Modification of the Arash Method using Facet Analysis

Mustapha Daruwana Ibrahim

Submitted to the
Institute of Graduate Studies and Research
in partial fulfilment of the requirements for the degree of

Master of Science
in
Industrial Engineering

Eastern Mediterranean University
July 2015
Famagusta, North Cyprus

Approval of the Institute of Graduate Studies and Research

Prof. Dr. Serhan Çiftçioğlu
Acting Director

I certify that this thesis satisfies the requirements as a thesis for the degree of Master of Science in Industrial Engineering.

Asst. Prof. Dr. Gökhan Izbirak
Chair, Department of Industrial Engineering

We certify that we have read this thesis and that in our opinion it is fully adequate in scope and quality as a thesis for the degree of Master of Science in Industrial Engineering.

Asst. Prof. Dr. Sahand Daneshvar
Supervisor

Examining Committee

1. Prof. Dr. Bela Vizvari
2. Asst. Prof. Dr. Sahand Daneshvar
3. Asst. Prof. Dr. Gökhan Izbirak

ABSTRACT

Data Envelopment Analysis, one of the most popular disciplines in operations research, it is a technique used to estimate the performance of Decision Making Units (DMUs). Technical efficient DMUs and Efficient DMUs are difficult to differentiate without the availability of additional information in the form of weight restrictions or the use of statistical technique and supper efficiency method. The Arash method (2013) distinguishes between Technical efficiency and Efficiency by introducing a small error in input values even if the values are accurate, the efficiency scores of the efficient DMUs does not change, only that of the technical efficient DMUs, it also establishes that, for a DMU to be efficient, technical efficiency is one of the necessary conditions.

In this study we expand the Arash Method by using facet analysis to modify the PPS of the Arash method. The proposed modification places an upper bound only on the free variable of VRS Arash method. This modification on the Arash method gives the true efficiency score and rank for the weak efficient DMUs and DMUs which take their efficiency score when compared to the weak part, because, the use of facet analysis on the frontier of the Arash method deduced some essential details about the constructive hyper planes of the production possibility set (PPS), Particularly the weak part of the frontier.

Keywords: Data Envelopment Analysis, Arash Method, Efficiency, Technical Efficiency, Facet Analysis, Modified Arash method, rank.

ÖZ

Yöneylem araştırmalarında en bilinen disiplin veri zarflama analizidir. Bu analiz Karar Verme Birimleri'nin (KVB) performansını tahmin etmede kullanılan bir tekniktir. Teknik verimli KVB ve verimli KVB'yi birbirinden ayırmak ek bilgi olmadan zordur. Bu ek bilgiler ağırlık kısıtlamaları formundadır veya istatistik tekniği ve süper verimlilik metodu kullanılarak elde edilir. Arash metodu (2013) teknik verimlilik ve verimliliği birbirinden ayırmada giriş değerindeki küçük bir hatanın, değerler doğru olsa bile verimli KVB'deki verimlilik değerini değiştirmeyeceğini, teknik verimli KVB'yi değiştireceğini sunmuştur. Ayrıca, KVB'nin verimli olması için teknik verimlilik gerekli bir durumdur.

Bu çalışmada Arash metod, Arash metodun üretim imkanları setini değiştirmek için yön analizi kullanılarak genişletilmiştir. Öngörülen değişim üst sınırda Arash metodun ölçek değişken dönüşünün serbest değişkeninde yapılmıştır. Arash metoddaki bu değişim KVB'lerin gerçek verimlilik hesabını ve zayıf verimlilik derecelerini göstermektedir. KVB'lerin verimlilik hesabı zayıf verimli taraf ile, sınırın özellikle zayıf parçası, karşılaştırıldığında Arash metodun sınırında yön analizi kullanımı sebebiyle, üretim imkanları setinin yapıcı hiperdüzlemleri hakkında bazı temel ayrıntılar ortaya çıkmıştır.

Anahtar Kelimeler: Veri Zarflama Analizi, Arash Metod, Verimlilik, Teknik Verimlilik, Yön Analizi, Değiştirilmiş Arash Metod, Derece.

To my late Dad

Ibrahim Abubakar Dalahs

ACKNOWLEDGMENT

I would like say thank you to the entire academic staff of Industrial engineering department, especially my supervisor, Asst. Prof. Dr Sahand Daneshvar for his immeasurable support and guidance throughout my programme, also, Assoc. Prof. Dr. Orhan Korhan for keeping his doors open whenever i needed advice on matters beyond academics, and the chairman of the department, Asst. Prof. Dr. Gökhan Izbirak for setting a standard for excellence.

A special gratitude goes to my family for supporting me beyond their means; I love you all.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

DEA	Data Envelopment Analysis
DMU	Decision making units
PPS	Production Possibility Set
VRS	Variable Return to Scale
AM	Arash Method
MAM	Modified Arash Method

Chapter 1

INTRODUCTION

1.1 Preamble

Performance evaluation and assessment of organizations efficiency and productivity is one of the fundamental aspects in economics and management, also maintaining the increase of sustainable growth and increase in efficiency, productivity and quality of output cannot be overemphasized. As the industrial world continues to be competitive, so do organizations try to grow and achieve global dominance, competition between organizations performing similar services grow stronger day by day. For an organization to have a competitive edge, their subsidiaries or stations need to perform efficiently. Performance evaluation is a necessary tool used to identify the strength and weaknesses of an organization. Operations Research is a discipline that deals with optimizing (maximizing) sales, profit, performance and minimizing cost, risk and other forms that reduce efficiency of a system.

It is an important technique in evaluating performance of an organization. It is a performance measurement technique that has been successfully implemented in a wide range of areas. Data Envelopment Analysis is gaining recognition as a key evaluating tool, because the primary objective of evaluation is to have an accurate and exact assessment of the Decision Making Unit (DMU).

It is used as an effective tool in performance comparison of organization efficiency, capabilities of sectors and determination of productivity improvement. It is now a pivotal assessment tool for the performance of comparable Decision Making Units (DMUs) like organizations and systems such as, the education sector, energy sector, defence sector and banks industry, schools and university departments, energy companies, electricity distribution and generation. DEA offers improvement options and help in decision making for managers and offers an insight to the level of improvement that can be attainable in an organization.

1.2 Problem Description

DEA, as a mathematical model for performance evaluation has its drawbacks, some of which has been addressed by researchers. The continual use of DEA points out areas of improvement in the models, papers are proposed towards improving these models and erasing their difficulties. In situations where the DMUs are not enough, that is, the numbers of DMUs are too small compared to the number output and input amount, DEA sometimes cannot offer the efficient DMUs a detail and comprehensive ranking of efficient DMUs.

The ranking of DMUs that are technically efficient or inefficient are meant to come after the efficient DMUs. Misplace ranking of technical efficient DMUs is one of the drawbacks of most DEA models, sometimes, a technical efficient DMU is ranked above an inefficient DMU when it is more inefficient than some inefficient DMUs. (Khezrimotlagh et al. 2012), recently, identified the inadequacy of Pareto definition of efficiency which is one of the foundations of DEA and showed a shortcoming in the basic DEA technique in benchmarking and ranking DMUs. They presented a strong method called the Arash method to remove the shortcoming of Pareto definition and give a practical definition of efficiency, and also rank both “technical efficient” and “efficient” DMUs. Although the Arash method does not successfully avoid the effects of the weak part of the efficiency frontier which proposes a bias efficiency score to DMUs located at the weak part of the frontier or DMUs that get their efficiency score when compared to DMUs on the weak part of the frontier. This is the drawback of the Arash method that is studied in this thesis.

The Arash Method is based on The Additive DEA model. This research attempts to achieve an improved result to that of the Arash method by using facet analysis, this will expand the scope and reliability of the model. Similar modification was achieved on the BCC model by (Daneshvar S., 2009). Called modified variable return to scale VRS.

The modified VRS model takes a strong look at the weak part of the efficient frontier and DMUs that take their efficiency score when compared to DMUs on the weak part of the frontier. Banker and Thrall (Banker R. D. and Thrall R. M., 1988) developed a strong structure to allow a possibility to have more than one optimal solutions and consider the subsequent problems in estimating return to scale RTS, the method tried to estimate the bounds for free variable u_o of the BCC model. Most papers make use of non-Archimedean number as the lower bound of factor weights in DEA models, specially BCC model, this bounds upset the weak part of the frontier, as a result, weak efficient DMUs appear weak and take an efficiency score less than 1. The modified BCC model points out that is not the true representation of the efficiency score. Using facet Analysis, the modified BCC model evaluate the exact efficiency of DMUs which belong to weak part of the frontier or DMUs that compare with this parts of the frontier by using (ε) as the upper bound of (u_o).

Using similar technique of the modified VRS model in the Arash method, we expect to achieve the following as the results of our research, first giving the true efficiency score to DMUs on the weak part of the frontier or DMUs that compare to this part of frontier, secondly, reaffirming the definition of technical efficiency and efficiency by the Arash method, thirdly, comparing the "modified Arash method", the Arash method, BCC model with other models to establish that the modification on the Arash method is truly effective.

1.3 Assumptions

DMUs and their inputs and outputs are the data in DEA literature and they must express the desires and purpose of the observer, managers or analyst. The data are organized in a system that will effectively present the goals of the organization. Higher outputs and lower inputs are usually the most preferred method of efficiency evaluation. It is not necessary for the measurement units of input and output to be the same.

Input orientation and output orientation are the two basic efficiency evaluation technique for the observed DMUs in DEA. Input orientation tries to maximize the output with the same level of input. This research in conducted using input orientation conditions, future research can be conducted using output orientation. Computation and analysis are done using WinQSB, linear programming software. The results obtained from the computations are presented in Appendix section.

1.4 Structure of Thesis

The thesis structure is as follows, in Chapter 2 short presentation of DEA literature, then, chapter 3 explains the concept of the Arash method which is the model that is modified in this thesis, and chapter 4 illustrates facet analysis and modified variable return to scale. In chapter 5, we propose a modified Arash method using the facet analysis of chapter 4. Finally, conclusion and suggestion for future study comes in chapter 6. Figure 1.1 illustrates the composition of the thesis main sections.

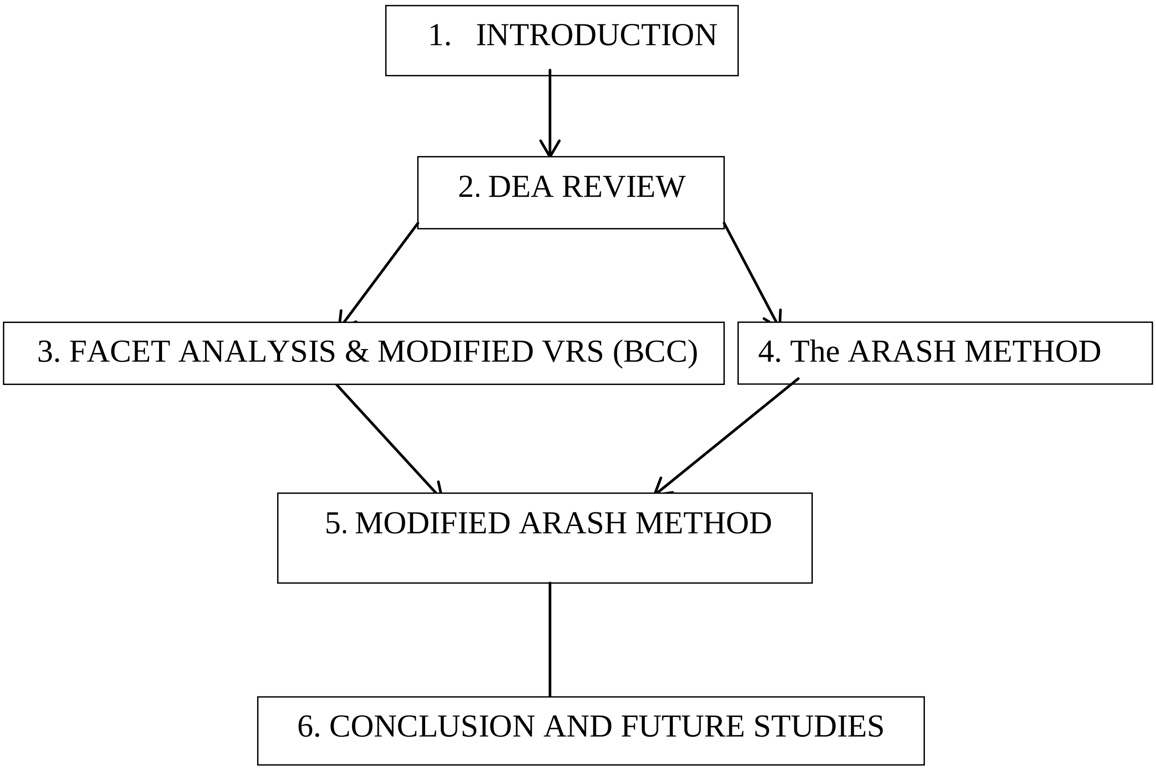

Figure 1.1: Structure of main sections of Thesis.

Chapter 2

DEA REVIEW

2.1 Data Envelopment Analysis (DEA)

Data envelopment analysis (DEA) is a highly powerful technique used in service management and benchmarking developed by (Charnes et al., 1978) to evaluate the economic and non-profit organizations". Since its inception, it has shown ways of improving services not visible by other techniques that were used, it is used as an evaluation tool for entities called DMUs with a collective inputs and outputs, it is also a decision making tool that measures relative efficiency of comparable unit.

Efficiency can simply be defined as the ratio of output to input in a single input single output case. More output per unit of input reveals relatively better efficiency. Apart from measuring relative efficiency, DEA has the capabilities of identifying the sources and level of inefficiency attributed to every input and its corresponding observed output, in contrast to central tendency approach, which is a statistical approach that evaluates DMUs efficiency relative to the average DMUs. DEA compares each DMU with one of the best DMUs. In some cases DEA might not be the best option to evaluate the entities, while sometimes; it is the most appropriate method.

In recent years, Data Envelopment Analysis has grown popular in evaluating relative efficiency of organizations, because, the efficiency any business is one of the important principle for the survival of the business, where the best possible economic result (Output) is obtained with little economic cost (Input). Efficiency can be defined as trying to achieve the best outcome with minimum use of available resources.

DEA is a technique of mathematical programming that helps you calculate efficiency based using inputs and outputs of the entities and compares it to other units under evaluation. DEA is regarded as data-oriented because it affects performance evaluation and other interferences directly and with minimal assumptions.

DEA is described as a non-parametric method because it does not require any assumptions about functional forms like, a production function or regression model. The DEA methodology is directed towards the frontier rather than the central tendencies. It is considered as a process of extremities. It was observed that financial service businesses have identified ways of decreasing operation cost (Input) by about 30% without decrease in service level or customer satisfaction with the use of DEA. Effective productivity in manufacturing activities in industrial and research applications have improved significantly with the use of DEA. In various industries, DEA help managers identify the pros and cons of new technologies that are designed to improve their system. DEA offers potential insight for research to managers and engineers.

2.2 How Does DEA Work

Data Envelopment Analysis is focused on evaluation of performance, mostly evaluating the activities of organizations such as government agencies, business firms, educational institutions, hospitals, and utility companies' etc. the evaluations might be inform of satisfaction per unit, cost per unit, and profit per unit and so on. The measure of the evaluation takes a ratio form like, $Output/Input$. The ratio is a common measure of efficiency for one input one output form, and the evaluation of productivity also takes a ratio form when evaluating employee performance. In Data Envelopment Analysis, for the evaluation of the DMUs, mathematical models are used for the data and the relationship between each DMU is identified.

The evaluation procedure for each DMU is considered a set of inputs to produce a set of outputs. For instance, consider a bank with many branches; each branch operates with tellers, functions around square footage of office space, and has a manager (the inputs). The output can be considered as the cheque cashed, amount deposited per day, number of loan application that is processed etc. Data Envelopment Analysis uses mathematical models to attempt to find which branch of the bank is most efficient and which is inefficient and also, which area the branches are inefficient, it also proposes possible areas of improvement to help increase the efficiency of both the efficient and inefficient branches.

The basic and central assumption behind this methodology is that, given a DMU, (DMU A) is capable of yielding output y (A) using input x (A), then other DMUs should be capable of producing similar outputs if they are to perform efficiently, the same goes for DMU B, if DMU B is able to produce y (B) output using x (B) inputs, then other DMUs should also be capable of doing the same. DMUs A, B and others can be combined to form a composite DMU with composite input and output, since this composite DMU does not necessarily exists, it's called a virtual DMU.

The core of this analysis lies in finding the best virtual DMU for each actual DMU, assuming the virtual DMU performance better than the original DMU by either producing more output with same amount of input or producing the same output with less amount of input, then the original DMU is presumed inefficient. The intricacies of DEA are introduced in ways that the DMUs A and B can be scaled up or down or combined.

2.3 What Does DEA Do?

1. Data Envelopment Analysis compares DMUs by considering the inputs used and outputs of each DMU and identifies the most efficient DMU (branches, departments, sales point, schools, and government ministries) and inefficient DMUs that real improvements are possible. This is achieved by precise comparison of the outputs achieved and input used for each DMU. In short DEA is a very powerful benchmarking system.

2. DEA calculates the amount of cost savings achievable if the inefficient DMUs are made efficient.

3. DEA estimates the additional improvement an inefficient DMU can provide without the need to use additional input, in addition the changes in inefficient DMUs are identified with which management can implement to achieve savings in resources.

4. Information on performance of each DMU is received that can be used to help transfer managerial expertise from better DMUs to less efficient DMUs. This results in improvement in productivity of inefficient DMUs thereby decreasing operational cost and increasing efficiency and profit.

The above stated information identifies relationships that are not identifiable in other techniques that are commonly used in performance evaluation. As a result, improvement in operations and performances of evaluated DMUs extend beyond any improvement achievable by other techniques.

DEA technique is focused on frontier analyses. This analysis compares relative efficiency of organizational units (DMUs). Frontier analysis creates room to evaluate the entire significant element that affect the DMU, and provide a comprehensive assessment of efficiency, efficient in the sense that, they make the most of their available resources.

DEA generates efficiency scores for the DMUs under evaluation; it shows how inefficient DMUs can become efficient by either reducing its inputs or increasing its outputs. Data Envelopment Analysis answers the question of “How well a DMU is performing” but also” How to improve a DMU”. It shows the best performing DMU and how they achieved that.

The tasks that are covered by Data Envelopment Analysis include:

- Monitoring efficiency as time changes
- Identification of “best operations”
- Identification of “poor operations”

- Resource allocation: changing from inefficient to efficient
- Setting targets

With DEA, managing data, visualizing results and understanding the procedures has been made a lot easier. In comparison to other techniques, DEA handles multiple inputs and outputs more accurately, additional information on functional form related to inputs and outputs are not required. DEA also has its limitations, therefore when considering DEA as an evaluation tool, the limitations should be taken into consideration.

2.4 DEA Background

The occurrence of multiple inputs and multiple outputs makes comparison of decision making units difficult and impossible in some cases, Data envelopment analysis utilizes linear programming for evaluation of the decision making units which can handle large number variables and relations (constraints), which relaxes the requirement that occurs when one is limited to selecting a few inputs and outputs. The original form of DEA was to measure the efficiency of DMUs as an entity without considering its inner structure like a black box of an aircraft.

Data envelopment analysis is a data-oriented performance evaluation technique originally developed by (Charnes et al., 1978) and was then extended by (Banker et al., 1988) to include variable return to scale. Farrels' work (Farrell J. M., 1957) was the basics in which DEA was generalized. After the seminal work of (Charnes et al., 1978) DEA has been widely accepted as an effective performance evaluation technique for measuring relative efficiency of homogenous DMUs. This led to improved theoretical development and practical application in many fields (for example, the review of Cook & Seiford, (Cook W D and Seiford L M, 2009).

Data envelopment analysis is considered as a superior technique because others are limited when it comes to managing productivity and DEA is flexible when it comes to areas of applications like profit analysis.

2.5 Production Possibility Sets (PPS)

The production frontiers developed from mathematical programming is the method used by DEA for assessing relative efficiency. The efficiency surface is formed based on the inputs and outputs of the evaluated DMUs, DMUs that lie on the frontier are called efficient DMUs while those that don't are considered inefficient.

Production possibility set is the set of all inputs and outputs of DMUs in which the inputs can produce an output. Relative efficiency of the decision making units are implicitly evaluated using PPS by data envelopment analysis models. DEA models cannot present efficient frontiers of PPS but they determine the efficiency of DMUs. The inputs and outputs are assumed relaxed in PPS. The set of feasible activities of the data is called the production possibility set denoted by "T". The models and concepts stated below are the basis of DEA most of which are acquired from [Data Envelopment Analysis "Second Edition" by William W. Cooper, Lawrence M.S, Kaoru T.].

Assuming a DMU uses input $X_j = (x_{1j}, \dots, x_{mj}) \geq 0 \quad X_j \neq 0$ to produce output $Y_j = (y_{1j}, \dots, y_{sj}) \geq 0, \quad Y_j \neq 0$. The production possibility set of the entity is represented as follows;

$T = \{(X, Y) \mid \text{output vector } Y \geq 0 \text{ can be produced from input vector } X \geq 0\}$

Properties of T (production possibility set)

1. The observed semi positive input (x) and output (y) belongs to T; i.e.

$(x_j y_j) \in T \qquad j = 1, \ldots, n.$

2. If (x, y) ∈ T, then, the, $(tx, ty) \in T$ for any $t > 0$, this postulate is called constant return to scale postulate.

3. For any input and output $(x, y) \in T$, any semi positive input and output $(\bar{x}, \bar{y})$ with $\bar{x} \geq x$, $\bar{y} \leq y$ is included in T.

4. T is closed and convex

The data sets are arranged in matrices X = (x_j) and Y = (y_j), j = 1, …, n.

Considering the postulates 1, 2, 3 and 4 the PPS (T) can be defined as:

$$T = \{(X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j \; Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j\} \qquad (1)$$

It can be proven that T satisfies 1 to 4.

T is built on the assumption of constant return to scale of inputs and outputs (x, y) belonging to T and for every t>0 which belong to (tx, ty). If we are to build T based on variable return to scale, postulate 2 will be omitted.

2.6 BCC Model

The first DEA model was proposed by Charnes, Cooper and Rhodes (CCR) (Charnes et al., 1978) which was based on constant return to scale (RTS), since then researches has been done to improve the model among which is the BCC model by Banker, Charnes, and Cooper (Banker R. D. and Thrall R. M., 1988). The BCC model frontier has a piecewise linear and concave characteristics, this leads to variable return to scale. The BCC model is different from the CCR model on convexity constraint ($\sum_{j=1}^{n} \lambda_j = 1 \;\; \lambda_j \geq 0, \forall j$).

The production possibility set (PPS) of the BCC model is denoted by P_B which include the following properties:

(P1) All observed input and output (x_j, y_j) included in P_B (j= 1, …, n)

(P2) If the inputs and outputs (x_j, y_j) belongs to P_B , then the convex combination of these data $\sum_{j=1}^{n} \lambda_j X_j, \sum_{j=1}^{n} \lambda_j Y_j), \sum_{j=1}^{n} \lambda_j = 1 \;\; \lambda_j \geq 0$ j=1,2, …, n also belongs to P_B.

(P3) For all inputs and outputs (X, Y) included in P_B any combination of input and output $(\bar{X}, \bar{Y})$ with $\bar{X} \geq X$ and $\bar{Y} \leq Y$ belongs to P_B.

(P4) All linear combination of inputs and outputs in P_B are included in P_B.

Banker, Charnes and Cooper (1984) published the BCC model with PPS (P_B) is defined by:

$$P_B = \{(X,Y) | X \geq \sum_{j=1}^{n} \lambda_j X_j , Y \leq \sum_{j=1}^{n} \lambda_j Y_j , \sum_{j=1}^{n} \lambda_j = 1 \; \lambda_j \geq 0, \forall j\} \qquad (2.1)$$

Evaluating the efficiency of DMU_0 which belong to PPS (P_B), the linear program represent the input orientation form:

$$
\begin{aligned}
& b_o^* = Min\ b_o \qquad (2.2)\\
& subject\ to\\
& -\sum_{j=1}^{n} \lambda_j X_J + b_o X_o \geq 0\\
& \sum_{j=1}^{n} \lambda_j Y_j \geq Y_o\\
& \sum_{j=1}^{n} \lambda_j = 1\\
& \lambda_j \geq 0,\ j = 1, \ldots, n\\
& b_o\ \ free
\end{aligned}
$$

b_0^* , Is the technical efficiency of the evaluated DMU " DMU_0" , The dual of the problem (multiplier side) is given by:

$$
\begin{aligned}
& z_o^* = Max UY_o + u_o \qquad (2.3)\\
& subject\ to\\
& UY_j - VX_j + u_o \qquad j = 1, \ldots, n\\
& VX_J = 1\\
& U \geq 0, V \geq 0\\
& u_o\ \ free
\end{aligned}
$$

Relationship between the primal-dual constraints and variables is described in Table 2.1. The major modification of the CCR model which resulted in the BCC is related to the variable u_0 and it is related to the convexity condition in PPS as its dual variable.

Table 2.1: Primal and Dual relations in BCC model

Envelopment form constraints	Multiplier form Variables	Multiplier form constraints	Envelopment form Variables
$-\lambda_j X_j + b_0 X_0 \geq 0$	$V \geq 0$	$VX_0 = 1$	b_0
$\lambda_j Y_j \geq Y_0$	$U \geq 0$	$UY_j - VX_j + u_0 \leq 0$	$\lambda_j \geq 0$
$\lambda_j = 1$	u_0		

Definition 2.1 (BCC-Efficiency)

If the evaluation presents an optimal solution (b_0^*,$\lambda^*, s^{-*}, s^{+*}$) satisfies b_0^*=1 and has no slacks ($s^{-*} = 0 , s^{+*} = 0$), then, DMU_0 is called BBC-Efficient, otherwise it is BCC-Inefficient.

$$\max_{\lambda, s^-, s^+} \quad z = es^- + es^+ \tag{2.4}$$
$$\text{subject to}$$
$$X\lambda + s^- = x_o$$
$$Y\lambda - s^+ = y_o$$
$$e\lambda = 1$$
$$\lambda \geq 0, s^- \geq 0, \quad s^+ \geq 0$$

2.6 The Additive Model

The CCR and BCC model necessitates us to distinguish between input-oriented and output-oriented models. The Additive model however combines both orientations in a single model. There are several types of additive models; the Basic Additive model is illustrated as follows:

The dual problem of the classic Additive model can be expressed as follows:

$$\min_{v,u,u_o} \quad w = vx_o - uy_o + u_o \qquad (2.5)$$

$$\begin{aligned} & subject\ to \\ & vX - uY + u_o e \geq 0 \\ & v \geq e \\ & u \geq e \\ & u_o \ free \end{aligned}$$

The Production possibility set (PPS) of the Additive model stated above has the same (PPS) as the BCC Model. The ADD- efficiency of observed DMUs is illustrated as follows:

Let the optimal solution of model (2.5) be ($\lambda^*, s^{-*}, s^{+*}$)

Definition 2.2: A DMU is ADD efficient if and only if s^{+*}=0, s^{-*}=0.

Theorem 2.1: A DMU is ADD-efficient if and only if it is BCC-efficient. It avails to note that the efficiency score of a DMU is not measured explicitly but rather implicitly in the slacks, s^{-*} *and* s^{+*} .

Theorem 2.2: let's define $\hat{x}_0 = x_0 - s^{-*}$ *and* $\hat{y}_0 = y_0 + s^{+*}$, then $(\hat{x}_0, \hat{y}_0)$ is ADD-efficient.

According to this theorem, the following formulae, (projection for the Additive model) offers an improvement to any efficient activity is attained by:

$$\hat{x}_0 \Leftarrow x_0 - s^{-*}$$
$$\hat{y}_0 \Leftarrow y_0 + s^{+*}$$

$(\hat{x}_o, \hat{y}_o)$ serves as the coordinates point on the efficient frontier use to evaluate a DMU.

2.7 Non-Archimedean Element Epsilon

The non-Archimedean element epsilon was introduced in DEA to distinguish between non-negative and positive values by (Charnes et al., 1978). Evaluating a less efficient DMU as an efficient DMU is a problem when some of the weights of inputs and outputs are equal to zero. It was changed to ensure that the weights must be strictly positive. (Ali and Seiford, 1993) proposed that epsilon be used as an upper bound to ensure feasibility on the multiplier side and boundedness for the envelopment side of the CCR and BCC models.

2.8 Ranking Methods Review

Evaluating decision making units in DEA has its limitations, one of which is ranking of DMUs, and ranking is an important issue in DEA studies. The efficiency score of the evaluated DMUs is from zero to one, with the efficient DMUs taking a score of one. A unique objectives of DEA is to find the most efficient DMU among the homogenous evaluated DMUs, this prove difficult because multiple DMUs among the evaluated DMUs take a score of one, which leads researchers to develop methods of distinguishing or ranking the DMUs that are efficient after evaluation. A model that prioritizes the ranking of only efficient units was developed by Cook et al., recently numerous papers have been published on how to rank both efficient and inefficient DMUs for assessment and improving the capabilities of DMUs.

In this study we consider that DEA ranking methods can be divided into six somewhat overlapping areas according to (Adler et al., 2002).

The first group of the ranking method is cross-efficiency technique by (Sexton et al, 1986) this established the business of ranking in DEA, in this technique they elaborated that the DMUs are both self and peer evaluated, certainly, (Doyle and Green, 1994) debated that reasonable mechanism in which to choose assurance regions are not always readily available for decision-maker. The method of cross efficiency ranking in DEA utilizes the results of cross efficiency matrix in ranking the DMUs. However, a draw back in this technique is that the reversal phenomenon occurs when there are changes in cross-efficiencies of some target when some candidates are included or eliminated.

The cross efficiency method appears to be a very reasonable method, but when there is some alternative solution in the linear problems of DEA, there is a disadvantage using this method. Super efficiency is the second method, it was proposed by (Andersen P. and Petersen N.C., 1993). The methodology allows an extreme efficient DMU to achieve an efficiency score greater than one by removing the kth constraint in the primal function. There are three main drawbacks in this methodology; first Ander and Petersen refer to the object function of DEA as rank score for all units even with the fact that each unit is evaluated with unique weights. Secondly super efficiency has the tendency of giving specialized DMUs an extremely high ranking. The third problem is an infeasibility issue, which if it occurs means this method is unable to offer a ranking of the DMUs. The third group is the benchmark ranking technique, (Torgersen, 1996) achieved a complete ranking of efficient DMUs by measuring their significance as a benchmark for inefficient DMUs. If a unit is chosen as a reference target for other DMUs, then it is ranked highly. The benchmark technique is a two stage procedure, first using the additive model to determine the value of the slacks, efficient units has slack values equal to zero, in the second stage a mathematical model is applied to all DMUs to rank the efficient DMUs and determine which is particularly important to the institution. A complete ranking cannot be assured in this methodology, because some DMUs may receive the same ranked score. The fourth group is ranking with multivariate statistics in DEA context, this method involve the use of statistical technique in coalition with DEA to achieve complete ranking of DMUs. Creating a relation between classical statistic technique and DEA was one of the main aims of this methodology. DEA is much more of frontier analysis technique than a central tendency. DEA focuses on each unit separately while regression tries to fit in a single function into a collection of data on the basis of average behaviour. (Adler et al., 2002) stated three ranking processes.

1. Canonical correlation analysis for ranking.

2. Linear discriminant analysis for ranking.

3. Discriminant analysis of ratios for ranking.

The literature of the methodology showed a high statistical importance between the statistical analysis and the results from DEA evaluation. The fifth group is the ranking of inefficient decision-making units. So far the techniques discussed non have ranked inefficient DMUs outside the efficiency score from the standard DEA models. (Bardhan, 1996), derived a concept which attempt to rank inefficient DMUs using a Measure of Inefficiency Dominance (MID). This technique is based on slack adjusted DEA model which an overall measure of inefficiency can be computed. The MID index uses the average proportional inefficiency of all inputs and outputs to rank the inefficient DMUs; however, as the benchmarking ranks efficient DMUs, the MID index ranks only inefficient DMUs. The last group is DEA and multi-criteria decision-making methods (MCDM). The successful combination of DEA and multi-objective linear programming by Golany 1988 produced MCDM, complete ranking is not a priority in MCDM, and however, it uses preference information to further clarify the biased nature of DEA models, that way they can specify which input and output has greater influence in the model solution. This approach can be considered a drawback in this manner, since additional information is needed from the decision makers. Researchers explained that MCDM and DEA are two distinct approaches, they explained that MCDM are applicable in ex ante problems when data are unavailable., example, discussion of future technology that doesn't exist, DEA, on the other hand, gives an ex post analysis of the past which we can use as reference for the future, (Belton V. and Stewart T.J., 1999). Recently a new ranking technique was proposed by Khodabakhshi and Aryavash for assessing a common fixed cost or revenue among units.

2.9 Super-Efficiency Ranking Technique

The super-efficiency ranking technique developed by (Andersen and Petersen, 1993) opened a unique method of ranking DMUs in DEA, this methodology gives an exceedingly efficient unit an efficiency score greater than one by eliminating the kth constraint in the primal objective formulation. Model (2.6) shows the AP super-efficiency ranking model.

$$h_k = Max \quad \sum_{r=1}^{s} u_r y_{rk} \tag{2.6}$$

$$subject\ to$$

$$\sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \geq 0 \qquad for \quad j = 1,..,n.\ j \neq k,$$

$$\sum_{i=1}^{m} v_i x_{ik} = 1$$

$$u_r \geq \varepsilon \quad for\ r = 1,...,s$$

$$v_i \geq \varepsilon \quad for\ i = 1,...,m$$

The dual function of model (2.6) that is model (2.7) computes the distance between Pareto frontier and the unit itself without unit k

$$Min \quad f_k \tag{2.7}$$

$$subject\ to:$$

$$\sum_{j \in J} L_{kj} x_{ij} \leq f_k x_{ik} \quad for\ i = 1,...,m$$

$$\sum_{j \in J} L_{kj} y_{rj} \geq y_{rk} \quad for\ r = 1,...,s$$

$$L_{kj} \geq 0 \quad for\ j = 1, ..., n.$$

There are three main drawbacks related to this method, first, Anderson and Peterson refer to the objective function value as a rank score for all units, which is not true because each unit is examined in accordance with a different weight. Secondly, specialized DMUs are given and excessively high ranking this methodology. (Sueyoshi, 1999) attempted rectifying this problem by introducing specific bounds on the weights in a super-efficiency ranking model (Andersen P. and Petersen N.C., 1993).

The last problem attributed to this model is an infeasibility issue. Suggesting that, the super-efficiency ranking technique sometimes cannot give a complete ranking of all the evaluated DMUs. Seiford and Zhu (1999) showed the various conditions that super-efficiency model can be infeasible. (Mehrabian S., 1999) made a suggestion to the dual function to ensure feasibility.

Each technique focuses on a separate aspect of ranking and can be used in a specific area of preference. None of them answers the question of complete ranking problem due to certain areas of limitation and weakness in technique.

Chapter 3

FACET ANALYSIS AND MODIFIED VARIABLE RETURN TO SCALE (VRS)

3.1 Introduction

Etymologically, facet refers to “little face” and in ordinary language it is the cut side of a diamond. Literally, facet analysis is the survey of facets. It is the process of breaking a body into its integral part with the selection of appropriate terminology to express those parts by means of notational device. (Ranganathan, 2nd ed. 1957, 3rd ed.1967) was the pioneer of this method, used in describing the colon classification, a faceted classification scheme.

In DEA context, facets analysis is the analysis of facets of the defining hyper plane. The efficiency frontier estimated by production function in input-output space takes the shape of diamond edges, especially in greater than two dimensional space, therefore, “facet analysis in DEA anchors on the hyper planes of PPS frontier for classic DEA models. The frontier is constructed by hyper planes which supports the PPS of efficient DMUs”. Facet analysis analyses and provides detail information about these hyper planes.

The facet analysis phrase in DEA was first used by (Bessent et al., 1988) and Chung & Guh. They utilized this concept in CCR model. In a polyhedral of n-dimensional space, facet is the face that has n-1 degree of freedom which refers to a face with n-1 dimension. Facet analysis provides us with a correlation between algebraic and geometric view of point of a DEA model. (Charnes et al., 1978) characterized the facet structure of CCR model, while (Banker et al., 1884) did the same for BCC model. Thrall (1996) introduced a distinction between interior and exterior facets. (Daneshvar S., 2009) used facet analysis to develop a modified VRS model based on (BCC) model (Banker et al., 1884)

3.2 Importance of Facet Analysis

In DEA efficiency evaluation, facet is an essential subject in achieving the true efficiency score of an evaluated DMU. This allows the analyst to discover areas of improvement of the DMU, whether it is by reduction of input to achieve the same amount of output or increase of output while maintaining the same amount of input. The part of the frontier responsible for evaluating efficiency score is called the facet.

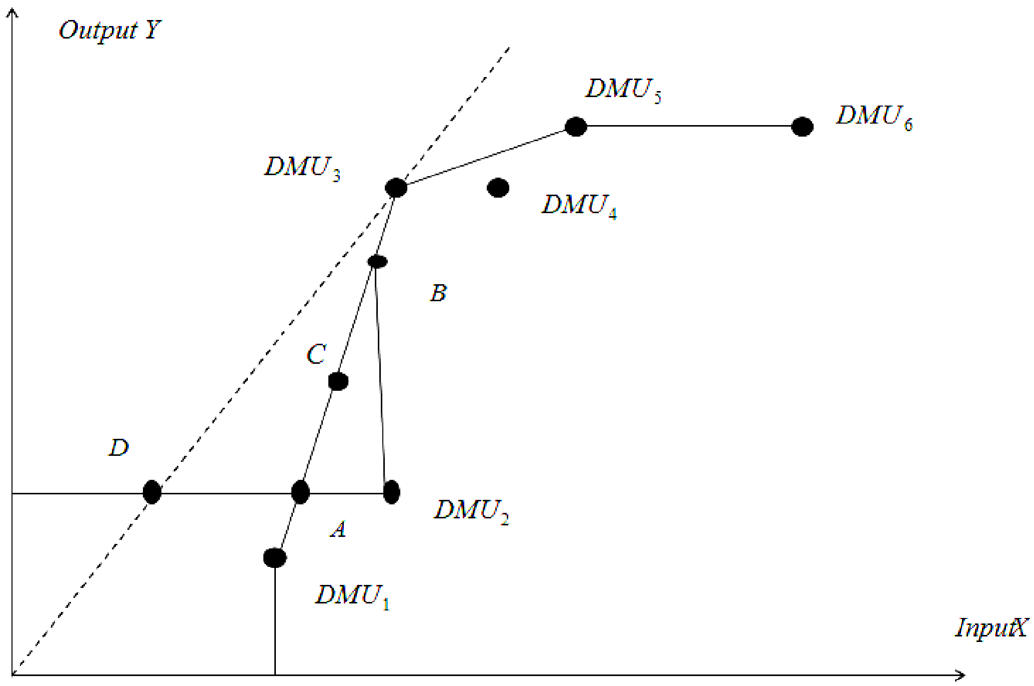

Figure 3.1: Efficiency Frontier

For example, in Figure 3.1, the facet, from DMU_1 to DMU_3 is exclusively responsible for evaluating the efficiency of DMU_2, similarly the facet from DMU_3 to DMU_5 is also responsible for evaluating DMU_4.

The essence of facet in expediting decisions for managers and analyst is clearly shown in Figure 3.1, the efficiency of DMU_2 can be improved in two ways, either by reducing the input while maintaining the same output to point A, or maintain the same input and increase the output to point B, either way the efficiency of DMU_2 will improve. Similar operation can be done for any DMU located within the facet of an efficient DMU.

3.3 Facet Analysis on Variable Return to Scale

Maintaining efficiency score of efficient DMUs is of great importance, especially in economical point of view, hence, the prioritization of sensitivity analysis by most researches. (Daneshvar S., 2009), developed a modified VRS model using facet analysis on (BCC) model. He generated and extended stability region for DMUs placed on the intersection of efficient and weak efficient frontier. Sensitivity analysis has been performed by previous researchers on other models such as, Charnes and Noralic (1990) investigating the sensitivity of DEA-additive model, such that adequate conditions for maintaining efficiency are determined. Charnes, Cooper, Lewin, and Morey performed the first DEA sensitivity analysis paper (Daneshvar et al., 2014), achieved the BCC model modification by finding a new stability region for DMUs in the production possibility set. They based their work (Jahanshahloo et al, 2005).

They identified that, (Jahanshahloo et al., 2005) , work was not sufficient for weak efficient DMUs.

Using facet analysis on BCC model, they achieved a modified variable return to scale model, taking (X_o, Y_o) as the evaluated DMU, examine the intersection of the production possibility set and the plane $P = \{(X,Y) \mid X = \beta Y_O, Y = \beta Y_{O,} \alpha, \beta \geq 0\}$ It is illustrated as follows:

$$P \cap T = \{(X,Y) \mid X = \alpha X_O \geq \sum_{j=1}^{n} \lambda_j X_j, Y = \beta Y_O \leq \sum_{j=1}^{n} \lambda_j Y_j\} \qquad (3.1)$$

$$\sum_{j=1}^{n} \lambda_j$$

$$\lambda_j \geq 0, \{\forall (j = 1.....n) \forall \alpha, \beta \geq 0\}$$

Figure 3.2, represents the model (3.1), consider the new axes α and β in the plane P, the plane P cut through the three dimensional figure of model (3.1), the corresponding set of model (3.1) can be illustrated as follows:

$$\bar{T}(X_O, Y_O) = (\alpha X_o, \beta Y_O) \mid \alpha X_O \geq \sum_{j=1}^{n} \lambda_j X_j, \beta Y_O \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1 \qquad (3.2)$$

$$\lambda_j \geq 0 \ \forall (j = 1,.....n) \forall \alpha, \beta \geq 0$$

The efficient point is $b_o^* = 1$ with (U, V^*, u_O^*) representing the optimal solution for the BCC model, therefore $(U^{*t} Y_0 + U^* = 1 = V^* X_0)$ in input and output space with the supporting hyperplane $(U^{*t} Y_0 + U^* = 1 = V^* X_0)$ passing through point (X_o, Y_o)

Definition (3.1): (Daneshvar S., 2009) " A hyper plane is a strong defining hyper plane of PPS if and only if it is supporting at least (m + s) strong efficient DMUs of PPS which lie on it and in its gradient vector components corresponding with output vector are non-negative and the components corresponding with input vector are non-positive."

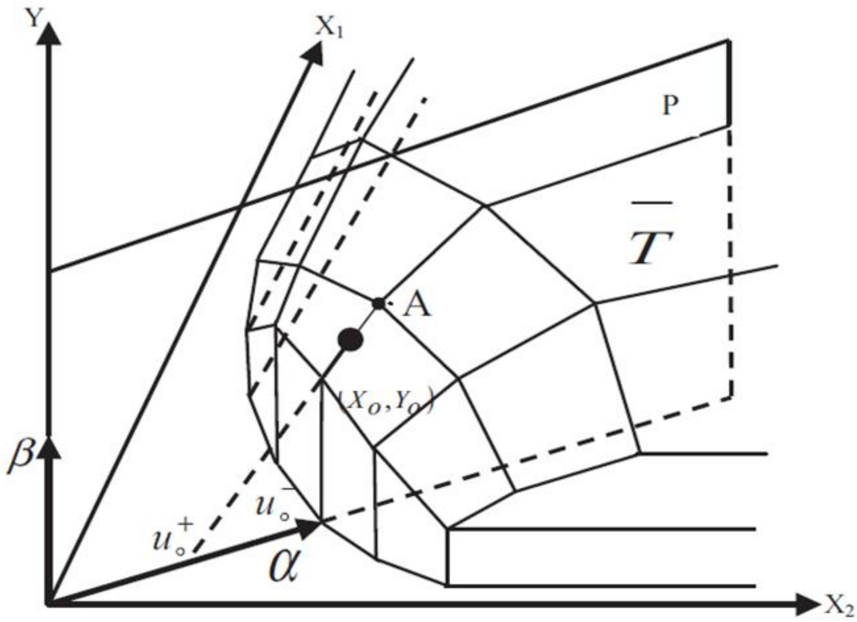

Figure 3.2: The intersection of Tv and P

Banker and Thrall (Banker R. D. and Thrall R. M., 1988) emphasized that the production possibility set may have more than one supporting hyper planes at any efficient point, for example in Figure 3.2 there are many binding hyper planes in A, the value of (u_0^*) at such point are not unique. The values of $(u_o^+ \text{ and } u_o^-)$ upper and lower bounds for all supporting hyper planes that pass through $(\alpha \text{ and } \beta)$ can be computed as follows:

$$
\begin{aligned}
& Max\ u_0 \qquad\qquad (3.3)\\
& Subject\ \ to\\
& UY_0 + u_0 = 1\\
& UY_j - VX_j + u_0 \leq 0 \quad j = 1,2,....n\\
& VX_0 = 1\\
& U \geq 0, \quad V \geq 0 \quad u_0\ free
\end{aligned}
$$

$$
\begin{aligned}
& Min\ u_0 && (3.4)\\
& Subject\ \ to \\
& UY_0 + u_0 = 1 \\
& UY_j - VX_j + u_0 \le 0 \quad j = 1,2,....n \\
& VX_0 = 1 \\
& U \ge 0, \quad V \ge 0 \quad u_0\, free
\end{aligned}
$$

$(u_o^+\ and\ u_o^-)$ Denotes the optimal solution of model (3.3) and (3.4) respectively, u_0^- may resolve to (-∞) for some DMUs. The following inequality holds for for classical BCC model $\left(u_0^+ \le u_0^* \le u_0^-\right)$.

Definition (3.2): (Daneshvar S., 2009). The supporting hyper planes generated by u_0^* which satisfy the inequalities $(u_o^+ \le u_o^* \le u_o^-)$ and passes through (X_o, Y_o) i.e. $(U^{*t}Y_o + U^* - V^* X_o = 0)$ are called admissible supporting hyper planes for T_v.

The modified variable return to scale model is achieved as follows, by restricting the free variable u_0 (Daneshvar et al., 2014) illustrated that, in input orientation case of BCC model. By using model (3.4) for all the efficient DMUs and taking the maximum of the values other than one, use that value and assign it as the upper bound for the free variable in the BCC model. This restriction makes the value of u_0 in the optimal solution to avoid the weak efficient frontier in the PPS. The restriction must be within the supporting hyper planes replaced by the constructed hyper planes, to ensure this use (3.4) for all efficient DMUs.

$$
\varepsilon = Max\{u_0^- \mid u_0^- \ne 1\ for\ efficienct\ \ DMUs\} \qquad (3.5)
$$

The classical BCC model is modified by using ε as an upper bound for the free

$$
\begin{aligned}
& z_0^* = Max\ UY_0 + u_0 \qquad (3.6) \\
& Subject\ \ to \\
& UY_j - VX_j + u_0 \le 0 \quad j = 1,2,....n \\
& VX_0 = 1 \\
& U \ge 0, \\
& V \ge 0 \\
& u_0 \le \varepsilon
\end{aligned}
$$

Theorem 3.1: (Daneshvar S., 2009) Model (3.6) “does not change the efficiency value of efficient and strong efficient DMUs, changes are only in the efficiency value of weak efficient DMUs and DMUs compared with weak frontier.” For further explanation of modified variable return to scale (VRS) model see (Daneshvar S., 2009)

Chapter 4

THE ARASH METHOD

4.1 Introduction

The ranking of efficient and inefficient DMUs together in DEA eluded researchers for a while, numerous methods for ranking has been proposed but so far most have not satisfied the broad vision of DEA philosophy in ranking, although the methods proposed has its applications and are beneficial to specific areas, examples are the AP Super-efficiency ranking method, Cross-efficiency ranking method, Benchmark ranking method, none of the above stated ranking techniques strongly distinguish between technical efficiency, efficiency and inefficiency of DMUs.

The inefficient DMUs are arranged after the technical efficient once by DEA models; nevertheless, it is possible for a technical efficient DMU to be less efficient than an inefficient DMU. The Arash method is a new model developed by (Khezrimotlagh et al., 2012) to estimate the production possibilities of Decision Making Units (DMUs) using flexible linear programming based on Additive DEA Model (ADD). They identified the inadequacy in the definition of efficiency of Pareto and illustrated the limitations in DEA technique to bench and rank DMUs. The Arash method is capable of differentiating between technical efficient and/or inefficient DMUs without additional information in the form of weight restriction or statistical technique and super-efficiency, it also point out that, technical efficiency is a necessary condition for becoming efficient but it is not enough to call it efficient. In the absence of cost information, the Arash method is also capable of measuring the cost efficiency of DMUs. The extension of Arash method into non-linear programming has the characteristics of Slack Based Measure (SBM) model but still possess the properties of linear Arash method. The AM score finds the best technical efficient DMU amongst the observed DMU by introducing a minor error in the values of input. It also shows that a minor error in the input values does not produce to significant errors in the calculation of the efficiency index which encouraged the introduction of axioms of continuity.

4.2 Efficiency, Technical Efficiency and Problem Statement

Efficiency or doing the job right can be defined as the ratio of $Output/Input$. A DMU ($x^{'},y^{'}$) does the job better than DMU (x,y) if the amount $y^{'}/x^{'}$ is greater than y/x. When a set of homogenous DMUs are considered for evaluation, the input and output variables will be identified for the DMUs, the PPS for the DEA axioms and its frontier which is the Farrell frontier is used to estimate the production frontier. The efficiency of a DMU is calculated by comparing the location of the DMU in the PPS to the frontier, it is also bench marked and ranked at the same time.

Pareto definition of efficiency states that a DMU is to be rated as fully (100%) efficient (referred to as technical efficiency in economics) on the basis of available evidence if and only if the performances of other DMUs do not show that some of its inputs or outputs can be improved without worsening some of its other inputs or outputs. Hence, by this definition, DMUs on the Farrell frontier are called fully (100%) efficient and others are inefficient. (Khezrimotlagh et al., 2013) pointed out a flaw in this definition and identified that, it is inappropriate to call a technical efficient DMU “100%” efficient, they pointed out that Pareto-Koopmans definition of efficiency is valuable for identifying only technical efficient DMUs and show that technical efficient DMUs that are considered efficient may be more inefficient than the inefficient DMUs, To better illustrate the differences between the terms "technical efficiency" and "efficiency" consider Table 4.1 from. (Khezrimotlagh et al., 2012) , using two inputs and one constant output, no other information is given.

Table 4.1: Three DMUs along with One Output and Two Inputs

DMUs	Input 1	Input 2	Output	CCR Score	AP Rank
A	2	55	10	1.000	1.500
B	3	3	10	1.000	9.500
C	55	2	10	1.000	1.500

Using the Pareto-Koopmans definition of efficiency, DMUs A, B and C are technically efficient because none of the input and output for each DMU can be improved without worsening some other input or output. The last column of Table 4.1 shows the AP ranking as follows: B > A = C.

Consider the addition of two inefficient DMUs in Table 4.2 to the ones in Table 4.1, the resulting AP Ranking are as follows. A = C > B > D = E

The Production Possibility Set (PPS) of DMU in Table 4.1 and Table 4.2 are the same, but the Ranking of technical efficient DMUs using the AP technique is sometimes ambiguous.

Table 4.2: Five DMUs along with One Output and Two Inputs

DMUs	Input 1	Input 2	Output	CCR Score	AP Rank
A	2	55	10	1.000	1.500
B	3	3	10	1.000	1.167
C	55	2	10	1.000	1.500
D	3	4	10	1.000	0.994
E	4	3	10	1.000	0.994

For example in Table 4.1 DMU B has the first ranking among the DMUs, but in Table 4.2, DMU B has the third ranking, this is misleading, pointing to the fact that Ranking with AP method may not be very significant. Looking at the inefficient DMUs D and E they are close to DMU B and removing DMU B in AP may not have substantial effect on the PPS of Table 4.2. Besides, DMUs D and E are inefficient compared to B and other Technical efficient DMUs, but the technical efficient DMUs do not dominate the inefficient DMUs, therefore, it is possible that an inefficient DMU be more efficient than an efficient one that does not dominate over it.

The Pareto-Koopmans definition is capable of identifying DMUs on the Farrell frontier, but the DMUs on the Farrell frontier may neither do the job right nor be more efficient than some inefficient DMUs (Khezrimotlagh et al., 2012). Models and techniques in DEA can be classified into two groups (Khezrimotlagh et al., 2012). Group one as those that does not detail information from the analyst, example, Super-efficiency and cross evaluation models (Khezrimotlagh et al., 2012), the second group on other hand, require some details about the data such allocation of weights and weight restriction. The Pareto-Koopmans definition of efficiency is upheld by the second group. To further illustrate the shortcomings of Pareto-Koopmans definition of efficiency consider Table 4.3, Figure 4.1 and Figure 4.2 with five DMUs one output and one input in Variable Return to Scale (VRS).

Table 4.3: Five DMUs one output and one input

DMU	X	Y	Paretor-Koopman Definition	Efficiency (y/x)
A	2	2	100% Efficient	1
B	3	9	100% Efficient	3
C	10	10	100% Efficient	1
D	3	8.7	Inefficient	2.9
E	3.3	9	Inefficient	3.7

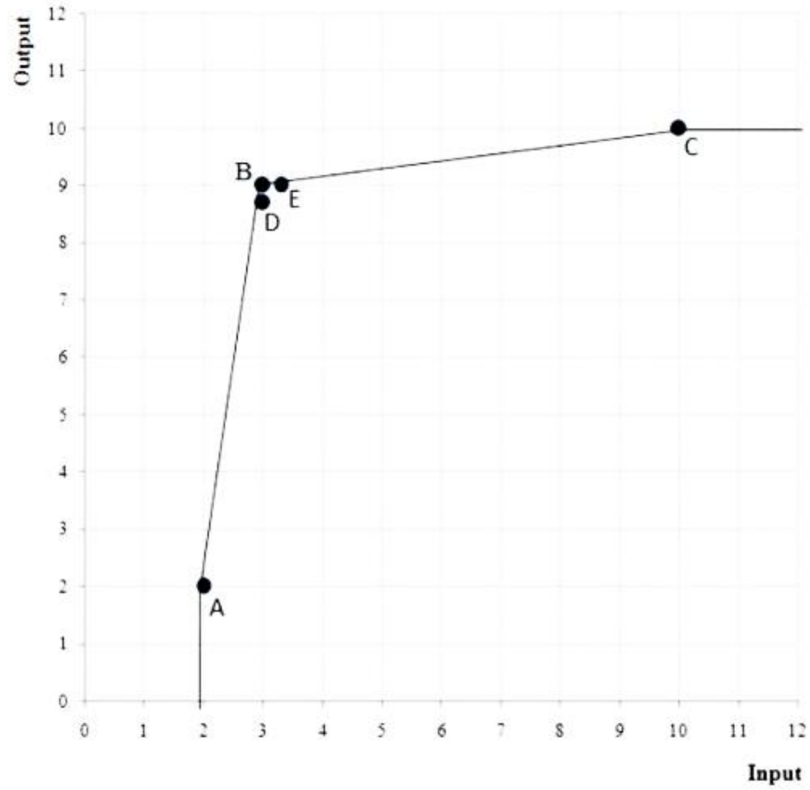

Figure 4.1: The VRS Farrel frontier.

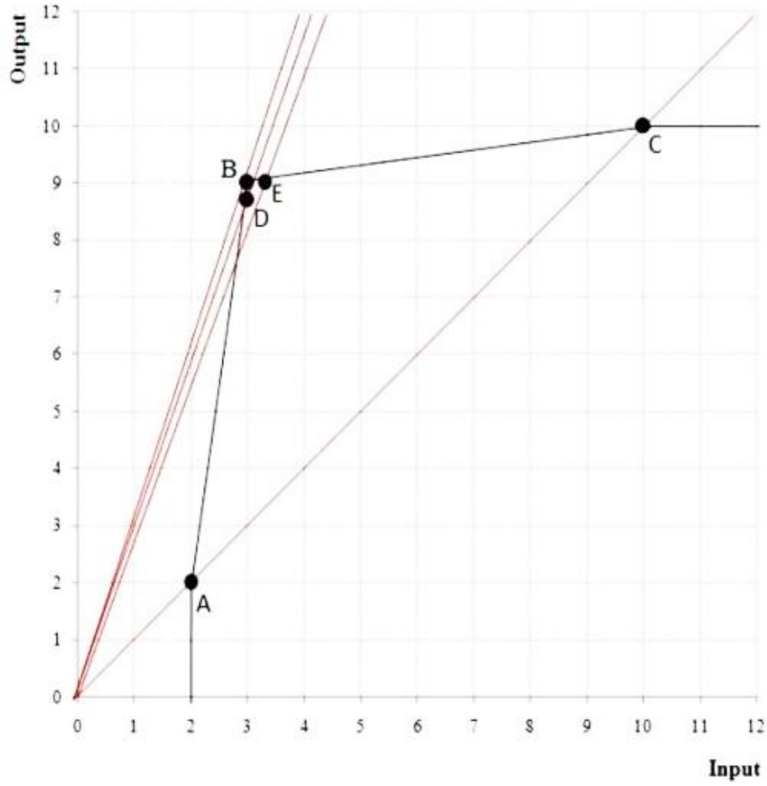

Figure 4.2: The measurement of DMUs efficiency

From efficiency definition of pareto, the technical efficient DMUs A, B, and C are fully (100%) efficient from Table 4.3 and Figure 4.1, but DMUs A and C are not more efficient than the inefficient DMUs D and E as noticed from Figure 4.2 and the last column of Table 4.3. The elaborated examples simply states and shows that, Pareto-Koopmans definition of efficiency is capable of identifying technical efficient DMUs but not efficient DMUs, therefore, (Khezrimotlagh et al., 2012), presented a new method called the Arash Method and a current definition of efficiency to construct a new DEA structure and at the same time cover the purpose of both DEA groups.

4.3 The Arash Method (AM)

The Arash method was proposed by (Khezrimotlagh et al., 2012) to examine the Farrell frontier and evaluate DMUs that do the job right and remove the drawbacks of arranging DMUs with linear programming using Additive DEA model they achieved that by introducing a small error into the inputs of the observed DMUs. To illustrate the method:

Assume there are n DMUs

$DMU_i = (i = 1,2,.....n)$

Inputs $x_{ij} \quad j = (1,2,.....m) \quad m\,(non-negative\ inputs)$

Outputs $y_{ik} \quad k = (1,2,......p) \quad p(non-negative\ inputs)$

For each DMU which has at least one of its inputs and one its outputs that is non-zero. The input-orientation case of $\varepsilon - AM$ is as follows.

$DMU_l\ (l = 1,2,.....n)$ is evaluated and $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_m) \quad \varepsilon_j \geq 0.$

$$Max \quad \sum_{j=1}^{m} w_j^- s_j^- + \sum_{k}^{p} w_k^+ s_k^+ \tag{4.1}$$

Subject to

$$\sum_{i=1}^{n} \lambda_i x_{ij} + s_{j=1}^- = x_{lj} + \varepsilon_j \qquad j = 1,2, ..., m$$

$$\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk} \qquad k = 1,2, ..., p$$

$$\sum_{i=1}^{n} \lambda_i = 1 \qquad i = 1,2, ..., n$$

$$s_j^- \geq 0 \qquad j = 1,2, ..., m$$

$$s_k^+ \geq 0 \qquad k = 1,2, ..., p$$

The $\varepsilon - AM$ targets its scores as follows:

$$x_{lj}^* = x_{lj} + \varepsilon_j - s_j^- \qquad j = 1,2, ..., m$$

$$y_{lk}^* = y_{lk} + s_k^+ \qquad k = 1,2, ..., p$$

$$A^* = \frac{\sum_{k=1}^{p} w_k^+ y_k \Big/ \sum_{j=1}^{m} w_j^- x_j}{\sum_{k=1}^{p} w_k^+ y_k^* \Big/ \sum_{j=1}^{m} w_{j=1}^- x_j^*} \tag{4.2}$$

For the weight definition:

$$Input \quad j = 1,2,.....m \ (w_j^-) \qquad w_j^- = \begin{cases} N_j & x_j = 0 \\ \dfrac{1}{x_j} & x_j \neq 0 \end{cases}$$

$$(4.3)$$

$$Output \ k = 1,2,.....p\left(w_k^+\right) \qquad w_k^+ = \begin{cases} M_k & y_k = 0 \\ \dfrac{1}{y_k} & y_k \neq 0 \end{cases}$$

N_j *and* M_j can be a positive real number selected by the evaluator to represent each goal proportionally to the value of the resource. The score of $\varepsilon - AM$ is marked by A_ε^* where $\varepsilon = \varepsilon_1, \varepsilon_2, ..., \varepsilon_n$.

The evaluated technical efficient DMU is compared with a technical efficient target DMU with a slightly different amount suggested by the model and it decides if the evaluated technical efficient DMU is efficient or not by using the real definition of efficiency which is $Output/Input$. The input constraint in the model ascertains that the corresponding virtual DMU of the observed DMU_l is under evaluation and how much of an epsilon error in input values changes the technical efficiency score. For instance, suppose that $x_j \neq 0$ *and* $y_k \neq 0$. The 0.1-AM examines that only one tenth error in each input of a DMU which is a DMU with this input Values $x_j + \varepsilon_j x_j$ *for* $j = 1,2,...,m$ which shows how much change it affects the efficiency score which is calculated as follows:

$$A_\varepsilon^* = \frac{\frac{p}{m}}{\sum_{k=1}^{p}\left(\frac{y_k^*}{y_k}\right) \Big/ \sum_{j=1}^{m}\frac{x_j^*}{x_j}} \tag{4.4}$$

The above model (4.4) clearly shows that it is independent of units and it assumes the input and output values of the evaluated DMU. When $A_\varepsilon^* < 1$ for an observed DMU, $\varepsilon - AM$ suggest that the observed DMU changes its input and output values to that of of $\varepsilon - AM$ target, otherwise, if $A_\varepsilon^* > 1$, the $\varepsilon - AM$ suggest the DMU to remain the same, showing that it has a good combination of input and output values in the PPS and preventing it from decreasing its efficiency score. Also, the $\varepsilon - AM$ is always feasible for $\varepsilon \geq 0$ because the virtual DMUs are always dominated by the real once, so if the real DMU is feasible so will the result. Furthermore when A_ε^* is equal for two DMUs A and B, it means, when ε error occurs in the input, both DMUs A and B are equal in combination of their data. A practical definition to define technical efficient DMUs is as follows (Khezrimotlagh et al., 2012)

Definition 4.1: A technically efficient DMU is efficient with ε degree of freedom $\varepsilon - DF$ in inputs if $\left(A_0^* - A_\varepsilon^* \leq \delta\right)$, Otherwise, it is inefficient with $\varepsilon - DF$ in inputs. The proposed amount for δ is $10^{-1}\varepsilon \; or \; \varepsilon/_m$.

Example 4.1 shows the effectiveness of the AM using Data from Table 4.3, from the table the least values of input and output is 2, therefore, $\varepsilon_j = 2\varepsilon$. Table 4.4 illustrates the results of AM from the data in Table 4.3 when ε is 0, 0.1 and 0.

Table 4.4 The Result of ε-AM from Table 4.3 data.

DMU	0-AM	O.1-AM	0.5-AM
A	1.0000	0.5882	0.2222
B	1.0000	0.9333	0.6667
C	1.0000	0.9800	0.9000
D	0.9667	0.9022	0.6444
E	0.9091	0.8485	0.6061

The result of $\varepsilon - AM$ in Table 4.4 clearly shows that DMUs A, B and C are technically efficient according to 0-AM, and the result of 0.1-AM shows that the technical efficient DMUs A and C are more inefficient than the inefficient DMUs D and E. this shows that technical efficient DMU A, when compared to other technical efficient DMUS B and C should increase its input to find a better efficient value and place on the Farrel frontier [6].

Chapter 5

MODIFICATION OF THE ARASH METHOD

5.1 Introduction

In this chapter, a modification of The Arash method presented in chapter 3 is introduced. The Arash method uses Additive DEA model to evaluate efficiency of DMUs, the use of small amount of error in input values helps differentiate between technical efficient and efficient DMUs, thereby presenting a new platform for the entire DEA.

For the proposed modified Arash method, we attempt to achieve similar modification by (Daneshvar S., 2009) on BCC model, to the PPS of The Arash method using facet analysis. This proposed modification, attempts to fix the weak part of the efficient frontier in the Arash method that gives a bias efficiency score to DMUs located at the weak part of the frontier or DMUs that get their efficiency score when compared to the weak part of the frontier. The proposed modified Arash method thereby gives the true efficiency score to DMUs at the targeted region, ranking weak efficient DMUs is equally as important as ranking the efficient DMUs because in the practical application of DEA, the economical or financial implication of misplaced ranking of a DMU might have a devastating effect on the organization.

5.2 Modification Assumptions

The modification is based on the assumption that, the technical efficient DMUs identified by the Arash method are DMUs located at the weak part of the frontier or get their efficiency score when compared to the weak part of the frontier, therefore, the efficiency score of the efficient DMUs remain the same, only that of the technical efficient DMUs changes. Furthermore, the PPS of the Arash method is the same as the PPS of BCC model, because the primary model, in which the Arash method is based on, i.e Additive DEA model, has the same PPS as the BCC model. Reaffirming our assumption that similar modification achieved on the BCC model to get the modified VRS model by (Daneshvar et al., 2014) which fixes the weak part of the frontier of the BCC model is possible on the Arash method. We attempt to improve the Arash method by simultaneously assigning the real efficiency score and rank to the DMUs on the weak part of the frontier and differentiate between technical efficiency and efficiency, hence, combining the achievements of the Arash method and Modified VRS model.

5.3 Problem Definition

Differentiating between technical efficiency and efficiency is of great importance in the practical application of DEA. A little difference in efficiency evaluation can have a drastic impact on decision making for a decision maker; therefore, sensitivity analysis on the efficiency frontier of DEA models is imperative. Technical efficient DMUs identified as efficient DMUs by previous DEA models such as the AP super-efficiency model are questionable, because, some technical efficient DMUs are more inefficient than some inefficient DMUs, (Khezrimotlagh et al., 2012) attempt rectifying the drawbacks by using a small error in input values of data using Additive DEA model. Although this technique (AM) proves logical and practical but it does not take into consideration the weak part of the efficient frontier or DMUs that take their efficiency score when compared to the weak part of the efficient frontier, we approach this drawback in this modification by placing an upper bound on the free variable of the dual VRS Arash method. This upper bound on the free variable will not interfere with the achievement of the Arash method, rather, it takes into consideration the weak part of the frontier, thereby, giving the DMUs related to the weak part of the frontier their real efficiency score and rank, thus creating a robust technique for efficiency evaluation. We introduce the characteristics of modified VRS model into the Arash method.

This modification is presumed to have the following characteristics:

- Find DMU which do the job and remove previous shortcomings of arranging DMUs (Ranking)
- Modify PPS by restricting free variable
- Give the real efficiency score for weak efficient DMUs or DMUs that get their score when compared to DMU on the weak part of the frontier
- Simultaneously suggest to the evaluated DMU to increase input by some units or decrease output by some units so the efficiency can improve sharply, the conventional DEA techniques are not able to offer this option.

The proposed modification in the Arash method like the basic Arash method should be able to address the shortcomings whether the information of data is available or not.

5.4 Modification of Arash Method using Facet Analysis

In this section, we try to modify the PPS of the Arash method using facet analysis by restricting the free variable u_o only. To illustrate the proposed modified Arash method, suppose there are n DMU, DMU_i, $i = 1,2,.......n$ with m non-negative inputs, x_{lj} $(j = 1,2,.......m)$ and p non-negative outputs, y_{ik} $(k = 1,2,........p)$ With at least one input and one output for each DMU not equal to zero

First compute the efficiency of the DMUs using the standard BCC model model (2.3). Then use model (3.4) to compute u_0^- for all efficient DMUs identified by model (2.3). The upper bound for the proposed Modified Arash method is β

$$\beta = Max\left[u_0^- \mid u_0^- \neq 1 \ \ for \ efficient \ \ DMUs\right] \tag{5.1}$$

The standard Arash method is modified by computing the dual of the Arash method model (4.1) and placing β as and upper bound for the free variable u_0 as follows:

$$Min \sum_{j=1}^{n} V_j \left(x_{lj} + \varepsilon_j^-\right) + \sum_{k=1}^{r} U_k y_{lk} + u_0 \tag{5.2}$$

$$\begin{aligned}
&subject\ to\\
&\sum_{j=1}^{m} V_j x_{ij} + \sum_{k=1}^{r} U_k y_{ik} + u_0 \geq 0\\
&\quad V_j \geq w_j^-\\
&-U_k \geq w_k^+\\
&\quad u_0 \leq \beta\\
&\quad V_j : free\\
&\quad U_k : free
\end{aligned}$$

The dual of model (5.2) illustrates the Proposed Modification to the Arash method

$$Max\ \sum_{j=1}^{m} w_j^- s_j^- + \sum_{k=1}^{p} w_k^+ s_k^+ + \eta\beta \qquad (5.3)$$

$$subject\ to$$

$$\sum_{i=1}^{n} \lambda_i x_{ij} + s_j^- = x_{lj} + \varepsilon_j^-$$

$$\sum_{i=1}^{n} \lambda_i y_{ik} - s_k^+ = y_{lk}$$

$$\sum_{i=1}^{n} \lambda_i + \eta = 1$$

$$\lambda_i \geq 0 \qquad i = 1..........n$$

$$s_j^- \geq 0 \qquad j = 1.........m$$

$$s_k^+ \geq 0 \qquad k = 1,.........p$$

$$\eta \geq 0$$

The weights for the model w_j^- *and* w_k^+ are defined as follows:

$$\begin{bmatrix} \varepsilon_j^- = \varepsilon \times \min(x_j, y_k) \\ w_j^- = \begin{cases} \frac{1}{x_{lj}} & x_{lj} \neq 0 \\ N_j & x_{lj} = 0 \end{cases} \\ w_k^+ = \begin{cases} \frac{1}{ylk} & y_{lj} \neq 0 \\ M_k & y_{lj} = 0 \end{cases} \end{bmatrix} \begin{pmatrix} N_J \text{ and } M_k \text{ can be selected from} \\ \text{a positive real number set depending} \\ \text{on the goals of the DMUs resources} \\ \text{and production} \end{pmatrix} \qquad (5.4)$$

$$T\arg ets : x_{lj}^* = \begin{cases} x_{lj} + \varepsilon_j^- - s_j^*, \forall j, \\ y_{lk}^* = y_{lk} + s_k^*, \forall k, \end{cases}$$

$$Score: \quad A^* = \frac{\sum_{k=1}^{p} w_k^+ y_{lk} \Big/ \sum_{j=1}^{m} w_j^- x_{lj}}{\sum_{k}^{+} w_k^+ y_{lk}^* \Big/ \sum_{j=1}^{m} w_j^- x_{lj}^*} \qquad (5.5)$$

5.5 Numerical Examples

In this section, we illustrate the proposed modified Arash method with an example, the example present a one input one output case to clearly state the achievement of the modified model.

We first determined the efficiency of the DMUs using BCC modal (2.3), and then used model (3.4) to determine the upper bound β for the free variable. Table 5.1 shows the input and output of the DMUs with their corresponding BCC efficiency and u_0^- values

Table 5.5.1: Nine DMUs with BCC efficiency

DMUs	Input	Output	BCC	u_o^-
A	2	2	1.0000	0.8570
B	3	9	1.0000	-20.0000
C	10	10	0.9800	*****
D	3	8.7	0.9670	*****
E	3.3	9	0.9091	*****
F	10.3	10	0.9514	*****
G	9.8	10	1.0000	$-\infty$
H	2	1	1.0000	1.0000
I	2	1.5	1.0000	1.0000

From Table 5.5.1, DMUs A, B and G are BCC efficient, therefore we computed the u_0^- for the efficient DMUs to get the upper bound for the free variable of the proposed modified Arash modal. From the table the β value is 0.8571 for the set of evaluated DMUs.

Table 5.5.2: The results of ε-AM and ε-MAM

DMUs	0-AM	Rank	0-MAM	Rank	0.1-AM	Rank	0.1-MAM	Rank
A	1.0000	1	-1.2380	7	0.6500	7	-1.4050	7
B	1.0000	1	1.0000	1	0.9667	3	0.9667	3
C	0.9800	4	0.9800	3	0.9700	2	0.9700	2
D	0.9656	5	0.9383	5	0.9323	5	0.9050	5
E	0.9091	7	0.9091	6	0.8788	6	0.8788	6
F	0.9515	6	0.9515	4	0.9417	4	0.9417	4
G	1.0000	1	1.0000	1	0.9898	1	0.9898	1
H	0.0000	9	-4.2833	9	-0.7000	9	-4.5550	9
I	0.6667	8	-2.2830	8	0.2008	8	-2.4547	8

We considered the five DMUs used by (Khezrimotlagh et al., 2013) to illustrate the finding of the Arash method, we added four more DMUs to show the shortcoming of the Arash method and further illustrate the improvement of the proposed modified

Arash method. Table 5.5.2 and Table 5.5.3 summarize the results and ranking of the evaluated DMUs.

Table 5.5.3: The results of 0.5-AM and 0.5-MAM

DMUs	0.5-AM	Rank	0.5-MAM	Rank
A	-0.7500	7	-1.8917	7
B	0.8334	4	0.8334	4
C	0.9300	2	0.9300	2
D	0.7989	6	0.7717	6
E	0.7576	5	0.7576	5
F	0.9029	3	0.9029	3
G	0.9490	1	0.9490	1
H	-3.5000	9	-5.6417	9
I	-1.6640	8	-3.1413	8

As can be seen from Table 5.5.2 column 2, 0-AM showed three efficient DMUs and six inefficient DMUs, column four of the same table shows the scores of 0-MAM, which immediately disagrees with the values of 0-AM, suggesting that DMU A is more inefficient that the inefficient DMUs C,D E and F. The 0-AM ranked DMU A as one while 0-MAM ranks it as seven. This is clearly logical because, if DMU B uses three units of inputs to produce nine units of outputs, then, DMU A can improve its efficiency immediately by increasing its input, this clearly shows the improvement in the proposed modified modal to point out that DMU A is not as efficient as it is. Table 5.5.2 column six shows that, DMUs A, B and G are technically efficient and not fully efficient by reducing their efficiency values, column eight of Table 5.5.2 gives the real efficiency values of all technically efficient DMUs.

DMUs A, D, H and I as highlighted in Table 5.5.2 and Table 5.5.3 shows the changes in the values of the DMUs, suggesting that they are at the weak part of the frontier or are compared to DMUs located at the weak part of the frontier. Other unchanged values are located at the strong part of the frontier or are compared to those located at the strong part of the frontier. This valuable finding examines the PPS of the Arash method and created an extended region for the weak efficient DMUs. The proposed modification ranks the technically efficient and inefficient DMUs together.

Chapter 6

CONCLUSION AND FUTURE STUDY

6.1 Conclusion

In this thesis we introduced Data envelopment analysis and its practical application, highlighting the robustness of this technique in measuring efficiency and evaluating performances of DMUs. We pointed out its areas of application in improving productivity and offer subjective decision making alternatives to managers, business owners and the entire economic platform. Chapter 2 presents a comprehensive review of the DEA subject and ranking methods. The concept of facet analysis and its importance is covered in chapter 3 together with its application in the modification of the BCC model. In chapter 4 the basis of the research "The Arash method" is explained. Its achievement in distinguishing between technically efficiency and efficiency is emphasized; the drawback in the method is also pointed out. Applying facet analysis on the efficiency frontier of the Arash method and using an upper bound on the free variable of the variable return to scale Arash method is in chapter 5, putting forward a new method of ranking that possesses the characteristics of the Arash method and taking into account the weak part of the efficiency frontier.

The results of Table 5.5.2 and Table 5.5.3 clearly shows that the Arash method gives a biased efficiency value to DMUs located at the weak part of the efficiency frontier or DMUs that take their efficiency score by comparing to the weak part of the frontier otherwise known as technically efficiency and weak efficient DMUs. By placing an upper upper bound on the free variable of the Arash method, we fix this discrepancy by extending the stability region of the efficiency frontier. This extension was achieved using facet analysis to identify the planes associated to the weak part of the frontier. The values of the modified Arash method (MAM) is justified and clearly effective, because it shows that a little difference in input or output is significant in identifying the entities that do the job right and those that can improve their performance.

The proposed modal can be considered as a pessimistic modal, because it focuses on the weak part of the efficiency frontier. A pessimistic point of view should be the view of all managers and decision makers, because the risk of losing finance and resources is reduced. Therefore, the proposed modified Arash method shows the true performance of a technically efficient DMU as suppose to the overly exaggerated performance proposed by the Arash method.

6.2 Suggestion for Future Study

Ranking is very important aspect of Data Envelopment Analysis, the recent publication in this area of research clearly shows that there is still room for improvement on the existing models; this study focuses on improving the achievement of the Arash method. The proposed medication on the Arash method is in input-oriented case, however, a case of out-put orientation can be considered in the future. Also, facet analysis is a highly essential technique in identifying areas improving due to its robust correlation between algebraic and geometric point of view of mathematical models, therefore, applying facet analysis in other ranking techniques can be useful in DEA. Its success in modifying the VRS model and now the Arash method shows that it is possible.

REFERENCES

Adler N., Friedman L. & Siniuany-Stern Z. (2002). Review of ranking methods in data envelopment analysis context. *European Journal of Operation Research, 140*, 249-265.

Ali Agha Iqbal, L. M. (1993). Computational Accuracy and Infinitesimal in Data Envelopment Analysis,. *INFOR, 31*(24), 290-297.

Andersen P. & Petersen N.C. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science, 39*(10), 1261-1294.

Banker R. D. (1884). Estimating Most Productive Scale Size Using Data Envelopment Analysis. *European Journal of Operational Research*, *17*, 35-44.

Banker R. D. & Thrall R. M. (1988). Estimation of returns to scale using data envelopment analysis.,. *European Journal of Operational Research*(62,), 74–84.

Bardhan I., Bowlin W.F., Cooper W.W., & Sueyoshi, T. (1996). Models for efficiency dominance in data envelopment analysis. Part I: Additive models and MED measures. . *Journal of the Operations Research Society of Japan , 39*, 322-332.

Belton V. & Stewart T.J. (1999). DEA and MCDA: Competing or complementary approaches? In: Meskens, N., Roubens, M. (Eds.), Advances in Decision Analysis. *Kluwer Academic Publishers, Norwell.*

Bessent A., Bessent W., Elma J. & Clark T. (1988). Efficiency Frontier Determination by Constrained Facet Analysis. *Operational Research, 36*(5), 785-793.

Charnes A., Cooper W. W. & Rhodes E. (1978). Measuring the inefficiency of decision making units. *European Journal of Operational Research, 2*(6), 429-444.

Cook W D & Seiford L M. (2009). Data envelopment analysis (DEA)-Thirty years on. *European Journal of Operational Research, 192*(1), 1-17.

Cooper W. T. (1997). Measures of inefficiency in data envelopment analysis and stochastic frontier estimation. . *European Journal of Operational Research* , *99*, 72-88.

Daneshvar S. (2009). The modification of BCC model using facet analysis. *In American Conference on Applied Mathematics,Harvard University, Cambridge,, Mathematics and Computer in Science and Engineering,* , . 635–641.

Daneshvar S., Izbirak G. & Jevadi A. (2014). Sensitivity analysis on modified variable returns to scale model in Data Envelopment Analysis using facet analysis. *Computers & Industrial Engineering, 76*, 32-39.

Doyle J.R. & Green R. (1994). Efficiency and cross-efficiency in data envelopment analysis: Derivatives, meanings and uses. . *Journal of the Operational Research Society, 45*(5), 567-578.

Farrell J. M. (1957). The measurement of productive efficiency.1957. *Journal of the Royal Statistical Society, Series A 120*(3), 253-281.

Jahanshahloo G. R. (2005). Sensitivity and stability analysis in DEA . *Elsevier Inc., 169*, 897-904.

Khezrimotlagh D, Salleh S & Mohsenpour Z. (2012). A new method in data envelopment analysis to find efficient decision making units and rank both technical efficient and inefficient DMUs together. *Applied Mathematical Sciences, 6*(93), 4609-4615.

Khezrimotlagh D., Parviz M., Shaharuddin S. & Zahra M. (2013). A Review of Arash Method in Data Envelopmenr Analysis. *DEA2013.* Samsun Turkey.

Mehrabian S., Alirezaee M.R. & Jahanshahloo G.R. (1999). A complete efficiency ranking of decision making units in data envelopment analysis. *Computational Optimization and Applications, 14*, 261-266.

Ranganathan S. R. (2nd ed. 1957, 3rd ed.1967). Prolegomena to library classification. *London, Asia Publishing House. Digital version. <http://arizona.openrepository.com/arizona/handle/10150/106370>*.

Sexton T.R., Silkman R.H. & Hogan A.J. (1986). Data envelopment analysis. *Critique and extensions. In: Silkman, R.H. (Ed.)*, 73-105.

Sueyoshi T. (1999). Data envelopment analysis non-parametric ranking test and index measurement: Slack-adjusted DEA and an application to Japanese agriculture cooperatives. *Omega International Journal of Management Science, 27*, 315-326.

Torgersen A.M., Forsund, F.R. & Kittelsen S.A.C. (1996). Slack-adjusted efficiency measures and ranking of efficient units. . *The Journal of Productivity Analysis* , *7*, 379-398.

APPENDICES

Appendix A: Optimal Coding Solutions of Arash method summarized in Table 5.5.2 and Table 5.5.3

Appendix A.1.Optimal Solution for 0-AM (model 3.1)

DMU A:

	12:20:15		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	1.0000	0	0	0	basic	-0.2500	3.0000
2	B	0	0	0	0	basic	-0.1500	M
3	C	0	0	0	-24.0000	at bound	-M	24.0000
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	-1.0500	at bound	-M	1.0500
6	F	0	0	0	-25.0500	at bound	-M	25.0500
7	G	0	0	0	-23.3000	at bound	-M	23.3000
8	H	0	0	0	-0.5000	at bound	-M	0.5000
9	I	0	0	0	-0.2500	at bound	-M	0.2500
10	sj	0	0.5000	0	-3.0000	at bound	-M	3.5000
11	sk	0	0.5000	0	0	basic	0.0714	M
	Objective	Function	(Max.) =	0				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	3.5000	2.0000	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	2.0000
3	u0	1.0000	=	1.0000	0	-6.0000	0.6667	1.0000

DMUB:

	12:21:16		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-15.5318	at bound	-M	15.5318
2	B	1.0000	0	0	0	basic	-0.1000	M
3	C	0	0	0	-0.0667	at bound	-M	0.0667
4	D	0	0	0	-0.6799	at bound	-M	0.6799
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-0.1666	at bound	-M	0.1666
7	G	0	0	0	0	basic	-0.0667	2.1553
8	H	0	0	0	-17.7982	at bound	-M	17.7982
9	I	0	0	0	-16.6650	at bound	-M	16.6650
10	sj	0	0.3333	0	0	basic	0.0163	M
11	sk	0	0.1111	0	-2.1553	at bound	-M	2.2664
	Objective	Function	(Max.) =	0				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.0000	=	3.0000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-2.2664	9.0000	9.0000
3	u0	1.0000	=	1.0000	0	19.3981	1.0000	1.0000

DMU C:

	12:22:00		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.6600	at bound	-M	4.6600
2	B	0	0	0	0	basic	-0.0300	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	-0.2040	at bound	-M	0.2040
5	E	0	0	0	-0.0300	at bound	-M	0.0300
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	0.5800
8	H	0	0	0	-5.3400	at bound	-M	5.3400
9	I	0	0	0	-5.0000	at bound	-M	5.0000
10	sj	0.2000	0.1000	0.0200	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5800	at bound	-M	0.6800
	Objective	Function	(Max.) =	0.0200				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.0000	=	10.0000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6800	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.8200	1.0000	1.1111

DMU D:

	12:22:48		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0	basic	-0.0573	0.4696
2	B	1.0000	0	0	0	basic	-0.0344	M
3	C	0	0	0	-5.5056	at bound	-M	5.5056
4	D	0	0	0	-0.0344	at bound	-M	0.0344
5	E	0	0	0	-0.2409	at bound	-M	0.2409
6	F	0	0	0	-5.7465	at bound	-M	5.7465
7	G	0	0	0	-5.3450	at bound	-M	5.3450
8	H	0	0	0	-0.1147	at bound	-M	0.1147
9	I	0	0	0	-0.0573	at bound	-M	0.0573
10	sj	0	0.3333	0	-0.4696	at bound	-M	0.8029
11	sk	0.3000	0.1147	0.0344	0	basic	0.0476	M
	Objective	Function	(Max.) =	0.0344				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.0000	=	3.0000	0	0.8029	2.9571	3.0000
2	U	8.7000	=	8.7000	0	-0.1147	-M	9.0000
3	u0	1.0000	=	1.0000	0	-1.3764	1.0000	1.0250

DMU E:

12:23:53		Friday	June	26	2015			
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4740	at bound	-M	0.4740
2	B	1.0000	0	0	0	basic	-0.0333	M
3	C	0	0	0	-2.0100	at bound	-M	2.0100
4	D	0	0	0	-0.0333	at bound	-M	0.0333
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.1009	at bound	-M	2.1009
7	G	0	0	0	-1.9494	at bound	-M	1.9494
8	H	0	0	0	-0.5850	at bound	-M	0.5850
9	I	0	0	0	-0.5295	at bound	-M	0.5295
10	sj	0.3000	0.3030	0.0909	0	basic	0.0163	0.7770
11	sk	0	0.1110	0	0	basic	0.0433	2.0604
	Objective	Function	(Max.) =	0.0909				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.3000	=	3.3000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1110	-M	9.0000
3	u0	1.0000	=	1.0000	0	0.0900	1.0000	1.1000

DMU F:

12:25:09		Friday	June	26	2015			
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7574	at bound	-M	M
2	B	0	0	0	0.6603	at bound	-M	M
3	C	0	0	0	-0.0194	at bound	-M	0.0194
4	D	0	0	0	0.6603	at bound	-M	M
5	E	0	0	0	0.6312	at bound	-M	M
6	F	0	0	0	-0.0485	at bound	-M	0.0485
7	G	1.0000	0	0	0	basic	-0.0194	M
8	H	0	0	0	0.7574	at bound	-M	M
9	I	0	0	0	0.7574	at bound	-M	M
10	sj	0.5000	0.0971	0.0485	0	basic	0	M
11	sk	0	0.1110	0	0.1110	at bound	-M	M
	Objective	Function	(Max.) =	0.0485				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.3000	=	10.3000	0	0.0971	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9516	0	1.0000

DMU H:

	12:27:11		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	1.0000	0	0	0	basic	-0.5000	6.5000
2	B	0	0	0	0	basic	-0.3000	M
3	C	0	0	0	-48.0000	at bound	-M	48.0000
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-2.1000	at bound	-M	2.1000
6	F	0	0	0	-50.1000	at bound	-M	50.1000
7	G	0	0	0	-46.6000	at bound	-M	46.6000
8	H	0	0	0	-1.0000	at bound	-M	1.0000
9	I	0	0	0	-0.5000	at bound	-M	0.5000
10	sj	0	0.5000	0	-6.5000	at bound	-M	7.0000
11	sk	1.0000	1.0000	1.0000	0	basic	0.0714	M
	Objective	Function	(Max.) =	1.0000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	7.0000	2.0000	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	2.0000
3	u0	1.0000	=	1.0000	0	-12.0000	0.6667	1.0000

DMU I:

	12:28:11		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	1.0000	0	0	0	basic	-0.3333	4.1662
2	B	0	0	0	0	basic	-0.2000	M
3	C	0	0	0	-31.9968	at bound	-M	31.9968
4	D	0	0	0	-0.2000	at bound	-M	0.2000
5	E	0	0	0	-1.3999	at bound	-M	1.3999
6	F	0	0	0	-33.3967	at bound	-M	33.3967
7	G	0	0	0	-31.0636	at bound	-M	31.0636
8	H	0	0	0	-0.6666	at bound	-M	0.6666
9	I	0	0	0	-0.3333	at bound	-M	0.3333
10	sj	0	0.5000	0	-4.1662	at bound	-M	4.6662
11	sk	0.5000	0.6666	0.3333	0	basic	0.0714	M
	Objective	Function	(Max.) =	0.3333				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	4.6662	2.0000	3.0000
2	U	1.5000	=	1.5000	0	-0.6666	-M	2.0000
3	u0	1.0000	=	1.0000	0	-7.9992	0.6667	1.0000

Appendix A.2 Optimal Solution for 0.1-AM (model 3.1)

DMU A:

	14:39:00		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.9000	0	0	0	basic	-0.2500	3.0000
2	B	0.1000	0	0	0	basic	-0.1500	M
3	C	0	0	0	-24.0000	at bound	-M	24.0000
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	-1.0500	at bound	-M	1.0500
6	F	0	0	0	-25.0500	at bound	-M	25.0500
7	G	0	0	0	-23.3000	at bound	-M	23.3000
8	H	0	0	0	-0.5000	at bound	-M	0.5000
9	I	0	0	0	-0.2500	at bound	-M	0.2500
10	sj	0	0.5000	0	-3.0000	at bound	-M	3.5000
11	sk	0.7000	0.5000	0.3500	0	basic	0.0714	M
	Objective	Function	(Max.) =	0.3500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	3.5000	2.0000	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	2.7000
3	u0	1.0000	=	1.0000	0	-6.0000	0.7000	1.0500

DMU B:

	14:40:49		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4444	at bound	-M	0.4444
2	B	1.0000	0	0	0	basic	-0.0333	M
3	C	0	0	0	-2.2220	at bound	-M	2.2220
4	D	0	0	0	-0.0333	at bound	-M	0.0333
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.3220	at bound	-M	2.3220
7	G	0	0	0	-2.1553	at bound	-M	2.1553
8	H	0	0	0	-0.5555	at bound	-M	0.5555
9	I	0	0	0	-0.5000	at bound	-M	0.5000
10	sj	0.1000	0.3333	0.0333	0	basic	0.0163	0.7777
11	sk	0	0.1111	0	0	basic	0.0476	2.2664
	Objective	Function	(Max.) =	0.0333				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.1000	=	3.1000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1111	-M	9.0000
3	u0	1.0000	=	1.0000	0	0	1.0000	1.0333

DMU C:

	14:41:59		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.6600	at bound	-M	4.6600
2	B	0	0	0	0	basic	-0.0300	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	-0.2040	at bound	-M	0.2040
5	E	0	0	0	-0.0300	at bound	-M	0.0300
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	0.5800
8	H	0	0	0	-5.3400	at bound	-M	5.3400
9	I	0	0	0	-5.0000	at bound	-M	5.0000
10	sj	0.3000	0.1000	0.0300	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5800	at bound	-M	0.6800
	Objective	Function	(Max.) =	0.0300				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.1000	=	10.1000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6800	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.8200	1.0000	1.1111

DMU D:

	14:43:02		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4699	at bound	-M	0.4699
2	B	1.0000	0	0	0	basic	-0.0344	M
3	C	0	0	0	-2.2163	at bound	-M	2.2163
4	D	0	0	0	-0.0344	at bound	-M	0.0344
5	E	0	0	0	-0.0999	at bound	-M	0.0999
6	F	0	0	0	-2.3162	at bound	-M	2.3162
7	G	0	0	0	-2.1497	at bound	-M	2.1497
8	H	0	0	0	-0.5846	at bound	-M	0.5846
9	I	0	0	0	-0.5272	at bound	-M	0.5272
10	sj	0.1000	0.3330	0.0333	0	basic	0.0169	0.8029
11	sk	0.3000	0.1147	0.0344	0	basic	0.0476	2.2644
	Objective	Function	(Max.) =	0.0677				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.1000	=	3.1000	0	0.3330	3.0000	M
2	U	8.7000	=	8.7000	0	-0.1147	-M	9.0000
3	u0	1.0000	=	1.0000	0	0.0333	0.9667	1.0333

DMU E:

	14:44:59		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4747	at bound	-M	0.4747
2	B	1.0000	0	0	0	basic	-0.0333	M
3	C	0	0	0	-2.0099	at bound	-M	2.0099
4	D	0	0	0	-0.0333	at bound	-M	0.0333
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.1008	at bound	-M	2.1008
7	G	0	0	0	-1.9493	at bound	-M	1.9493
8	H	0	0	0	-0.5858	at bound	-M	0.5858
9	I	0	0	0	-0.5303	at bound	-M	0.5303
10	sj	0.4000	0.3030	0.1212	0	basic	0.0163	0.7777
11	sk	0	0.1111	0	0	basic	0.0433	2.0604
	Objective	Function	(Max.) =	0.1212				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.4000	=	3.4000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1111	-M	9.0000
3	u0	1.0000	=	1.0000	0	0.0909	1.0000	1.1333

DMU F:

	14:56:24		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7573	at bound	-M	M
2	B	0	0	0	0.6602	at bound	-M	M
3	C	0	0	0	-0.0194	at bound	-M	0.0194
4	D	0	0	0	0.6602	at bound	-M	M
5	E	0	0	0	0.6311	at bound	-M	M
6	F	0	0	0	-0.0485	at bound	-M	0.0485
7	G	1.0000	0	0	0	basic	-0.0194	M
8	H	0	0	0	0.7573	at bound	-M	M
9	I	0	0	0	0.7573	at bound	-M	M
10	sj	0.6000	0.0971	0.0583	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
	Objective	Function	(Max.) =	0.0583				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.4000	=	10.4000	0	0.0971	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9515	0	1.0000

DMU G:

	14:47:21		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.7532	at bound	-M	4.7532
2	B	0	0	0	0	basic	-0.0306	M
3	C	0	0	0	-0.0204	at bound	-M	0.0204
4	D	0	0	0	-0.2081	at bound	-M	0.2081
5	E	0	0	0	-0.0306	at bound	-M	0.0306
6	F	0	0	0	-0.0510	at bound	-M	0.0510
7	G	1.0000	0	0	0	basic	-0.0204	0.5936
8	H	0	0	0	-5.4468	at bound	-M	5.4468
9	I	0	0	0	-5.1000	at bound	-M	5.1000
10	sj	0.1000	0.1020	0.0102	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5936	at bound	-M	0.6936
	Objective	Function	(Max.) =	0.0102				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	9.9000	=	9.9000	0	0.1020	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6936	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.9364	1.0000	1.1111

DMU H:

	14:48:50		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.9000	0	0	0	basic	-0.5000	6.5000
2	B	0.1000	0	0	0	basic	-0.3000	M
3	C	0	0	0	-48.0000	at bound	-M	48.0000
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-2.1000	at bound	-M	2.1000
6	F	0	0	0	-50.1000	at bound	-M	50.1000
7	G	0	0	0	-46.6000	at bound	-M	46.6000
8	H	0	0	0	-1.0000	at bound	-M	1.0000
9	I	0	0	0	-0.5000	at bound	-M	0.5000
10	sj	0	0.5000	0	-6.5000	at bound	-M	7.0000
11	sk	1.7000	1.0000	1.7000	0	basic	0.0714	M
	Objective	Function	(Max.) =	1.7000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	7.0000	2.0000	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	2.7000
3	u0	1.0000	=	1.0000	0	-12.0000	0.7000	1.0500

DMU I:

	14:50:29		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.9000	0	0	0	basic	-0.3330	4.1620
2	B	0.1000	0	0	0	basic	-0.1998	M
3	C	0	0	0	-31.9680	at bound	-M	31.9680
4	D	0	0	0	-0.1998	at bound	-M	0.1998
5	E	0	0	0	-1.3986	at bound	-M	1.3986
6	F	0	0	0	-33.3666	at bound	-M	33.3666
7	G	0	0	0	-31.0356	at bound	-M	31.0356
8	H	0	0	0	-0.6660	at bound	-M	0.6660
9	I	0	0	0	-0.3330	at bound	-M	0.3330
10	sj	0	0.5000	0	-4.1620	at bound	-M	4.6620
11	sk	1.2000	0.6660	0.7992	0	basic	0.0714	M
	Objective	Function	(Max.) =	0.7992				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	4.6620	2.0000	3.0000
2	U	1.5000	=	1.5000	0	-0.6660	-M	2.7000
3	u0	1.0000	=	1.0000	0	-7.9920	0.7000	1.0500

Appendix A.3 Optimal Solution for 0.5-AM (model 3.1)

DMU A:

	15:04:14		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.5000	0	0	0	basic	-0.2500	3.0000
2	B	0.5000	0	0	0	basic	-0.1500	M
3	C	0	0	0	-24.0000	at bound	-M	24.0000
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	-1.0500	at bound	-M	1.0500
6	F	0	0	0	-25.0500	at bound	-M	25.0500
7	G	0	0	0	-23.3000	at bound	-M	23.3000
8	H	0	0	0	-0.5000	at bound	-M	0.5000
9	I	0	0	0	-0.2500	at bound	-M	0.2500
10	sj	0	0.5000	0	-3.0000	at bound	-M	3.5000
11	sk	3.5000	0.5000	1.7500	0	basic	0.0714	M
	Objective	Function	(Max.) =	1.7500				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	3.5000	2.0000	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	5.5000
3	u0	1.0000	=	1.0000	0	-6.0000	0.8333	1.2500

DMU B:

	15:05:39		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4437	at bound	-M	0.4437
2	B	1.0000	0	0	0	basic	-0.0333	M
3	C	0	0	0	-2.2221	at bound	-M	2.2221
4	D	0	0	0	-0.0333	at bound	-M	0.0333
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.3221	at bound	-M	2.3221
7	G	0	0	0	-2.1554	at bound	-M	2.1554
8	H	0	0	0	-0.5547	at bound	-M	0.5547
9	I	0	0	0	-0.4992	at bound	-M	0.4992
10	sj	0.5000	0.3333	0.1666	0	basic	0.0163	0.7770
11	sk	0	0.1110	0	0	basic	0.0476	2.2664
	Objective	Function	(Max.) =	0.1666				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.5000	=	3.5000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1110	-M	9.0000
3	u0	1.0000	=	1.0000	0	-0.0009	1.0000	1.1667

DMU C:

	15:06:32		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7800	at bound	-M	M
2	B	0	0	0	0.6800	at bound	-M	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	0.6800	at bound	-M	M
5	E	0	0	0	0.6500	at bound	-M	M
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	M
8	H	0	0	0	0.7800	at bound	-M	M
9	I	0	0	0	0.7800	at bound	-M	M
10	sj	0.7000	0.1000	0.0700	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
	Objective	Function	(Max.) =	0.0700				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.5000	=	10.5000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9800	0	1.0000

DMU D:

	15:07:23		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4710	at bound	-M	0.4710
2	B	1.0000	0	0	0	basic	-0.0345	M
3	C	0	0	0	-2.2182	at bound	-M	2.2182
4	D	0	0	0	-0.0345	at bound	-M	0.0345
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.3182	at bound	-M	2.3182
7	G	0	0	0	-2.1515	at bound	-M	2.1515
8	H	0	0	0	-0.5859	at bound	-M	0.5859
9	I	0	0	0	-0.5285	at bound	-M	0.5285
10	sj	0.5000	0.3333	0.1666	0	basic	0.0169	0.8043
11	sk	0.3000	0.1149	0.0345	0	basic	0.0476	2.2664
	Objective	Function	(Max.) =	0.2011				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.5000	=	3.5000	0	0.3333	3.0000	M
2	U	8.7000	=	8.7000	0	-0.1149	-M	9.0000
3	u0	1.0000	=	1.0000	0	0.0342	0.9667	1.1667

DMU E:

	15:08:52		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-0.4747	at bound	-M	0.4747
2	B	1.0000	0	0	0	basic	-0.0333	M
3	C	0	0	0	-2.0099	at bound	-M	2.0099
4	D	0	0	0	-0.0333	at bound	-M	0.0333
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.1008	at bound	-M	2.1008
7	G	0	0	0	-1.9493	at bound	-M	1.9493
8	H	0	0	0	-0.5858	at bound	-M	0.5858
9	I	0	0	0	-0.5303	at bound	-M	0.5303
10	sj	0.8000	0.3030	0.2424	0	basic	0.0163	0.7777
11	sk	0	0.1111	0	0	basic	0.0433	2.0604
	Objective	Function	(Max.) =	0.2424				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.8000	=	3.8000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1111	-M	9.0000
3	u0	1.0000	=	1.0000	0	0.0909	1.0000	1.2667

DMU F:

	15:10:12		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7574	at bound	-M	M
2	B	0	0	0	0.6603	at bound	-M	M
3	C	0	0	0	-0.0194	at bound	-M	0.0194
4	D	0	0	0	0.6603	at bound	-M	M
5	E	0	0	0	0.6312	at bound	-M	M
6	F	0	0	0	-0.0485	at bound	-M	0.0485
7	G	1.0000	0	0	0	basic	-0.0194	M
8	H	0	0	0	0.7574	at bound	-M	M
9	I	0	0	0	0.7574	at bound	-M	M
10	sj	0.7000	0.0971	0.0680	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
	Objective	Function	(Max.) =	0.0680				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.5000	=	10.5000	0	0.0971	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9516	0	1.0000

DMU G:

	15:12:29		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7956	at bound	-M	M
2	B	0	0	0	0.6936	at bound	-M	M
3	C	0	0	0	-0.0204	at bound	-M	0.0204
4	D	0	0	0	0.6936	at bound	-M	M
5	E	0	0	0	0.6630	at bound	-M	M
6	F	0	0	0	-0.0510	at bound	-M	0.0510
7	G	1.0000	0	0	0	basic	-0.0204	M
8	H	0	0	0	0.7956	at bound	-M	M
9	I	0	0	0	0.7956	at bound	-M	M
10	sj	0.5000	0.1020	0.0510	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
	Objective	Function	(Max.) =	0.0510				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.3000	=	10.3000	0	0.1020	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9996	0	1.0000

DMU H:

	15:13:41		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.5000	0	0	0	basic	-0.5000	6.5000
2	B	0.5000	0	0	0	basic	-0.3000	M
3	C	0	0	0	-48.0000	at bound	-M	48.0000
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-2.1000	at bound	-M	2.1000
6	F	0	0	0	-50.1000	at bound	-M	50.1000
7	G	0	0	0	-46.6000	at bound	-M	46.6000
8	H	0	0	0	-1.0000	at bound	-M	1.0000
9	I	0	0	0	-0.5000	at bound	-M	0.5000
10	sj	0	0.5000	0	-6.5000	at bound	-M	7.0000
11	sk	4.5000	1.0000	4.5000	0	basic	0.0714	M
	Objective	Function	(Max.) =	4.5000				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	7.0000	2.0000	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	5.5000
3	u0	1.0000	=	1.0000	0	-12.0000	0.8333	1.2500

DMU I:

	15:14:30		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0.5000	0	0	0	basic	-0.3333	4.1662
2	B	0.5000	0	0	0	basic	-0.2000	M
3	C	0	0	0	-31.9968	at bound	-M	31.9968
4	D	0	0	0	-0.2000	at bound	-M	0.2000
5	E	0	0	0	-1.3999	at bound	-M	1.3999
6	F	0	0	0	-33.3967	at bound	-M	33.3967
7	G	0	0	0	-31.0636	at bound	-M	31.0636
8	H	0	0	0	-0.6666	at bound	-M	0.6666
9	I	0	0	0	-0.3333	at bound	-M	0.3333
10	sj	0	0.5000	0	-4.1662	at bound	-M	4.6662
11	sk	4.0000	0.6666	2.6664	0	basic	0.0714	M
	Objective	Function	(Max.) =	2.6664				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	4.6662	2.0000	3.0000
2	U	1.5000	=	1.5000	0	-0.6666	-M	5.5000
3	u0	1.0000	=	1.0000	0	-7.9992	0.8333	1.2500

Appendix B: Optimal Coding solutions of Modified Arash method summarized in Table 5.5.2 and Table 5.5.3

Appendix B.1: Optimal solution for 0-AM (model 5.3)

	16:59:21		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.2833	at bound	-M	2.2833
2	B	0.6667	0	0	0	basic	-0.1500	M
3	C	0	0	0	-8.0167	at bound	-M	8.0167
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	0	at bound	-M	0
6	F	0	0	0	-8.3817	at bound	-M	8.3817
7	G	0	0	0	-7.7733	at bound	-M	7.7733
8	H	0	0	0	-2.7833	at bound	-M	2.7833
9	I	0	0	0	-2.5333	at bound	-M	2.5333
10	sj	0	0.5000	0	-0.7167	at bound	-M	1.2167
11	sk	4.0000	0.5000	2.0000	0	basic	0.2611	M
12	eta	0.3333	0.8500	0.2833	0	basic	-6.0000	3.0000
	Objective	Function	(Max.) =	2.2833	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	1.2167	0.6667	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	6.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.6667	M

DMU B:

	17:01:04		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-15.5318	at bound	-M	15.5318
2	B	1.0000	0	0	0	basic	-0.5230	M
3	C	0	0	0	-0.0667	at bound	-M	0.0667
4	D	0	0	0	-0.6799	at bound	-M	0.6799
5	E	0	0	0	0	at bound	-M	0
6	F	0	0	0	-0.1666	at bound	-M	0.1666
7	G	0	0	0	0	basic	-0.0667	2.0609
8	H	0	0	0	-17.7982	at bound	-M	17.7982
9	I	0	0	0	-16.6650	at bound	-M	16.6650
10	sj	0	0.3333	0	0	basic	0.0163	M
11	sk	0	0.1111	0	-2.1553	at bound	-M	2.2664
12	eta	0	0.8500	0	-18.5481	at bound	-M	19.3981
	Objective	Function	(Max.) =	0	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.0000	=	3.0000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-2.2664	9.0000	9.0000
3	u0	1.0000	=	1.0000	0	19.3981	1.0000	1.0000

DMU C:

	17:02:45		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.6600	at bound	-M	4.6600
2	B	0	0	0	0	basic	-0.1569	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	-0.2040	at bound	-M	0.2040
5	E	0	0	0	0	at bound	-M	0
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	0.5522
8	H	0	0	0	-5.3400	at bound	-M	5.3400
9	I	0	0	0	-5.0000	at bound	-M	5.0000
10	sj	0.2000	0.1000	0.0200	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5800	at bound	-M	0.6800
12	eta	0	0.8500	0	-4.9700	at bound	-M	5.8200
	Objective	Function	(Max.) =	0.0200	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.0000	=	10.0000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6800	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.8200	1.0000	1.1111

DMU D:

	17:04:10		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.1055	at bound	-M	1.1055
2	B	0.9667	0	0	0	basic	-0.0638	0.8158
3	C	0	0	0	-2.1276	at bound	-M	2.1276
4	D	0	0	0	-0.0617	at bound	-M	0.0617
5	E	0	0	0	0	at bound	-M	0
6	F	0	0	0	-2.2275	at bound	-M	2.2275
7	G	0	0	0	-2.0609	at bound	-M	2.0609
8	H	0	0	0	-1.3111	at bound	-M	1.3111
9	I	0	0	0	-1.2083	at bound	-M	1.2083
10	sj	0.1000	0.3333	0.0333	0	basic	0.0614	M
11	sk	0	0.1149	0	-0.0906	at bound	-M	0.2055
12	eta	0.0333	0.8500	0.0283	0	basic	0.0342	19.3981
	Objective	Function	(Max.) =	0.0617	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.0000	=	3.0000	0	0.3333	2.9000	M
2	U	8.7000	=	8.7000	0	-0.2055	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.9667	M

DMU E:

	17:19:51		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.0651	at bound	-M	1.0651
2	B	1.0000	0	0	0	basic	-0.0607	0.7591
3	C	0	0	0	-1.9256	at bound	-M	1.9256
4	D	0	0	0	-0.0586	at bound	-M	0.0586
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.0165	at bound	-M	2.0165
7	G	0	0	0	-1.8650	at bound	-M	1.8650
8	H	0	0	0	-1.2606	at bound	-M	1.2606
9	I	0	0	0	-1.1628	at bound	-M	1.1628
10	sj	0.3000	0.3030	0.0909	0	basic	0.0500	M
11	sk	0	0.1111	0	-0.0843	at bound	-M	0.1954
12	eta	0	0.8500	0	0	basic	0.0909	17.6346
	Objective	Function	(Max.) =	0.0909				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.3000	=	3.3000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1954	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	1.0000	M

DMU F:

	17:08:33		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7574	at bound	-M	M
2	B	0	0	0	0.6603	at bound	-M	M
3	C	0	0	0	-0.0194	at bound	-M	0.0194
4	D	0	0	0	0.6603	at bound	-M	M
5	E	0	0	0	0.6312	at bound	-M	M
6	F	0	0	0	-0.0485	at bound	-M	0.0485
7	G	1.0000	0	0	0	basic	-0.0194	M
8	H	0	0	0	0.7574	at bound	-M	M
9	I	0	0	0	0.7574	at bound	-M	M
10	sj	0.5000	0.0971	0.0485	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
12	eta	0	0.8500	0	1.8016	at bound	-M	M
	Objective	Function	(Max.) =	0.0485				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.3000	=	10.3000	0	0.0971	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9516	0	1.0000

DMU G:

	17:10:28		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.7532	at bound	-M	4.7532
2	B	0	0	0	0	basic	-0.0306	M
3	C	0	0	0	-0.0204	at bound	-M	0.0204
4	D	0	0	0	-0.2081	at bound	-M	0.2081
5	E	0	0	0	-0.0306	at bound	-M	0.0306
6	F	0	0	0	-0.0510	at bound	-M	0.0510
7	G	1.0000	0	0	0	basic	-0.0204	0.5652
8	H	0	0	0	-5.4468	at bound	-M	5.4468
9	I	0	0	0	-5.1000	at bound	-M	5.1000
10	sj	0	0.1020	0	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5936	at bound	-M	0.6936
12	eta	0	0.8500	0	-5.0864	at bound	-M	5.9364
	Objective	Function	(Max.) =	0				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	9.8000	=	9.8000	0	0.1020	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6936	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.9364	1.0000	1.1111

DMU H:

	17:12:50		Thursday	June	25	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.2833	at bound	-M	4.2833
2	B	0.6667	0	0	0	basic	-0.3000	M
3	C	0	0	0	-18.0167	at bound	-M	18.0167
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-0.8150	at bound	-M	0.8150
6	F	0	0	0	-18.8317	at bound	-M	18.8317
7	G	0	0	0	-17.4733	at bound	-M	17.4733
8	H	0	0	0	-5.2833	at bound	-M	5.2833
9	I	0	0	0	-4.7833	at bound	-M	4.7833
10	sj	0	0.5000	0	-2.2167	at bound	-M	2.7167
11	sk	5.0000	1.0000	5.0000	0	basic	0.2611	M
12	eta	0.3333	0.8500	0.2833	0	basic	-12.0000	7.5000
	Objective	Function	(Max.) =	5.2833				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	2.7167	0.3333	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	6.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.6667	M

DMU I:

17:14:38		Thursday	June	25	2015		
Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.9497	at bound	-M	2.9497
2	B	0.6667	0	0	0	basic	-0.2000	M
3	C	0	0	0	-11.3487	at bound	-M	11.3487
4	D	0	0	0	-0.2000	at bound	-M	0.2000
5	E	0	0	0	-0.5149	at bound	-M	0.5149
6	F	0	0	0	-11.8636	at bound	-M	11.8636
7	G	0	0	0	-11.0054	at bound	-M	11.0054
8	H	0	0	0	-3.6163	at bound	-M	3.6163
9	I	0	0	0	-3.2830	at bound	-M	3.2830
10	sj	0	0.5000	0	-1.2165	at bound	-M	1.7165
11	sk	4.5000	0.6666	2.9997	0	basic	0.2611	M
12	eta	0.3333	0.8500	0.2833	0	basic	-7.9992	4.4994
	Objective	Function	(Max.) =	3.2830				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.0000	=	2.0000	0	1.7165	0.5000	3.0000
2	U	1.5000	=	1.5000	0	-0.6666	-M	6.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.6667	M

Appendix B.2: Optimal solution for 0-MAM (model 5.3)

DMU A:

	11:28:23		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.2833	at bound	-M	2.2833
2	B	0.7000	0	0	0	basic	-0.1500	M
3	C	0	0	0	-8.0167	at bound	-M	8.0167
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	-0.3650	at bound	-M	0.3650
6	F	0	0	0	-8.3817	at bound	-M	8.3817
7	G	0	0	0	-7.7733	at bound	-M	7.7733
8	H	0	0	0	-2.7833	at bound	-M	2.7833
9	I	0	0	0	-2.5333	at bound	-M	2.5333
10	sj	0	0.5000	0	-0.7167	at bound	-M	1.2167
11	sk	4.3000	0.5000	2.1500	0	basic	0.2611	M
12	eta	0.3000	0.8500	0.2550	0	basic	-6.0000	3.0000
	Objective	Function	(Max.) =	2.4050				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	1.2167	0.6667	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	6.3000
3	u0	1.0000	=	1.0000	0	0.8500	0.7000	M

DMU B:

	11:29:40		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.1055	at bound	-M	1.1055
2	B	1.0000	0	0	0	basic	-0.0638	0.8509
3	C	0	0	0	-2.1276	at bound	-M	2.1276
4	D	0	0	0	-0.0617	at bound	-M	0.0617
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.2275	at bound	-M	2.2275
7	G	0	0	0	-2.0609	at bound	-M	2.0609
8	H	0	0	0	-1.3111	at bound	-M	1.3111
9	I	0	0	0	-1.2083	at bound	-M	1.2083
10	sj	0.1000	0.3333	0.0333	0	basic	0.0497	M
11	sk	0	0.1110	0	-0.0945	at bound	-M	0.2055
12	eta	0	0.8500	0	0	basic	-0.0009	19.3981
	Objective	Function	(Max.) =	0.0333				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.1000	=	3.1000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-0.2055	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	1.0000	M

DMU C:

	11:34:12		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.6600	at bound	-M	4.6600
2	B	0	0	0	0	basic	-0.0300	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	-0.2040	at bound	-M	0.2040
5	E	0	0	0	-0.0300	at bound	-M	0.0300
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	0.5522
8	H	0	0	0	-5.3400	at bound	-M	5.3400
9	I	0	0	0	-5.0000	at bound	-M	5.0000
10	sj	0.3000	0.1000	0.0300	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5800	at bound	-M	0.6800
12	eta	0	0.8500	0	-4.9700	at bound	-M	5.8200
	Objective	Function	(Max.) =	0.0300				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.1000	=	10.1000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6800	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.8200	1.0000	1.1111

DMU D:

	11:39:18		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.1055	at bound	-M	1.1055
2	B	0.9667	0	0	0	basic	-0.0638	0.8158
3	C	0	0	0	-2.1276	at bound	-M	2.1276
4	D	0	0	0	-0.0617	at bound	-M	0.0617
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.2275	at bound	-M	2.2275
7	G	0	0	0	-2.0609	at bound	-M	2.0609
8	H	0	0	0	-1.3111	at bound	-M	1.3111
9	I	0	0	0	-1.2083	at bound	-M	1.2083
10	sj	0.2000	0.3333	0.0667	0	basic	0.0614	M
11	sk	0	0.1149	0	-0.0906	at bound	-M	0.2055
12	eta	0.0333	0.8500	0.0283	0	basic	0.0342	19.3981
	Objective	Function	(Max.) =	0.0950				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.1000	=	3.1000	0	0.3333	2.9000	M
2	U	8.7000	=	8.7000	0	-0.2055	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.9667	M

DMU E:

	11:30:52		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.0651	at bound	-M	1.0651
2	B	1.0000	0	0	0	basic	-0.0607	0.7600
3	C	0	0	0	-1.9256	at bound	-M	1.9256
4	D	0	0	0	-0.0586	at bound	-M	0.0586
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.0165	at bound	-M	2.0165
7	G	0	0	0	-1.8650	at bound	-M	1.8650
8	H	0	0	0	-1.2606	at bound	-M	1.2606
9	I	0	0	0	-1.1628	at bound	-M	1.1628
10	sj	0.4000	0.3030	0.1212	0	basic	0.0497	M
11	sk	0	0.1110	0	-0.0844	at bound	-M	0.1954
12	eta	0	0.8500	0	0	basic	0.0900	17.6346
	Objective	Function	(Max.) =	0.1212				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.4000	=	3.4000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1954	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	1.0000	M

DMU F:

	11:31:50		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7800	at bound	-M	M
2	B	0	0	0	0.6800	at bound	-M	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	0.6800	at bound	-M	M
5	E	0	0	0	0.6500	at bound	-M	M
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	M
8	H	0	0	0	0.7800	at bound	-M	M
9	I	0	0	0	0.7800	at bound	-M	M
10	sj	0.6000	0.1000	0.0600	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
12	eta	0	0.8500	0	1.8300	at bound	-M	M
	Objective	Function	(Max.) =	0.0600				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.4000	=	10.4000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9800	0	1.0000

DMU G:

	11:43:08		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.7532	at bound	-M	4.7532
2	B	0	0	0	0	basic	-0.0306	M
3	C	0	0	0	-0.0204	at bound	-M	0.0204
4	D	0	0	0	-0.2081	at bound	-M	0.2081
5	E	0	0	0	-0.0306	at bound	-M	0.0306
6	F	0	0	0	-0.0510	at bound	-M	0.0510
7	G	1.0000	0	0	0	basic	-0.0204	0.5652
8	H	0	0	0	-5.4468	at bound	-M	5.4468
9	I	0	0	0	-5.1000	at bound	-M	5.1000
10	sj	0.1000	0.1020	0.0102	0	basic	0.0147	M
11	sk	0	0.1000	0	-0.5936	at bound	-M	0.6936
12	eta	0	0.8500	0	-5.0864	at bound	-M	5.9364
	Objective	Function	(Max.) =	0.0102				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	9.9000	=	9.9000	0	0.1020	9.8000	M
2	U	10.0000	=	10.0000	0	-0.6936	9.0000	10.0000
3	u0	1.0000	=	1.0000	0	5.9364	1.0000	1.1111

DMU H:

	11:44:06		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.2833	at bound	-M	4.2833
2	B	0.7000	0	0	0	basic	-0.3000	M
3	C	0	0	0	-18.0167	at bound	-M	18.0167
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-0.8150	at bound	-M	0.8150
6	F	0	0	0	-18.8317	at bound	-M	18.8317
7	G	0	0	0	-17.4733	at bound	-M	17.4733
8	H	0	0	0	-5.2833	at bound	-M	5.2833
9	I	0	0	0	-4.7833	at bound	-M	4.7833
10	sj	0	0.5000	0	-2.2167	at bound	-M	2.7167
11	sk	5.3000	1.0000	5.3000	0	basic	0.2611	M
12	eta	0.3000	0.8500	0.2550	0	basic	-12.0000	7.5000
	Objective	Function	(Max.) =	5.5550				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	2.7167	0.3333	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	6.3000
3	u0	1.0000	=	1.0000	0	0.8500	0.7000	M

DMU I:

	11:45:15		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.9497	at bound	-M	2.9497
2	B	0.7000	0	0	0	basic	-0.2000	M
3	C	0	0	0	-11.3487	at bound	-M	11.3487
4	D	0	0	0	-0.2000	at bound	-M	0.2000
5	E	0	0	0	-0.5149	at bound	-M	0.5149
6	F	0	0	0	-11.8636	at bound	-M	11.8636
7	G	0	0	0	-11.0054	at bound	-M	11.0054
8	H	0	0	0	-3.6163	at bound	-M	3.6163
9	I	0	0	0	-3.2830	at bound	-M	3.2830
10	sj	0	0.5000	0	-1.2165	at bound	-M	1.7165
11	sk	4.8000	0.6666	3.1997	0	basic	0.2611	M
12	eta	0.3000	0.8500	0.2550	0	basic	-7.9992	4.4994
	Objective	Function	(Max.) =	3.4547				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.1000	=	2.1000	0	1.7165	0.5000	3.0000
2	U	1.5000	=	1.5000	0	-0.6666	-M	6.3000
3	u0	1.0000	=	1.0000	0	0.8500	0.7000	M

Appendix B.3 Optimal solution for 0.5-M-AM (model 5.3)

DMU A:

	12:01:43		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.2833	at bound	-M	2.2833
2	B	0.8333	0	0	0	basic	-0.1500	M
3	C	0	0	0	-8.0167	at bound	-M	8.0167
4	D	0	0	0	-0.1500	at bound	-M	0.1500
5	E	0	0	0	-0.3650	at bound	-M	0.3650
6	F	0	0	0	-8.3817	at bound	-M	8.3817
7	G	0	0	0	-7.7733	at bound	-M	7.7733
8	H	0	0	0	-2.7833	at bound	-M	2.7833
9	I	0	0	0	-2.5333	at bound	-M	2.5333
10	sj	0	0.5000	0	-0.7167	at bound	-M	1.2167
11	sk	5.5000	0.5000	2.7500	0	basic	0.2611	M
12	eta	0.1667	0.8500	0.1417	0	basic	-6.0000	3.0000
	Objective	Function	(Max.) =	2.8917				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	1.2167	0.6667	3.0000
2	U	2.0000	=	2.0000	0	-0.5000	-M	7.5000
3	u0	1.0000	=	1.0000	0	0.8500	0.8333	M

DMU B:

	12:02:48		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.1055	at bound	-M	1.1055
2	B	1.0000	0	0	0	basic	-0.0638	0.8500
3	C	0	0	0	-2.1276	at bound	-M	2.1276
4	D	0	0	0	-0.0617	at bound	-M	0.0617
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.2275	at bound	-M	2.2275
7	G	0	0	0	-2.0609	at bound	-M	2.0609
8	H	0	0	0	-1.3111	at bound	-M	1.3111
9	I	0	0	0	-1.2083	at bound	-M	1.2083
10	sj	0.5000	0.3333	0.1666	0	basic	0.0500	M
11	sk	0	0.1111	0	-0.0944	at bound	-M	0.2055
12	eta	0	0.8500	0	0	basic	0.0000	19.3981
	Objective	Function	(Max.) =	0.1666				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.5000	=	3.5000	0	0.3333	3.0000	M
2	U	9.0000	=	9.0000	0	-0.2055	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	1.0000	M

DMU C:

	12:03:58		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7800	at bound	-M	M
2	B	0	0	0	0.6800	at bound	-M	M
3	C	0	0	0	-0.0200	at bound	-M	0.0200
4	D	0	0	0	0.6800	at bound	-M	M
5	E	0	0	0	0.6500	at bound	-M	M
6	F	0	0	0	-0.0500	at bound	-M	0.0500
7	G	1.0000	0	0	0	basic	-0.0200	M
8	H	0	0	0	0.7800	at bound	-M	M
9	I	0	0	0	0.7800	at bound	-M	M
10	sj	0.7000	0.1000	0.0700	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
12	eta	0	0.8500	0	1.8300	at bound	-M	M
	Objective	Function	(Max.) =	0.0700				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.5000	=	10.5000	0	0.1000	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9800	0	1.0000

DMU D:

	12:05:16		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.1055	at bound	-M	1.1055
2	B	0.9667	0	0	0	basic	-0.0638	0.8158
3	C	0	0	0	-2.1276	at bound	-M	2.1276
4	D	0	0	0	-0.0617	at bound	-M	0.0617
5	E	0	0	0	-0.1000	at bound	-M	0.1000
6	F	0	0	0	-2.2275	at bound	-M	2.2275
7	G	0	0	0	-2.0609	at bound	-M	2.0609
8	H	0	0	0	-1.3111	at bound	-M	1.3111
9	I	0	0	0	-1.2083	at bound	-M	1.2083
10	sj	0.6000	0.3333	0.2000	0	basic	0.0614	M
11	sk	0	0.1149	0	-0.0906	at bound	-M	0.2055
12	eta	0.0333	0.8500	0.0283	0	basic	0.0342	19.3981
	Objective	Function	(Max.) =	0.2283				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.5000	=	3.5000	0	0.3333	2.9000	M
2	U	8.7000	=	8.7000	0	-0.2055	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	0.9667	M

DMU E:

	12:06:24		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-1.0651	at bound	-M	1.0651
2	B	1.0000	0	0	0	basic	-0.0607	0.7591
3	C	0	0	0	-1.9256	at bound	-M	1.9256
4	D	0	0	0	-0.0586	at bound	-M	0.0586
5	E	0	0	0	-0.0909	at bound	-M	0.0909
6	F	0	0	0	-2.0165	at bound	-M	2.0165
7	G	0	0	0	-1.8650	at bound	-M	1.8650
8	H	0	0	0	-1.2606	at bound	-M	1.2606
9	I	0	0	0	-1.1628	at bound	-M	1.1628
10	sj	0.8000	0.3030	0.2424	0	basic	0.0500	M
11	sk	0	0.1111	0	-0.0843	at bound	-M	0.1954
12	eta	0	0.8500	0	0	basic	0.0909	17.6346
	Objective	Function	(Max.) =	0.2424				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	3.8000	=	3.8000	0	0.3030	3.0000	M
2	U	9.0000	=	9.0000	0	-0.1954	0	9.0000
3	u0	1.0000	=	1.0000	0	0.8500	1.0000	M

DMU F:

	12:07:54		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7574	at bound	-M	M
2	B	0	0	0	0.6603	at bound	-M	M
3	C	0	0	0	-0.0194	at bound	-M	0.0194
4	D	0	0	0	0.6603	at bound	-M	M
5	E	0	0	0	0.6312	at bound	-M	M
6	F	0	0	0	-0.0485	at bound	-M	0.0485
7	G	1.0000	0	0	0	basic	-0.0194	M
8	H	0	0	0	0.7574	at bound	-M	M
9	I	0	0	0	0.7574	at bound	-M	M
10	sj	1.0000	0.0971	0.0971	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
12	eta	0	0.8500	0	1.8016	at bound	-M	M
	Objective	Function	(Max.) =	0.0971				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.8000	=	10.8000	0	0.0971	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9516	0	1.0000

DMU G:

	12:09:04		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	0.7956	at bound	-M	M
2	B	0	0	0	0.6936	at bound	-M	M
3	C	0	0	0	-0.0204	at bound	-M	0.0204
4	D	0	0	0	0.6936	at bound	-M	M
5	E	0	0	0	0.6630	at bound	-M	M
6	F	0	0	0	-0.0510	at bound	-M	0.0510
7	G	1.0000	0	0	0	basic	-0.0204	M
8	H	0	0	0	0.7956	at bound	-M	M
9	I	0	0	0	0.7956	at bound	-M	M
10	sj	0.5000	0.1020	0.0510	0	basic	0	M
11	sk	0	0.1000	0	0.1000	at bound	-M	M
12	eta	0	0.8500	0	1.8496	at bound	-M	M
	Objective	Function	(Max.) =	0.0510				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	10.3000	=	10.3000	0	0.1020	9.8000	M
2	U	10.0000	=	10.0000	0	0	10.0000	M
3	u0	1.0000	=	1.0000	0	-0.9996	0	1.0000

DMU H:

	12:10:19		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-4.2833	at bound	-M	4.2833
2	B	0.8333	0	0	0	basic	-0.3000	M
3	C	0	0	0	-18.0167	at bound	-M	18.0167
4	D	0	0	0	-0.3000	at bound	-M	0.3000
5	E	0	0	0	-0.8150	at bound	-M	0.8150
6	F	0	0	0	-18.8317	at bound	-M	18.8317
7	G	0	0	0	-17.4733	at bound	-M	17.4733
8	H	0	0	0	-5.2833	at bound	-M	5.2833
9	I	0	0	0	-4.7833	at bound	-M	4.7833
10	sj	0	0.5000	0	-2.2167	at bound	-M	2.7167
11	sk	6.5000	1.0000	6.5000	0	basic	0.2611	M
12	eta	0.1667	0.8500	0.1417	0	basic	-12.0000	7.5000
	Objective	Function	(Max.) =	6.6417				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	2.7167	0.3333	3.0000
2	U	1.0000	=	1.0000	0	-1.0000	-M	7.5000
3	u0	1.0000	=	1.0000	0	0.8500	0.8333	M

DMU I:

	12:11:18		Friday	June	26	2015		
	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	A	0	0	0	-2.9497	at bound	-M	2.9497
2	B	0.8333	0	0	0	basic	-0.2000	M
3	C	0	0	0	-11.3487	at bound	-M	11.3487
4	D	0	0	0	-0.2000	at bound	-M	0.2000
5	E	0	0	0	-0.5149	at bound	-M	0.5149
6	F	0	0	0	-11.8636	at bound	-M	11.8636
7	G	0	0	0	-11.0054	at bound	-M	11.0054
8	H	0	0	0	-3.6163	at bound	-M	3.6163
9	I	0	0	0	-3.2830	at bound	-M	3.2830
10	sj	0	0.5000	0	-1.2165	at bound	-M	1.7165
11	sk	6.0000	0.6666	3.9996	0	basic	0.2611	M
12	eta	0.1667	0.8500	0.1417	0	basic	-7.9992	4.4994
	Objective	Function	(Max.) =	4.1413				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	V	2.5000	=	2.5000	0	1.7165	0.5000	3.0000
2	U	1.5000	=	1.5000	0	-0.6666	-M	7.5000
3	u0	1.0000	=	1.0000	0	0.8500	0.8333	M