

# **Medical Image Enhancement through Intuitionistic Fuzzy Sets**

**Amar Mahdi Mustafa**

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Approval of the Institute of Graduate Studies and Research

---

Prof. Dr. Mustafa Tümer  
Acting Director

I certify that this thesis satisfies the requirements as thesis for the degree of Master of Science in Computer Engineering.

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Prof. Dr. Işık Aybay  
Chair, Department of Computer Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Computer Engineering.

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Asst. Prof. Dr. Adnan Acan  
Supervisor

---

Examining Committee

1. Asst. Prof. Dr. Adnan Acan

2. Asst. Prof. Dr. Mehmet Bodur

3. Asst. Prof. Dr. Ahmet Ünveren

## **ABSTRACT**

A contrast enhancement of medical, color and Grayscale, images via intuitionistic fuzzy sets on different types of entropy – based methods have been studied. Fuzzy set concept counts vagueness in the formula of the membership functions. Intuitionistic fuzzy sets count fuzziness in the membership and non-membership functions. Various entropy – based methods are applied as enhancement operators, and the enhanced image is the one that is interpreted based on the used intuitionistic fuzzy membership function. As medical images include too much ambiguity, this study demonstrated that the intuitionistic fuzzy sets are shown to be useful tools implemented for medical image enhancement. To determine the efficiency of the studied methods, experimental results associated with the handled entropy methods are presented in thesis. Experiments on several image libraries indicate that the spatial entropy method among several applications performs better than it is alternatives.

**Keywords:** Intuitionistic Fuzzy Set; Fuzzy Entropy, Spatial Entropy, Contrast Enhancement, Image Quality Measurement.

## ÖZ

Renkli veya gri tonlu medikal görüntülerin karışıklık iyileştirilmesi için entropi tabanlı yöntemlere dayalı sezgisel bulanık kümelerin kullanımı çalışıldı. Bulanık küme mantığı belirsizliği üyelik işlevi üzerinden modeller. Sezgisel bulanık kümeler ise belirsizliği üyelik ve üye olmama işlevleri üzerinden modeller. Çeşitli entropi tabanlı yöntemler karışıklık iyileştirme operatörleri olarak uygulandı ve elde edilen görüntü sezgisel bulanık üyelik işlevleri kullanılarak yorumlandı. Medikal görüntüler çok fazla belirsizlik içerdiğinden, bu çalışma sezgisel bulanık kümelerin medikal görüntülerin iyileştirilmesinde kullanışlı araçlar olduğunu gösterdi. Kullanılan yöntemlerin etkinliğini göstermek için ele alınan entropi tabanlı yöntemlere yönelik deneysel sonuçlar sunuldu. Değişik medikal görüntü kütüphaneleri kullanılarak yapılan deneyler alansal entropi yönteminin diğer kullanılan yöntemlerden daha başarılı olduğunu gösterdi.

**Anahtar Kelimeler:** Sezgisel Bulanık Küme, Bulanık Entropi, Mekansal Entropi, Kontrast Geliştirme, Görüntü Kalitesi Ölçüm.



## DEDICATION

To my beloved *Mom* and my precious *Dad*  
The reason of what I become today

&

To my dear *brothers* and *sisters*  
I am really grateful for your encouragement

&

To my lovely *wife*  
You have been my inspiration, and my soul mate

&

To My angel (*Mohammed*)

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# Chapter 1

## INTRODUCTION

### 1.1 Background Study

The field of medical imaging had great advancements with the transition from analog to digital technology [1]. Producing remarkable outcomes, for instance, lower time spent by consultants, decreasing differences in intra-and inter-inspector, additional thoughts to non-experts and instructional systems, lead to the automation of image analysis tasks. In every automated system, for the conception of images and analysis, the recognition of items and sometimes the relationship between them are required. Thus the automated recognition of assemblies is an exploration region of image processing that is still a hot research area.

The computer's processing power is exceptionally higher than the humans in tasks that involve primarily activities of calculation. In contrast, in analysis and recognition tasks, the human brain has an incredible capacity that has carefully been achieving through any computational system. Taking into account, humans have a unique capability to recognize the items that are critical and, among them, those that denote the core of interest for a certain condition. For a given scene, people have a unique capacity to recognize the items that are critical and, among them, those that speak to the center of enthusiasm for a specific circumstance [2].

Transforming the images into the numerical format is the primary assignment of image processing. A multiplicity of issues in image processing ranges from image understanding to image analysis. Low- scale image processing is interested with the pixels, which primarily enhance and or filter the image, and it covers the preliminary processing of the image. The following scale of the image processing is interested with larger areas in the image. The afterward process is high-scale processing where the entire or sections of the image or chains of the images are considered. The images had divided into segments for organization and identification and lastly explanation [1].

Traditionally, specialists sometimes transact with images in which the visible conception is not adequate for recognition of various specific structure. Accordingly, at every new image examined, a doctor utilized an enormous quantity of cumulative information gained over the years of medical preparation for conceptual schemes so as to perform this task. This manner is a natural response and a situation base logic system, as every new image evaluated provides the further specialist information about a particular problem. A system of medical support for recognition and examination of structures must have the ability to consolidate and employ information about the problem range [2].

Medical image analysis is a broad concept that includes several processing and analysis methods applied to some different imaging modalities [3]. The most common imaging modalities would briefly be presenting below:

- X-rays

X-rays are electromagnetic radiation that attenuates differently within different substances. There is a wide attenuation in bones which means that a smaller value of x-rays reaches the detector behind the bones, accordingly, appears on the images. Various soft tissues in the body attenuate an equivalent quantity of x-ray, that is why it is hard to recognize e.g. organs from each other. The lungs are visible because of the low attenuation in the air contrasted to the attenuation in tissue. Some samples of examinations are skeletal x-ray, mammography, and chest x-ray.

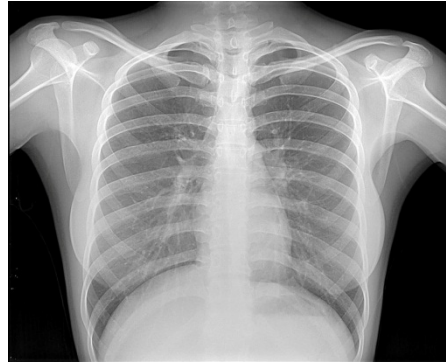
- Computed Tomography

Computed tomography (*CT*) is an imaging technique where the x-ray tube rotates around the body, and the rays are detected by a stationary circular array of detectors. The images are reconstructed using measurements of the transmitted x-rays through the body and mathematical models; see Figure 1.1(b) for an example of a *CT* image. In this approach, the body can be inspected in slices from many directions. There is a greater contrast resolution in *CT* images than in ordinary x-ray images which means that various organs could be separated in *CT* images. *CT* could be used for exposing tumors, head infarctions, and intestinal diseases.

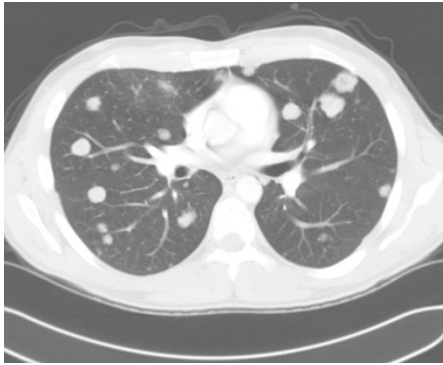
- Magnetic Resonance Imaging

In magnetic resonance imaging (*MRI*) the body must be placed inside a magnetic field. By entering an electromagnetic field to stimulate a reverberation among the protons (fundamentally in water molecules inside the body), a radio frequency signal is emitted and can be detected via a receiver coil. Through mathematical models, these signals are transformed into cross-sectional images of the body. Mainly soft

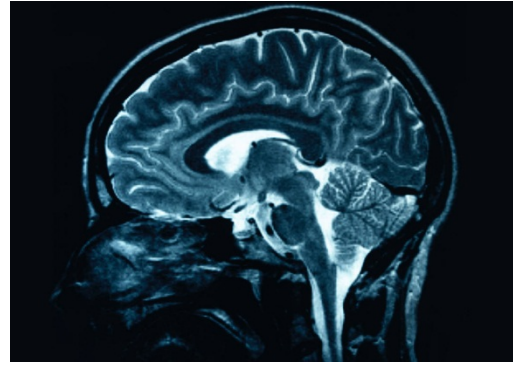
tissue is attractive in these images because of the high contrast resolution. In Figure 1.1(c) an MR image of the brain can be seen [6].



(a)



(b)



(c)

Figure 1.1: Examples of Images from Different Modalities.  
(a) X-Ray image [22], (b) CT image [22], (c) MR image [32]

## 1.2 Statement of the Problem

In Medical images, due to poor contrast, lots of structures are usually not visible. Many areas or borders are unclear or fuzzy in origin. Thus, medicinal image enhancement is an essential operation. In an improved image, it suits better for professionals or clinicians to promote the exceptions in x-rays, computed tomography scans, mammograms or *MR* images [4]. Therefore, medical images need the successive application of numerous image processing operations, for instance, restoration, enhancement, regularization, segmentation and registration, to be applied

for quantification and analysis of proposed characteristics. The features extracted during image processing operations are particular components of the image, like specified tissues, tumors, or lesions, along with some statistical property over the whole image domain or parts of it [1]. Fundamentally, image enhancement contains contrast enhancement and rim enhancement. The aim of enhancement methods is to enhance the complete visible contrast of the image that enables the human eye to envisage obviously and be extremely appropriate for extra tests. It is advantageous while the density of critical areas of images like tissues. Contrast enhancement highlights the regions of low strength, therefore enhancing the readability. Edge enhancement highlights the ends of the irregular lesions or every texture in the images, specifically the images where the ends are not evidently detectible. The preliminary processing phase of image enhancement includes the elimination every noise extant from the images.

There are numerous crisp procedures on image development, and one of the exceedingly common processes is histogram equalization. However, due to uncertainties in pixel interpretation, crisp enhancement methods usually do not improve the image quality satisfactorily. Fuzzy set based methods already are proposed by various authors to tackle vagueness in pixel interpretation. However, fuzzy enhancement methods do not also provide satisfactory results for image enhancement. Primarily for real-time images including noise and device artifacts, this deficiency can be because of fuzzy methods, in general, estimate solely one uncertainty that is a sort of a membership function. Thus, following fuzzy set concepts, for instance, Intuitionistic Fuzzy Sets and Type II fuzzy sets that examine more ambiguities should be utilized in image enhancement to acquire superior outcomes [4]. There are several publications on image enhancement via the

Intuitionistic fuzzy sets. In this respect, intuitionistic fuzzy contrast enhancement was also suggested by Vlachos [7] where intuitionistic fuzzy entropy has also been applied. In this study, utilization of intuitionistic fuzzy sets with different types of entropy methods to enhance the contrast of medical images is studied. The proposed entropy methods were tested on several (Medical; Color and Gray) images, and the outcomes are comparatively evaluated.

### **1.3 Purpose of the Study**

The aim of this study is to convert the inventive medical images into a different image that is more appropriate for additional processing by using image enhancement through intuitionistic fuzzy sets, as a result of this, physicians will have the ability to diagnose illnesses more accurately, and this will benefit the field of medicine as a whole.

### **1.4 Significance of the Study**

Image enhancement, which had necessary preprocessing on every image, performs a fundamental part in image processing where human professionals make crucial judgments center of image knowledge. The main objective of image enhancement is to convert an image to an alternative form that is more appropriate for advanced processing and interpretation [4].

## Chapter 2

### RELATED WORKS TO MEDICAL IMAGE ENHANCEMENT

#### 2.1 Introduction

Medical imaging is a general name for the widely-used techniques developed to create images of the human body for medical purposes. As acquired images could involve complete human body, they can also span it partially. Medical imaging data had used for revealing normal or abnormal physiological and anatomical structures. Medical imaging techniques had also been employing in diagnosis and treatment planning processes of patients suffering from many health problems. Professionals from the field of medicine make use of medical imaging data to guide or avoid medical intervention.

#### 2.2 Medical Image Processing

Image processing is a subfield of signal processing, for which the input signal is an image and the outputs are again an image and/or various parameters defining the characteristics of the image and applied operations. Medical image processing has been applied to the images acquired by medical imaging techniques, such as *CT*, *MRI*, and X-Ray [8, 9].

Quantitative analysis of medical images is crucial for diagnosis and prognosis stages of many diseases and abnormalities. Quantification of radiographic information includes various features such as linear measurements, estimation of cross section

and surface areas, volume quantization, estimate tissue density, monitoring tumor growth, verification of treatment, and comparison of patient's data with anatomical atlases. Medical images are post-processed for many purposes, such as denoising, restoration, segmentation, registration, and enhancement [10, 11].

As briefly detailed in the first chapter, the main aim of this thesis is to convert medical images into an enhanced form that is more appropriate for additional processing through using intuitionistic fuzzy sets combined with different types of entropy methods. The main objective is that physicians will be able to diagnose illnesses more accurately. Following sections of this chapter introduce stages that are essential in medical image handling and previously specified modules of operations used in medical image processing in the characteristics of mathematical methodology in the literature.

### **2.2.1 Contrast Enhancement of Images**

The purpose of image enhancement is to modify the raw image into a different image that is further proper for extra processing. Through making the dark pixels more dusky and luminous pixels more luminous, the contrast of the image is incremented. As the images, particularly the medical images, are not brightened correctly and several textures are not obviously noticeable, contrast enhancement emerges the textures in the image. Initially, the images are filtered to erase any distortion existing.



### **2.2.2 Segmentation of Image**

“Segmentation is one of the most important steps in medical image processing that has usually done after enhancement. It extracts any clot/abnormal lesion or blood cells/blood vessels present in an image” [1]. Every area in a segmented image owns comparable characteristics on features, for example, gray scale, structure or color, and the property are dissimilar for diverse areas in an image. It divides the image into split sets equivalent to items in the sight, which are critical in recognizing various kinds of leukocytes or calculating Blood Vessels or resulting in the magnitude of the tumor or some further defects existent in the human body.

### **2.2.3 Detecting of Boundary**

“Boundary detection is an important process in medical image processing. It finds the structural information of the image, thus drastically reducing the data to be processed” [1]. Edge is existent while there is a variation in gray scale density. Edge image might have labeled as a gradient image. The borders specify the position and the character of the items, for instance, unusual lesions or tumor or Blood cells or Vessels or some further structures existent in an image. The ends in medical images are not identical because of poor contrast and various illuminations. Therefore, in the beginning, the edges are occasionally improved by executing edge detection methods.

### **2.2.4 Morphology**

“Morphology is a non-linear image processing technique that deals with the shape of the image features” [1]. Sometimes, there is a need to obtain a skeleton of an image or to dilate/erode an image or close/open an image to remove or fill any holes or gaps present in the image. It may be used to find the gradient of an image. It uses a

structuring element of any size and shape which is applied on an image to perform the operation.

### **2.3 Some Methods on Medical Image Enhancement**

It has well known that much vagueness is existent in the image. Particularly for medical images, due to poor illumination and various arrangements, many of the image borders/sections/areas are unclear. So, it becomes difficult to segment or to expose the borders of the structures in the image for proper diagnosis. There are various techniques of image enhancement, and one of the most known non-fuzzy techniques is histogram equalization.

### **2.4 Contrast Enhancement of Images**

Contrast is a field that has relied on the personal view. A likely explanation of contrast is

$$C = \frac{(N - M)}{(N + M)} \quad (2.1)$$

where  $N$  and  $M$  are the median gray scales of the two areas where the variation has calculated. Contrast enhancement is implemented on images where the contrast among the items and the background is faint, which is when the items are adequately analogous with the background. The aim of contrast enhancement is to move shady areas more dark and luminous areas more luminous, but no contrast enhancement is essential while the contrast of the image is enhanced.

Contrast enhancement of an image using fuzzy sets based methods use gray-scale plotting from a crisp (gray) scale to a fuzzy scale applying a particular membership transformation. Focused on a user-defined threshold,  $T$ , the contrast is prolonged in

such a manner that the gray scales of the threshold  $T$  have decreased, and the gray scales overhead the threshold  $T$  have grown in a non-linear fashion. This expansion procedure causes congestion at both ends (gray scales) [25]. Fuzzy image processing is a collection of different fuzzy approaches to image processing that can understand, represent, and process the images. It has three main stages, namely, image fuzzification, modification of membership values (Exponential, Triangular, Gaussian, Gamma, etc.), and if necessary, image defuzzification, as shown in Figure 2.1.

Fuzzification and defuzzification are the important steps that are required in processing the images with fuzzy techniques. The main part of fuzzy image processing lies in the membership plane, as shown in Figure 2.2.

Membership degree is the degree of belongingness of the pixels in an image. Fuzzification is the transformation of the gray level of the image to a membership function using some mathematical function. After fuzzification, depending on the user's requirements, the membership values of the gray level are modified using an appropriate fuzzy technique. Techniques for modifying the membership values of the pixels in an image may involve fuzzy rule-based method, fuzzy linguistic hedges, fuzzy integration, and so on [5].

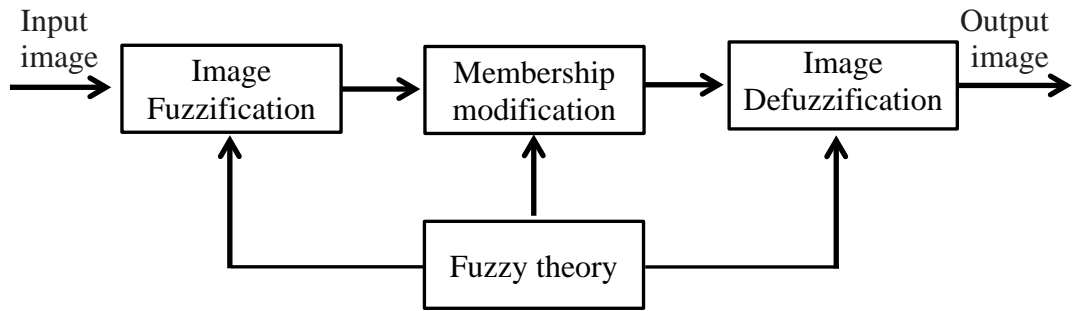


Figure 2.1: Structure of Fuzzy Image Enhancement [5]

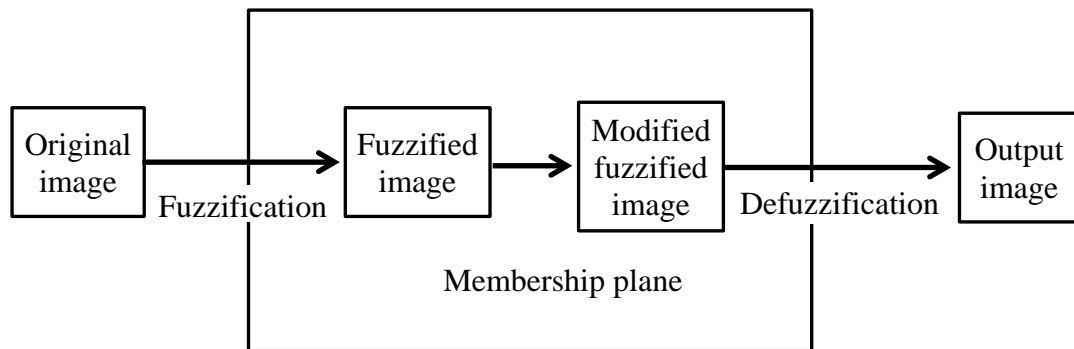


Figure 2.2: Steps of Fuzzy Image Enhancement [5]

## 2.4.1 Fuzzy Techniques in Contrast Enhancement

For contrast enhancement, different fuzzy techniques are explained briefly in this part.

### 2.4.1.1 Contrast Enhancement Using the Intensification Operator

In this approach, the membership values are changed by applying an intensifier. Primarily, the membership function is chosen that discovers the membership values of the pixels of an image. Then the conversion of the membership values above 0.5 to greater values and membership values lower than 0.5 lower values is carried out in a non-linear fashion to achieve a good contrast in an image [15].

#### **2.4.1.2 Contrast Enhancement Using Fuzzy Histogram Hyperbolization**

The concept of fuzzy histogram hyperbolization was discussed by Tizhoosh and Fochem [19]. Initially, a membership function is selected that finds the membership values of the pixels of an image. A fuzzified beta,  $\beta$ , which is a linguistic hedge, is set to modify the membership function. Hedges [17, 18] may be brilliant, medium bright, etc., and the selection has made by the user's needs. The value of beta may be in the range  $\beta \in [0.5, 2]$ . Depending on the value of  $\beta$ , the operation may be dilution or concentration. If the image is a low-intensity image, then the fuzzified  $\beta$  after operating on the membership values will produce slightly bright or quite bright images.

#### **2.4.1.3 Contrast Enhancement Using IF-THEN Rules**

The fuzzy rule-based approach is such a method that incorporates human intuitions which are non-linear in nature, and these have not been easily characterized by traditional modeling. As it was challenging to define a precise or crisp condition under which enhancement had applied, the fuzzy set theoretic approach is well studied to this solution. The rule-based approach incorporates fuzzy rules into the conventional methods. A set of conditions on the pixel values and the pixel neighborhood (if it requires) are defined, and these conditions will form the antecedent part of the IF-THEN rules.

#### **2.4.1.4 Contrast Enhancement Using Fuzzy Expected Value**

The utility of the fuzzy expected value (FEV) in image enhancement had proposed by Schneider and Craig [20] where the image quality is improved in terms of the distance between the gray levels and the FEV. The fuzzy expected value by Friedman [21] replaces the mean and median value with a more representative value

or 'typical value' when treating with fuzzy sets. This value would indicate an average grade of membership to a fuzzy set.

Figure 2.3 illustrate that the contrast enhancement of blood vessels applying four fuzzy methods for image enhancement on low-contrast blood vessel images.

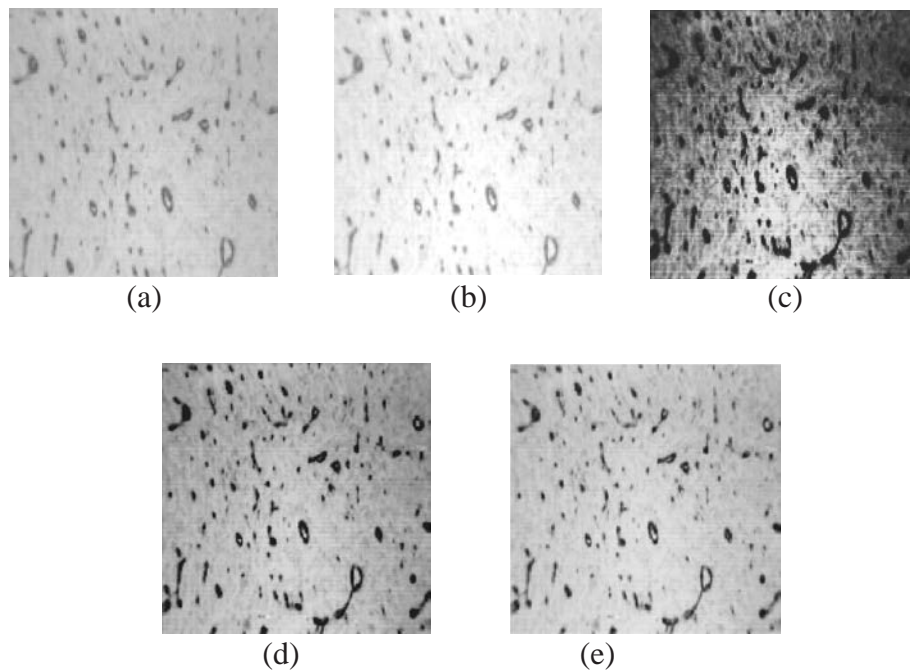


Figure 2.3: Contrast Enhancement of Blood Vessel [1]  
(a) Blood Vessel Picture, (b) Enhancement applying the Fuzzy Expected value (c) Enhancement employing the INT operator, (d) Enhancement applying the NINT operative and (e) enhancement applying histogram hyperbolization

## Chapter 3

### THE RESEARCH PROCEDURES

This section will provide an illustration of procedures of this thesis; they are basically the reasons why the particular methodology was selected, the choice of medical image enhancement techniques, image quality assessment methods, measuring the quality of distorted images and removal the distortions from images.

#### 3.1 Image Enhancement

In Chapter 1 and Chapter 2, it has been explained that the crucial role of image enhancement for image processing is to help the physician to make critical resolutions built on image facts. As it is well-known, medical images contain uncertainties, many region/boundaries are fuzzy in nature, and many structures are not obviously detectable. Consequently, crisp enhancement methods such as, histogram equalization may not improve the images correctly and fuzzy set theory will not perform satisfactory results as well, especially in the case of real-time images, since the statistic that fuzzy procedures consider is a generally based on one ambiguity which is in the method of a membership function. Therefore, progressive fuzzy set theory, for instance, Intuitionistic Fuzzy Sets and type II Fuzzy Sets, achieve better results by using different types of entropy to improve medical images so as to obtain better results.

### 3.2 Intuitionistic Fuzzy Sets

Based on Zadeh's description of fuzzy sets concept [13], where ambiguity or fuzziness is measured solely in the method of the membership function, various concepts of higher-order fuzzy sets have been offered by different scientists. Among them, Intuitionistic Fuzzy Sets suggested via Atanassov, has been an appropriate tool for modeling the indecision rising from vague/inadequate knowledge. This hesitation is because of the deficiency of information or the individual mistake in determining the membership function.

Intuitionistic Fuzzy Sets are represented by two specific features, called the membership and the non-membership, explaining the membership or non-membership of a component sequentially [24].

One of the Basic definitions associated with Intuitionistic fuzzy sets is as follows:

A fuzzy set  $A$  on a universal set  $X = \{x_1, x_2, \dots, x_n\}$  was representing as:

$$A = \{(x, \mu_A(x), x \in X) \tag{3.1}$$

where the function  $\mu_A(x): X \rightarrow [0, 1]$  is a degree of the range of belongingness or membership function of a factor  $x$  in the universal set  $X$ , and the quantity of non-membership is

$$1 - \mu_A(x) \tag{3.2}$$



Atanassov suggested that while defining the membership degree, there may be some hesitation, which arises due to the lack of knowledge [24]. Hence, using the preface of hesitation grade,  $\pi_A(x)$ , the non- membership grade is not the supplement of the membership degree as in a fuzzy set, relatively less than or equivalent the correlate of membership degree. An Intuitionistic Fuzzy Set  $A$  in a limited set  $X$  was representing as:

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\} \quad (3.3)$$

Where,  $\mu_A(x), \nu_A(x): X \rightarrow [0, 1]$  are sequentially the membership and the non-membership functions of an component  $x$  with the essential condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad (3.4)$$

and

$$\pi_A(x) + \mu_A(x) + \nu_A(x) = 1 \quad (3.5)$$

Some basic operations on intuitionistic fuzzy sets

Let  $A = (\mu_A, \nu_A)$  and  $B = (\mu_B, \nu_B)$  be IFSs of  $X$ . Then:

$$(1) \text{ [Inclusion]} A \subseteq B \leftrightarrow \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x), \forall x \in X$$

$$(2) \text{ [Intersection]} A \cap B = \{(x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)) \mid x \in X\}$$

$$(3) \text{ [Union]} A \cup B = \{(x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)) \mid x \in X\}$$

$$(4) \text{ [Addition]} A \oplus B = \{x, \mu(x) + \mu_B(x) - \mu_A(x) \mu_B(x), \nu_A(x) \nu_B(x) : x \subseteq X\}$$

$$(5) \text{ [Multiplication]} A \otimes B = \{x, \mu(x) \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \nu_B(x) : x \subseteq X\}$$

$$(6) \text{ [Complement]} A^c = \{x, \nu_A(x), \mu(x) : x \in X\}$$

For our convenience we shall use the notation  $A(x) \geq B(x)$ , when  $\mu_A(x) \geq \mu_B(x)$  and  $\nu_A(x) \leq \nu_B(x)$  for all  $x \in X$ .

### **3.3 Image Enhancement using Intuitionistic Fuzzy Methods**

There is more than one technique for improving medical images by using Intuitionistic Fuzzy as listed below:

- Entropy – Based Enhancement Methods [7]
- Two – Dimensional Entropy – Based *IF* Enhancement (Method II) [53]
- Entropy – Based Enhancement Method by Chaira (Method III) [12]
- Contrast Enhancement by Chaira (Method IV) [57]
- Hesitancy Histogram Equalization [26]

In this thesis, various entropy – based methods are applied as enhancement operators, and the enhanced image is the one that is interpreted based on the used intuitionistic fuzzy membership function, to determine the efficiency of the studied methods, experimented results associated with the handled entropy methods are presented, as illustrated in details in next section.

### **3.4 Intuitionistic Fuzzy Entropy**

Entropy is a measure of fuzziness in a fuzzy set. Zadeh first introduced as the term of fuzzy entropy in [13]. Kaufmann used the distance measure to define fuzzy entropy [27], while Yager defined entropy like distance from a fuzzy set and its complement [28]. Likewise, for the IFS, Intuitionistic Fuzzy Entropy (IFE) provides the quantity of fuzziness or uncertainty in a set. Several writers assigned IFE in various styles. Two descriptions of the entropy of IFS were supplied by Burillo and Bustince and Szmidt and Kacprzyk. These two descriptions have various structures. Burillo and Bustince defined entropy for the first time concerning the degree of intuitionism of

IFS [29]. Szmidt and Kacprzyk defined entropy regarding the non-probabilistic type of entropy [30]. In IFS, three factors must be considered with

$$\mu_A + \nu_A + \pi_A = 1, \quad 0 \leq \mu_A, \nu_A, \pi_A \leq 1 \quad (3.6)$$

The attributes of IFE by Burillo and Bustince [29] are

A real function  $IFE = IFSs(X) \rightarrow [0, 1]$  is called IFE on IFSs(X) if

1.  $IFE(A) = 0, \forall A \in FS(X)$ .
2.  $IFE(A) = \text{Cardinal}(X) = n$ , if  $\mu_A(x_i) = \nu_A(x_i) = 0, \forall x_i \in X$ , that is, the entropy is extreme if the set is completely intuitionistic.
3. If the membership and non-membership of every component increment, leading to incrementing the value; in this manner, the vagueness will rise, and the entropy will decrement. It can be written as

$$IFE(A) \geq IFE(B) \text{ if } \mu_A(x_i) \leq \mu_B(x_i) \text{ and } \nu_A(x_i) \leq \nu_B(x_i) \quad (3.7)$$

4.  $IFE(A) = IFE(A^C)$ .

Entropy can describe as

$$IFE(A) = \sum_{i=1}^n \pi_A(x_i) \quad (3.8)$$

Where  $\pi_A(x_i)$  is called the intuitionistic index or degree of indeterminacy of  $x_i$  to  $A$ .

### 3.4.1 Entropy – Based Enhancement Techniques

In *IF* enhancement, both membership and non-membership values of an *IF* image are required to determine. To find the degrees, an optimum value of the constant parameter is required. Entropy-based methods used *IF* entropy to find the optimum value of the constant term.

One of these methods was proposed by Vlachos and Sergiadis [7]. The image is primarily fuzzified. Thus an Intuitionistic Fuzzy image is formed applying the membership and non- membership functions. An Intuitionistic Fuzzy image has formulated as

$$A_{IFS} = \{x, \mu_A(g), \nu_A(g)\}, g \in \{0, 1, 2, \dots, L-1\} \quad (3.9)$$

Where  $g$  is the gray level and  $L$  is a maximum of gray level.

An image (say  $X$ ) of size  $M \times N$  is initially fuzzified using the following formula:

$$\mu_A(g) = \frac{g - g_{\min}}{g_{\max} * g_{\min}} \quad (3.10)$$

$g_{\min}$  and  $g_{\max}$  are the minimum and maximum values of the gray levels of the image respectively, based on the fuzzy set, the membership degree of the intuitionistic fuzzy image is calculated as

$$\mu_{IFS}(g; \lambda) = 1 - (1 - \mu_A(g))^{\lambda-1} \quad (3.11)$$

As  $\lambda$  is not fixed for all the images, the optimum value of  $\lambda$  is obtained using *IF* entropy. The optimum values are calculated using different entropies. Using standard fuzzy negation,  $\varphi(x) = (1 - x)^\lambda$ ,  $\lambda \geq 1$ , the non-membership function is given as

$$v_{IFS}(\mathbf{g}; \lambda) = (1 - \mu_A(\mathbf{g}; \lambda))^{\lambda(\lambda-1)} \quad (3.12)$$

The hesitation degree is

$$\pi_{IFS}(\mathbf{g}; \lambda) = 1 - \mu_{IFS}(\mathbf{g}; \lambda) - v_{IFS}(\mathbf{g}; \lambda) \quad (3.13)$$

There are many types of entropies suggested by different authors; some of them are listed below:

#### 3.4.1.1 Various Types of Entropies

- 1- Burillo and Bustince Entropy I (method I) [29]

$$E_1(A_{IFS}) = \frac{1}{M \times N} \sum_{j=0}^{M-1} \sum_{i=0}^{N-1} (1 - \mu_A(\mathbf{g}_{ij}) - \nu_A(\mathbf{g}_{ij})) e^{1 - (\mu_A(\mathbf{g}_{ij}) + \nu_A(\mathbf{g}_{ij}))} \quad (3.14)$$

- 2- Vlachos and Sergiadis Entropy (method II) [7]

$$E_2(A_{IFS}) = \frac{1}{M \times N} \sum_{j=0}^{N-1} \sum_{i=0}^{M-1} \frac{2\mu_A(\mathbf{g}_{ij}) \nu_A(\mathbf{g}_{ij}) + \pi_A^2(\mathbf{g}_{ij})}{\pi_A^2(\mathbf{g}_{ij}) + \mu_A^2(\mathbf{g}_{ij}) + \nu_A^2(\mathbf{g}_{ij})} \quad (3.15)$$

- 3- Burillo and Bustince Entropy II (Method III)

$$E_3 (A_{IFS}) = \frac{1}{M \times N} \sum_{j=0}^{N-1} \sum_{i=0}^{M-1} \pi_A(g_{ij}) e^{(1-\pi_A(g_{ij}))} \quad (3.16)$$

Intuitionistic fuzzy entropy (*IFE*) is calculated from any of the entropies for all the  $\lambda$  values. The optimum value of  $\lambda$  that corresponds to the maximum value of the entropy values is written as

$$\lambda_{opt} = \max(\text{IFE}(A_{IFS}; \lambda)) \quad (3.17)$$

Where  $\lambda_{opt}$  is the optimum value of  $\lambda$ . So, in the *IF* domain, the image is represented as

$$A_{IFS\_opt} = \{g, m_A(g; \lambda_{opt}), v_A(g; \lambda_{opt}) \mid g \in \{0, 1, \dots, L-1\}\} \quad (3.18)$$

Atanassov's operator is applied to  $A_{IFS\_opt}$  to deconstruct an *IF* image to a fuzzy image. With different values of  $\alpha$ , different images are obtained in the fuzzy domain.

Atanassov's operator is written as [31]

$$D_\alpha (A_{IFS\_opt}) = \{x, \mu_A(x) + \alpha \pi_A(x) + \mathcal{V}_A(x) + (1-\alpha) \pi_A(x) \mid x \in X\}, \alpha \in [0, 1] \quad (3.19)$$

Where  $D_\alpha$  can be called as the Atanassov's operator

The maximum index of fuzziness intuitionistic defuzzification [33] is used to select the optimum value of  $\alpha$ . In computing the maximum index of defuzzification, the linear index of fuzziness is required. The linear index of fuzziness of fuzzy set  $A$  is

$$\gamma_I(x) = \frac{1}{2|X|} \sum_{i=1}^n \min(\mu_A(x_i), 1 - \mu_A(x_i)) \quad (3.20)$$

Where  $|X|$  is the cardinality of  $X$ ,  $n = |X|$ . Substituting min t-norm with the product operator, the modified index of fuzziness is written as

$$\gamma_i(x) = \frac{1}{2|X|} \sum_{i=1}^n \mu_A(x_i)(1 - \mu_A(x_i)) \quad (3.21)$$

To find  $\alpha_{opt}$ , the maximization index of fuzziness has desired:

$$\alpha_{opt} = \max_{\alpha \in [0,1]} \{\gamma(D_\alpha(A_{IFS_{opt}}))\} \quad (3.22)$$

where

$$\gamma_{D_\alpha}(A_{IFS_{opt}}) = \frac{1}{4MN} \sum_{g=0}^{L-1} h_A(g) (\mu_{D_\alpha(A_{IFS_{opt}})}(g) \cdot (1 - \mu_{D_\alpha(A_{IFS_{opt}})}(g))) \quad (3.23)$$

Where  $h_A$  is the crisp histogram of the image after fuzzification

$$= \frac{1}{4MN} \sum_{g=0}^{L-1} h_A(g) (\mu_A(g; \lambda_{opt}) + \alpha \cdot \pi_A(g; \lambda_{opt})) (1 - \mu_A(g; \lambda_{opt}) - \alpha \cdot \pi_A(g; \lambda_{opt})) \quad (3.24)$$

Finally, the image in the grey-level domain is written as

$$g' = (L - 1) \mu_{D_{\alpha_{opt}}(A_{opt})}(g) \quad (3.25)$$

$$\mu_{D_{\alpha(A_{IFS_{opt}})}(g) = \alpha_{opt} + (1 - \alpha_{opt}) \mu_A(g; \lambda_{opt}) - \alpha_{opt} \nu_A(g; \lambda_{opt}) \quad (3.26)$$

$g$  and  $g'$  are the initial and final intensity levels of the image, respectively

An illustration of enhancement a medical image by employing the intuitionistic fuzzy entropies technique is displayed in Figure 3.1 to explain the efficiency of the methods. The way we measure image enhancement will be explained in details in section 3.6.

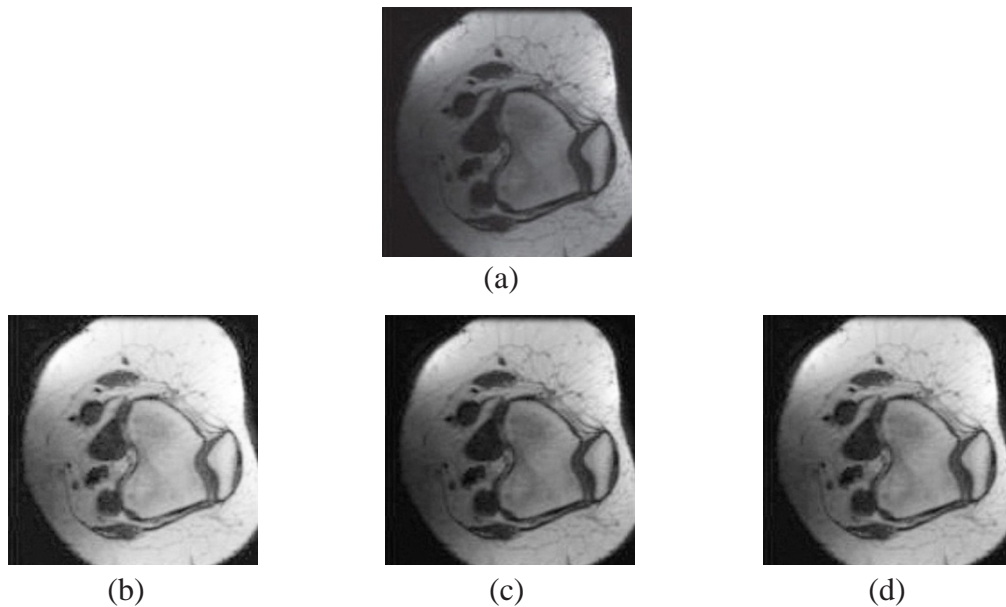


Figure 3.1: Knee Patella Image Enhancement [1]  
(a) Image of Knee Patella, (b) Enhancement utilizing the *IF* Entropy (Method I) (c) using the *IF* Entropy (Method II) and (d) using the *IF Entropy* (Method III)



### 3.5 Spatial Entropy - Based Contrast Enhancement

Another method of Entropy that we compared with Fuzzy Entropy methods (shown in details in the previous section) is Spatial Entropy - based Contrast Enhancement.

The mutual information between spatial location distributions of gray levels of an image is used to obtain a function which is further mapped to a uniform distribution to achieve contrast enhancement. The proposed global contrast enhancement algorithm is named as “Spatial Entropy-based Contrast Enhancement (SECE)”. SECE produces naturally looking global contrast enhancement without any parameter selection. Furthermore, the algorithm does not alter the information structure of the processed histogram with respect to the original histogram. Thus, it results in naturally looking enhancement given that the original image has a margin for contrast improvement with no apparent distortions on it. In order to achieve both global and local contrast enhancements at the same time as shown below in Figure 3.2. Transform domain (two-dimensional discrete cosine transform (2D-DCT)) coefficients of an image globally enhanced by SECE is further weighted followed by inverse transform (inverse 2D-DCT) to obtain an output image which is contrast enhanced both globally and locally. This method is able to perform both global and local contrast enhancement without any visual artifacts. This algorithm is named as “Spatial Entropy-based Contrast Enhancement in DCT (SECEDCT)”. SECEDCT is a generalization of SECE and allows controlling the level of local contrast enhancement. Thus, zero-level of local contrast enhancement transforms SECEDCT to SECE [34].

Spatial histogram of a gray-level of an image: Let  $X = \{x_1, x_2, \dots, x_K\}$  be the sorted set of all possible  $K$  gray-levels that exist in an input image  $X$  where  $x_1 < x_2 < \dots < x_K$

, where  $K$  is the number of the distinct gray-scales. The 2D spatial histogram of the gray-level  $x_k$  on the spatial grid of  $X$  is computed as

$$h_k = \{ h_k(m, n) \mid 1 \leq m \leq M, 1 \leq n \leq N \} \quad (3.27)$$

Where  $m, n \in \mathbb{Z}^+$ ,  $h_k(m, n) \in [0, \mathbb{Z}^+]$  is the number of occurrences of the gray-level  $x_k$  in the spatial grid located on the image region of  $\left[ (m-1)\frac{H}{M}, m\frac{H}{M} \right] \times \left[ (n-1)\frac{W}{N}, n\frac{W}{N} \right]$ . The total number of the grids on 2D histogram is  $MN$  which is dynamically estimated using the number of distinct gray-levels  $K$  and the aspect ratio  $r = \frac{M}{N} = \frac{H}{W}$ , i.e.

$$N = \left\lceil \left( \frac{K}{r} \right)^{1/2} \right\rceil, M = \lfloor (K_r)^{1/2} \rfloor \quad (3.28)$$

Where the operator  $\lfloor \cdot \rfloor$  rounds its argument toward the nearest integer. In forming 2D spatial histogram  $h_k$  of gray-level  $x_k$ , the aspect ratio of the original image is protected on spatial grids. In this way, spatial characteristics of pixels are protected informing of 2D spatial histogram [34].

Spatial Entropy and distribution function by using the 2D spatial histogram  $h_k$ , entropy measure  $S_k$  is computed for gray-level  $x_k$  according to

$$S_K = - \sum_{m=1}^M \sum_{n=1}^N h_k(m, n) (\log_2)(h_k(m, n)) \quad (3.29)$$

which is used to compute a discrete function  $f_k$  according to

$$f_k = S_k / \sum_{l=1, l \neq K}^K S_l \quad (3.30)$$

The discrete function  $f_k$  measures the relative importance of the gray-scale  $x_k$  with respect to the rest of the gray-scales  $x_l$ ,  $l \neq k$ ,  $l = 1 \dots K$ . The discrete function  $f_k$  is further normalized according to

$$f_k \leftarrow f_k / \sum_{l=1}^K f_l \quad (3.31)$$

Moreover, accumulative apportionment function  $F_k$  is written as follows

$$F_k = \sum_{l=1}^K f_l \quad (3.32)$$

An illustration of enhancement of a gray-scale image performed from *IF* Entropy (method I) and Spatial Entropy Contrast Enhancement is shown in Figure 3.3 to clarify the efficiency of the latter method.

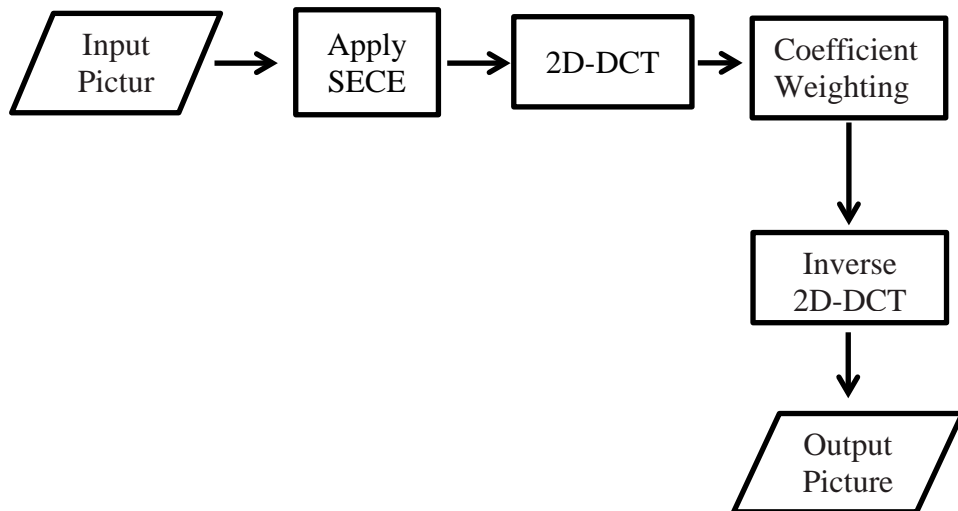


Figure 3.2: The Planned Demonstration of the SECEDCT Algorithm [34]



(a)



(b)



(c)

Figure 3.3: Gray Image Enhanced by IF Entropy I and Spatial Entropy  
(a) Gray-Scale Image, (b) *IF* Entropy (method I) and (c) Spatial Entropy - Contrast Enhancement [34]

## **3.6 Image Quality Assessment Methods**

Each operation utilized to an image may result in a major waste of data or feature. Image feature estimation approaches could be subdividing into thematic and particular techniques [35, 36]. Particular techniques were based on the human decision and function without reference to exact measures [37]. Thematic techniques were basing on comparisons utilizing precise numerical principles [38, 39], and many references are tolerable for instance the ground accuracy or prior information expressed regarding statistical parameters and tests [40-41]. In this thesis we introduced three different types of image quality measurement, to estimate the achievement of our suggested methods.

### **3.6.1 Peak Signal-to-Noise Ratio (PSNR)**

“The phrase peak signal-to-noise ratio is an expression for the ratio of the maximum potential value of an indication and the power of deforming noise that affects the quality of its representation” [42]. As various signals have a very vast active domain, the *PSNR* is regularly represented concerning the logarithmic decibel measure.

Image enhancement or enhancing the optical quality of a digital image can be subjective. Stating that single technique offers a well quality image could differ from one to another. Consequently, it is essential to create quantitative/empirical means to match the impacts of image enhancement algorithms on image quality.

Through utilizing the identical set of tested images, various image enhancement algorithms may regularly be matched to classify if a specific algorithm yields enhanced outcomes of tested images. Thus, the metric under examination is the peak-

signal-to-noise ratio. If we can confirm that an algorithm or set of algorithms can improve a corrupted known image to match the original more closely, then we can more accurately conclude that it is a better algorithm.

For the following implementation, let us suppose we are dealing with a standard  $2D$  array of data or matrix. The dimensions of the correct image matrix and the dimensions of the degraded image matrix must be identical.

The mathematical description of the *PSNR* is as follows:

$$PSNR(f, g) = 10 \log_{10} (255^2 / MSE(f, g)) \quad (3.33)$$

Where *MSE* is

$$MSE(f, g) = \frac{1}{M \times N} \sum_{i=1}^M \sum_{j=1}^N (f_{ij} - g_{ij}) \quad (3.34)$$

$$i = 1, 2, \dots, M, \quad j = 1, 2, \dots, N$$

$f$  represents the matrix statistics of the original image

$g$  represents the matrix statistics of a corrupted image in question

$M$  represents the records of rows of pixels of the images, and  $i$  represent the index of that row

$N$  represents the value of columns of pixels of the image, and  $j$  represents the index of that column

The peak signal-to-noise ratio value approaches infinity as the Mean Square Error approximates zero; this display that a top *PSNR* value indicates a top image quality.

At the other side of the measure, a slight value of the *PSNR* denotes significant numeral variations among images [42].

### 3.6.2 Structural Similarity Index (SSIM)

The *SSIM* is a well-known quality metric used to measure the similarity between two images. It was developed by Wang et al. [43], and is considered to be correlated with the quality perception of the human visual system. Instead of using traditional error summation methods, the SSIM is designed by modeling any image distortion as a combination of three factors that are a loss of correlation, luminance distortion, and contrast distortion. The SSIM is defined as:

$$SSIM(f, g) = l(f, g)c(f, g)s(f, g) \quad (3.35)$$

Where

$$\begin{cases} l(f, g) = \frac{2\mu_f\mu_g + C_1}{\mu_f^2 + \mu_g^2 + C_1} \\ c(f, g) = \frac{2\sigma_f\sigma_g + C_2}{\sigma_f^2 + \sigma_g^2 + C_2} \\ s(f, g) = \frac{\sigma_{fg} + C_3}{\sigma_f\sigma_g + C_3} \end{cases} \quad (3.36)$$

The first term in (3.36) is the luminance comparison function which measures the closeness of the two images' mean luminance ( $\mu_f$  and  $\mu_g$ ). This factor is maximal and equal to 1 only if  $\mu_f = \mu_g$ . The second term is the contrast comparison function which measures the closeness of the contrast of the two images. Here the contrast is measured by the standard deviation  $\sigma_f$  and  $\sigma_g$ . This term is maximal and equal to 1 only if  $\sigma_f = \sigma_g$ . The third term is the structure comparison function which measures the correlation coefficient between the two images  $f$  and  $g$ . Note that  $\sigma_{fg}$  is the covariance between  $f$  and  $g$ . The positive values of the SSIM index are in  $[0, 1]$ . A value of 0 means no correlation between images, and 1 means that  $f = g$ . The positive constants  $C_1$ ,  $C_2$  and  $C_3$  are used to avoid a null denominator [42].

### 3.6.3 Image Enhancement Metric (IEM)

Modifications in sharpness and contrast indicate density variance among a pixel and its neighbors. Thus, it is a clear idea to compare the exact value of density variance among a pixel and its neighbors according to the reference and improved images.

Image Enhancement Metric estimates the contrast and sharpness of an image by separating an image into non-overlapping blocks. The mean value of the actual variation among the middle pixel and its eight neighbors for all local windows according to the original and improved image will provide an indication of the modification in contrast and sharpness. The window size of 3x3 is adequate as the metric employs solely eight neighbors.

Full-reference metric, *IEM* is acquainted as the ratio of quantity of positive values of the modification of every pixel from its 8-neighbors of the improved image to the reference image and is arithmetically stated as

$$IEM_{8n} = \frac{\sum_{m=1}^{k_1} \sum_{l=1}^{k_2} \sum_{n=1}^8 |I_{e,c}^{l,m} - I_{e,n}^{l,m}|}{\sum_{m=1}^{k_1} \sum_{l=1}^{k_2} \sum_{n=1}^8 |I_{r,c}^{l,m} - I_{r,n}^{l,m}|} \quad (3.37)$$

Where the image has divided into  $k_1 k_2$  blocks of size 3x3 and  $I_{e,c}^{l,m}$ ,  $I_{r,c}^{l,m}$  are the density of the center pixel in  $(l, m)$  block of the enhanced and original images sequentially.  $I_n^{l,m}$ ,  $n = [1, 2, \dots, 8]$  specify the eight neighbors of the center pixel.

While the original image and improved image are equal,  $IEM=1$ .  $IEM > 1$  means that the image is enhanced. However, there is retrogradation otherwise. The top value of *IEM* improved the enhancement in image contrast and sharpness [44].



### 3.7 Image Noise and Noise Removal Methods

In this project, we assessed the performance of proposed methods after added the noise and evaluated the performance as well when removal this distortion by a particular filter, as described in briefly below.

#### 3.7.1 Gaussian Noise

Such a deformity images additive in nature [45] and follow Gaussian spreading. The significance that every pixel in the deformity image is the value of the actual pixel value and a casual, Gaussian spread deformity value. The deformity is unsupported of the density of pixel value at every point.



Figure 3.4: Original Image without Distortion [22]

The PDF of Gaussian random variable is given by:

$$P(x) = 1 / (\sigma\sqrt{2\pi}) * e^{-(x-\mu)^2 / 2\sigma^2} \quad -\infty < x < \infty \quad (3.38)$$

Where:  $P(x)$  is the Gaussian spread deformity in an image;  $x$  is a real argument,  $\mu$  and  $\sigma$  are the means and standard deviation sequentially. Figure 3.5, illustrations the influence of adding Gaussian noise to Figure 3.4, with zero means [46].



Figure 3.5: Original Image with Gaussian Noise

### 3.7.2 Image De-noising (Median Filter)

The median filter is the excellent command fixed, non-linear filter, whose reply has relied on the rating of pixel values included in the clean area. The median filter is very common for decrease several species of deformity.

Because the median filter is a nonlinear filter, its mathematical analysis is relatively complex for the image with random noise. For an image with zero mean noise under normal distribution, the noise variance of the median filtering is approximately [55]

$$\sigma_{med}^2 = \frac{1}{4nf^2(\bar{n})} \approx \frac{\sigma_i^2}{n + \frac{\pi}{2} - 1} \cdot \frac{\pi}{2} \quad (3.39)$$

Where  $\sigma_i^2$  is input noise power (the variance),  $n$  is the size of the median filtering mask (3x3, 5x5, 7x7.....etc.),  $f(\bar{n})$  is the function of the noise density. And the noise variance of the average filtering is

$$\sigma_0^2 = \frac{1}{n} \sigma_i^2 \quad (3.40)$$

Comparing of (3.39) and (3.40), the median filtering effects depend on two things: the size of the mask, and the distribution of the noise. The median filtering performance of random noise reduction is better than the average filtering performance, but to the impulse noise, especially narrow pulses are farther apart and the pulse width is less than  $n / 2$ , the median filter is very effective. The median filtering performance should be improved if the median filtering algorithm, combined with the average filtering algorithm, can adaptively resize the mask according to the noise density.

Here replacing the mean value of the pixel by the average of the pixel value below the filter area [47] [48]. Fig 3.6 displays the influence of the median filter on Gaussian noise.



Figure 3.6: Median Filter of Original Image Used for Gaussian Noise

The median filter utilized very vast as softer for image processing, also to indicator processing. A significant benefit of the median filter over linear filters is that the median filter can reduce the influence of participation distortion values with excessively great magnitudes [46].

## Chapter 4

# IMPLEMENTATION AND PERFORMANCE EVALUATION

### 4.1 Implementation

The experiments are have implemented on a laptop with CPU 2.10 GHz processor, 4 GB RAM, and Windows 7 as an operating system under 64-bit as system type. We selected MATLAB 2016 to be the platform for implement a whole new application for medical image enhancement by using more than one data set of images in the process.

### 4.2 Data Description

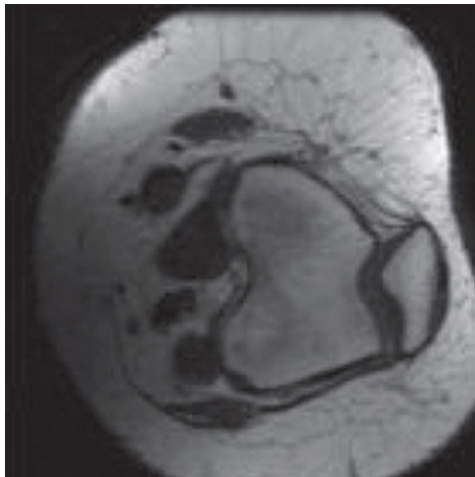
For the testing purpose, we applied our new application to many different sets of images. The first set is Medical Images contains eight input images, each image with a different size of resolution (e.g. 501x501, 400x400; 787x787 and ..... etc.) as shown in Figure 4.1. The Second set is color images (RGB) contains seven input images, all images of this set not in the same resolution size, for instance, 361x361, 244x244; 256x256 and .... etc., as demonstrated in Figure 4.2. The last set is Gray-scale scale images holds seven input images, the sizes of this set images are in the range (247x247 to 500x500) in resolutions, as displayed in Figure 4.3.



(a) Dataset 1: image 1 [22]



(b) Dataset 1: image 2 [22]



(c) Dataset 1: image 3 [1]



(d) Dataset 1: image 4 [22]



(e) Dataset 1: image 5 [22]



(f) Dataset 1: image 6 [22]



(g) Dataset 1: image 7 [22]



(h) Dataset 1: image 8 [22]

Figure 4.1: The First Group (Medical) of 8 Input Images



(a) Dataset 2: image 1[50]



(b) Dataset 2: image 2[51]



(c) Dataset 2: image 3[34]

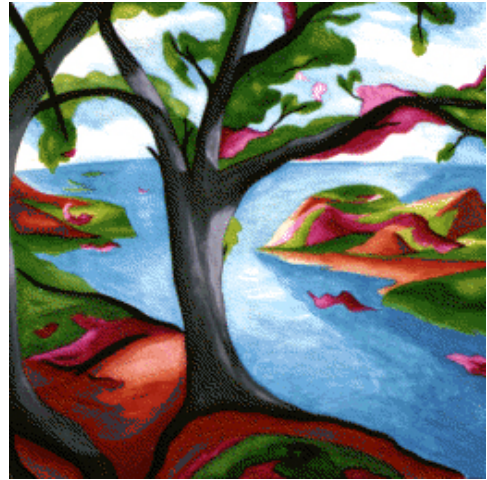


(d) Dataset 2: image 4[34]





(e) Dataset 2: image 5 [51]



(f) Dataset 2: image 6 [52]



(g) Dataset 2: image 7 [51]

Figure 4.2: The Second Group (Color) of 7 Input Images



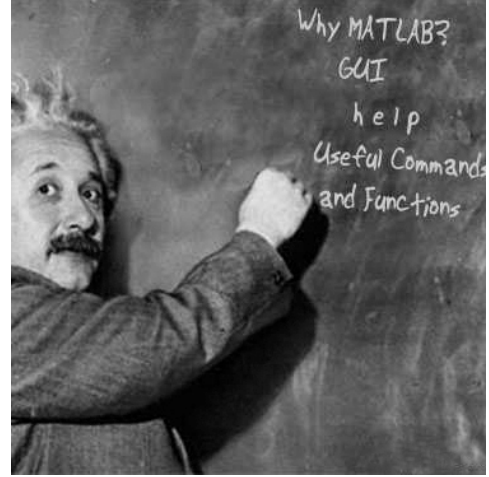
(a) Dataset 3: image 1 [54]



(b) Dataset 3: image 2 [54]



(c) Dataset 3: image 3 [58]



(d) Dataset 3: image 4 [54]



(e) Dataset 3: image 5 [54]



(f) Dataset 3: image 6 [49]



(g) Dataset 3: image 7 [56]

Figure 4.3: The Third Group (Gray) of 7 Input Images



### 4.3 Image Histogram

In an image processing context, the histogram of an image normally refers to a histogram of the pixel intensity values. This histogram is a graph showing the number of pixels in an image at each different intensity value found in that image. For an 8-bit gray-scale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those gray scale values. Histograms can also be taken of color images and brightness at each point representing the pixel count. The exact output from the operation depends on upon the implementation, it may simply be a picture of the required histogram in a suitable image format, or it may be a data file of some sort representing the histogram statistics.

A Histogram has two axes the  $x$  axis and the  $y$  axis.

The  $x$  axis contains intensity level.

The  $y$  axis contains frequency count.

The  $x$  axis of the histogram shows the range of pixel values. Since it is an 8-bit gray scale image that means it has 256 levels of gray or shades of gray in it. That is why the range of  $x$  axis starts from 0 and end at 255, whereas, on the  $y$  axis, is the count of these intensities.

The different heights of the bar show different frequency of occurrence of data, for a histogram of the dark image is always on the left side as black is the first value and white is the last value, in contrast, the corresponding histogram of the light image will be on the right side. On the other hand, in the case for a histogram of the low-contrast image the intensity of pixels is not formally distributed, while, a histogram

of the high-contrast image has the uniform distribution of intensity level which provides the best results [23], as shown in Figure 4.4 below.

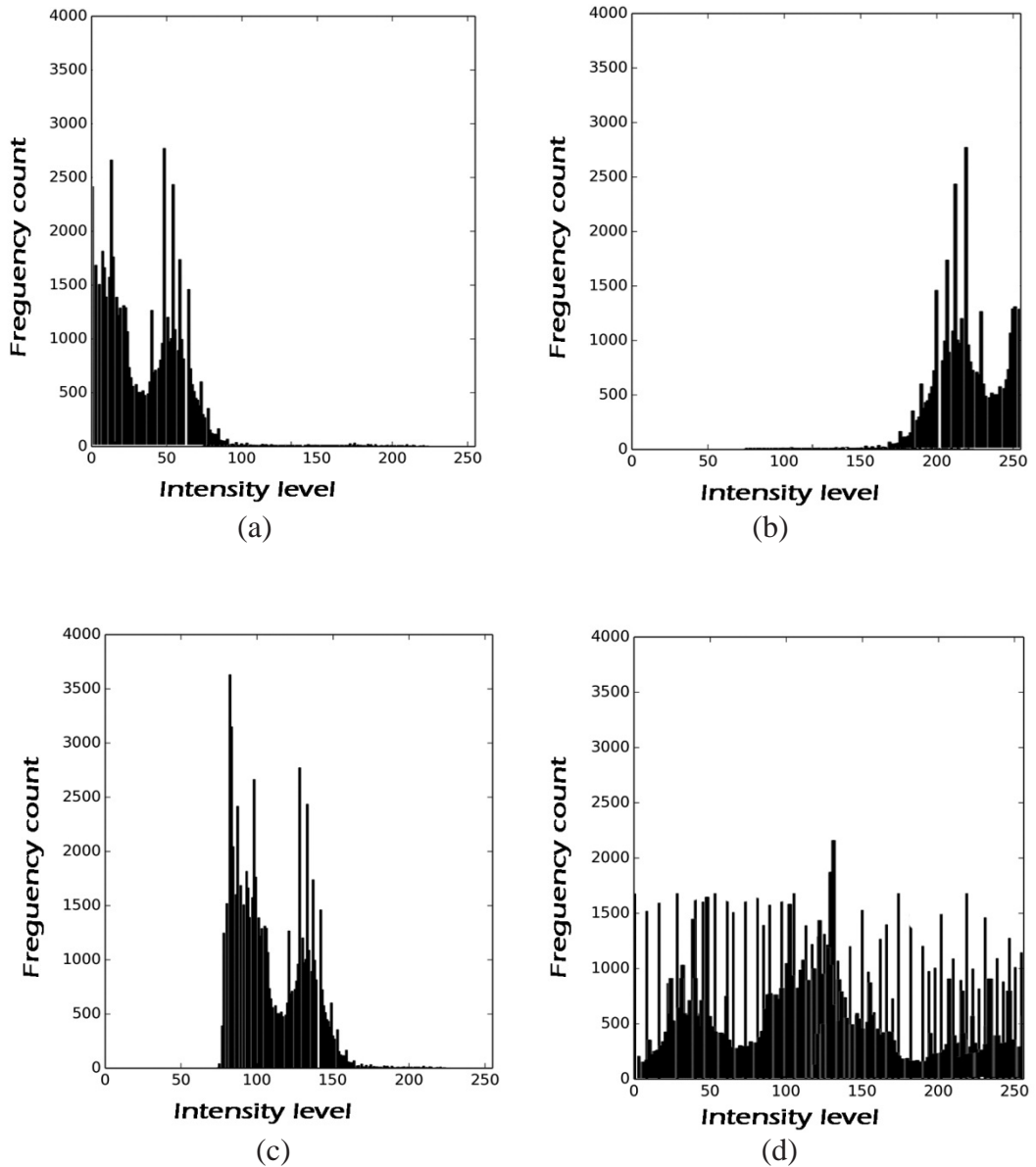


Figure 4.4: Different Intensity Levels of Histogram Image [62]  
 (a) Histogram of dark image, (b) Histogram of light image, (c) Histogram of low-contrast image and (d) Histogram of high-contrast image

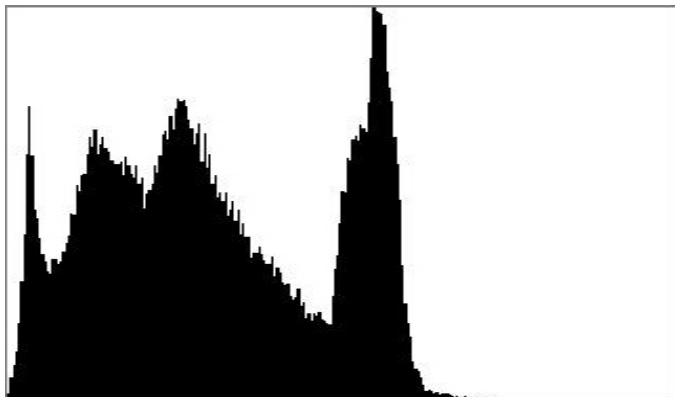
#### 4.4 Results of Entropy Method I with the Corresponding Histogram



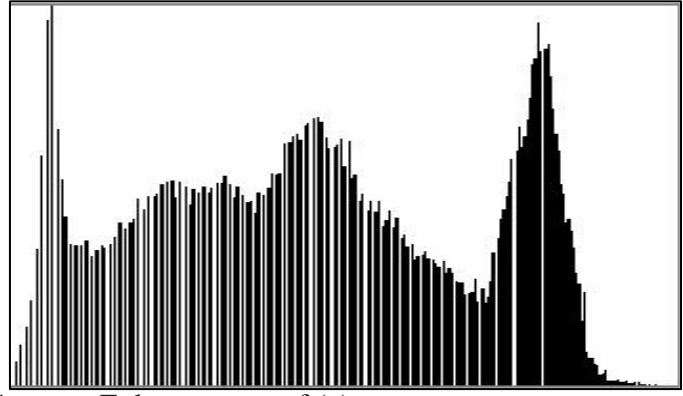
(a) Medical Image (Cervical vertebrae)



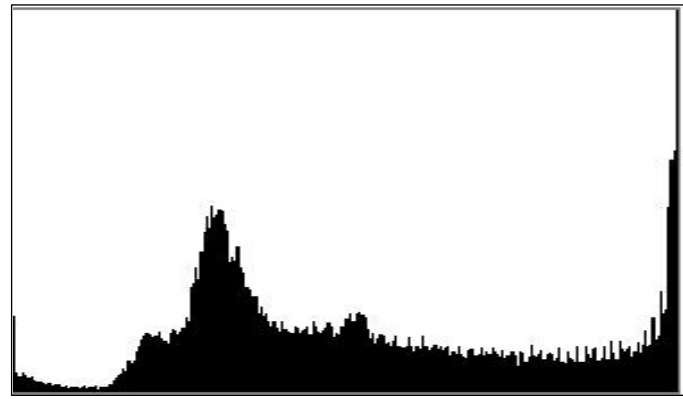
(b) Contrast Enhancement of (a)



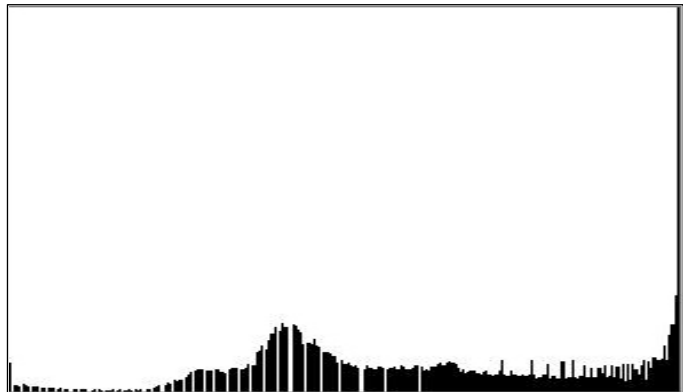
(c) RGB Image (arch)



(d) Contrast Enhancement of (c)



(e) Gray Image (Dog)



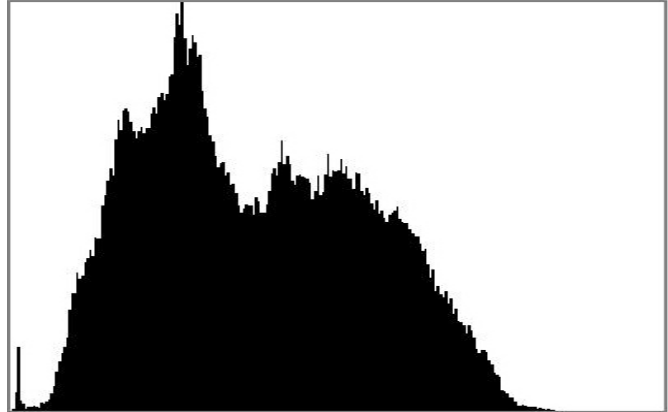
(f) Contrast Enhancement of (e)

Figure 4.5: Contrast Enhancement Results on Three Datasets Using IF Method I  
(a), (c), (e) input image results of (b), (d), (f) enhanced by IF Entropy I

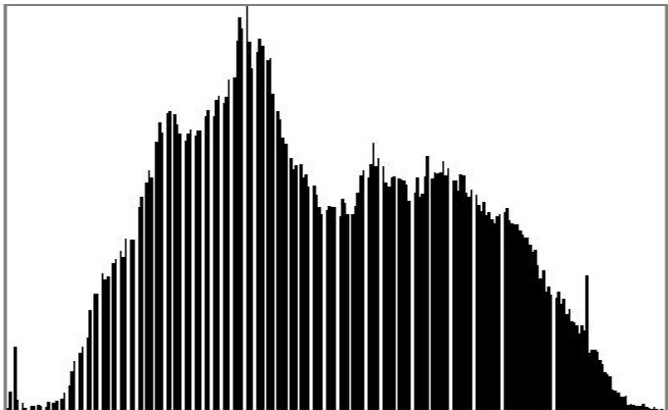
First, qualitative estimations on medical, color and gray-scale images are performed. The first experiment image from the medical data set is cervical vertebrae image, for the color data set arch image is selected. Finally, the dog image has been picked for the gray-scale data set, and corresponding contrast enhancement results of IF entropy method (I) are showing in Fig. 4.5.

From the results of the corresponding histograms of tested images after processing it is evident that the contrast enhancement of color image illustrates better results followed by the gray-scale image, whereas, the enhanced medical image is not much.

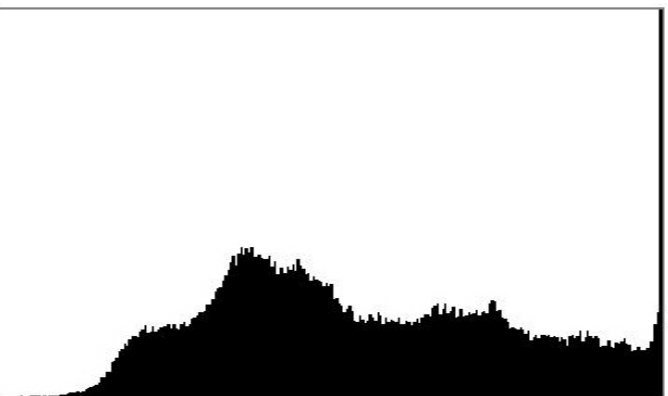
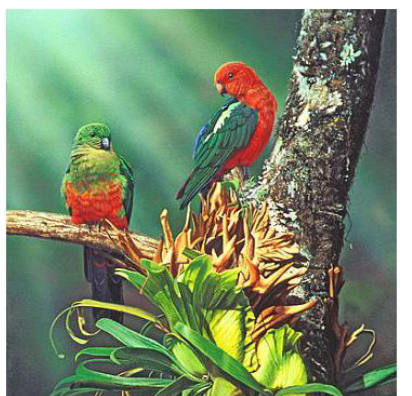
## 4.5 Results of Entropy Method II with the Corresponding Histogram



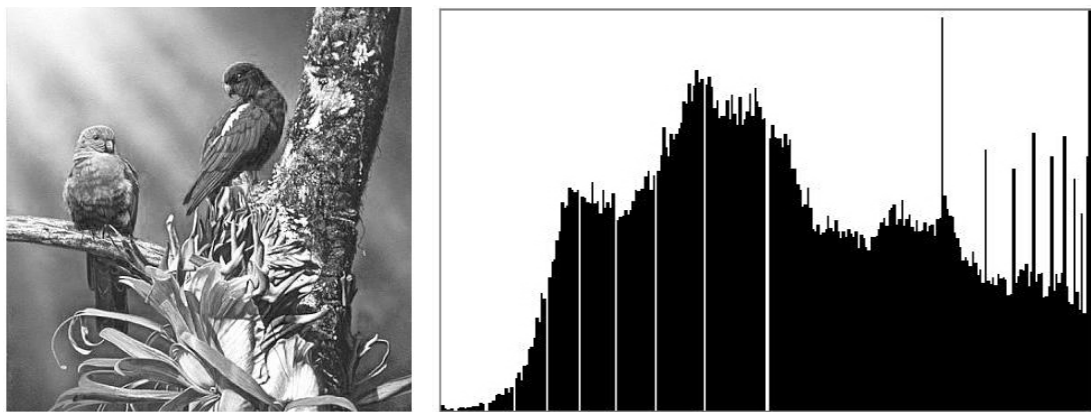
(a) Medical Image (Chest)



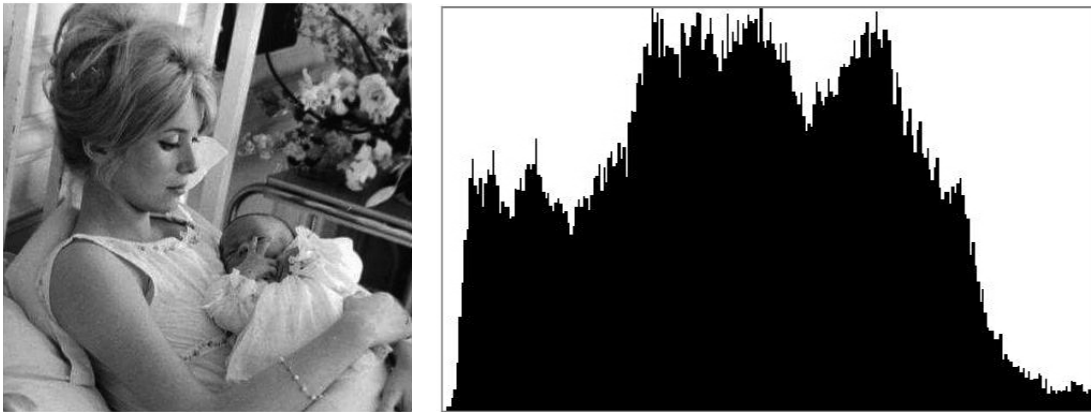
(b) Contrast Enhancement of (a)



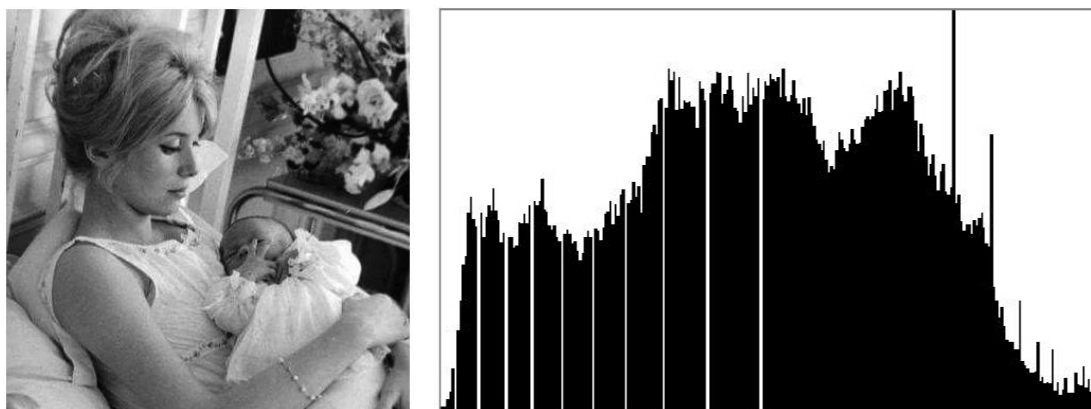
(c) RGB Image (love birds)



(d) Contrast Enhancement of (c)



(e) Gray Image (Catherine Deneuve)



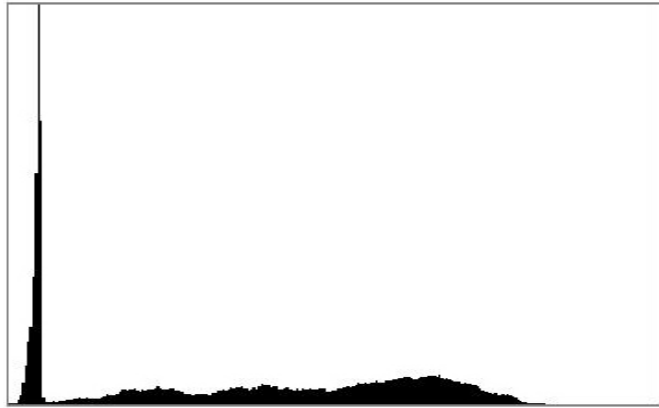
(f) Contrast Enhancement of (e)

Figure 4.6: Contrast Enhancement Results on Three Datasets Using IF Method II  
 (a), (c), (e) input image results of (b), (d), (f) enhanced by IF Entropy II

The results for the second method have implemented on our data sets with different input images. There is a slight over – enhancement from the image of entry to the tested image for both color and gray-scale images, on the other hand, a small under enhancement overhead the image caused by a medical image, as shown in Figure 4.6 above.



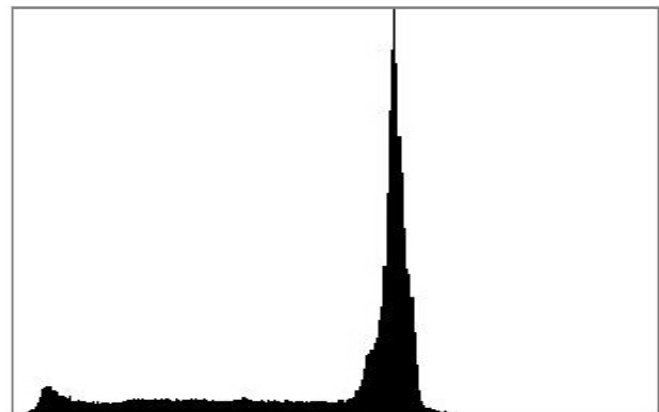
## 4.6 Results of Entropy Method III with the Corresponding Histogram



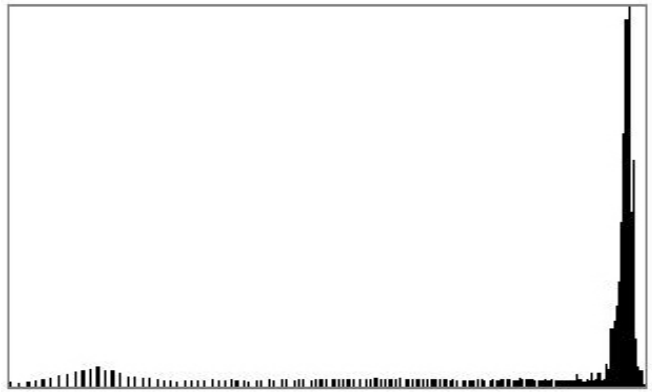
(a) Medical Image (Ankle)



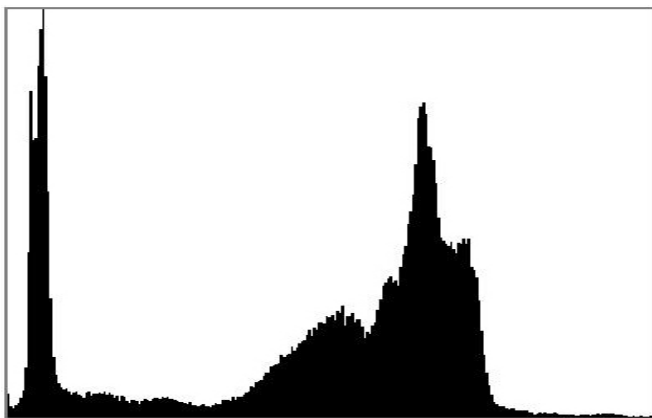
(b) Contrast Enhancement of (a)



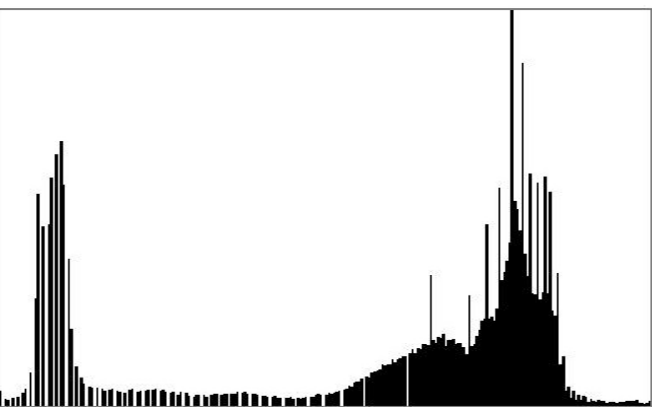
(c) RGB Image (Seagull )



(d) Contrast Enhancement of (c)



(e) Gray Image (Cameraman)

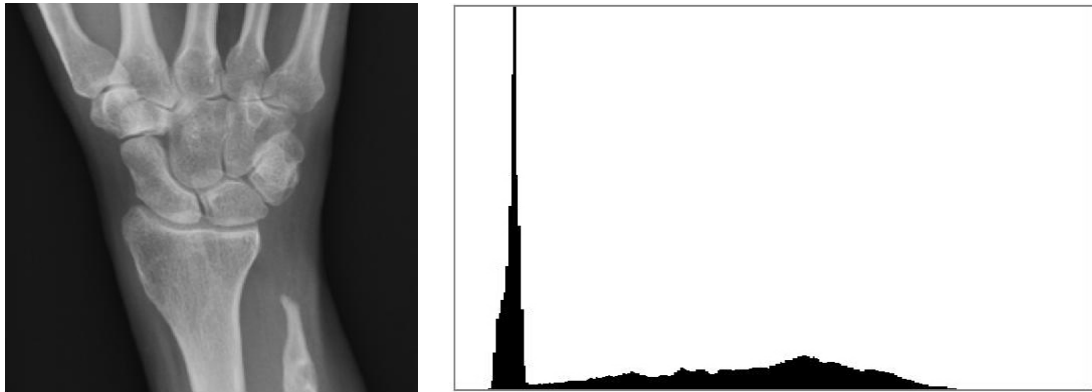


(f) Contrast Enhancement of (e)

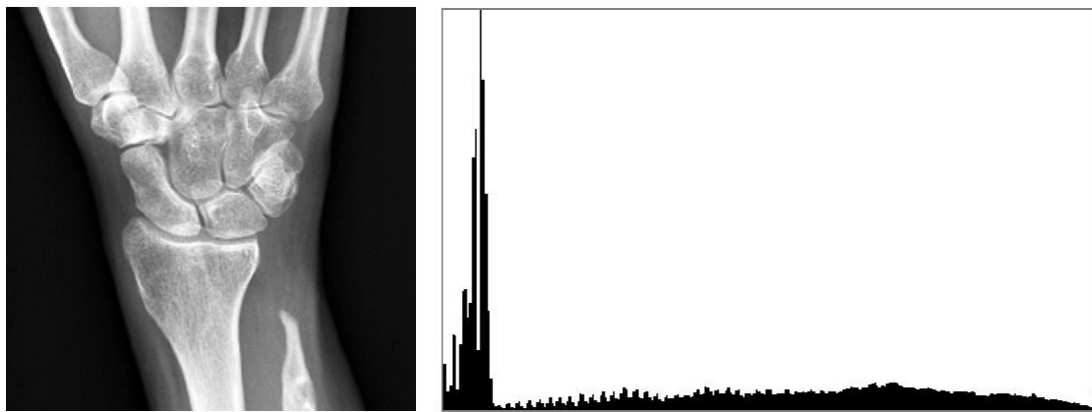
Figure 4.7: Contrast Enhancement Results on Three Datasets Using IF Method III  
(a), (c), (e) input image results of (b), (d), (f) enhanced by IF Entropy III

As it can be easily observing from the enhanced images above (Figure 4.7), the global contrast incremented of the three tested images, especially for both the medical image (Ankle) and the color image (Bird) where many regions have been removing. In contrast, the gray – scale image (Cameraman) showing a better result. Moreover, this is mostly as a consequence of the object that reference image's histogram is primarily focusing nearby the median value of the dynamic domain with a greater trend to the lower end of the median -value, which effects in darkening and over enhancement on particular areas of the image.

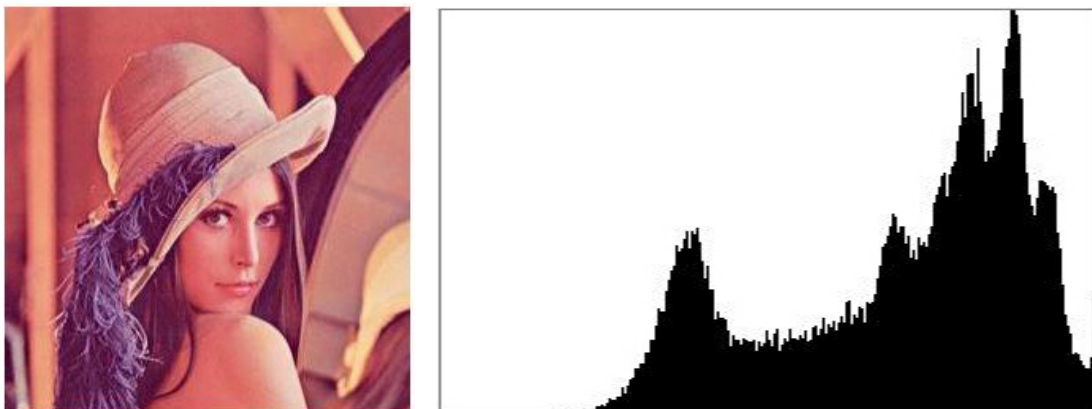
## 4.7 Outcomes of Spatial Entropy Method with the Corresponding Histogram



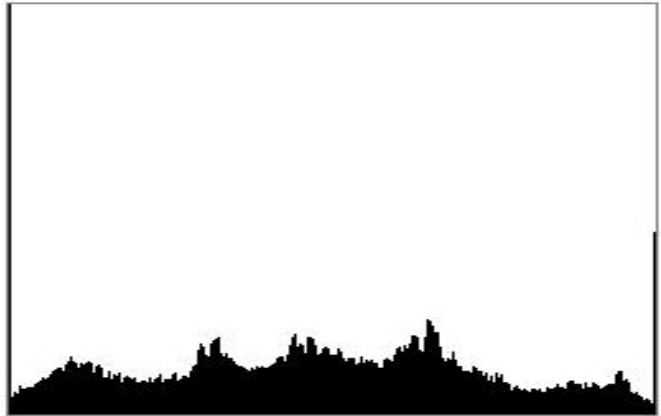
(a) Medical Image (Metacarpus)



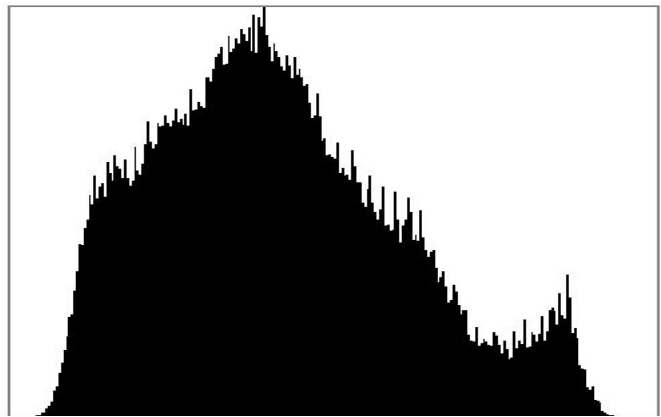
(b) Contrast Enhancement with  $\gamma=25$  of (a)



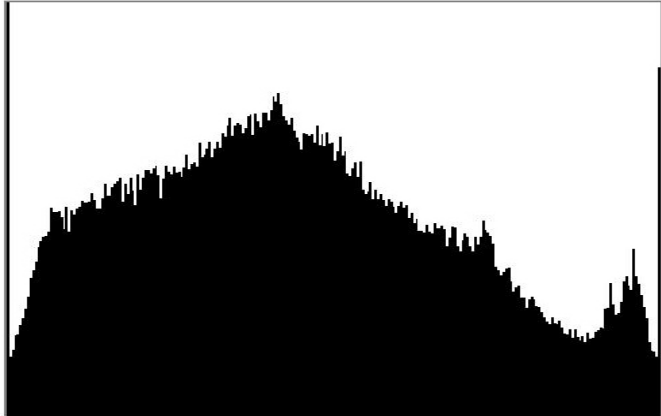
(c) RGB Image(Lena)



(d) Contrast Enhancement of (c)



(e) Gray Image (Elaine)



(f) Contrast Enhancement of (e)

Figure 4.8: Contrast Enhancement Results on Three Datasets Using Spatial Entropy  
(a),(c),( e) input image results of (b),(d),(f) enhanced by Spatial Entropy

The outcomes of all data sets (Medical, Color, and Gray – scale images) that has been applying to this technique extending us with best findings equally for all different type of images.

This method is providing us better results according to the input images with corresponding histograms comparable with the rest IF entropy methods, as it is apparently showing in figure 4.8, this is due to that Spatial Entropy achieving both global and local contrast enhancement simultaneously without visible deformation.

## 4.8 Image Quality Assessment Techniques

Performance evaluation is a crucial task in image enhancement process.

We examine the performance of our new application according to the following measures: PSNR, SSIM, and IEM, as they explained in Chapter three in details.

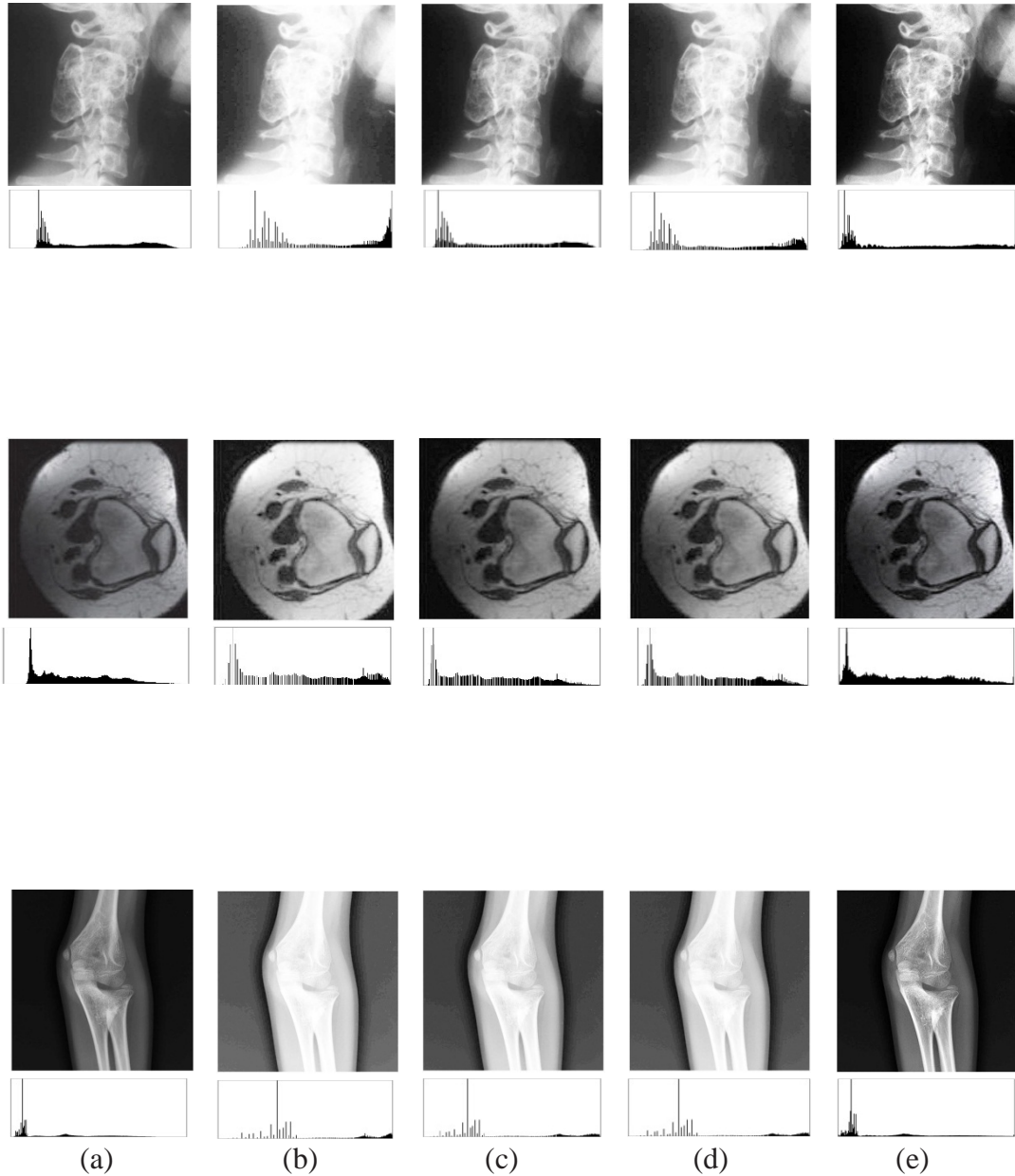
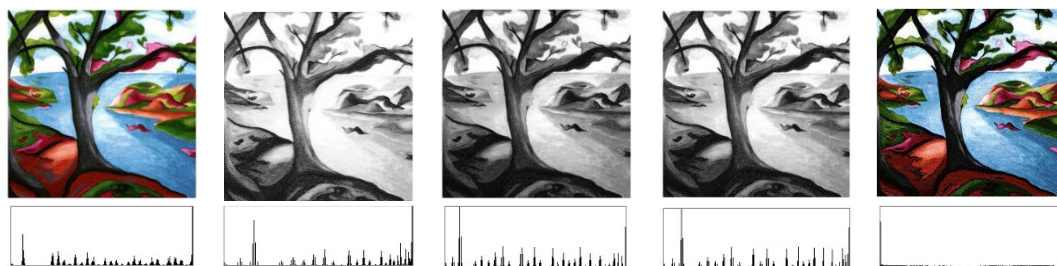


Figure 4.9: Medical Images Enhancement

(a) Reference image; outcomes of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.





(a)

(b)

(c)

(d)

(e)

Figure 4.10: Color Images Enhancement

(a) Reference image; outcomes of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.



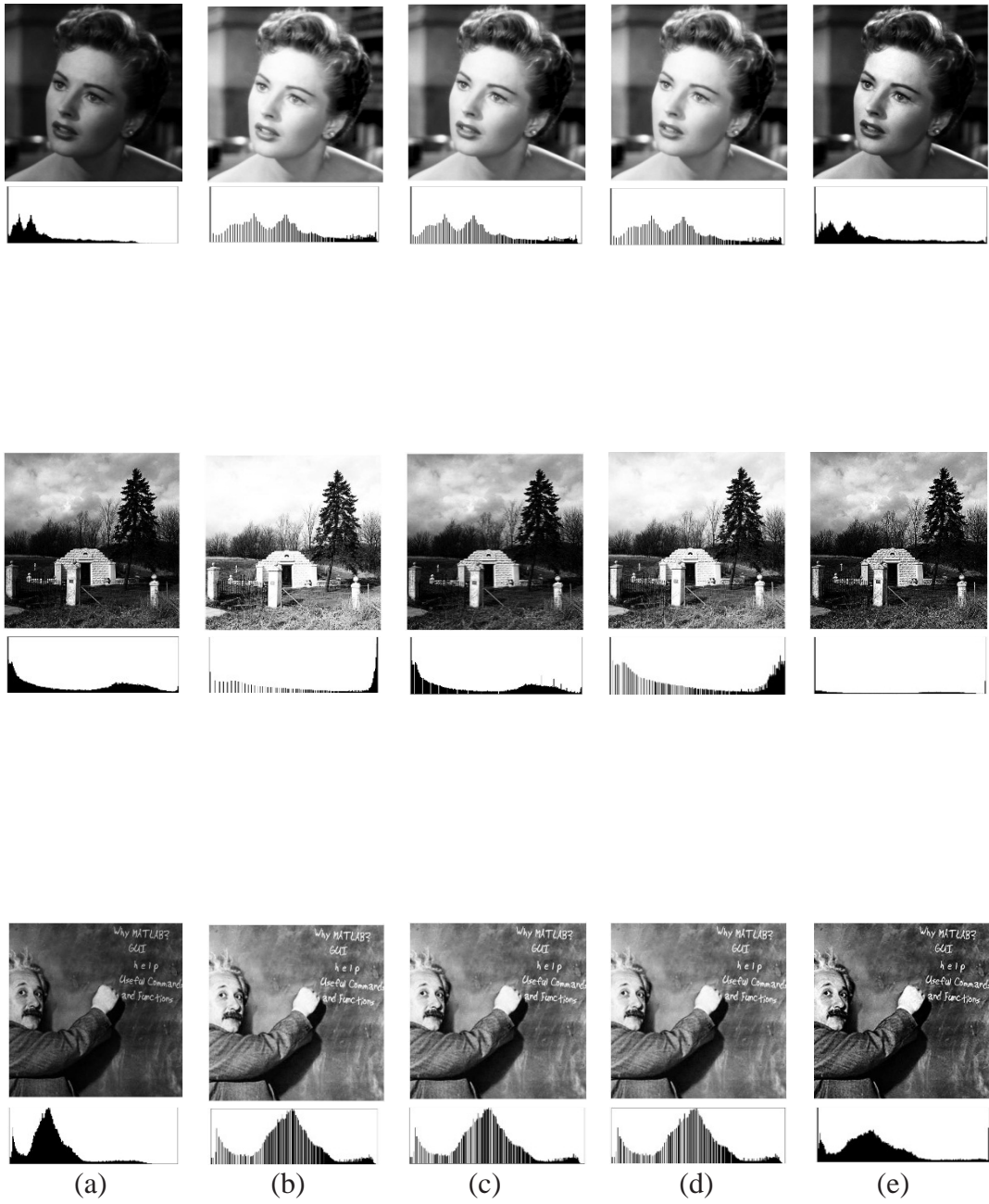


Figure 4.11: Gray Images Enhancement  
(a) Reference image; outcomes of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.

Table 4.1: Performance Analysis of Contrast Enhancement

Data set	Original Image			IF Entropy I Burillo & Bustince			IF Entropy II Vlachos & Sergiadis			IF Entropy III Burillo & Bustince			Spatial Entropy		
	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM
<b>Medc1</b>	Inf.	1	1	24.099	0.840	1.522	24.105	0.944	0.568	24.099	0.940	0.977	24.645	0.820	0.295
<b>Medc2</b>	Inf.	1	1	24.104	0.827	0.787	24.324	0.921	0.393	24.178	0.906	0.515	24.633	0.884	0.242
<b>Medc3</b>	Inf.	1	1	24.1	0.458	5.375	24.102	0.553	3.937	24.100	0.517	4.375	24.421	0.981	1.125
<b>Colr1</b>	Inf.	1	1	24.132	0.726	2.120	24.201	0.977	1.224	24.180	0.929	1.431	24.205	0.993	1
<b>Colr2</b>	Inf.	1	1	24.100	0.965	1.263	24.100	0.986	1.157	24.100	0.965	1.263	24.114	0.976	1.015
<b>Colr3</b>	Inf.	1	1	24.116	0.923	1.213	24.118	0.957	1.140	24.116	0.932	1.196	24.135	0.995	1.056
<b>Gry1</b>	Inf.	1	1	24.330	0.576	4	24.352	0.660	3	24.338	0.605	3.615	24.560	0.913	1.384
<b>Gry2</b>	Inf.	1	1	24.324	0.523	1.233	24.943	0.995	1.038	24.454	0.683	1.213	24.865	0.988	1.033
<b>Gry3</b>	Inf.	1	1	24.206	0.737	1.903	24.228	0.763	1.831	24.210	0.743	1.891	24.367	0.931	1.385

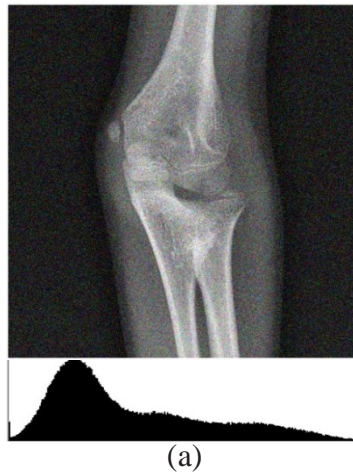
Table 4.1. Contains the results of implementation for the PSNR, SSIM, and IEM measurements, applying on three various data sets namely Medical, Color and Grayscale images contrast enhancement outcomes and consistent histograms employing various entropy techniques. It is evident from outcomes of PSNR for the spatial entropy has highest values acquired on all data sets of tested images; followed by IF entropy II (Vlachos & Sergiadis). Whereas, IF entropy I consider the lowest PSNR value.

The accuracy performance of SSIM measurement relies on how the output value approximates to one. Therefore a certain variation can be seen between data sets image, for instance, computing SSIM measurement on color image data sets for all kinds of entropy enhancement providing best us results comparable with remaining data sets.

Finally, the last measurement IEM which is originally affected by the illumination of pixels from input images. This lead to losing many critical regions, as it is clearly presenting in Medical, gray-scale image data sets especially, when implemented on IF entropy I, II and III techniques, as shown in parts b, c, and d in Figures (4.9, 4.10 and 4.11). On the other hand, the outcomes that obtained from the spatial entropy method are more accurate, as shown in part e in Figures (4.9, 4.10 and 4.11).

## 4.9 Comparison of Performance after Adding Distortion and after Noise Removal

### 4.9.1 Gaussian Noise





(d)

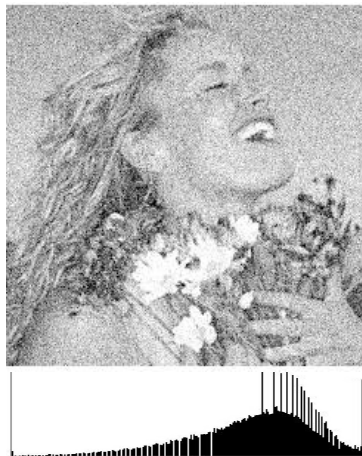


(e)

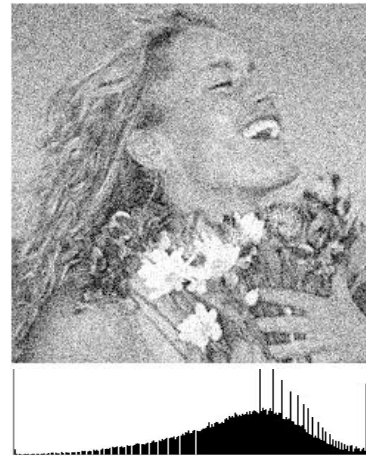
Figure 4.12: Medical Image after Noise  
 (a) Input image with Gaussian noise - zero means; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.



(a)

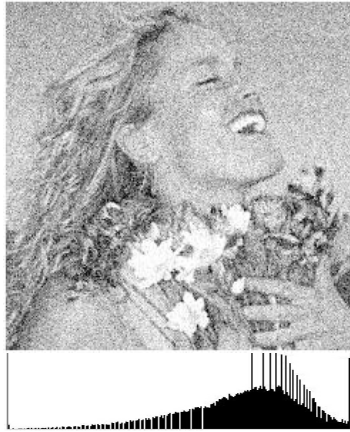


(b)

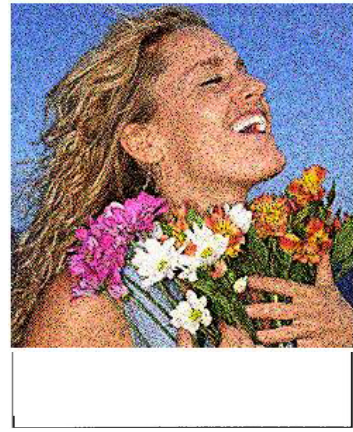


(c)





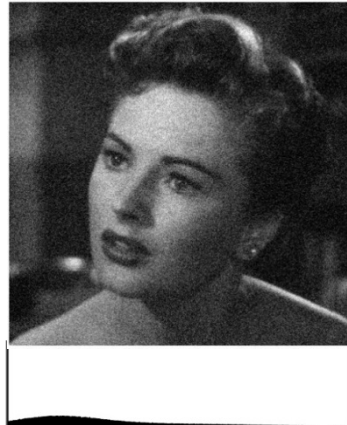
(d)



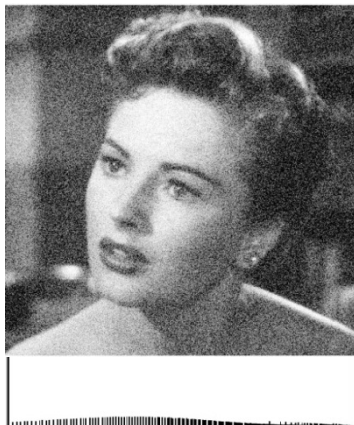
(e)

Figure 4.13: Color Image after Noise

(a) Input image with Gaussian noise – zero means; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.



(a)



(b)



(c)



(d)



(e)

Figure 4.14: Gray Image after Noise

(a) Input image with Gaussian noise – zero means; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.

Table 4.2: Performance Analysis of Contrast Enhancement with Gaussian Noise

Data set	Original Image			IF Entropy I Burillo & Bustince			IF Entropy II Vlachos & Sergiadis			IF Entropy III Burillo & Bustince			Spatial Entropy		
	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM
<b>Medc3</b>	Inf.	1	1	24.591	0.664	2.785	24.647	0.824	1.8571	24.60	0.716	2.428	24.741	0.980	1.214
<b>Colr2</b>	Inf.	1	1	24.105	0.947	1.217	24.106	0.975	1.1470	24.105	0.947	1.217	24.117	0.983	1.005
<b>Gry1</b>	Inf.	1	1	24.548	0.667	2.567	24.581	0.757	2.207	24.556	0.693	2.103	24.674	0.961	1.03

#### 4.9.2 Median Filter (De-Noising)

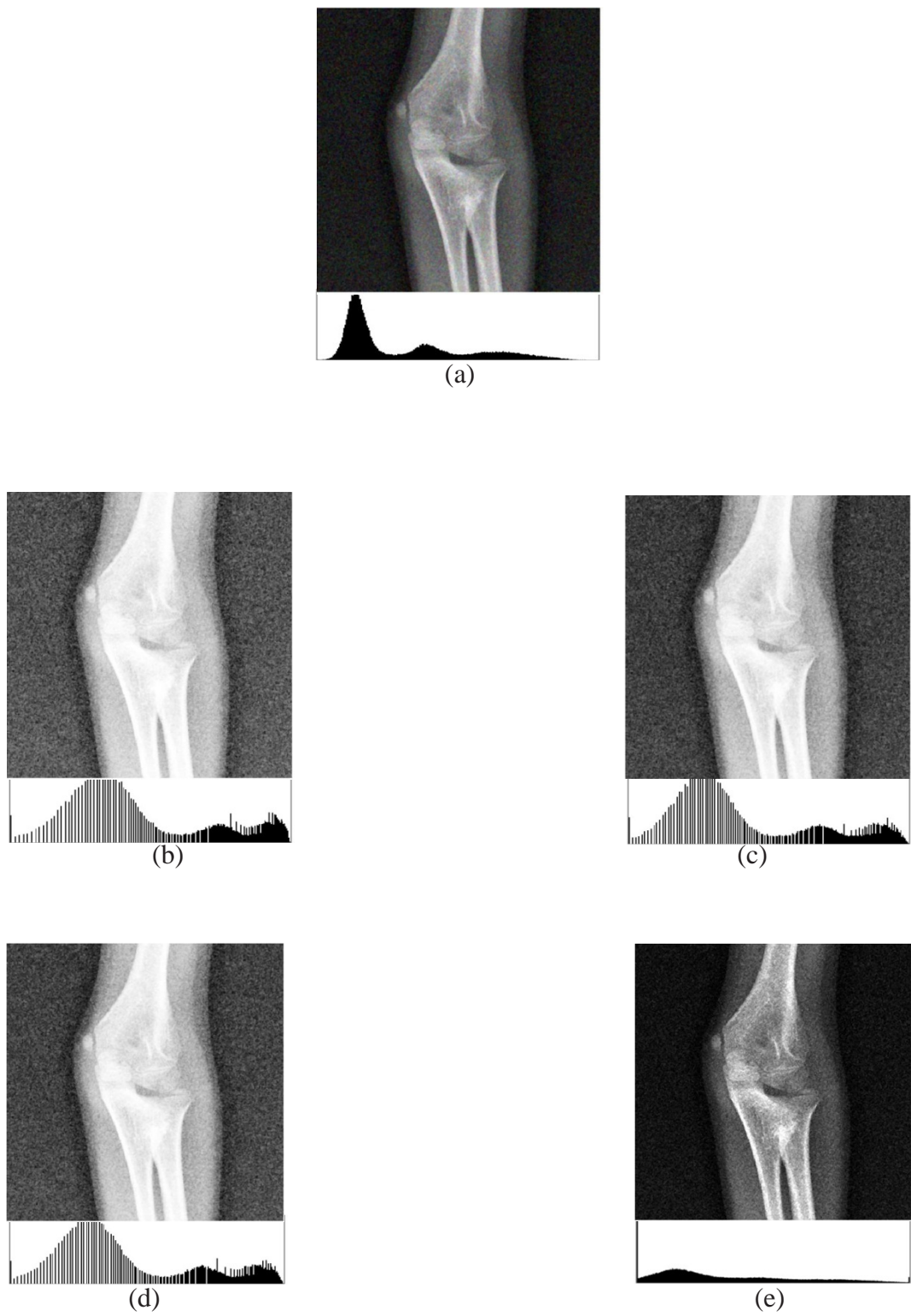
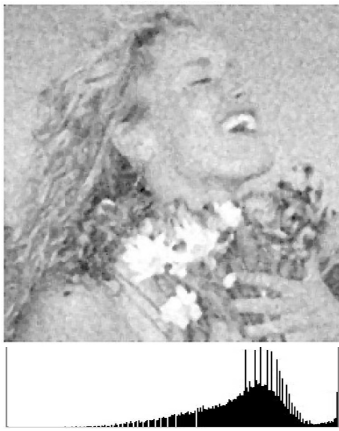


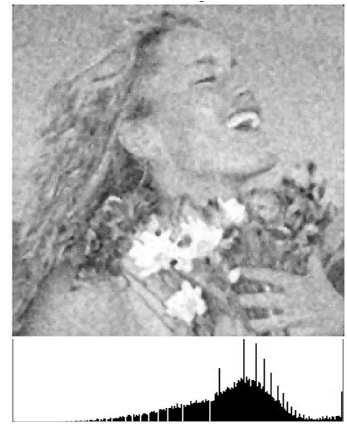
Figure 4.15: Medical Image after De-noising  
(a) Input image with Median filter; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.



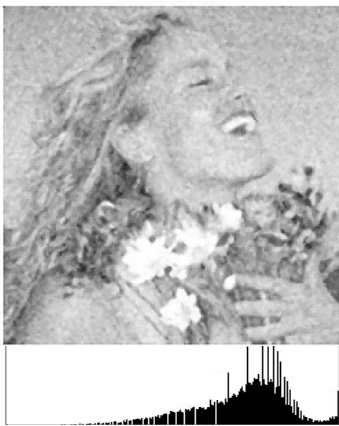
(a)



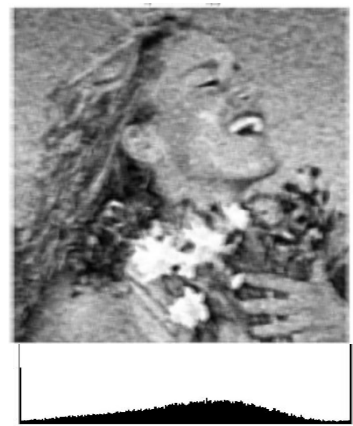
(b)



(c)



(d)



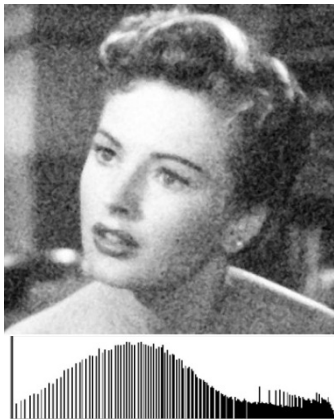
(e)

Figure 4.16: Color Image after De-noising  
(a) Input image with Median filter; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.

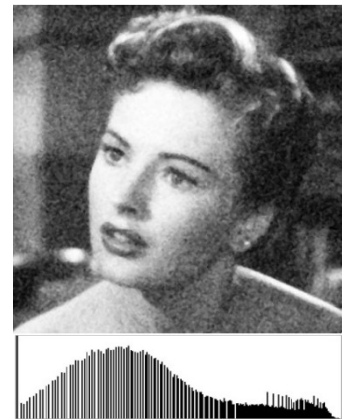




(a)



(b)



(c)



(d)



(e)

Figure 4.17: Gray Image after De-noising  
(a) Input image with Median filter; results of (b) IF Entropy I; (c) IF Entropy II; (d) IF Entropy III; and (e) Spatial Entropy.

Table 4.3: Performance Analysis of Contrast Enhancement after Removal of Noise

Data set	Original De-noising Median filter			IF Entropy I Burillo & Bustince			IF Entropy II Vlachos & Sergiadis			IF Entropy III Burillo & Bustince			Spatial Entropy		
	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM	PSNR	SSIM	IEM
<b>Medc3</b>	Inf.	1	1	24.153	0.561	3.062	24.165	0.666	2.437	24.161	0.624	2.656	24.233	0.946	1.375
<b>Colr2</b>	Inf.	1	1	24.099	0.946	1.346	24.099	0.975	1.228	24.099	0.946	1.346	24.122	0.923	0.842
<b>Gry1</b>	Inf.	1	1	24.208	0.610	3	24.225	0.687	2.428	24.214	0.631	2.809	24.307	0.917	1.333

Noise is an irregular difference of image density and visual as a mixture in the image. It can present at the time of taking or image converting. Distortion means the pixels in the image display many density values substituted for actual pixel types. The de-noise algorithm is the procedure of eliminating or decreasing the noise from the image.

In this thesis, Gaussian noise has been applied on one image for each data set randomly, as shown in Table 4.2. To illustrate the performance of our entropy enhancement methods through examining the efficiency of these entropies by executing one of the best De-noising filters (Median filter).

After applying the non-linear median filter on the noisy images the outcomes of spatial entropy performing better enhancement comparable with other entropy methods, as shown in Table 4.3.

## **Chapter 5**

### **CONCLUSION**

In this thesis, a new application of contrast image enhancement utilizing Intuitionistic Fuzzy Set theory on medical; color and gray-scale images has been proposed where different types of entropies are used for the enhancement of the contrast of images. The approach employs window based enhancement and for every window, the image is improved accordingly. The results of the handle methods that are presented in this project have been examined, and it has been observed that the spatial entropy method provides images with a better contrast where all the areas are evident and notable, as well as removing noise. The execution of the offered application has approved visibly and quantitatively. So as to display the enhancement of the application process, contrast images are enhanced, based on the results of several image quality assessment metrics. As each type of these images contains distinct properties (resolution, size, intensity and illumination), getting precise results when combining various types of images in one application is complicated.

This process demonstrates that the offered enhanced application is beneficial in medical image enhancement for post processing which will be helpful for doctors or pathologists to analyze the disease precisely. Also, it will be useful for further image processing implementation.

The new application must be modified to provide more precision in terms of quality, especially for medical images. In the future, this application could be improved by adding various types of contrast entropy beside fuzzy entropy models and spatial entropy.

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