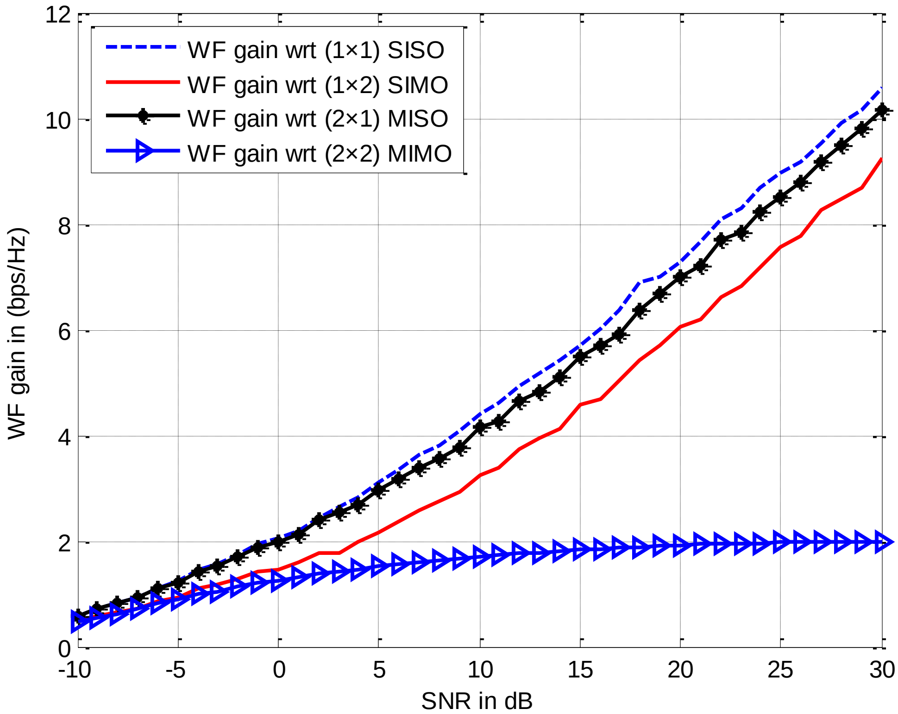

Figure 4.5: WF gain w.r.t SISO, SIMO, MISO and MIMO system

Figure 4.5 shows the differences as mention in equation (4.4), (4.5), (4.6) and (4.7). According to the graph, there is a very minor difference between (2×2) MIMO system and (2×2) MIMO with Water-filling approach as well as we also noted that after 10 (dB) SNR the difference between them is becoming minor. While the gain of (2×2) MIMO system with water filling gradually increased w.r.t SISO, SIMO, MISO except (2×2) MIMO system.

4.2.2 Testing Results with Different Power Budget

We evaluated the results by allocate the different value of the power budget. We noted that the capacity of the channel increases by increasing the power budget as shown in Figure 4.6.

At Power budget =3 kW

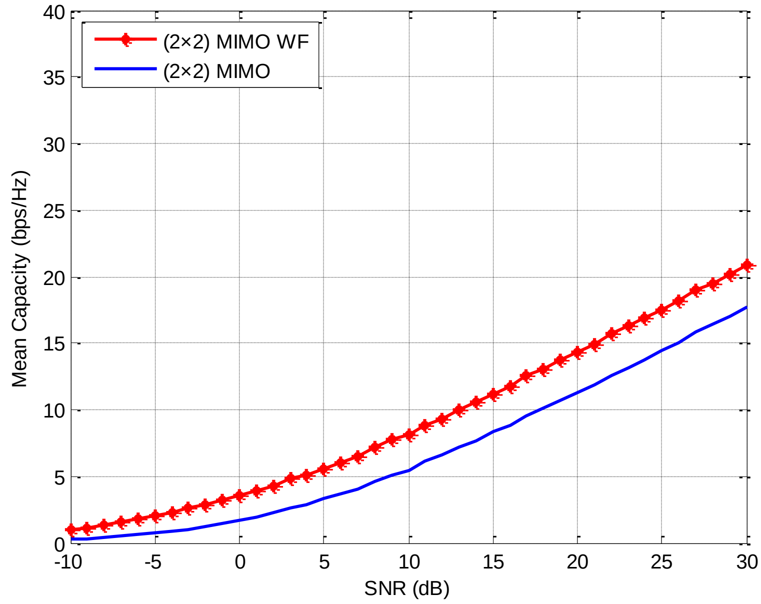

Figure 4.6: Mean Capacity Comparison of (2×2) MIMO with WF at 3 kW power budget

Figure 4.6 shows the difference of the mean capacity between WF strategy and without WF over (2×2) MIMO system at 3 kW power budget. We noted, when we apply 3 kW power budget over (2×2) MIMO system. The difference between two lines increased as power increased that can be compare with Figure 4.4. So we can see that after increasing the power budget, the capacity will also increase.

When we apply 3.5 kW power budget over (2×2) MIMO system. We observed that deference between two line gradually increased as SNR increase as shown in Figure 4.7. Alternatively, we can say that the gain between two lines increased as SNR increase.

At Power budget =3.5 kW

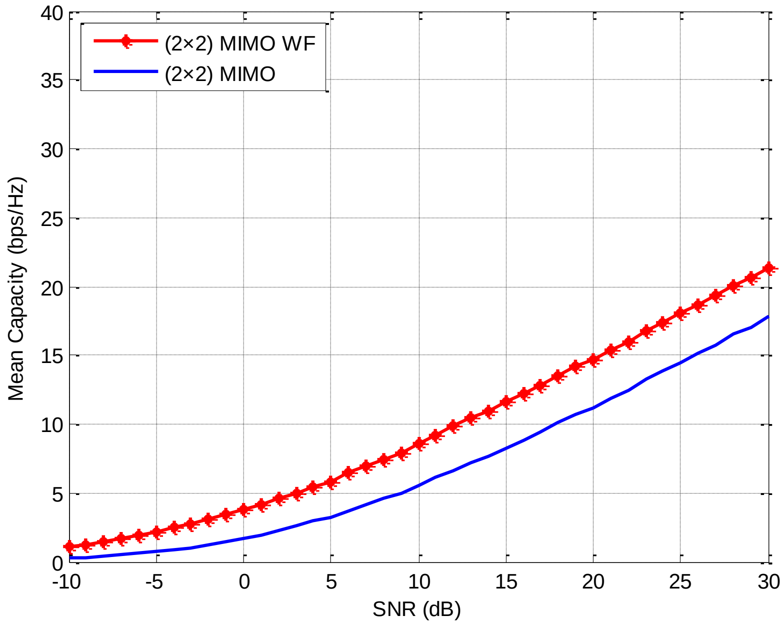

Figure 4.7: Mean Capacity Comparison of (2×2) MIMO with WF at 3.5 kW power budget

Afterward, we tested capacity by using different power budget as shown in Table 4.2. We noted, there is significant differences between values of the capacity that have been calculated with different power budgets. The values of every column increases gradually as SNR increase.

Table 4.2: Mean capacity bps/Hz for (2×2) MIMO at different power budget using water-filling strategy

Mean capacity bps/Hz of (2X2) MIMO system at different power budget					
SNR(dB)	**(2×2) MIMO without WF**	**2 kW**	**2.5 kW**	**3 kW**	**3.5 Kw**
-10	0.25969396	0.735812	0.866588	0.987514	1.110566684
-9	0.330419874	0.868672	1.026042	1.170556	1.278358738
-8	0.407331425	1.016324	1.195535	1.364409	1.481715444
-7	0.497064027	1.195842	1.367023	1.545953	1.74006239
-6	0.585910476	1.371407	1.584896	1.791931	1.950255233
-5	0.747756664	1.59026	1.849589	2.020044	2.168157152
-4	0.863053881	1.830739	2.103567	2.283027	2.504564046
-3	1.032454689	2.091478	2.361915	2.561421	2.797778588
-2	1.233577108	2.374362	2.614633	2.850318	3.096151202
-1	1.430551043	2.639504	2.907196	3.191659	3.383825103
0	1.653210816	2.9549	3.238374	3.508126	3.79910034
1	1.961853764	3.300639	3.597256	3.916743	4.197145175
2	2.245499751	3.622819	3.9821	4.291287	4.573447311
3	2.592138898	3.991337	4.36353	4.715635	5.019171532
4	2.921384434	4.367618	4.820875	5.123326	5.407861866
5	3.322518206	4.862653	5.207353	5.655567	5.883062915
6	3.7173753	5.167114	5.702547	6.094608	6.490341407
7	4.159439951	5.692767	6.19203	6.626729	6.975856928
8	4.529695701	6.155535	6.779334	7.118074	7.504213128
9	5.077959161	6.705737	7.293772	7.566944	8.01898598
10	5.606611587	7.237823	7.76012	8.172172	8.563230877
11	6.035432417	7.808767	8.342381	8.820784	9.21720896
12	6.585233655	8.364446	8.899518	9.310726	9.817616067
13	7.09333477	8.927028	9.449454	9.963022	10.37772726
14	7.609940085	9.470046	10.17187	10.61237	10.99190737
15	8.291888718	10.03523	10.63989	11.17592	11.59817474
16	8.891179429	10.66072	11.35087	11.7045	12.20904686
17	9.410861753	11.29367	11.84697	12.40911	12.81224833
18	10.00318688	11.98975	12.67191	13.03763	13.64669525
19	10.60775688	12.65816	13.10749	13.68042	14.09379886
20	11.3604139	13.25272	13.71703	14.30894	14.86851181
21	11.96769355	13.77849	14.54228	15.14267	15.44313249
22	12.56213112	14.38167	15.24346	15.56097	15.99943462
23	13.14131288	15.12059	15.7558	16.22106	16.78876933
24	13.85402582	15.75015	16.46614	16.91278	17.35259479
25	14.47838235	16.4395	17.13501	17.64079	17.97516108
26	15.04831553	17.12682	17.60953	18.21524	18.65023475
27	15.7468847	17.65675	18.28524	18.89178	19.42801746
28	16.36196942	18.38744	19.19987	19.51162	19.99534402
29	17.14156364	19.07293	19.74768	20.10699	20.59147362
30	17.72776084	19.74873	20.34479	20.92739	21.40863209

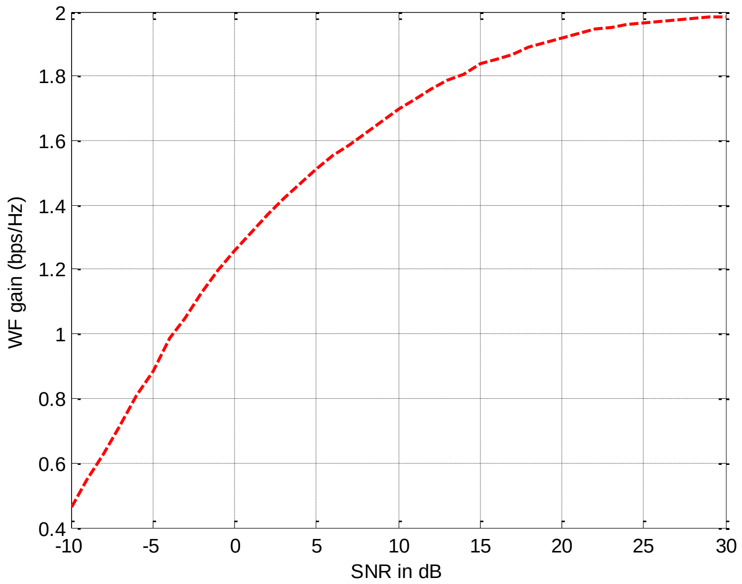

Figure 4.8: WF gain over (2×2) MIMO at 2 kW Power budget

Figure 4.8 shows the differences or gain of WF and without WF over (2×2) MIMO system at 2 kW power budget. Initially, the difference is very minor at -10 dB SNR and gradually increased as SNR increase. Alternatively, we can say that WF gain of (2×2) MIMO system is directly proportional to the power budget of the system.

4.2.3 Results of WF Over (4×4) MIMO System

Now, we will apply Water-filling strategy over (4×4) MIMO system. When we apply same 2 kW power budget over (4×4) MIMO system as shown in Figure 4.9. We noted that there is significant improvement in capacity as compare to (2×2) MIMO system with same power budget. Its mean that we can enhance the capacity of the system by increasing the transmitters and receivers.

Afterward, when we increased the power budget from 2 kW to 3 kW. We noted that increase in power budget can also cause of enhancement in capacity as shown in Figure 4.10.

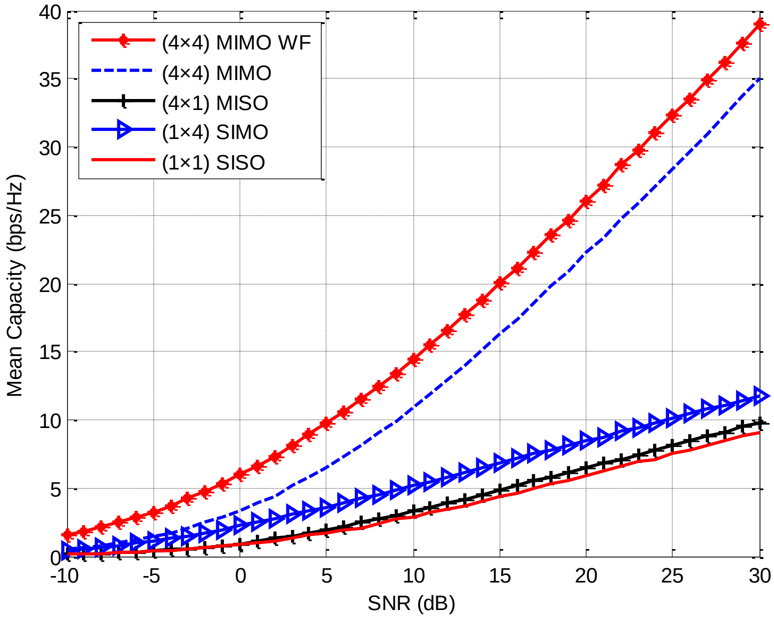

Figure 4.9: Mean Capacity Comparison of (4×4) MIMO with WF at 2 kW Power budget

Power budget at 3 kW

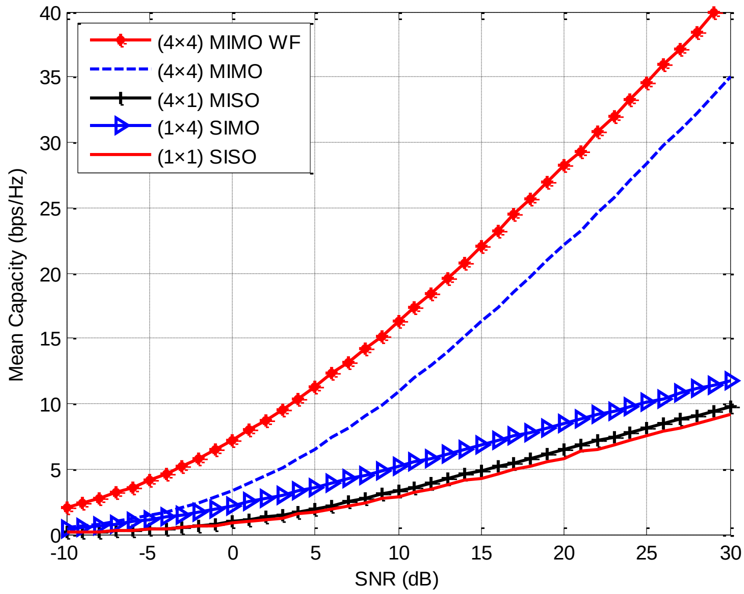

Figure 4.10: Mean Capacity Comparison of (2×2) MIMO with WF at 3 kW power budget

Now after optimal power allocation in (4×4) MIMO system, we can observe the significant enhancement in capacity as shown in Table 4.3. According to the table, water filling algorithm is efficient for low signal to noise ratio (SNR) but it’s depend on Power budget.

Table 4.3: Mean capacity comparison with (4×4) MIMO WF

Mean capacity in bps/Hz					
SNR(dB)	**(1×1) SISO**	**(4×1) MISO**	**(1×4) SIMO**	**(4×4) MIMO**	**(4×4) MIMO WF**
-10	0.131260324	0.133126532	0.472810064	0.532342052	2.072111678
-9	0.163206764	0.168094932	0.569273245	0.655016705	2.384367164
-8	0.197391659	0.209700203	0.680695065	0.794311334	2.754909972
-7	0.238291015	0.261625014	0.803469044	0.980629874	3.176480398
-6	0.290923146	0.323437809	0.956644148	1.188848028	3.580462323
-5	0.341307611	0.387375984	1.128765251	1.437563148	4.0730427
-4	0.419490528	0.475606009	1.307912464	1.722506535	4.618939762
-3	0.543014746	0.572524351	1.508767731	2.051985205	5.216168683
-2	0.607058638	0.689718598	1.719055993	2.455097675	5.806151501
-1	0.74259595	0.81822983	1.972215826	2.860543776	6.48131821
0	0.888956735	0.961739008	2.19845432	3.352432133	7.184547402
1	0.999826732	1.112021735	2.490392478	3.883452469	7.904858253
2	1.139986886	1.296418972	2.740043054	4.480966301	8.708582417
3	1.308993705	1.510538941	3.012194665	5.072323128	9.487021175
4	1.520878228	1.725496644	3.280047574	5.836540055	10.46220085
5	1.704165708	1.928336718	3.617768311	6.544870792	11.31232572
6	1.906978406	2.197507815	3.925349908	7.355304092	12.2681565
7	2.063688693	2.486708062	4.229925096	8.192313035	13.2244494
8	2.357993563	2.715500336	4.524953827	9.054153531	14.24917066
9	2.700721136	3.022119452	4.87081911	9.92061207	15.26458888
10	2.893317816	3.30459857	5.212905346	10.98579731	16.18621404
11	3.202971472	3.595736853	5.523729822	11.85291948	17.39006541
12	3.486641891	3.949282303	5.812253651	12.9868345	18.47328967
13	3.780908829	4.20270264	6.174225111	14.068347	19.66198857
14	3.997743697	4.505804978	6.476360676	15.21459612	20.80949555
15	4.308937394	4.870037196	6.84653679	16.22419317	22.06176193
16	4.715988519	5.160365365	7.13054571	17.34472254	23.29064321
17	4.923333597	5.529523146	7.476529071	18.45423256	24.42008032
18	5.267279728	5.801009917	7.774649945	19.88783021	25.69418507
19	5.474010796	6.104359825	8.158175831	20.83390675	26.96587275
20	5.8097992	6.472134865	8.468864896	22.15409728	28.1958571
21	6.310466884	6.788276701	8.79325955	23.32571643	29.49533759
22	6.480255531	7.122665717	9.116246403	24.60402159	30.82177664
23	6.856815607	7.449324504	9.51388841	25.93545723	32.07958134
24	7.129766277	7.784640396	9.783624665	27.25706412	33.31244737
25	7.523570113	8.090710162	10.05527687	28.49375935	34.58799443
26	7.765014369	8.447780721	10.4867305	29.6398704	35.98440776
27	8.099522954	8.778536467	10.76107448	31.0732341	37.19756858
28	8.340228361	9.094217213	11.10177204	32.29639642	38.58437592
29	8.703214089	9.436916824	11.48597807	33.60580236	39.86102953
30	9.20576988	9.786581818	11.77279618	34.80231062	41.2731111

According to our simulation results, the capacity can be increased by increasing the power budget of the system but in real life every equipment can bear a fix level of power. In theoretically, the more power can cause of enhancement in capacity of the system.

Chapter 5

CONCLUSION AND FUTURE WORK

5.1 Conclusion

This thesis presents an enhancement in capacity using water-filling strategy that is based on optimal power allocation to each channel. If we use simple MIMO system rather than SISO system then we can increase the data rate and mean capacity according to the results of our simulations. Furthermore, we can increase data rate and mean capacity by using water filling algorithm over MIMO system. Simulation results indicate that applying water-filling strategy on (2×2) and (4×4) MIMO will help increase the capacity by 1.74323 bps/Hz and 5.838 bps/Hz respectively at SNR 15 dB with 2 kW power budget.

5.2 Future Work

In the literature there are multiple power allocation algorithms. Some of these include conventional water filling (CWF), constant power water filling, inverse water filling (IWF) and adaptive iterative water filling (AIWF) techniques. In the thesis only CWF and constant power water filling algorithms have been investigated. Future work can involve developing codes for IWF and AIWF and obtaining system capacity in bits/s/Hz for each technique.

Furthermore water-filling strategy can also be applied in coded OFDM systems. The information bits can be coded and then send over a *L*-tap frequency selective

AWGN channel using OFDM technique. If we know the channel state information then we can also apply the WF strategy over fast fading channels.

Finally capacity for MU-MIMO broadcast channels should also be investigated. In MU-MIMO broadcast channels, the transmit information for each user is emitted simultaneously, and each user can receive the information of all the users. Therefore, the transmit beamforming technique is essential for users to eliminate the interference. Suitability of using with MU-MIMO should also be investigated.

REFERENCES

[1] Khalighi, M. A., Brossier, J. M., Jourdain, G., & Raoof, K. (2001, September). Water filling capacity of Rayleigh MIMO channels. In *Personal, Indoor and Mobile Radio Communications, 12th IEEE International Symposium on* (Vol. 1, pp. A-155).

[2] Foschini, G. J., & Gans, M. J. (1998). On limits of wireless communications in a fading environment when using multiple antennas. *Wireless personal communications*, *6*(3), 311-335.

[3] Khalighi, M. A., Raoof, K., & Jourdain, G. (2002). Capacity of wireless communication systems employing antenna arrays, a tutorial study. *Wireless Personal Communications*, *23*(3), 321-352.

[4] Gesbert, D., Bölcskei, H., Gore, D., & Paulraj, A. (2000). MIMO wireless channels: Capacity and performance prediction. In *Global Telecommunications Conference, 2000. GLOBECOM'00. IEEE* (Vol. 2, pp. 1083-1088).

[5] Garg, V. (2010). *Wireless communications & networking*. Morgan Kaufmann.

[6] Scholtz, R. (1993, October). Multiple access with time-hopping impulse modulation. In *Milcom. IEEE* (Vol. 93, No. 1993.2, pp. 447-450).

[7] Paulraj, A. J., Gore, D., Nabar, R. U., & Bölcskei, H. (2004). An overview of MIMO communications-a key to gigabit wireless. *Proceedings of the IEEE*, *92*(2), 198-218.

[8] Paulraj, A., Nabar, R., & Gore, D. (2006). Introduction to space-time wireless communications. Cambridge university press.

[9] Biglieri, E., Calderbank, R., Constantinides, A., Goldsmith, A., Paulraj, A., & Poor, H. V. (2007). MIMO wireless communications. Cambridge university press.

[10] Belfiore, J. C., Rekaya, G., & Viterbo, E. (2005). The Golden code: A 2×2 full-rate space-time code with nonvanishing determinants. *IEEE Transactions on information theory*, *51*(4), 1432-1436.

[11] Alamouti, S. M. (1998). A simple transmit diversity technique for wireless communications. *Selected Areas in Communications, IEEE Journal on*, *16*(8), 1451-1458.

[12] Heath Jr, R. W., & Paulraj, A. J. (2005). Switching between diversity and multiplexing in MIMO systems. *Communications, IEEE Transactions on*, *53*(6), 962-968.

[13] Muquet, B., Biglieri, E., & Sari, H. (2007, March). MIMO link adaptation in mobile WiMAX Ssystems. In *Wireless Communications and Networking Conference, 2007. WCNC 2007. IEEE* (pp. 1810-1813).