Heuristic Solutions for Electric Vehicle Routing Problem with Time Windows and Recharging Stations

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ABSTRACT

Due to the regulation and laws concerning the emission of greenhouse gasses, carriers are starting to use electric vehicles for last-mile deliveries. The limited battery capacities of these vehicles necessitate visits to recharging stations during delivery tours of industry-typical length, which have to be considered in the route planning to avoid inefficient vehicle routes with long detours. This thesis seeks to propose new heuristic solution methods for Electric Vehicle Routing Problem with Time Windows (E-VRPTW), which incorporates the possibility of recharging at any of the available charging stations to minimize the total travel distance based on the Clarke and Wright (CW) Saving Heuristic. The solution method focuses on the construction of routes according to waiting time of vehicles, determined priority of customers with regard to their earliest starting service time, customers' demands and etc. Moreover, the recharging rate, vehicle freight capacity, battery capacity, time windows and recharging time are considered to make it close to real-life logistics problems. Numerical tests are performed on newly designed instances by Schneider and performances of proposed methods are discussed.

Keywords: Vehicle Routing Problem, Clarke and Wright Saving Heuristic, Green Logistic, Electric Vehicles, Combinatorial Optimization

ÖZ

Sera gazlarının salınımı ile ilgili kural ve yasalardan dolayı taşımacılar şehir içi taşımalarını elektrikle çalışan araçlarla yapmaya başlamışlardır. Bu araçların sınırlı olan batarya kapasiteleri, taşıma sırasında zorunlu olarak batarya dolum istasyonlarında şarj edilmeleri gereksinimini doğurmakta ve bu durum da uzun ve verimsiz rotaların önlenmesi için rota planlamasında gözönüne alınmalıdır. Bu tezin amacı, alınan toplam mesafeyi enküçüklemek amacıyla şarjın tüm dolum istasyonlarında mümkün olduğu ve Clarke & Wright tasarruf algoritmasını esas alan Electric Vehicle Routing Problem with Time Windows (E-VRPTW) problemine yeni bir sezgisel çözüm yöntemi önermektir. Önerilen çözüm yöntemi öncelikle araçların bekleme sürelerine, müşterilerin zaman önceliklerine, taleplerine vs. göre rotalar oluşturmaya odaklanmaktadır. Problemin lojistik olarak daha da gerçekçi olmasını sağlamak üzere araçların taşıma kapasiteleri, batarya kapasiteleri, zaman aralıkları, şarj zamanları ve hızları da gözönüne alınmıştır. Schneider'in değerleri üzerinden sayısal deneyler de yapılmış ve önerilen yöntemlerin performansları da tartışılmıştır.

Anahtar Kelimeler: Araç rotalama sorunu, Clarke ve Wright tasarruf sezgiseli, yeşil lojistik, elektrikli araçlar, birleşi eniyilemesi

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LIST OF ABBREVIATIONS

AFS	Alternative Fuel Stations
AFV	Alternative Fuel Vehicles
BEV	Battery Electric Vehicles
CVRP	Capacitated Vehicle Routing Problem
CW	Clarke and Wright
E-VRPTW	Electric Vehicle Routing Problem with Time Windows
FMVRP	Fleet size and Mixed Vehicle Routing Problem
GHG	Green-House Gasses
GVRP	Green Vehicle Routing Problem
LRP	Location Routing Problem
MCW	Modified Clarke and Wright
MDVRP	Multi-Depot Vehicle Routing Problem
PRP	Pollution Routing Problem
PVRP	Periodic Vehicle Routing Problem
SPS	Small Package Shipping
SVRP	Stochastic Vehicle Routing Problem
TDVRPTW	Time-Dependent Vehicle Routing Problem with Time Windows
TS	Tabu Search
TSP	Travelling Salesperson Problem
VNS	Variable Neighborhood Search
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Windows

Chapter 1

INTRODUCTION

Through the history, many variants of Vehicle Routing Problems (VRP) are introduced of which Routing of Vehicles with Green Fuel is a subgroup member receiving close and increasing attention from business companies and governments. The importance of GVRP originates from the fact that current distribution and production strategies are not sustainable. Besides the conventional costs at designing level of logistics policies, the social, ecological and environmental effects should be taken into consideration. Among the environmental effects, the emission of greenhouse gasses, particularly CO_2, is the most concerning one due to its hazardous impacts. Freight transport comprises 21% of total CO_2 emissions from the transport division in the United Kingdom and this rises to 28% of national greenhouse gas emissions in the United States.

The implementation of green logistics itself requires change in transportation scheme and usage of sustainable distribution network with less negative effect on environment. Many alternatives are provided for shifting onto sustainable logistic, such as using the green intelligent transportation systems, promotion of alternative fuels, electronic vehicles and other eco-friendly infrastructures. Many Small Package Shipping (SPS) companies have started to substitute their current fleets of vehicles to include Alternative Fuel Vehicles (AFVs) with greener fuels, such as CNG, LPG, Hydrogen, Biodiesel and electricity to meet new laws by governments.

The best fuel alternative for medium and heavy duty vehicles is promoted alternative fuels, like biodiesel, and for small package shipping vehicles, electricity is the best choice. However, using such vehicles leads to other challenges in route planning like meeting Alternative Fuel Stations (AFSs) due to fuel capacity constraint of green vehicles, while number of available AFSs and recharging stations are still scarce.

Agencies consider various factors in selection of a specific vehicle type, including fuel availability, distribution of fuel stations, fuel efficiency, fuel cost and vehicle driving range. Several SPS companies such as Royal Mail, DHL, DPD, UPS and Japan Post started using Battery Electric Vehicles (BEVs) in urban area transports for last-mile deliveries by considering all the criteria. Concerning the utilization of electric vehicles, incorporation of electric vehicle's specifications in route planning is required, such as driving range of BEVs, which is insufficient to meet all customers in one run or to travel to customers who are far from depot. Since reduction of customers in each tour is not an economical option, insertion of recharging stations along the tours is required to visit more customers in each departure. An insufficient integration of these characteristics in route planning methods can cause long detours, specifically if available recharging stations are scarce.

The Capacitated Vehicle Routing Problem (CVRP) was introduced by Dantzing and Ramser (Dantzig & Ramser, 1959) and since then many extensions and varieties of the VRP have been proposed to add more real-life logistic constraints to the original problem so as to make it more applicable. One of the most famous extensions of VRP is a Vehicle Routing Problem with Time Windows (VRPTW) (Russell, 1977), where each customer should be reached in a predefined time interval. Recently, Electric Vehicle Routing Problem with Time Windows (E-VRPTW) and Recharging Stations is proposed by Schneider et al. (Schneider, Stenger, & Goeke, 2014). E-VRPTW is an extension of VRPTW which deals with routing of electric vehicles while capacity constraint and time windows are taken into consideration.

Vehicle Routing Problem is a well-known NP-hard problem with a primary objective of total travel distance minimization. Exact algorithms such as branch-and-cut, branch-and-bound and branch-and-cut-and-price are not able to find optimal solutions for large number of customers. Computation time of exact algorithms for large number of customers is huge and this is more considerable in E-VRPTW when recharging stations are involved. However, heuristic and meta-heuristic algorithms compute appropriate solutions close to optimal in much less computation time. In general, NP-hard problems are those of which solution is not verifiable in polynomial time.

Heuristic Algorithms find feasible solutions among all feasible ones in much less computation time. Saving Algorithms (Gajpal & Abad, 2009) and Sweep Algorithm (Dondo & Cerdá, 2013) are the most popular and efficient algorithms among all Classical Heuristic Algorithms for VRP and other proposed heuristics for VRP are as follows: (1) Sequential Insertion Algorithm; (2) Petal Algorithm; (3) Cluster-First-Route-Second Algorithm; (4) k-opt Heuristic; (5) Two-Phase Insertion Algorithm; (6) 2-Petal Algorithm; (7) λ-Interchanges; (8) OR-Exchanges and etc. All mentioned heuristic solution methods can be implemented to find a feasible (close to optimal) set of routes out of all possibilities, while there is no guarantee for optimality.

CW Saving Algorithm uses saving values by merging pairs of customers in the same route. To the best of author knowledge, CW Saving Algorithm is the most implemented heuristic method for solving VRP due to its efficient computation time and generation of reliable solution. Many enhancements are proposed for the CW Saving Algorithm to improve its performance to generate a better set of solutions.

In recent years, Erdogan et al (Erdoğan & Miller-Hooks, 2012) and Schneider et al (Schneider et al., 2014) have investigated logistic problems concerning green vehicles. Erdogan used modified Clarke and Wright Saving Algorithm (MCW) to solve GVRP.

This thesis investigates the efficiency of MCW which is proposed by Erdogan and Miller-Hooks (Erdoğan & Miller-Hooks, 2012) as a solution method for Routing of Green Vehicles for Green Fuels while it is adapted for capacitated E-VRPTW and Recharging Stations. Validity of the solution methodologies is investigated by small instances which are solved by both exact and heuristic methods. Then, further modifications are applied on MCW Algorithm to improve its performance and similar test instances are solved by enhanced model. Three of the most studied modifications on CW Algorithm are presented by Doyuran & Catay (Doyuran & Catay, 2009), Yellow (Yellow, 1970), Altinel and Öncan (Altinel & Öncan, 2005) and Paessens (Paessens, 1988). The aforementioned modifications are adapted for E-VRPTW to increase the solutions' quality by expanding exploration ability of the algorithm in the least computation time and these algorithms are tested on the same benchmark instances to compare their results.

The computational results were obtained using Solomon's benchmark (Solomon, 1987) which is modified by Schneider et al. (Schneider et al., 2014) for electric vehicles and classified by customers' geographical distribution. The solution methods which performed better for small instances are also tested for bigger population size instances.

The thesis results are not for only E-VRPTW but also logistics service providers with a fleet of AFVs and limited refueling stations in their service area can implement the proposed techniques to find a good near to optimal set of solutions in an efficient computation time.

The thesis has the following structure: Section 2 briefly reviews the basic concepts of Vehicle Routing Problem, existing research and studies up to now. Chapter 3 defines the problem characteristics and explains related works, while Chapter 4 focuses on the solution approaches. The computational results for small and large instances by proposed methodologies are discussed in Chapter 5. Lastly, conclusion and discussion are presented in Chapter 6 besides possible future studies.

Chapter 2

LITERATURE REVIEW

In order to grasp the idea behind VRP, the existence terminology and definitions should be provided. This chapter demonstrates the required terminology to be able to both understand the idea clearly and be consistent with the existing literature. On top of that, summary of the existing literature is also provided for representing the wideness of the studies and defining scopes of this study. Finally, a summary for heuristic methodologies on VRPs is indicated.

2.1 Vehicle Routing Problem and Its Variants

VRP is a well-known integer programming problem in combinatorial optimization and it dates back to the end of the fifties when Dantzing et al. (Dantzig & Ramser, 1959) proposed a mathematical and an algorithmic formulation as a solution method for delivery of gasoline to service stations. VRP is derived by Travelling Salesman Problem (TSP), while capacity constraint is considered.

VRP is a combination of TSP and Bin Packing Problem (BPP) and it is applied in various fields. Some of the real-world applications of VRP are: laundry and mail distribution, delivery of goods in department store, tour planning, newspaper deliveries, picking up students by school buses, maintenance inspection tours and scheduling problems and so on. In VRP, there are m vehicles initially located at a depot to deliver predefined quantities of freights to n customers. Determining the optimal set of routes which are used by vehicles to satisfy customers' demand is known as VRP. The objectives of VRPs can be minimization of total travel distance and pollution, maximization of customers' satisfaction and etc. Every vehicle in VRP has a limited capacity which may vary with others. VRP with freight capacity constraint for vehicles is known as CVRP (Dantzig & Ramser, 1959). Capacity constraint is the most studied constraint which makes restriction in route construction for vehicles and each vehicle can meet limited number of customers with predefined demands that may vary from each other. Classical VRP assumes that vehicles leave a common depot and they return back in a single depot. However, Tillman (Tillman, 1969) introduced Multi-depot Vehicle Routing Problem (MDVRP), which contains more than one depot and customers' demands can be satisfied by a vehicle which is assigned to one of these depots. This variant of VRP is also originated from real-life distribution problems such as the delivery of packaged foods, chemical materials, soft drinks, industrial gasses and etc.

An important extension of VRP is Vehicle Routing Problem with Time Windows (VRPTW), which was introduced by Russell (Russell, 1977). In the proposed problem by Russell, all customers must be visited in predefined time intervals. There are two variants of VRPTW which are mainly studied in literatures and they can be stated as follows.

1) Soft Time Windows where customers can be serviced after their due-time with a price of predefined penalties.
2) Hard Time Windows, where a vehicle must arrive and be ready to serve the customer before or right before the specified time interval. Late arrival is not allowed. If the vehicle arrives earlier than the time window, it has to wait.

The traditional VRP considers Euclidean distance between the customers, however this assumption throws down the real transportations on real road networks where travel time between customers is dependent on both road distance and time of the day. Musolino et al. (Musolino, Polimeni, Rindone, & Vitetta, 2013) considered other circumstances which affects travel time such as whether conditions, rush hour and etc., and they also introduced Time-dependent Vehicle Routing Problem (TDVRP) and Time-dependent Traveling Sales-man Problem. One of the most studied extensions of TDVRP is Time-dependent Vehicle Routing Problem with Time Windows (TDVRPTW). Based on traditional benchmark instances by Solomon (Solomon, 1987), Figliozzi (Andres Figliozzi, 2012) introduced a set of benchmark problems to compare the results in terms of computation time and quality.

Watson-Gandy and Dohm (Watson-Gandy & Dohrn, 1973) not only applied modifications on VRP to be able to solve the grocery distribution, parcel delivery and waste collection problems, but also have introduced Location Routing Problem (LRP). In LRP, decisions are related to travel cost and opening cost of a set of depots or a depot among all available locations so as to satisfy customers.

VRP can be applied for multi-period deliveries in different day combinations and number of visits during a week or longer period of time. The application of VRP for multi-period deliveries was proposed by Beltrami and Bodin (Beltrami & Bodin, 1974) as Periodic Vehicle Routing Problem (PVRP).

Uncertainty in operational environments is included for VRP by Psaraftis (Psaraftis, 1980). In real-world logistics problems, vehicles break down and traffic control can take place and Dynamic Vehicle Routing Problem (DVRP) considers all these circumstances in the dynamic world.

The main question, which logistics companies seek to answer, is how many vehicles with what characteristics are needed to satisfy customers' demands with minimum cost. Clark and Wright (Clarke & Wright, 1964) considered this real-life issue and they developed Fleet Size and Mixed Vehicle Routing Problem (FMVRP). In FMVRP, each vehicle has its own characteristics such as speed, freight capacity and fuel capacity. Green Vehicle Routing Problem (GVRP) is a new aspect of VRP which has recently been introduced in three main scopes, including VRP in Reverse Logistics (VRPRL) (Beullens, Van Oudheusden, & Van Wassenhove, 2004), Pollution Routing Problem (PRP) (Bektaş & Laporte, 2011) and Green-VRP (Erdoğan & Miller-Hooks, 2012). PRP focuses on minimization of emitted Greenhouse Gasses (GHG) by vehicles. VRPRL investigates reverse logistics distribution aspects in four main categories including Waste Collection, Selective Pickup with Pricing, Simultaneous Distribution and Collection and End-of-life Goods Collection. A Green Vehicle Routing Problem was firstly introduced by Erdogan et al. (Erdoğan & Miller-Hooks, 2012) for dealing with problems caused by the usage of new vehicles with alternative fuels such as biodiesel which are needed by vehicles to meet fuel stations through the routes.

All the above mentioned VRPs can be solved in deterministic or stochastic environment. The notion of stochastic VRP was introduced by Gendreau et al. (Gendreau, Laporte, & Séguin, 1996). Stochastic Vehicle Routing Problem (SVRP) assigns random values to customers' demands, location and travel time to make Classical VRP applicable in uncertain environments.

2.2 Solution Methods

As VRP is an NP-hard Combinatorial Optimization Problem, computation of solutions by exact algorithms in polynomial time is only possible for small instances in long computation time. As a consequence of this property of VRP, all exact algorithms such as direct search methods, dynamic programming, integer linear programming and etc., become useless. Many Heuristic and Meta-heuristic Algorithms, whether designed or adapted to solve various VRPs in short computation time and much research investigates on decreasing the computation time of such these algorithms and increasing output quality of them.

In general, approximate algorithms are categorized into two main sub-groups which are classical heuristic algorithms and meta-heuristic algorithms. The fastest way to reach the solution is implementation of classical heuristic algorithms. These algorithms are developed to achieve the best solution for large VRPs in efficient ways as fast as possible, while they can get equipped with improvement approaches to utilize the primary solution and improve the solutions as much as possible, therefore, heuristic algorithms with improvement part can generate solutions with higher quality, but in longer computation time.

Many heuristic solution methods have been proposed since 1959, such as k-opt Heuristic, Petal Algorithm, λ-interchanges, OR-exchanges, Sequential Insertion Algorithm, Sweep Algorithm, Saving Algorithms and etc., which can produce a feasible set of routes in a short computation time. Among all aforementioned algorithms, Sweep and CW Saving Algorithms are the most studied methods, since they generate results with higher qualities.

There are two versions of CW Saving Algorithm which are Parallel CW Saving Algorithm and Sequential CW Saving Algorithm. The Sequential CW Saving Algorithm only constructs one route at a time, whereas the parallel version may construct more than one route at a time. The CW Saving Algorithm calculates saving values for each pair of nodes and it starts to merge nodes in a way to satisfy constraints and minimize total travel distance. Erdogan et al. (Erdoğan & Miller-Hooks, 2012) used sequential CW Saving Algorithm to solve a GVRP, while many other authors like Cao (Cao, 2012), Pichpibul et al. (Pichpibul & Kawtummachai, 2012) and etc., also implement CW Saving Algorithm to solve classical VRP with various properties.

Sweep Algorithm is another well-studied heuristic that constructs routes based on the angles of customers with depot and another arbitrary line (Schneider et al., 2014). However, to the best of my knowledge, it is not as efficient as CW Saving Algorithm in terms of both quality of solutions and implementation complexity.

Meta-heuristic Algorithms can be categorized into two sub-groups (Lin, Choy, Ho, Chung, & Lam, 2014) as follows.

1) Local Search Algorithms such as Tabu Search (TS) (Brandão, 2004), Variable Neighborhood Search (VNS) (Wen, Krapper, Larsen, & Stidsen, 2011), Large Neighborhood Search (LNS) (Mattos Ribeiro & Laporte, 2012), Simulated Annealing (SA) (Baños, Ortega, Gil, Fernández, & de Toro, 2013) and etc., explore solution space iteratively from a solution in a current neighborhood to another solution in other neighborhoods.

2) Population Search Algorithms, like Ant Colony Optimization (ACO) (Yu & Yang, 2011), Genetic Algorithm (GA) (Vidal, Crainic, Gendreau, & Prins, 2013) and many others of which most are inspired from natural phenomena. Population Search Algorithms keep a pool of good parent solutions and by sequential selecting of parent solutions, they generate a new reliable offspring and hence updating the pool.

Chapter 3

PROBLEM DESCRIPTION

Using Electric Vehicles (EVs) for last-mile deliveries in Small Package Shipping companies (SPS) caused new restrictions for routing problems for EVs such as battery capacity constraint, various recharging rates depending on the charge level and etc. All these restrictions are raised from EVs in logistic companies, encouraged new investigations for determination of better routes according to fleet specifications. VRPTW (Russell, 1977) is a well-studied variant of VRP which is close to real-life problems. Schneider et al. (Schneider et al., 2014) proposed Electric Vehicle Routing Problem with Time Windows (E-VRPTW) and Recharging Stations which is an extension of VRP for electric vehicles. The first step in the definition of E-VRPTW is presentation of mathematical model, however one should not that without presenting mathematical model for CVRP and VRPTW, it is incomplete.

3.1 Capacitated Vehicle Routing Problem

CVRP is a variant of VRP that known number of vehicles with uniform capacity must serve customers with minimum transit cost, while all vehicles leave a common depot (Dantzig & Ramser, 1959). Total demand of each route must be less than the vehicle capacity and any violation in capacity is not allowed.

Figure 3.1 illustrates a CVRP. In Figure 3.1, a depot and a set of customers are defined, while a possible solution is proposed for better understanding of the problem. The following figure illustrates the classical VRP with freight capacity; however the real-world logistic problems are more complicated. Many variants of VRP are proposed to include real-life logistic problems and constraints.

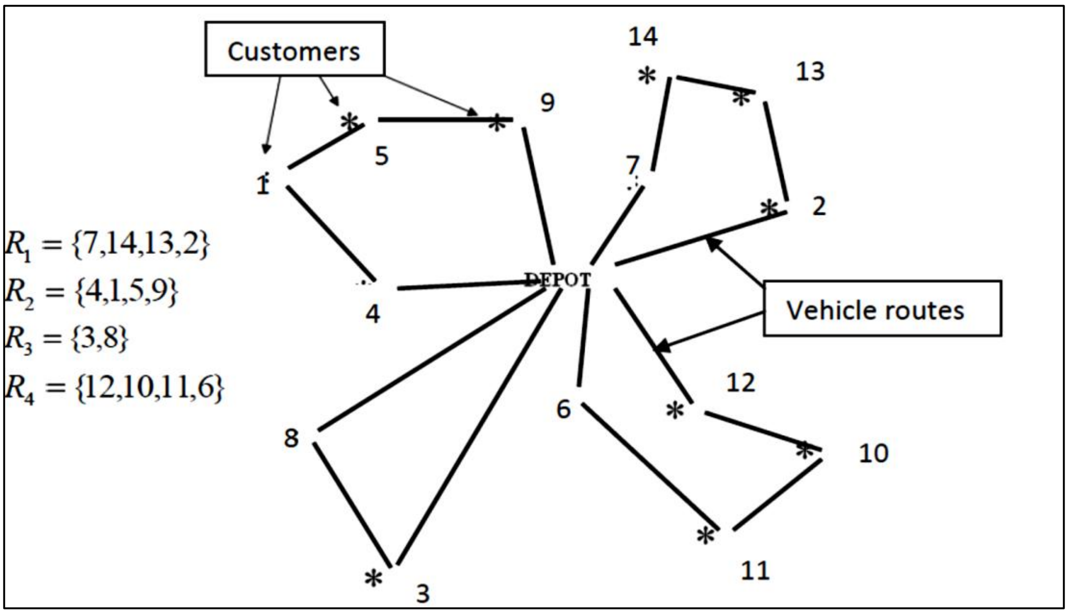

Figure 3.1. Possible Solution for a VRP

3.2 Vehicle Routing Problem with Time Windows

Besides CVRP, another variant of VRP is used in this thesis to have a better simulation of real-life logistic problems. In VRPTW, certain time intervals are defined for each customer in such a way that beginning of customer service must be within the intervals. In general, time window for customer *i* will be shown as (e_i,l_i) denoting the earliest time and the latest possible time that vehicles can start the service (Russell, 1977). VRPTW had dealt with Solomon case study (Solomon, 1987) and benchmarks are prepared according to Solomon's instances. Figure 3.2 shows the general concept of VRP with time windows for three customers, where e_i and l_i, represent the earliest time for beginning of customer *i* service and the latest time that vehicle can arrive to customer *i* and start the service respectively.

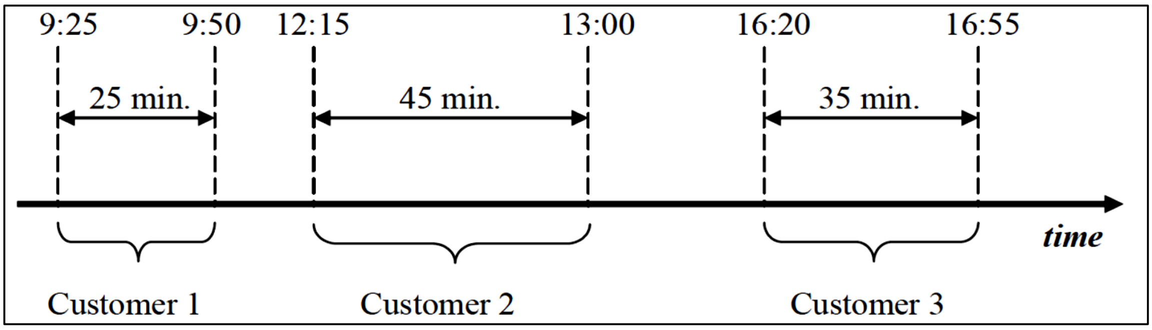

Figure 3.2. Time Windows for Three Customers

The formal definition of VRPTW is provided as mixed-integer program in this section. Let 0 and N+1 denote instances of the same depot and V =1, 2, …, N the set of N customers where all routes start at 0 and end at N + 1. Moreover, N + 1 and 0 indices implies that a set contains depot like V_0, which shows set of V contains 0. VRPTW can be shown as a complete directed graph $G = (V_{0,N+1}, A)$ which contains a set of arcs $A = \{(i, j) \mid i, j \in V_{0,N+1}, i \neq j\}$. For each pair of nodes d_{ij} and t_{ij} indicate distance and travel time from customer i to j . There are k number of homogeneous vehicles with a maximum capacity of C at depot and q_i is a positive demand of each node, which is 0 for depot. Furthermore, each vertex $i \in V_{0,N+1}$ has a time window $[e_i, l_i]$ and specific service time s_i , which is 0 for depot. Variable τ_j indicates the service starting time, while u_j specifies the remaining freight at arrival to customer $j \in V_{0,N+1}$. Variable $x_{ij}, i \in V_0, j \in V_{N+1}, i \neq j$ is a binary variable, which is equal to 1 if an arc (i, j) traveled and otherwise it is 0. According to descriptions above, the mathematical model for VRPTW is formulated as follows (Russell, 1977).

$$Min \sum_{i \in V_0, j \in V_{N+1}, i \neq j} d_{ij} x_{ij} \quad \textbf{(3.1)}$$

$$\sum_{j \in V_{N+1}, i \neq j} x_{ij} = 1 \qquad \forall i \in V \quad \textbf{(3.2)}$$

$$\sum_{j \in V_0, i \neq j} x_{ji} = 1 \qquad \forall i \in V \quad \textbf{(3.3)}$$

$$\tau_i + (t_{ij} + s_i)x_{ij} - l_0(1 - x_{ij}) \leq \tau_j \qquad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad \textbf{(3.4)}$$

$$e_j \leq \tau_j \leq l_j \qquad \forall j \in V_{0,N+1} \quad \textbf{(3.5)}$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \qquad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad \textbf{(3.6)}$$

$$0 \leq u_0 \leq C \quad \textbf{(3.7)}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in V_0, \forall j \in V_{N+1}, i \neq j \quad \textbf{(3.8)}$$

The objective of the above function (3.1) is the minimization of total distance. Constraint (3.2) guarantees that each node has only one successor. Equation (3.3) ensures that incoming and outgoing arcs into each customer are equal. Time feasibility is enforced by equation (3.4) for all customers and depot, while constraint (3.5) checks the arrival time of vehicles to customers which must be within the intervals. Capacity constraints are applied by (3.6) and (3.7). The subtours' information is prevented by equation (3.4) and (3.5). Constraint (3.9) describes a binary variable, which may be 1 or 0.

3.3 Electric Vehicle Routing Problem with Time Windows

E-VRPTW concerns with VRP which fleet vehicles' fuel is electricity. Since recharging facilities for EVs are limited through the world and limited battery capacity of vehicles, VRP needs further modifications based on EVs characteristics in order to be beneficial, i.e., maximum travel ranges of EVs are not usually sufficient for doing the typical deliveries and fuel constraint must be taken into consideration. In order to overcome the problem, EVs need to visit appropriate recharging stations before they run out of electricity to be able to complete routes and traverse minimum distance, while they visit in route customers. Another variant of routing of vehicles with green fuels is VRP for biodiesel vehicles which also needs refueling through the routes (Erdoğan & Miller-Hooks, 2012) and it is the same as E-VRPTW in origin, while EVs recharging is not as fast as other vehicles and recharging time needs to be considered in travel time.

The following figure shows E-VRPTW. The shaded regions of the cylinders show the battery level concerned. Customers are shown by circles and available charge stations in triangles. The vehicle leaves depot and meets customers till customer five; however as it does not have sufficient charge, it has to visit the closest recharging (F2) to complete the route. All the customers and depot have to be visited within a predefined time interval.

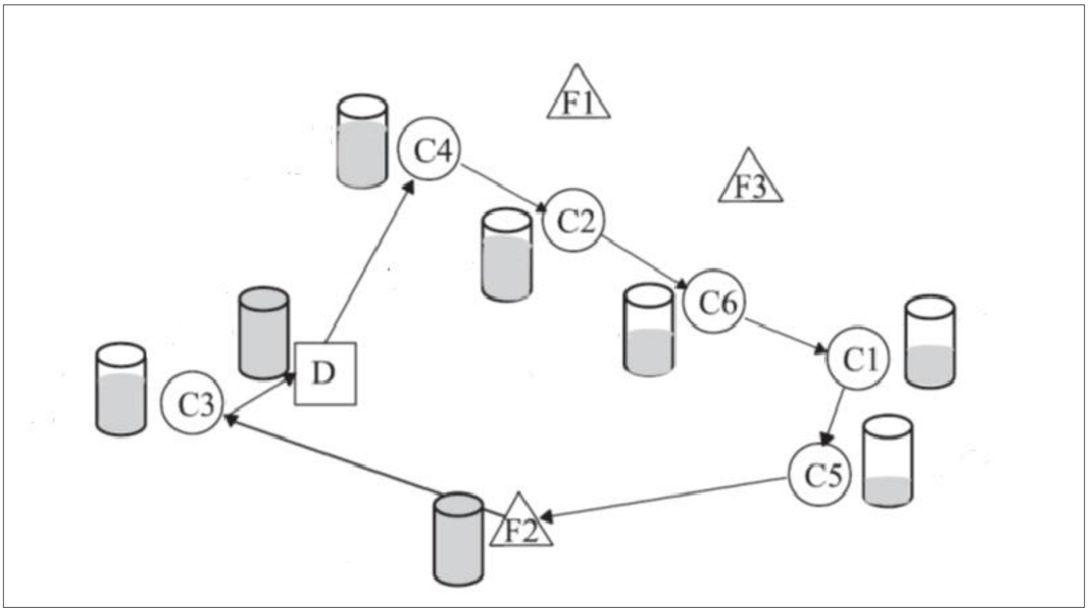

Figure 3.3. A sample solution for EVRPTW

In routing of vehicles with green fuels, Refueling Stations may be visited once, more than once or not visited at all (see Fig 3.4).

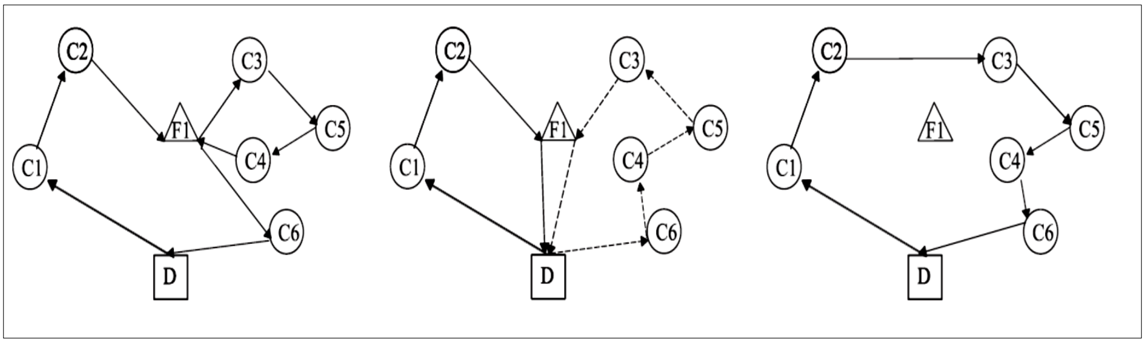

Figure 3.4. Possible Visits for Refueling Stations

The formal definition of capacitated E-VRPTW with Recharging Stations is provided as a mixed-integer program in this section. Let 0 and $N+1$ denote the same depot, where each route starts at 0 and ends at $N+1$. Let F be a set of recharging stations, while F' is a set of dummy vertices, which allows vehicles to visit recharging stations several times. One of the recharging stations is located at depot and vehicles leave depot when they are fully charged. Let $V = 1,2,...,N$ denote the set of N customers and $V' = V \cup F'$. The sets, which are subscripted by 0 or $N+1$ contain respective instances of the depot, such as $V_0' = V' \cup \{0\}$, $F_0' = F' \cup \{0\}$, $V_{0,N+1}' = V' \cup \{0\} \cup \{N+1\}$, $V_{N+1}' = V' \cup \{N+1\}$.

E-VRPTW can be presented as a completed directed graph $G = (V_{0,N+1}', A)$, which contains a set of arcs $A = \{(i, j) \mid i, j \in V_{0,N+1}, i \neq j \}$, where d_{ij} and t_{ij} are distance and travel time of each arc, while $h \cdot d_{ij}$ indicates required battery charge for travelling through each arc, h and g are charge consumption and recharging rates respectively. Each customer $i \in V_{0,N+1}'$ has demand q_i, time window $[e_i, l_i]$ and a unique service time

s_i, where homogeneous vehicles with freight capacity C and charge capacity Q must start the service after e_i and no later than l_i. Service starting time for each vertex is shown by τ_i, while remaining cargo and battery charge are shown by u_i and y_i respectively. Variable $x_{ij}, i \in V_0, j \in V_{N+1}, i \neq j$ is a binary variable, which is equal to 1 if an arc (i, j) is travelled and otherwise it is 0. According to above descriptions, the mathematical model for E-VRPTW is formulated as below (Schneider et al., 2014).

$$Min \sum_{i \in V'_0, j \in V'_{N+1}, i \neq j} d_{ij} x_{ij} \qquad \textbf{(3.9)}$$

$$\sum_{j \in V'_{N+1}, i \neq j} x_{ij} = 1 \qquad \forall i \in V \qquad \textbf{(3.10)}$$

$$\sum_{j \in V'_{N+1}, i \neq j} x_{ij} \leq 1 \qquad \forall i \in F' \qquad \textbf{(3.11)}$$

$$\sum_{j \in V'_{N+1}, i \neq j} x_{ji} - \sum_{j \in V'_0, i \neq j} x_{ij} = 0 \qquad \forall j \in V' \qquad \textbf{(3.12)}$$

$$\tau_i + (t_{ij} + s_i) x_{ij} - l_0 (1 - x_{ij}) \leq \tau_j \qquad \forall i \in V_0, \forall j \in V'_{N+1}, i \neq j \qquad \textbf{(3.13)}$$

$$\tau_i + t_{ij} x_{ij} + g(Q - y_i) - (l_0 + gQ)(1 - x_{ij}) \leq \tau_j \qquad \forall i \in F', \forall j \in V'_{N+1}, i \neq j \qquad \textbf{(3.14)}$$

$$e_j \leq \tau_j \leq l_j \qquad \forall j \in V'_{0,N+1} \qquad \textbf{(3.15)}$$

$$0 \leq u_j \leq u_i - q_i x_{ij} + C(1 - x_{ij}) \qquad \forall i \in V'_0, \forall j \in V'_{N+1}, i \neq j \qquad \textbf{(3.16)}$$

$$0 \leq u_0 \leq C \qquad \textbf{(3.17)}$$

$$0 \leq y_j \leq y_i - (h \cdot d_{ij}) x_{ij} + Q(1 - x_{ij}) \qquad \forall j \in V'_{N+1}, \forall i \in V, i \neq j \qquad \textbf{(3.18)}$$

$$0 \leq y_j \leq Q - (h \cdot d_{ij}) x_{ij} \qquad \forall j \in V'_{N+1}, \forall i \in F'_0, i \neq j \qquad \textbf{(3.19)}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in V'_0, j \in V'_{N+1}, i \neq j \qquad \textbf{(3.20)}$$

Variables and parameters in the above mathematical formulation are defined in following table.

Table 3.1. Variables and parameter definitions

Variables and Parameter Definitions	
$0, N+1$	Depot instances
q_i	Demand of customer *i*
C	Vehicle capacity
V	Set of customers
V'_0	Set of customers and recharging visits including depot instances
V'	Set of customers including recharging stations
F'_0	Set of recharging stations including depot
e_i	Earliest start of service at customer *i*
s_i	Service time of customer *i*
l_i	Latest start of service at customer *i*
d_{ij}	Distance between customer *i* and *j*
t_{ij}	Travel time between customer *i* and *j*
r	Charge consumption rate
Q	Battery capacity

The objective of the above model is minimization of total travel distance (3.9). Equation (3.10) ensures that each node has one successor. Connectivity of Recharging Stations visits are handled by constraint (3.11). Equation (3.12) ensures that incoming and outgoing arcs into each customer are equal. Time feasibility is enforced by equation (3.13) for all customers and depot, while time feasibility for arcs leaving recharging stations is applied by constraint (3.14). Formation of subtours is prevented by constraints (3.13)-(3.15). Constraint (3.15) enforces time window constraint for all vertices. Demand satisfaction of customers is guaranteed by constraints (3.16) and (3.17). Constraints (3.18) and (3.19) check the battery level, which is never negative. Finally, Constraint (3.20) describes a binary variable, which may be 0 or 1.

Chapter 4

SOLUTION METHODOLOGY

In this chapter, CW Algorithm and existing modifications are introduced. And those which are implemented are discussed.

4.1 Clarke and Wright Saving Algorithm

CW Saving Algorithm (Clarke & Wright, 1964) generates relatively good near to optimal solutions as it is a heuristic algorithm. CW Saving Algorithm is based on cost saving values obtained by merging two routes into one route as it is represented in figure 4.1.

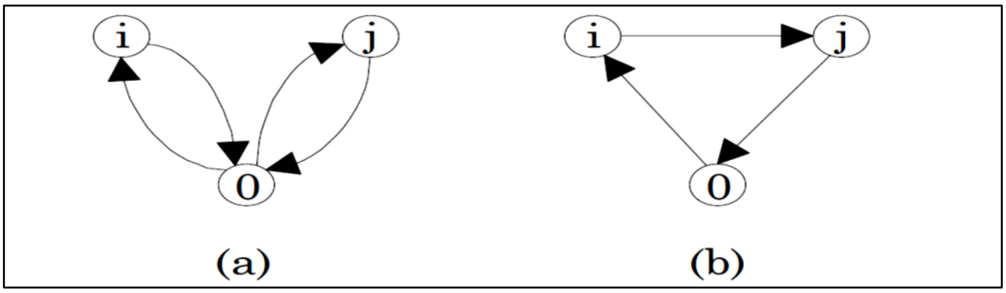

Figure 4.1. Illustration of the Savings Concept

In Figure 4.1(a) customers i and j are visited in two separate routes, while customers i and j are in the same route after saving values calculation. Transportation cost between i and j is represented by c_{ij} which is equal to d_{ij}. The transportation costs for figure 4.1(a) and 4.1(b) can be expressed as follows respectively.

$$D_a = c_{0i} + c_{i0} + c_{0j} + c_{j0} \tag{4.1}$$

$$D_a = c_{0i} + c_{ij} + c_{j0} \qquad \textbf{(4.2)}$$

By considering equation 4.1 and 4.2, saving value for each pair can be figured out as below.

$$S_{ij} = D_a - D_b = c_{i0} + c_{0j} - c_{ij} \qquad \textbf{(4.3)}$$

Five main steps of CW Saving Algorithm for classical VRP is as follows.

- Step 1: Calculating saving values for all pairs and construction of saving pair list.
- Step 2: Sorting out saving values in descending order.
- Step 3: Merging the customer’s route of nodes at the top of the saving pair list.
- Step 4: Checking the vehicle capacity to prevent occurrence of capacity violation.
- Step 5: The above steps should be repeated in the case if any unvisited customers are remained.

4.2 Clarke and Wright Saving Heuristic for Green Vehicle Routing Problem

Erdogan and Miller-Hooks (Erdoğan & Miller-Hooks, 2012) considered fuel constraint besides the other constraints such as time windows and capacity for Alternative Fuel Vehicles (AFVs) and applied required modifications on CW Saving Algorithm to adapt it for routing of vehicles with green fuels. The main five steps of modified CW Saving Algorithm can be illustrated as follows.

- Step 1: Assign each customer to each route which starts and ends at the depot, while it is meeting the corresponding customers.
- Step 2: Check route’s feasibility by calculating tour duration, distance and etc., and categorize routes into feasible and infeasible tours list.

- Step 3: For each route in the infeasible tour list, try to insert an Alternative Fuel Station (AFS) and if by AFS insertion the route, driving range and travel duration constraints are satisfied, add it to the feasible tour list. Only insertion of one AFS is allowed for starting tours. Insertion cost of an AFS (f) between vertex (i) and depot (0) can be expressed as below.

$$C_{i0}^{f} = c_{if} + c_{f0} - c_{i0} \quad \textbf{(4.4)}$$

- Step 4: Compute the saving values and sort them in a descending order in saving pair list.
- Step 5: If the saving pair list is not empty: select the first unvisited pair of vertices from the saving pair list, merge with associated tours and check the driving range and tour duration constraints. If both constraints are met, add the new route to the feasible tour list. However, if the driving range violates from maximum driving range, insert a new AFS with a least cost into the route and check the feasibility constraints again. Insertion cost of AFS (f) between customers (i) and (j) is as follows.

$$C_{ij}^{f} = c_{if} + c_{fj} - c_{i0} - c_{0j} \quad \textbf{(4.5)}$$

After insertion of AFSs, redundancy check should be applied to check the possibility of removing unnecessary AFSs in the route. Add the final tour to the feasible tour list after redundancy check. If any tour has been added to the feasible tours list, return to Step 4. Otherwise, stop.

The above steps are illustrated in Figure 4.2 which illustrates additional characteristics of this problem class that affect the merging process. E . in Fig. 4.2a, two tours that visit the same AFS can be merged with only a deletion in the links incident on the depot. No additional links are required. Moreover, tours that cannot be merged directly may be combined if an AFS is included as depicted in Fig. 4.2b. When a tour containing an AFS is included in a merge that involves an additional AFS visit, as in Fig. 4.2b, it may be that inclusion of an AFS from an original tour is redundant. This AFS can be dropped from the final post-merge tour, resulting in, for example, the tour depicted in Fig. 4.2c.

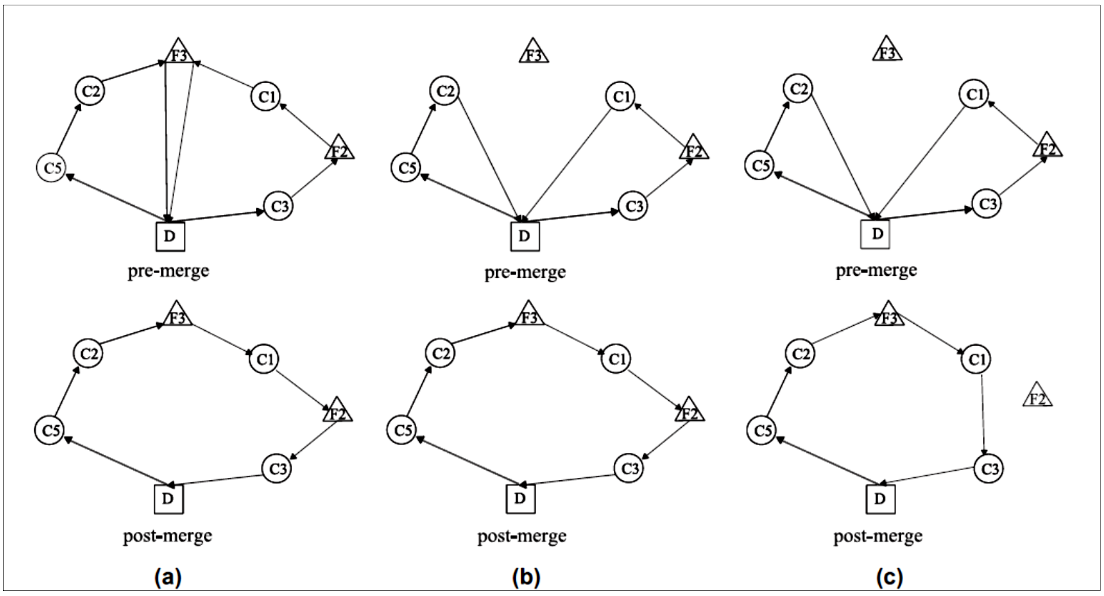

Figure 4.2. Merging Characteristics

4.2.1 Clarke and Wright Saving Heuristic with One and Two Parameters

The classical CW Saving Algorithm calculates saving values by equation 4.3 and sort saving values in descending order. The corresponding saving values for each pair is large whenever nodes are far from depot and the distance between them is short. One of the weaknesses of CW Saving Algorithm is that it starts to join pairs with higher saving values together and it causes circular shapes for constructed tours. Both Yellow (Yellow, 1970) and Gaskell (Gaskell, 1967) improved CW Saving Algorithm by adding a new parameter to equation 4.3, which causes change in pairs' priorities and in general shapes of constructed routes The modified version of CW Saving Algorithm is as below, which λ interval is $[0.1,2]$ and 0.1 is the step size. The best λ interval has already been defined by researchers.

$$S_{ij} = \lfloor c_{i0} + c_{0j} - \lambda c_{ij} \rfloor \quad \textbf{(4.6)}$$

Paeessens (Paessens, 1988) introduced another modification to CW Saving Algorithm by adding a second term to equation 4.7 in order to increase the reshaping ability of CW Saving Algorithm with one parameter. The second term of the proposed modification may exploit the asymmetry information of each pair of customers about their distances to the depot. The CW Saving Algorithm with two parameters is as follows.

$$S_{ij} = \lfloor c_{i0} + c_{0j} - \lambda c_{ij} \rfloor + \lfloor \mu | c_{i0} - c_{0j} | \rfloor \quad \textbf{(4.7)}$$

In equation 4.7, λ and μ intervals are $[0.1,2]$ and $[0,2]$ respectively, while their incremental step size is 0.1.

4.2.2 Clarke and Wright Saving Heuristic Algorithm with Three Parameters

Altinel and Öncan (Altinel & Öncan, 2005) proposed a new enhancement to CW Saving Algorithm. They considered customers' demands in each pair and they gave more priority to the pairs with higher demands. The proposed formula is as follows.

$$S_{ij} = \lfloor c_{i0} + c_{0j} - \lambda c_{ij} \rfloor + \lfloor \mu | c_{i0} - c_{0j} | \rfloor + \left\lfloor \nu \frac{d_i + d_j}{\bar{d}} \right\rfloor \quad \textbf{(4.8)}$$

In equation 4.8 λ, μ and ν intervals are $[0.1,2]$, $[0,2]$ and $[0,2]$ respectively, while their incremental step size is 0.1. d_i shows the demand of customer i in the formula.

Recently, Doyuran and Çatay (Doyuran & Catay, 2009) revealed that either promotion of pairs' priorities with less or high demands results in the same outcome and they drew the conclusion that the performance of (4.9) and (4.10) are almost the same which can be observed as illustrated below.

$$S_{ij} = \left[c_{i0} + c_{0j} - \lambda c_{ij}\right] + \left[\mu \left|c_{i0} - c_{0j}\right|\right] - \left[\nu \frac{d_i + d_j}{\bar{d}}\right] \tag{4.9}$$

$$S_{ij} = \left[c_{i0} + c_{0j} - \lambda c_{ij}\right] + \left[\mu \left|c_{i0} - c_{0j}\right|\right] + \left[\nu \frac{\bar{d}}{d_i + d_j}\right] \tag{4.10}$$

Doyuran et al. (Doyuran & Catay, 2009) proposed a new enhancement to Altinel formula for assigning customers with high and low demands to the same route. Moreover, they added a new term to saving formula to increase the priority of customers near to depot.

$$S_{ij} = \left[\frac{c_{ij} + c_{0j} - \lambda c_{ij}}{c^{\max}}\right] + \left[\mu \left(\frac{\cos\theta_{ij} \left|c^{\max} - \left(c_{i0} - c_{0j}\right)/2\right|}{c^{\max}}\right)\right] + \left[\nu \left(\frac{\left|\bar{d} - (d_i + d_j)/2\right|}{d^{\max}}\right)\right] \tag{4.11}$$

The second term of the formula above is originated from Sweep Algorithm characteristic, while the third one allows customers with low and high demands to assign to same routes. Figure 4.3 shows how the second term of the above formula improves the performance of the classical CW Saving Algorithm. From the figure, one can observe that the last route which is constructed by the saving value indicated in equation 4.11 is much better than the routes constructed by Sweep Algorithm and classical CW Saving Algorithm. The reason for that is considering customers' demands, saving values and angles between customers.

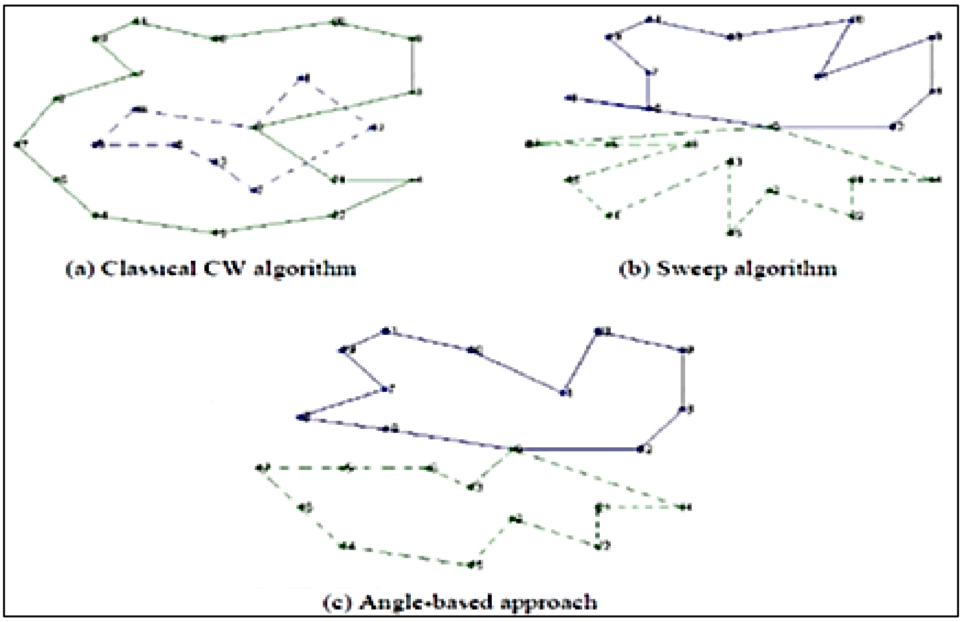

Figure 4.3. Illustration of CWs Heuristic with Three Parameters
(Doyuran & Catay, 2009)

4.3 Solution Methodologies for E-VRPTW

The E-VRPTW is solved by different variants of CW Saving Algorithm to evaluate their performance, while appropriate modifications are applied and they are adapted for E-VRPTW and Recharging Stations. Proposed solution methodologies for E-VRPTW are explained in the following sections.

4.3.1 MCW Saving Algorithm

Erdogan et al. (Erdoğan & Miller-Hooks, 2012) proposed Modified Clarke and Wright (MCW) Saving and Density Based Clustering Algorithm as solution methods for GVRP. The performance and solution qualities of these two algorithms are evaluated and compared by Erdogan and due to many similarities of the GVRP with E-VRPTW in principal concepts and admirable performance of MCW Saving Heuristic in comparison with Density Based Clustering Algorithm, it is chosen as one of the solution methodologies. The MCW Saving Algorithm has already been described in section 4.2, while the only difference of adapted MCW Saving Algorithm for E-VRPTW is that vehicles meet Recharging Stations instead of Alternative Fuel Stations for refueling. Recharging of electric vehicles compared to other AFVs requires more time for recharging dependent on the vehicle's battery capacity which should be taken into consideration. The following methodologies focus on improvement of solution qualities in terms of time and accuracy.

4.3.2 A New MCW Saving Heuristic for E-VRPTW

A new modification of CW Saving Heuristic is proposed by the implementation of further adaptions to basic CW Saving Algorithm and inspiration from Erdogan and Miller-Hooks' methodology. The new modifications on CW Saving Algorithm for E-VRPTW tries to insert customers with earlier ready time at the beginning of the routes so as to decrease the probability of violation in time windows constraint and visit more customers in each route. Moreover, in this methodology, insertion of Recharging Stations in suitable and necessary locations occurs simultaneously with customer insertion; thus, prior to the insertion of each customer, fuel feasibly for reaching next customer will be checked by the algorithm and in the case if violation of fuel constraint occurs, a new Recharging Station will be inserted. In general, all the following constraints must be checked by the algorithm before each customer insertion (Schneider et al., 2014).

$$q_v + q_w \geq C \qquad \forall v, w \in V \qquad \textbf{(4.12)}$$

$$e_v + s_v + t_{vw} \geq l_w \qquad \forall v \in V_0', \forall w \in V_{n+1}' \qquad \textbf{(4.13)}$$

$$e_v + s_v + t_{vw} + s_w + t_{wn+1} \geq l_0 \qquad \forall v \in V_0', \forall w \in V' \qquad \textbf{(4.14)}$$

$$h(d_{jv} + d_{vw} + d_{wi}) \geq Q \qquad i \in F_{n+1}' \qquad \textbf{(4.15)}$$

Equations (4.12) and (4.13) check load capacity and time window violations for each customer insertion and if violation occurs, the route can be labelled as infeasible. Equation (4.14) checks the time feasibility by considering service time, waiting time and travelling time. Insertion of a customer can be cancelled, if a vehicle cannot get back to the depot before its due time. Equation (4.15) is problem specific and refers to violations of the battery capacity.

4.3.2.1 Insertion of Recharging Stations

In adapted Erdogan and Miller-Hooks' methodology for E-VRPTW, insertion of Recharging Stations occurs while the next customer is unreachable by the remained charge for the vehicle. Recharging Stations would be inserted between correspondent customers in order to eliminate violation in fuel constraint. In the case when the vehicle is able to reach neither the next Recharging Station nor the next customer, even by the insertion of Recharging Station with minimum cost, a Recharging Station should be inserted into the route between the previous pair of customers for providing sufficient charge level to reach the assigned customer or the next recharging station.

4.3.3 Enhanced CW Saving Heuristic with One and Two Parameters

CW Saving Algorithm with modifications mentioned in Section 4.3.2 is used, while the saving value is computed by equations (4.6) and (4.7). The algorithm proceeds as follows.

- Step 1: Calculate saving values by using appropriate savings function.
- Step 2: Sort saving values and corresponding pairs in descending order.
- Step 3: Calculate insertion cost of Recharging Stations between each pair of customers by equation 4.5.
- Step 4: Arrange cost values in increasing order.
- Step 5: Select the starting pair of customers at the top of the saving list.

- Step 6: Check the feasibility of starting pair by constraints (4.12), (4.13), (4.14) and (4.15).If any violation occurs, get back to step 3 and select the next pair in the saving pair list.
- Step 7: If violation in the battery constraint occurs, place a new Recharging Station on the route with minimum cost.
- Step 8: If the remaining customers cannot be assigned to either constructed or new routes, they will be assigned individually to separate routes.
- Step 9: All routes have to start with visiting a customer with earlier ready time before visiting the other customer of the pair.
- Step 10: Find the first feasible link in the list which can be used to extend one of the two ends of the currently constructed route. Each step must be complied with the constraints.
- Step 11: If the number of marked customers is less than the total number of customers, return to step 10, otherwise start a new route and return to step 5.
- Step 12: Do the above steps while saving values are being calculated by new coefficients and return to step1 if any value is remained in the predefined intervals.
- Step 13: Check all the set of the routes which are constructed by various coefficient and chose the one with least travel distance as the best.

Equations 4.6 and 4.7 in the first step refers to CW Saving Algorithm with one and two parameters.

4.3.4 MCW Saving Heuristic Algorithm with Three Parameters

The proposed modification by Doyuran and Çatay (Doyuran & Catay, 2009) in computation of saving values is considered to evaluate the influence of customers' angles and their demands on the saving pair list construction (see section 4.2.2) , while CW Saving Algorithm has already been adapted for E-VRPTW and Recharging Stations. Saving values are computed by equation 4.11, while λ, μ and ν intervals are $[0.1,2]$, $[0,2]$ and $[-0.2,0.2]$ respectively and the incremental step size for λ and μ is 0.1 and 0.01 for ν.

4.3.5 MCW Saving Heuristic with Four Parameters

It has already been proven that by adding any extra parameter to the saving formula, an improvement in the solution quality can be obtained. The following additional parameters in the cost formula for the insertion of Recharging Stations are added to check how they can improve the solution quality.

$$C_{ij}^{f} = \lambda_2 c_{if} + \lambda_3 c_{fj} - c_{i0} - c_{0j} \qquad \textbf{(4.15)}$$

The above parameters in Equation 4.15 let those Recharging Stations which are worse in cost to be inserted where required. The benefit of parameters above are more obvious while a Recharging Station with minimum insertion cost cannot be reached by the vehicle from customer *i*, but the second best Recharging Station can be assigned to the route, since the Recharging Station is closer to customer *i* and farther from customer *j*. If any saving can be obtained by the insertion of the Recharging Stations that are worse than the ones with minimum cost, the algorithm will insert the Recharging Station between the pair of customers. It is worthwhile mentioning that two parameters are from the above formula and the other two from equation 4.7.

Chapter 5

COMPUTATIONAL STUDY

This chapter presents the results of E-VRPTW solved by former discussed methodologies, namely adapted Erdogan and Miller-Hooks' methodology, Modified Clarke and Wright (MCW), MCW with one parameter, MCW with two parameters, MCW with three parameters and MCW with four parameters for small instances. All tests are performed on a laptop computer equipped with an Intel Core i5 -2430M processor clocked at 2.4 GHz with 4 GB RAM, running Windows 7 Professional. The proposed methodologies are coded in C++ and Microsoft Visual Studio 2012 is used as the compiler.

5.1 Description of Benchmark Instances

5.1.1 Solomon Benchmark Instances

As priory mentioned, numerous scientists have worked on VRPTW and a few benchmark instances are proposed to evaluate new solution methodologies by comparing with other well-known results. The most reliable benchmark instances are designed by Solomon (Solomon, 1987) which 56 of the largest ones with 100 customers have received much more attention. In Solomon's instances and such benchmark instances time windows and coordinates of customers are defined. Moreover, the average speeds which are proposed for each set of instances have already been adjusted in such a way to take the traffic and road situation into the consideration. The proposed instances represented various problem specifications which are divided in 6 sub-groups namely, C1, R1, RC1, C2, R2, RC2 and they include between 8 and 12 test problems. The aforementioned categorizations are based on four different customer location distributions: uniform distribution, cluster distribution and combination of clustered and uniform. Some features vary in above groups, i.e. scheduling horizon, vehicles' capacity and number of customers. Figure 4.1 illustrates the geographical distribution of customers in each group with 100 customers.

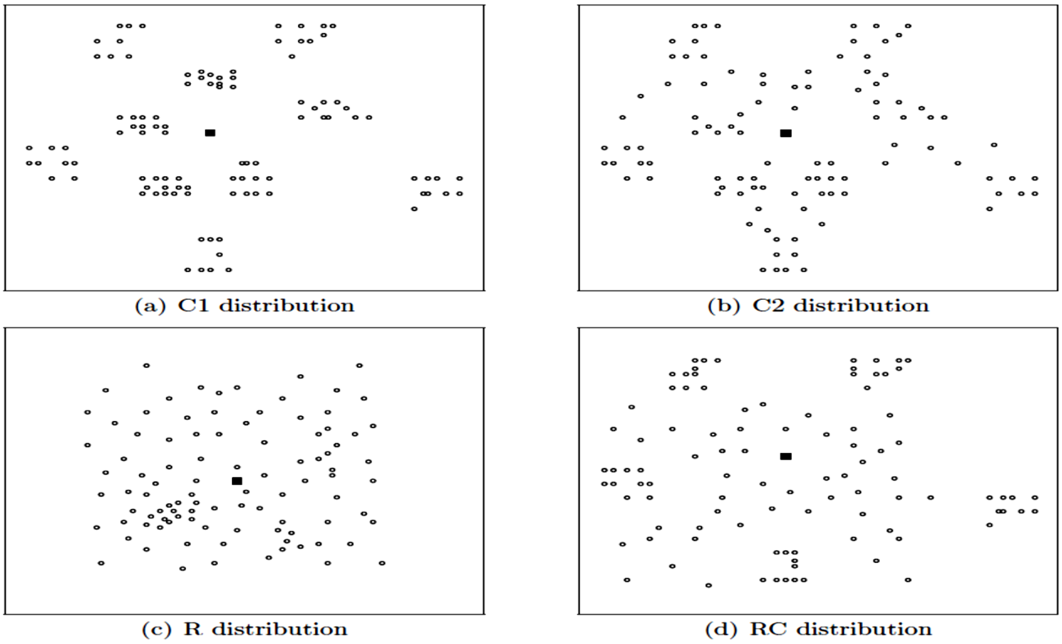

Figure 5.1. Distribution of Customers in Solomon's Benchmark Instances

5.1.2 Schneider's Benchmark Instances

Two set of benchmark instances are used for evaluation of proposed methodologies. A set of 36 instances with 5, 10 and 15 customers and a set of 56 large instances with 21 recharging stations and 100 customers per instance. All customers can be reached from depot by visiting at most two recharging stations, while a recharging station is located at the depot. The battery capacity has to be at least equal to the maximum value of the following conditions:

1) 60% of the average of best-known routes' length.

2) Two times more than the required battery charge to travel from a station to a customer*.

5.2 Results for Small Instances

All the proposed solution methods are examined in this section in order to evaluate their performance on small instances of Schneider benchmark. Furthermore, run times for small instances are mentioned in following tables besides the total number of routes and total travelling distance.

5.2.1 Adapted Erdogan's Methodology (MCW 1)

Small instances of Schneider benchmark which contain data for 5, 10 and 15 customers, totally 36 instances, are solved and the correspondent computation times are mentioned in the tables.

*The generated instances by Schneider are available for download at http://evrptw.wiwi.unifrankfurt.de

Table 5.1. Obtained results for small sized instances by MCW 1 and CPLEX

Best Known Results			MCW 1			
Instance	**m**	**Travel Distance**	**m**	**Travel Distance**	**Δ best (%)**	**T(ms)**
C101C5	3	247.15	3	250.03	1.16	0.09
C103C5	2	165.67	2	184.38	10.69	0.11
C206C5	2	236.58	1	245.96	3.89	0.12
C208C5	1	158.48	1	185.15	15.52	0.11
R104C5	2	136.69	2	185.21	30.15	0.10
R105C5	2	156.08	2	168.47	7.64	0.08
R202C5	1	128.78	2	159.51	21.32	0.10
R203C5	1	179.06	1	232.38	25.92	0.11
RC105C5	3	238.05	3	238.05	0.00	0.09
RC108C5	2	253.93	2	258.75	1.88	0.12
RC204C5	1	176.39	2	185.44	5.00	0.13
RC208C5	1	167.98	1	188.63	11.58	0.13
C101C10	3	393.76	5	440.53	11.21	0.21
C104C10	2	273.93	2	297.26	8.17	0.23
C202C10	2	243.20	3	264.57	8.42	0.21
C205C10	2	228.28	3	314.33	31.72	0.19
R102C10	3	249.19	5	334.00	29.08	0.22
R103C10	3	202.85	3	232.10	13.45	0.21
R201C10	3	217.67	3	276.03	23.64	0.19
R203C10	1	218.21	3	362.79	49.77	0.24
RC102C10	4	423.51	5	454.65	7.09	0.19
RC108C10	3	345.92	4	434.44	22.69	0.19
RC201C10	3	310.06	3	395.54	24.23	0.12
RC205C10	2	325.98	5	499.29	42.00	0.25
C103C15	4	371.70	4	436.86	16.12	0.26
C106C15	3	275.13	4	424.21	42.63	0.20
C202C15	3	376.79	5	578.57	42.24	0.29
C208C15	2	300.55	4	411.54	31.17	0.25
R102C15	5	413.93	7	518.27	22.39	0.35
R105C15	4	336.15	6	431.53	24.85	0.24
R202C15	2	358.00	4	517.09	36.36	0.35
R209C15	2	293.20	3	522.12	56.15	0.34
RC103C15	4	397.67	5	450.51	12.46	0.32
RC108C15	3	370.25	5	524.37	34.45	0.43
RC202C15	2	394.39	5	555.28	33.88	0.29
RC204C15	2	310.58	3	509.51	48.51	0.31
Average Deviation					**22.43**	
Average Time(sec)						**0.0002**

Table 5.1 shows the results by both Erdogan's methodology and CPLEX. There is a positive correlation between number of customers and its deviation from the best-known results by CPLEX. This is more evident among instances with 15 customers. Total deviation of the mentioned method from the best-known solutions for small instances is 22.43% which is reasonable for a heuristic solution in a very short computation time. As the computation times are extremely short, they are converted into millisecond to make it easier to read, while the total average time is shown in second to clarify the short computation time (0.0002). In each set of results, m demonstrates the number of required routes (vehicles) to satisfy all customers' demands in predefined time windows.

5.2.2 MCW Saving Heuristic Algorithm (MCW 2)

This methodology focuses on insertion of customers with earlier starting service time, while waiting time for starting the service is limited and vehicles will pass from those customers which should be serviced after a long waiting time. Most importantly, assignment of recharging stations to routes occurs with the customer insertion, simultaneously.

In Table 5.2, deviation from the best-known results is illustrated for each of the instances, while they are improved considerably compared to the Erdogan's solution method. Number of required vehicles and total travel distance are also included in the table besides the computation time.

Table 5.2. Obtained results for small sized instances by MCW 2 and CPLEX

Best Known Results			MCW 2			
Instance	**m**	**Travel Distance**	**m**	**Travel Distance**	**Δ best (%)**	**T(ms)**
C101C5	3	247.15	3	250.03	1.16	0.11
C103C5	2	165.67	2	165.67	0.00	0.13
C206C5	2	236.58	2	245.96	3.89	0.90
C208C5	1	158.48	1	185.15	15.52	0.14
R104C5	2	136.69	2	185.21	30.15	0.11
R105C5	2	156.08	2	168.47	7.64	0.10
R202C5	1	128.78	1	157.51	20.07	0.16
R203C5	1	179.06	1	179.06	0.00	0.15
RC105C5	3	238.05	3	238.05	0.00	0.09
RC108C5	2	253.93	3	313.64	21.04	0.10
RC204C5	1	176.39	1	185.44	5.00	0.14
RC208C5	1	167.98	1	188.63	11.58	0.13
C101C10	3	393.76	4	414.03	5.02	0.20
C104C10	2	273.93	2	297.26	8.17	0.24
C202C10	2	243.20	2	243.31	0.05	0.23
C205C10	2	228.28	2	269.99	16.74	0.21
R102C10	3	249.19	4	301.05	18.85	0.23
R103C10	3	202.85	3	230.53	12.77	0.21
R201C10	3	217.67	2	273.25	22.64	0.18
R203C10	1	218.21	1	240.39	9.67	0.23
RC102C10	4	423.51	4	459.33	8.11	0.19
RC108C10	3	345.92	4	430.33	21.75	0.18
RC201C10	3	310.06	3	388.46	22.45	0.13
RC205C10	2	325.98	2	405.3	21.69	0.24
C103C15	4	371.70	4	434.49	15.58	0.27
C106C15	3	275.13	5	402.62	37.62	0.19
C202C15	3	376.79	3	486.19	25.35	0.29
C208C15	2	300.55	2	334.59	10.72	0.27
R102C15	5	413.93	7	527.99	24.22	0.39
R105C15	4	336.15	5	364.43	8.07	0.23
R202C15	2	358.00	3	497.02	32.52	0.37
R209C15	2	293.20	2	401.01	31.06	0.36
RC103C15	4	397.67	5	450.52	12.46	0.40
RC108C15	3	370.25	4	426.43	14.10	0.46
RC202C15	2	394.39	4	621.97	44.78	0.35
RC204C15	2	310.58	2	424.27	30.94	0.39
Average Deviation					**15.87**	
Average Time(sec)						**0.00024**

It can be observed that outputs are improved roughly by 7%, while the computation time barely experienced any change and stabilized around 0.00024 seconds. In some cases, outputs reached to the best-known result, but the average deviation of results is still 15.87% compared to those by CPLEX.

The proposed solution maintained a reduction in the total travel distance and hence an improvement is achieved, since the main objective of the defined problem was to minimize the total travelling distance. To sum up, one can draw the conclusion that the implemented modifications on CW Saving Algorithm led to robust enhancement and with further modifications alongside, more significant improvements can also be obtained.

5.2.3 MCW with One Parameter (MCW 3)

In this case, one parameter is added to the saving value formula with the purpose of decreasing the total traveling distance. Adding a parameter acts as an improvement part for the MCW heuristic. Therefore, as can be seen in table 5.3, the total traveling distance compared to MCW and Erdogan's solution method decreased by 2.27% and approximately 9%, respectively. As expected, computation time increased in this case due to the addition of λ which is increasing by 0.1 as its step size between 0.1 and 2.

In general, outputs' quality increased slightly in this case with cost of an increase in computation time which is negligible. This proved that adding an improvement part to MCW can contribute to solutions' quality improvement in EVRP.

Table 5.3. Obtained results for small sized instances by MCW 3 and CPLEX

Best Known Results			MCW 3				
Instances	**m**	**Total Distance**	**m**	**Travel Distance**	**Δ best (%)**	**T(ms)**	**λ**
C101C5	3	247.15	3	250.03	1.16	0.21	0.1
C103C5	2	165.67	2	165.67	0.00	0.43	0.5
C206C5	2	236.58	2	245.96	3.89	0.33	0.1
C208C5	1	158.48	1	185.15	15.52	0.37	0.1
R104C5	2	136.69	2	185.22	30.15	0.38	0.6
R105C5	2	156.08	2	156.08	0.00	0.27	0.1
R202C5	1	128.78	1	157.55	20.10	0.61	0.3
R203C5	1	179.06	1	179.06	0.00	0.43	0.9
RC105C5	3	238.05	3	238.05	0.00	0.38	0.1
RC108C5	2	253.93	3	313.64	21.04	0.96	0.8
RC204C5	1	176.39	1	179.45	1.72	0.53	0.9
RC208C5	1	167.98	1	179.96	6.89	0.49	0.1
C101C10	3	393.76	4	427.76	8.28	0.86	0.3
C104C10	2	273.93	2	297.26	8.17	1.72	0.7
C202C10	2	243.20	2	243.31	0.05	1.92	0.6
C205C10	2	228.28	2	269.99	16.74	0.89	0.2
R102C10	3	249.19	4	301.05	18.85	3.31	0.1
R103C10	3	202.85	3	222.55	9.26	3.82	0.1
R201C10	3	217.67	3	259.26	17.44	1.18	1.2
R203C10	1	218.21	1	240.39	9.67	1.35	0.9
RC102C10	4	423.51	4	459.33	8.11	2.13	0.5
RC108C10	3	345.92	4	420.27	19.41	4.23	1.5
RC201C10	3	310.06	4	378.59	19.90	0.89	0.5
RC205C10	2	325.98	3	405.30	21.69	2.87	0.2
C103C15	4	371.70	4	430.83	14.74	5.11	1.1
C106C15	3	275.13	5	372.22	30.00	3.22	0.6
C202C15	3	376.79	3	486.19	25.35	5.27	1
C208C15	2	300.55	2	305.80	1.73	2.19	0.9
R102C15	5	413.93	7	508.65	20.53	6.73	0.1
R105C15	4	336.15	5	364.44	8.08	1.82	0.3
R202C15	2	358.00	3	459.61	24.86	9.41	0.3
R209C15	2	293.20	2	378.24	25.33	2.98	0.5
RC103C15	4	397.67	5	450.52	12.46	5.18	0.5
RC108C15	3	370.25	4	426.43	14.10	7.23	0.7
RC202C15	2	394.39	4	590.45	39.82	9.87	1.7
RC204C15	2	310.58	2	359.02	14.47	6.63	1.5
Average Deviation					**13.60**		
Average Time(sec)						**0.0027**	

5.2.4 MCW with Two Parameters (MCW 4)

In this case, two parameters are considered in the saving formula (see equation 4.7). Both two parameters try to consider more realistic route distance. Moreover, they bring about changes in constructed routes shapes. The parameters mentioned above improved the solution quality noticeably and this methodology produces results with 12.85% deviation from the best-known solutions, which shows that it is highly reliable (see table 5.4). It is worth to mention that values of parameters which are referred in the table show the first values that lead to the optimal output and many other values might be found for all parameters, which bring about the same result.

5.2.5 MCW with Three Parameters (MCW 5)

It is already proven that consideration of customers' demands in computation of saving values can improve the solution quality. Now, customers' demands are taken into consideration in computation of saving values to see how it affects E-VRPTW. According to table 5.5, the result's deviation fell to 12.04%, which shows the validity of the methodology, however the computation time rose slightly.

Table 5.4. Obtained results for small sized instances by MCW 4 and CPLEX

Best Known Results			MCW 4					
Instances	**m**	**Travel Distance**	**m**	**Travel Distance**	**T(ms)**	**λ**	**μ**	**Δ best (%)**
C101C5	3	247.15	3	250.03	3	0.1	0	1.16
C103C5	2	165.67	2	165.67	8	0.5	0	0.00
C206C5	2	236.58	2	245.96	7	0.1	0	3.89
C208C5	1	158.48	1	185.15	9	0.1	0	15.52
R104C5	2	136.69	2	185.22	6	0.5	1.1	30.15
R105C5	2	156.08	2	156.08	8	0.1	0	0.00
R202C5	1	128.78	1	144.67	6	0.1	1.1	11.62
R203C5	1	179.06	1	179.06	10	0.9	0	0.00
RC105C5	3	238.05	3	238.05	9	0.1	0	0.00
RC108C5	2	253.93	3	313.64	19	0.1	0.9	21.04
RC204C5	1	176.39	1	185.16	5	1	0.1	4.85
RC208C5	1	167.98	1	178.90	9	0.1	1.3	6.30
C101C10	3	393.76	4	420.19	15	0.9	0.5	6.49
C104C10	2	273.93	2	297.26	13	0.7	0	8.17
C202C10	2	243.20	2	243.31	19	0.6	0	0.05
C205C10	2	228.28	2	269.99	17	0.2	0	16.74
R102C10	3	249.19	4	293.78	22	1.2	0.3	16.42
R103C10	3	202.85	3	222.55	14	0.1	0	9.26
R201C10	3	217.67	3	251.13	19	1.3	0.6	14.27
R203C10	1	218.21	1	240.39	26	0.8	0.5	9.67
RC102C10	4	423.51	4	459.33	11	0.2	0.3	8.11
RC108C10	3	345.92	4	417.11	14	1.2	1.3	18.66
RC201C10	3	310.06	4	378.59	12	0.5	0	19.90
RC205C10	2	325.98	3	401.91	16	0.1	1.6	20.86
C103C15	4	371.70	4	430.83	19	1.3	0	14.74
C106C15	3	275.13	5	372.22	25	0.6	0	30.00
C202C15	3	376.79	3	486.19	29	1.4	0.5	25.35
C208C15	2	300.55	2	305.80	28	0.9	0	1.73
R102C15	5	413.93	7	502.57	19	0.1	0	19.34
R105C15	4	336.15	5	364.44	31	0.2	0.4	8.08
R202C15	2	358.00	3	459.61	26	0.3	0	24.86
R209C15	2	293.20	2	356.56	37	0.9	1.7	19.50
RC103C15	4	397.67	5	450.52	35	0.5	0	12.46
RC108C15	3	370.25	4	426.43	22	0.7	1.8	14.10
RC202C15	2	394.39	4	561.42	24	1.9	0.7	34.95
RC204C15	2	310.58	2	359.02	21	1.5	0	14.47
Average Deviation								**12.85**
Average Time(sec)					**0.613**			

Table 5.5. Obtained results for small sized instances by MCW 5 and CPLEX

Best Known Results			MCW 5						
Instances	**m**	**Travel Distance**	**m**	**Travel Distance**	**T(ms)**	**λ**	**μ**	**ν**	**Δ best (%)**
C101C5	3	247.15	3	250.03	151	0.1	0	-0.20	1.16
C103C5	2	165.67	2	165.67	369	0.1	0.2	-0.20	0.00
C206C5	2	236.58	2	245.96	245	0.1	0	-0.20	3.89
C208C5	1	158.48	1	183.59	208	0.1	0	0.10	14.68
R104C5	2	136.69	2	185.22	238	0.1	0.5	-0.20	30.15
R105C5	2	156.08	2	156.08	218	0.1	0	-0.20	0.00
R202C5	1	128.78	1	157.55	254	0.1	0.1	-0.12	20.10
R203C5	1	179.06	1	179.06	212	0.1	0.4	-0.20	0.00
RC105C5	3	238.05	3	238.05	211	0.1	0	-0.20	0.00
RC108C5	2	253.93	3	313.64	713	0.1	0.6	-0.20	21.04
RC204C5	1	176.39	1	185.16	297	0.1	1	-0.20	4.85
RC208C5	1	167.98	1	177.47	190	0.1	0.9	-0.20	5.49
C101C10	3	393.76	4	414.04	1276	0.1	1	-0.14	5.02
C104C10	2	273.93	2	292.47	1280	0.1	0.2	-0.19	6.55
C202C10	2	243.20	2	243.31	600	0.1	0.2	0.00	0.05
C205C10	2	228.28	2	269.99	518	0.1	0.1	-0.20	16.74
R102C10	3	249.19	3	282.03	2315	0.1	0	-0.20	12.36
R103C10	3	202.85	3	222.55	1715	0.1	0	-0.20	9.26
R201C10	3	217.67	3	241.69	559	0.3	0.2	0.11	10.46
R203C10	1	218.21	1	230.39	840	1.5	0.5	-0.20	5.43
RC102C10	4	423.51	4	459.33	1431	0.1	0.1	-0.20	8.11
RC108C10	3	345.92	4	413.80	2597	0.1	1.3	-0.20	17.87
RC201C10	3	310.06	3	365.24	672	0.1	0.1	-0.20	16.34
RC205C10	2	325.98	2	393.45	1835	0.1	0.6	0.17	18.76
C103C15	4	371.70	3	409.40	2420	2	0.3	0.00	9.65
C106C15	3	275.13	5	379.61	1697	0.1	0.4	0.10	31.91
C202C15	3	376.79	3	486.19	2268	0.1	0.4	-0.20	25.35
C208C15	2	300.55	2	305.80	1541	0.1	0.7	-0.20	1.73
R102C15	5	413.93	7	496.46	5476	0.1	0.4	-0.20	18.13
R105C15	4	336.15	5	364.44	1395	0.1	0.1	0.04	8.08
R202C15	2	358.00	3	456.48	4718	0.1	0.3	-0.20	24.18
R209C15	2	293.20	2	336.86	1949	0.2	0.3	-0.20	13.86
RC103C15	4	397.67	5	448.69	3095	0.4	0.1	-0.05	12.06
RC108C15	3	370.25	4	422.06	4382	0.1	0.3	-0.11	13.08
RC202C15	2	394.39	3	547.92	6706	0.2	0.2	-0.20	32.59
RC204C15	2	310.58	2	359.02	1980	1.5	0	-0.03	14.47
Average Deviation									**12.04**
Average Time(sec)					**1.57**				

Table 5.6. Obtained results for small sized instances by MCW 6 and CPLEX

Best Known Results			**MCW 6**							
Instances	**m**	**Travel Distance**	**m**	**Travel Distance**	**T(ms)**	**λ**	**λ1**	**λ2**	**μ**	**Δ best (%)**
C101C5	3	247.15	3	250.03	1086	0.1	0.1	0.1	0	1.16
C103C5	2	165.67	2	165.67	2640	0.5	0.1	0.1	0	0.00
C206C5	2	236.58	2	245.96	2530	0.1	0.1	0.1	0	3.89
C208C5	1	158.48	1	185.15	1602	0.1	0.1	0.1	0	15.52
R104C5	2	136.69	2	161.25	2889	0.1	0.7	0.8	1	16.49
R105C5	2	156.08	2	156.08	1596	0.1	0.1	0.1	0	0.00
R202C5	1	128.78	1	128.88	1779	0.1	0.1	0.6	1.1	0.08
R203C5	1	179.06	1	179.06	3994	0.9	0.1	0.1	0	0.00
RC105C5	3	238.05	3	238.05	2093	0.1	0.1	0.1	0	0.00
RC108C5	2	253.93	3	308.81	5182	0.1	0.1	1.7	1.2	19.50
RC204C5	1	176.39	1	176.39	1794	0.9	0.1	0.2	0	0.00
RC208C5	1	167.98	1	178.90	3006	0.1	0.1	0.1	1.3	6.30
C101C10	3	393.76	4	414.04	8214	0.4	0.1	0.1	0	5.02
C104C10	2	273.93	2	292.09	13638	1.9	0.2	0.1	0.7	6.42
C202C10	2	243.20	2	243.31	7798	0.6	0.1	0.1	0	0.05
C205C10	2	228.28	2	269.99	4891	0.1	0.1	0.1	0.4	16.74
R102C10	3	249.19	3	259.03	16662	0.1	0.2	0.1	0	3.87
R103C10	3	202.85	3	209.23	14746	0.5	1.4	1.9	0	3.10
R201C10	3	217.67	2	250.97	5545	0.1	0.1	0.1	1	14.21
R203C10	1	218.21	1	230.39	9286	0.6	0.2	0.1	0.8	5.43
RC102C10	4	423.51	4	423.51	13065	1.6	0.1	0.1	0.3	0.00
RC108C10	3	345.92	3	354.37	22502	0.1	0.2	0.1	0	2.41
RC201C10	3	310.06	3	337.82	5331	0.1	0.2	0.1	0.6	8.57
RC205C10	2	325.98	2	330.55	15798	1.4	0.2	0.3	0	1.39
C103C15	4	371.70	4	394.88	42622	0.8	0.2	0.1	0.8	6.05
C106C15	3	275.13	4	350.63	20727	1.6	0.1	0.1	1.8	24.13
C202C15	3	376.79	4	482.20	27622	1.3	0.4	0.1	0	24.54
C208C15	2	300.55	2	305.80	17284	0.9	0.1	0.1	0	1.73
R102C15	5	413.93	6	450.08	43707	0.8	0.2	0.1	1.1	8.37
R105C15	4	336.15	5	361.95	16524	0.1	0.1	0.1	0.6	7.39
R202C15	2	358.00	3	400.93	59511	0.1	0.1	0.3	0.4	11.31
R209C15	2	293.20	2	362.41	1586.41	0.5	1.7	0.7	0	21.11
RC103C15	4	397.67	5	450.52	55624	0.4	0.1	0.1	0.1	12.46
RC108C15	3	370.25	3	398.98	57654	0.1	0.6	0.1	0.5	7.47
RC202C15	2	394.39	4	512.92	142045	0.1	0.1	1.6	0.5	26.13
RC204C15	2	310.58	1	350.07	29974	0.6	0.1	0.4	1.3	11.95
Average Deviation										**8.13**
Average Time(sec)					**18.95**					

5.2.6 MCW with Four Parameters

The solution method by benefitting from four parameters which are added to both saving function of customer insertion and Recharging Stations bring about improvement in outputs by decrements in the total travel distance. The total travel distance deviation decreased to just above 8%, while the average run time boomed to 18.95 seconds (see table 5.6). It is noteworthy that λ and μ refer to equation 4.7, while λ_1 λ_2 was derived from equation 4.15.

5.3 Results for Large Instances

All the proposed solution methods are examined on small size instances and the results are discussed. The superiority of MCW was cleared, compared to adapted Erdogan and Miller-Hooks' methodology for E-VRPTW. Furthermore, the obtained results for small size instances evidenced that outputs quality has positive correlation with number of parameters that can be used in saving value formula, while it contributes to the increase in computation time. Now, among all aforementioned methods, three of them are chosen to solve big instances and evaluate the results. There are 56 set of large instances which contain 100 customers and 21 Recharging Stations.

As previously mentioned, MCW 2 focuses on insertion of customers with earlier starting service time, while waiting time for starting the service is limited and vehicles will pass from those customers who should be serviced after a long waiting time. Most importantly, placing Recharging Stations to routes occurs with the customer insertion, simultaneously. Computation times of large size instances are neglected and the results are shown in table A.1 in appendix. The deviation of the results from the best known solutions are shown in the table.

Many of the constructed routes by MCW have circular shape as it is the nature of CW Saving Algorithm. In MCW 3, one parameter is added with the purpose of changing the routes' shape and decreasing the total travel distance. The calculated results show about 5% improvement in total travel distance, compared to those obtained by MCW 2 (see table A.2, appendix).

The proposed method by Doyuran et al. (Doyuran & Catay, 2009) works well on large instances and the results support this claim (see table A.3 in appendix) where outputs are improved by approximately 13.5% with MCW3, compared to the obtained results by MCW 2.

5.3.1 Summary of Large Instances Results

All the results for large size instances are compared with the best known solution which was obtained by hybrid of LNS to check the validity of the results. The best result can be found in table A.4 in appendix (Keskin & Catay, 2014). Results for large size instances are summarized in table 5.7

Table 5.7. Summary of obtained results for large sized instances

Summary of Large Instances				
	Best	**MCW 2**	**MCW3**	**MCW 5**
Average Travel Distance	1032.06	1496.66	1427.11	1305.80
Average Number of Routes	9	12	11	10
Average Deviation from Best Known Solution	0.00%	36.96%	32.16%	23.69%

Chapter 6

CONCLUSION

Route planning for Electric Vehicles with Time Windows has been carried out for small and large size instances to determine the cost-optimal set of routes. The limited battery capacity of electric vehicles leads to vehicles meeting one of the available Recharging Stations through the routes to recharge the battery with a recharging time, which depends on the battery level at arrival and the recharging rate. Moreover, vehicles' capacity constraint and hard time windows constraint are incorporated into the problem to represent a real-life problem.

Several solution methods based on Clarke and Wright Saving Heuristic have been developed to evaluate the efficiency, where they are adapted for the E-VRPTW. The numerical studies are performed on Schneider's benchmark instances and the results have demonstrated that the applied modifications on classical CW work properly.

Therefore, all the aforementioned methodologies are able to generate close to optimal results by which they can be applicable in real-world problems. In this way, transportation companies can realize how profitable it is to shift their fleets' vehicles from conventional vehicles to green vehicles and they would be encouraged to support green logistic practices.

As the discussed topic is newly introduced, many subjects can be considered as a future study topic. Specifically, consideration of soft time windows instead of the hard time windows can be an interesting topic which allows time windows violation and by this way, customer satisfaction rate can be taken into consideration to approach the problem from a different perspective. Furthermore, inhomogeneous vehicle fleet can be considered instead of the current one, while more than one recharging scheme can be defined for the Recharging Stations.

REFERENCES

Altinel, I. K., & Öncan, T. (2005). A new enhancement of the Clarke and Wright savings heuristic for the capacitated vehicle routing problem. *Journal of the Operational Research Society, 56*(8), 954-961.

Andres Figliozzi, M. (2012). The time dependent vehicle routing problem with time windows: Benchmark problems, an efficient solution algorithm, and solution characteristics. *Transportation Research Part E: Logistics and Transportation Review, 48*(3), 616-636. doi: http://dx.doi.org/10.1016/j.tre.2011.11.006

Baños, R., Ortega, J., Gil, C., Fernández, A., & de Toro, F. (2013). A Simulated Annealing-based parallel multi-objective approach to vehicle routing problems with time windows. *Expert Systems with Applications, 40*(5), 1696-1707. doi: http://dx.doi.org/10.1016/j.eswa.2012.09.012

Bektaş, T., & Laporte, G. (2011). The Pollution-Routing Problem. *Transportation Research Part B: Methodological, 45*(8), 1232-1250. doi: http://dx.doi.org/10.1016/j.trb.2011.02.004

Beltrami, E. J., & Bodin, L. D. (1974). Networks and vehicle routing for municipal waste collection. *Networks, 4*(1), 65-94. doi: 10.1002/net.3230040106

Beullens, P., Van Oudheusden, D., & Van Wassenhove, L. (2004). Collection and Vehicle Routing Issues in Reverse Logistics. In R. Dekker, M. Fleischmann,

K. Inderfurth & L. Van Wassenhove (Eds.), *Reverse Logistics* (pp. 95-134): Springer Berlin Heidelberg.

Brandão, J. (2004). A tabu search algorithm for the open vehicle routing problem. *European Journal of Operational Research, 157*(3), 552-564. doi: http://dx.doi.org/10.1016/S0377-2217(03)00238-8

Cao, B. (2012) Solving vehicle routing problems using an enhanced clarke-wright algorithm: A case study. *Vol. 7555 LNCS. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)* (pp. 190-205).

Clarke, G., & Wright, J. W. (1964). Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. *Operations Research, 12*(4), 568-581. doi: doi:10.1287/opre.12.4.568

Dantzig, G. B., & Ramser, J. H. (1959). The Truck Dispatching Problem. *Management Science, 6*(1), 80-91. doi: doi:10.1287/mnsc.6.1.80

Dondo, R., & Cerdá, J. (2013). A sweep-heuristic based formulation for the vehicle routing problem with cross-docking. *Computers & Chemical Engineering, 48*(0), 293-311. doi: http://dx.doi.org/10.1016/j.compchemeng.2012.09.016

Doyuran, T., & Catay, B. (2009). A robust enhancement to the Clarke-Wright savings algorithm. *Journal of the operational research society*.

Erdoğan, S., & Miller-Hooks, E. (2012). A Green Vehicle Routing Problem. *Transportation Research Part E: Logistics and Transportation Review, 48*(1), 100-114. doi: http://dx.doi.org/10.1016/j.tre.2011.08.001

Gajpal, Y., & Abad, P. L. (2009). Saving based algorithm for multi-depot version of vehicle routing problem with simultaneous pickup and delivery. *International Journal of Enterprise Network Management, 3*(3), 201-222.

Gaskell, T. J. (1967). Bases for Vehicle Fleet Scheduling. *J Oper Res Soc, 18*(3), 281-295.

Gendreau, M., Laporte, G., & Séguin, R. (1996). Stochastic vehicle routing. *European Journal of Operational Research, 88*(1), 3-12. doi: http://dx.doi.org/10.1016/0377-2217(95)00050-X

Keskin, M., & Catay, B. (2014). An adaptive large neighborhood search approach for solving the electric vehicle routing problem. In Y. C. Toklu & G. Bekdas (Eds.), (pp. 31-36). Bilecik, Turkey: Bilecik Seyh Edebali University.

Lin, C., Choy, K. L., Ho, G. T. S., Chung, S. H., & Lam, H. Y. (2014). Survey of Green Vehicle Routing Problem: Past and future trends. *Expert Systems with Applications, 41*(4 PART 1), 1118-1138.

Mattos Ribeiro, G., & Laporte, G. (2012). An adaptive large neighborhood search heuristic for the cumulative capacitated vehicle routing problem. *Computers &*

Operations Research, *39*(3), 728-735. doi: http://dx.doi.org/10.1016/j.cor.2011.05.005

Musolino, G., Polimeni, A., Rindone, C., & Vitetta, A. (2013). Travel Time Forecasting and Dynamic Routes Design for Emergency Vehicles. *Procedia - Social and Behavioral Sciences*, *87*(0), 193-202. doi: http://dx.doi.org/10.1016/j.sbspro.2013.10.603

Paessens, H. (1988). The savings algorithm for the vehicle routing problem. *European Journal of Operational Research*, *34*(3), 336-344. doi: http://dx.doi.org/10.1016/0377-2217(88)90154-3

Pichpibul, T., & Kawtummachai, R. (2012). An improved Clarke and Wright savings algorithm for the capacitated vehicle routing problem. *ScienceAsia*, *38*(3), 307-318.

Psaraftis, H. N. (1980). A Dynamic Programming Solution to the Single Vehicle Many-to-Many Immediate Request Dial-a-Ride Problem. *Transportation Science*, *14*(2), 130-154. doi: doi:10.1287/trsc.14.2.130

Russell, R. A. (1977). An Effective Heuristic for the M-Tour Traveling Salesman Problem with Some Side Conditions. *Operations Research*, *25*(3), 517-524. doi: 10.2307/169938

Schneider, M., Stenger, A., & Goeke, D. (2014). The Electric Vehicle-Routing Problem with Time Windows and Recharging Stations. *Transportation Science, 0*(0), null. doi: doi:10.1287/trsc.2013.0490

Solomon, M. M. (1987). Algorithms for the Vehicle Routing and Scheduling Problems with Time Window Constraints. *Operations Research, 35*(2), 254-265. doi: doi:10.1287/opre.35.2.254

Tillman, F. A. (1969). The Multiple Terminal Delivery Problem with Probabilistic Demands. *Transportation Science, 3*(3), 192-204. doi: doi:10.1287/trsc.3.3.192

Vidal, T., Crainic, T. G., Gendreau, M., & Prins, C. (2013). A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. *Computers & Operations Research, 40*(1), 475-489. doi: http://dx.doi.org/10.1016/j.cor.2012.07.018

Watson-Gandy, C. D. T., & Dohrn, P. J. (1973). Depot location with van salesmen — A practical approach. *Omega, 1*(3), 321-329. doi: http://dx.doi.org/10.1016/0305-0483(73)90108-4

Wen, M., Krapper, E., Larsen, J., & Stidsen, T. K. (2011). A multilevel variable neighborhood search heuristic for a practical vehicle routing and driver scheduling problem. *Networks, 58*(4), 311-322.

Yellow, P. C. (1970). A Computational Modification to the Savings Method of Vehicle Scheduling. *Operational Research Quarterly (1970-1977), 21*(2), 281-283. doi: 10.2307/3008161

Yu, B., & Yang, Z. Z. (2011). An ant colony optimization model: The period vehicle routing problem with time windows. *Transportation Research Part E: Logistics and Transportation Review, 47*(2), 166-181. doi: http://dx.doi.org/10.1016/j.tre.2010.09.010

APPENDIX

Table A.1. Obtained results for large sized instances by MCW 2

MCW 2							
Instance	**m**	**Travel Distance**	**Δ best (%)**	**Instance**	**m**	**Travel Distance**	**Δ best (%)**
c101	21	2165.97	69.08%	**r112**	14	1357.65	27.78%
c102	13	1825.96	56.41%	**r201**	8	1590.32	36.43%
c103	16	1668.01	49.91%	**r202**	7	1363.77	31.33%
c104	15	1392.47	37.62%	**r203**	7	1205.31	32.95%
c105	18	1885.3	58.33%	**r204**	5	1044.69	36.69%
c106	16	1583.17	42.59%	**r205**	5	1347.31	34.54%
c107	15	1598.5	43.66%	**r206**	5	1276.68	34.98%
c108	16	1563.84	42.50%	**r207**	4	1117.67	33.07%
c109	13	1457.38	37.83%	**r208**	4	959.56	30.34%
c201	7	1107.99	53.05%	**r209**	5	1169.03	30.90%
c202	6	1042.63	47.10%	**r210**	5	1083.88	26.17%
c203	7	1188.05	59.25%	**r211**	3	983.11	25.40%
c204	5	957.66	40.30%	**rc101**	24	2415.4	33.05%
c205	6	1075.56	50.61%	**rc102**	21	2196.54	34.41%
c206	5	886.52	32.58%	**rc103**	18	2001.76	38.81%
c207	5	892.18	33.20%	**rc104**	15	1728.61	33.94%
c208	5	895.32	33.54%	**rc105**	20	2097.41	34.96%
r101	27	2063.69	22.51%	**rc106**	17	1879.59	28.20%
r102	25	1917.52	26.63%	**rc107**	15	1655.19	25.64%
r103	20	1847.98	37.34%	**rc108**	15	1688.64	33.16%
r104	15	1436.92	29.10%	**rc201**	9	1918.03	41.58%
r105	21	1806.14	26.52%	**rc202**	8	1734.73	41.20%
r106	17	1572.87	20.82%	**rc203**	6	1445.76	40.71%
r107	16	1498.95	26.48%	**rc204**	5	1308.45	44.78%
r108	14	1415.96	29.68%	**rc205**	6	1686.74	44.60%
r109	16	1549.04	23.51%	**rc206**	5	1720.88	46.35%
r110	15	1464.86	28.64%	**rc207**	4	1393.95	40.08%
r111	15	1478.76	28.83%	**rc208**	4	1203	40.27%
Average Deviation							**36.96%**

Table A.2. Obtained results for large sized instances by MCW 3

MCW 3									
Instance	**m**	**Travel Distance**	**λ**	**Δ best (%)**	**Instance**	**m**	**Travel Distance**	**λ**	**Δ best (%)**
c101	22	2060.86	2	64.66%	**r112**	13	1302.71	1.6	23.72%
c102	19	1799.51	1.9	55.06%	**r201**	8	1513.98	1.4	31.65%
c103	16	1668.01	1	49.91%	**r202**	7	1258.37	1.1	23.44%
c104	14	1386.63	1.1	37.21%	**r203**	7	1175.85	1.8	30.54%
c105	17	1760.24	1.3	51.99%	**r204**	5	968.84	1.8	29.36%
c106	15	1513.62	1.5	38.28%	**r205**	5	1254.11	1.5	27.55%
c107	15	1598.5	1	43.66%	**r206**	5	1217.2	1.5	30.33%
c108	15	1463.87	1.5	36.15%	**r207**	4	1002.88	1.9	22.45%
c109	13	1376.03	2	32.26%	**r208**	2	924.44	2	26.68%
c201	7	1044.77	1.2	47.54%	**r209**	5	1084.64	2	23.55%
c202	6	1026.06	1.1	45.58%	**r210**	5	1059.02	1.1	23.88%
c203	7	1173.98	1.7	58.17%	**r211**	3	949.9	1.6	22.01%
c204	5	935	1.3	38.00%	**rc101**	23	2381.95	0.5	31.70%
c205	6	971.39	2	40.96%	**rc102**	20	2146	0.5	32.15%
c206	5	882.48	1.1	32.13%	**rc103**	18	1912.05	1.7	34.38%
c207	5	886.66	1.1	32.59%	**rc104**	14	1683.62	0.5	31.37%
c208	5	858.9	1.4	29.49%	**rc105**	19	1977.56	0.3	29.23%
r101	27	2050.87	1.3	21.90%	**rc106**	17	1841.99	0.5	26.22%
r102	24	1906.08	0.5	26.04%	**rc107**	14	1599.52	1.1	22.26%
r103	20	1703.28	2	29.42%	**rc108**	15	1605.59	1.9	28.24%
r104	15	1405.25	0.7	26.91%	**rc201**	8	1799.63	1.9	35.44%
r105	21	1774.04	2	24.75%	**rc202**	8	1636.54	1.4	35.58%
r106	17	1572.87	1	20.82%	**rc203**	7	1270.18	0.8	28.15%
r107	15	1460.13	0.6	23.90%	**rc204**	5	1019.9	2	20.56%
r108	14	1335.32	1.9	23.92%	**rc205**	7	1555.88	1.3	36.86%
r109	17	1537.23	1.2	22.75%	**rc206**	6	1521.62	1.6	34.55%
r110	14	1428.15	1.1	26.15%	**rc207**	4	1187.48	1.9	24.48%
r111	14	1363.32	1.7	20.82%	**rc208**	4	1123.43	1.4	33.66%
Average Deviation									**32.16%**

Table A.3. Obtained results for large sized instances by MCW 5

MCW 5							
Instance	**m**	**Travel Distance**	**Δ best (%)**	**Instance**	**m**	**Travel Distance**	**Δ best (%)**
c101	18	1783.83	51.45%	**r112**	11	1089.12	5.92%
c102	17	1470.15	35.91%	**r201**	8	1510.93	31.45%
c103	14	1340.39	28.91%	**r202**	7	1226.98	20.94%
c104	12	1208.21	23.77%	**r203**	6	1156.54	28.92%
c105	15	1485.97	35.88%	**r204**	4	938.21	26.21%
c106	14	1456.86	34.59%	**r205**	5	1254.05	27.54%
c107	14	1365.80	28.45%	**r206**	5	1087.21	19.22%
c108	13	1326.81	26.56%	**r207**	4	1009.40	23.09%
c109	12	1236.73	21.79%	**r208**	3	853.32	18.78%
c201	6	973.04	40.78%	**r209**	5	1078.89	23.02%
c202	5	951.88	38.41%	**r210**	5	916.73	9.56%
c203	6	1001.90	43.35%	**r211**	3	903.13	17.01%
c204	5	834.54	26.94%	**rc101**	21	2293.21	27.98%
c205	5	844.14	27.34%	**rc102**	18	1783.42	13.90%
c206	5	847.42	28.17%	**rc103**	16	1747.21	25.57%
c207	4	829.43	26.06%	**rc104**	13	1497.01	19.82%
c208	5	765.22	18.11%	**rc105**	19	1843.79	22.34%
r101	24	1993.90	19.11%	**rc106**	15	1751.42	21.25%
r102	21	1587.52	7.90%	**rc107**	14	1411.96	9.88%
r103	20	1670.81	27.53%	**rc108**	14	1564.22	25.67%
r104	14	1308.57	19.89%	**rc201**	8	1584.58	22.99%
r105	17	1555.53	11.72%	**rc202**	8	1460.72	24.48%
r106	17	1572.87	20.82%	**rc203**	7	1252.94	26.81%
r107	15	1363.42	17.12%	**rc204**	5	998.00	18.41%
r108	12	1173.46	11.10%	**rc205**	7	1445.26	29.69%
r109	15	1457.20	17.46%	**rc206**	6	1369.92	24.28%
r110	12	1187.79	7.87%	**rc207**	4	1151.03	21.40%
r111	14	1281.79	14.71%	**rc208**	5	1070.46	28.95%
Average Deviation							**23.69%**

Table A.4. Results by Large Neighborhood Search (Keskin & Catay, 2014)

Instance	m	Travel Distance	Instance	m	Travel Distance
c101	12	1053.83	**r112**	11	1026.52
c102	12	1022.58	**r201**	7	1100.27
c103	11	1001.81	**r202**	6	994.35
c104	10	951.57	**r203**	5	864.32
c105	12	1033.93	**r204**	3	720.82
c106	12	1027.25	**r205**	6	950.45
c107	12	1025.63	**r206**	5	896.61
c108	11	1015.68	**r207**	4	800.48
c109	11	993.77	**r208**	3	706.81
c201	4	643.45	**r209**	4	856.13
c202	4	645.16	**r210**	5	833.08
c203	4	644.98	**r211**	3	761.56
c204	4	636.43	**rc101**	17	1730.26
c205	4	641.13	**rc102**	16	1551.61
c206	4	638.17	**rc103**	13	1351.15
c207	4	638.17	**rc104**	12	1227.05
c208	4	638.17	**rc105**	14	1473.24
r101	20	1646.07	**rc106**	14	1414.99
r102	19	1466.94	**rc107**	12	1279.08
r103	14	1266.45	**rc108**	12	1208.31
r104	12	1071.89	**rc201**	9	1257.83
r105	15	1383.29	**rc202**	7	1142.15
r106	14	1276.33	**rc203**	6	956.78
r107	12	1148.43	**rc204**	5	829.72
r108	11	1050.04	**rc205**	6	1071.62
r109	14	1223.17	**rc206**	6	1073.33
r110	12	1097.89	**rc207**	6	928.52
r111	12	1106.19	**rc208**	5	799.75