Modified Data Envelopment Analysis Model Based on Service Quality Concept for Vendor Selection

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ABSTRACT

Purchasing function is the key part of the logistics management in firms, and the prime responsibility for this function is the selection of appropriate vendors i.e. the most efficiently performing vendors. Many analytical and conceptual models for tackling the vendor selection problem have been established. Several criteria are to be considered in evaluating vendors' relative efficiencies, hence this problem is being recognized as multiple criteria decision making problem. Researchers developed techniques for tackling this multi criteria efficiency evaluation problems in recent years by applying Data Envelopment Analysis (DEA) being the most effective method for evaluating vendor efficiencies, but all their researches did not address the issue of weakly efficient vendors in the DEA. Therefore, this thesis introduce a modified method for figuring vendors efficiency with the issue of weak efficient vendors being properly addressed so that only truly efficient vendors are selected in the appropriation situation. The modified method uses facet analysis in modifying the standard DEA model employed by several researchers in evaluating the vendors' efficiencies. The criteria chosen in these models are service quality, rate of rejected items, late deliveries and price. The results and comparisons between the modified and standard DEA model shows that the modified DEA model gives a better and true efficiency scores of vendors, this greatly improve the vendor evaluation and selection methods.

Keywords: Modified Data Envelopment Analysis, Vendor Evaluation and Selection, Service Quality

ÖZ

Şirketlerde lojistik yönetiminin temel kısmı satın alma fonksiyonudur ve bu fonksiyonun başlıca sorumluluğu, uygun satıcıları, yani en verimli şekilde çalışan satıcıları seçmektir. Satıcı seçimi problemini çözmek için birçok analitik ve kavramsal model geliştirilmiştir. Satıcıların göreli verimliliklerinin değerlendirilmesinde çeşitli kriterler göz önüne alınmalıdır, bu nedenle satıcı seçim problemi çok kriterli karar verme problemi olarak kabul edilmektedir. Son yıllarda, araştırmacılar, bu çok ölçütlü verimlilik değerlendirme sorunlarını, Veri Zarflama Analizi'ni (VZA) en etkin değerlendirme yöntemi olarak uygulayarak, çözmek için teknikler geliştirmişlerdir ancak tüm bu araştırmalar, VZA'daki zayıf verimli satıcıların sorununu ele almamaktadır. Bu nedenle, bu tez, zayıf verimli satıcıların uygun bir şekilde ele alınması sorunu ile satıcıları değerlendirmek için modifiye edilmiş bir yaklaşım sunmakta ve böylece yalnızca bir tedarik durumunda gerçekten verimli satıcılar seçilmektedir. Değiştirilen yaklaşım, satıcıların verimliliklerini değerlendirirken birçok araştırmacı tarafından kullanılan standart VZA modelinin modifikasyonunda faset analizini kullanmaktadır. Bu modellerde göz önüne alınan kriterler, hizmet kalitesi, fiyat, geç teslimat ve reddedilen parçalardır. Modifiye ve standart VZA modeli arasındaki sonuçlar ve karşılaştırmalar, modifiye VZA modelinin satıcıların daha iyi ve gerçek etkinlik skorları verdiğini, bunun da satıcı değerlendirme ve seçim yöntemlerini büyük ölçüde iyileştirdiğini göstermektedir.

Anahtar Kelimeler: Veri Zarflama Analizi, Satıcı Seçimi, Servis Kalitesi

To my Late Mother (Hajiya Aisha Ibrahim)

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LIST OF ABBREVIATIONS

CCR	Charnes Cooper and Rhodes
BCC	Banker Charnes and Cooper
DEA	Data Envelopment Analysis
DMU	Decision Making Unit
CRS	Constant Return to Scale
VRS	Variable Returns to Scale
PPS	Production Possibility Set
LP	Linear Programming

Chapter 1

INTRODUCTION

1.1 Preamble

For the past decades, the literature of business management has earned extensive concern with regard to supply chain management and the vendor selection technique. The processes involved in reviewing, evaluating and ultimately selecting the best vendors is what we referred to as Vendor Selection. It is an important decision making issue, because to select effective vendors, this will significantly reduce the cost of purchasing which enhances competitiveness by improving the output quality, this directly has significant effects on firms concerned. Important factors to be considered are the various decisions managers make in the vendor selection which are naturally complicated for various reasons. Several approaches such as weigh scoring models and advance mathematical programming models were established and implemented to address this issue.

The vastly employed approach for addressing the analytical decision making problem in logistics management and purchasing is the Data envelopment analysis (DEA). DEA applies LP model to calculate the relative efficiencies as well as inefficiencies of DMUs possessing several inputs and outputs. The managers concerned with supplier selection decisions creates set of criteria that would be applied to evaluate and compare the effectiveness of various potential suppliers. The evaluation criteria are divided into inputs and outputs which are the driving tools for

DEA in evaluating the suppliers' efficiencies. Weber (1996) applies DEA by considering price, rate of rejected parts, allocations, etc. in evaluating vendors, where he identified the application of DEA according to this multiple criteria situation. Several other extended applications of DEA were employed for the same task which creates several interests in applying DEA particularly in recent years. This is because DEA has the ability of evaluating problems with multiple inputs and multiple outputs which cannot be used by other evaluating methods because of the complicated nature of such problems. Example of such problems may include the performances of bank branches in Cyprus, universities efficiencies in conducting educational research functions in Turkey, etc.

In this research, DEA is also employed in which service quality is considered as one output, percentage late deliveries, price, and percentage rate of rejected items are the utilized inputs of model. Vendors' efficiencies were evaluated by applying both traditional DEA model and then the modified DEA model to compare and figure out the most effective among the two methods. An empirical example is also presented with 12 vendors and results obtained from questionnaires are considered as data of the models to show the effectiveness of the modified DEA model over the standard DEA model in vendor selection. The results are useful in bringing out the exact and real efficiency value of each vendor, so organizations that apply the traditional DEA model in evaluating vendors may improve their results by not mistakenly selecting inefficient vendor as efficient.

1.2 Problem Description

DEA as a powerful benchmarking tool has few drawbacks. DEA indicates weak discriminatory power. Also some basic DEA model considers weight suppleness likely which often results in untrue weighting proposal in identifying a DMU as being efficient. These DMUs weigh greatly on certain favorable inputs and outputs and entirely disregarding others thus they accomplish an efficiency score of 1 in relative to other DMUs.

(Banker, Charnes and Cooper, 1984) (BCC), established a Linear Program for measuring efficiency. Many scholars developed models for vendor selection based on the BCC model which improves the above limitations of the DEA models but still their findings did not clearly address the issue of DMUs in the weak part of the frontier as well as the impact these DMUs may cause in decision making and improving results. Hence we propose a better methodology based on an extension of the BCC model that can evaluate vendors in a more efficient and reliable approach compared to the traditional methodology.

In many scholarly papers, Non-Archimedean number ε is used specifically for BCC model as the lower bound for variable weights in DEA models when trying to modify the standard BCC model. Applying the bound disturb the weak efficient frontier and in this manner weak efficient DMUs were detected, and weak efficient DMUs take an efficiency value less than 1 but these values are not real efficiency values. Therefore using ε as the lower bound of factor weights is not suitable.

BCC frontier modification is achieved by assimilating and putting into work many standards on ground. The BCC based model is modified using facet analysis. This is achieved by introducing bounds to free variables which also modifies the supporting hyperplanes of the convex production possibility set that passes through the efficient Decision Making Units. Using these bounds, the DMUs that appear on weak part of the efficiency frontier would be changed and this reduce the efficiency score of DMUs which, compare their efficiency scores with this weak frontier.

By implementing the above modifications in our research, we are expected to achieve better results which greatly enhance the discriminatory power of standard DEA model for vendor selection and also gives the exact efficiency value for DMUs in the weak frontier and the DMUs which compares with them and finally compare between the standard BCC model and the modified BCC model for selection of the most efficient vendor.

1.3 Thesis Structure

This work was separated into two major sections. The preliminary pages and the body which consists of six chapters. In chapter 1, we introduce the vendor selection and its advantages as well as the DEA. In chapter 2 review of the supplier selection, its methods and criteria, quality concepts and DEA literature was introduced. In chapter 3, basic model for vendor selection was discussed followed by modified BCC model in chapter 4. Chapter 5 combines the procedures of vendor selection model and that of the modified BCC model in modifying the model for vendor selection. Finally, chapter 6 presents our conclusion and suggestions for future studies. These processes are shown in the figure below:

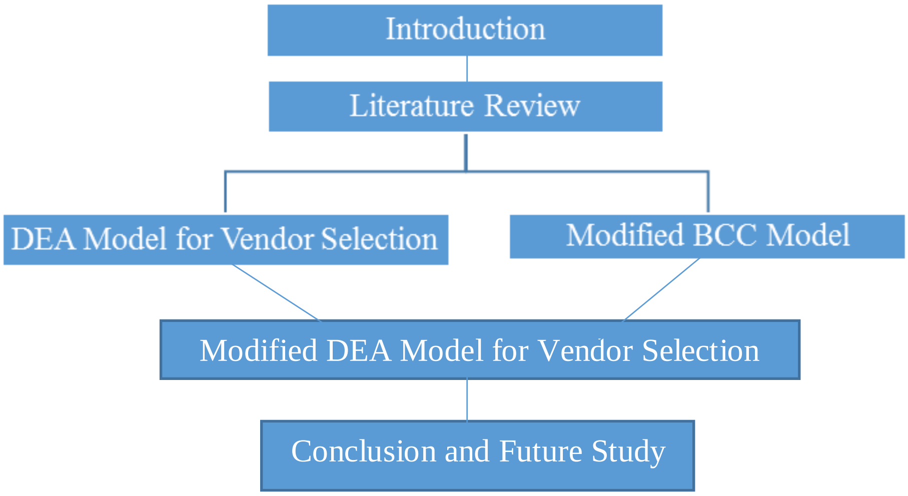

Figure 1.1: Thesis Structure

Chapter 2

LITERATURE REVIEW

2.1 Vendor Selection

A term vendor selection, refers to a state of pre-contractual relationship, when suppliers are being evaluated by management of a firm before doing business with them.

Selection of highly performing vendors in order to optimize cost, and improve products and services is one of the critical decisions and continuous task of great importance for all firms so as to match market requirements, especially with the recent advances where life cycle of products is short which ranges from 3 to 4 years, knowing that new plans necessitate new technologies and materials (Benyoucef, L., Ding, H., & Xie, X., 2003). This has a significant practical impact.

Such perception of selecting an appropriate vendor dates back to as far as 1940‘s in purchasing literature. Most scholars on purchasing idea agree that, in general, firm‘s purchases account for 50 % or even more of the total product costs.

Thus, selecting the best supplier depends on choosing suitable criteria and method.

2.2 Service Quality Concept for Vendor Selection

Several definitions of Service Quality (SQ) have been proposed by scholars even though there is no all-around definition for SQ (Chinh and Anh, 2008). Some important and well-known definitions are the definition by Crosby (1984), defined as conformance to requirements, and also defined as fitness for use by Juran (1988) similarly (Eiglier and Langeard, 1987) defines it as customer satisfaction. Quality is also considered as zero defects, or do it correct the first time, according to general Japanese philosophy. Crosby (1979) describes quality as being in line with the needs. German Institute for Quality also define quality as total characteristics and qualities of a products and services, with regard to fulfilling market requirements. Service concept is also described as operating ways which are not delivered to consumer and do not compel to obtaining tangible goods for consumer.

Service quality is assumed to be a connection of contrasting aspects, not restricted to tangibles but intangibles also, and individual aspects are advised in the concept of service quality.

Quality and its determinants are of great importance to firms and consumers. This contribute to market share and returns on investment, and also optimizing manufacturing costs and efficiency (Garvin, 1983).

2.2.1 Service Quality Background

Researches shows strong interests and greater attention on quality in vendor selection decisions between large and small companies. Several service quality literatures examined concepts, measurement, management and implementation of service quality. Quality of goods served account for more increasing concern in the service quality concept. Garvin (1988) was among the researches who earlier examine the quality concept to include goods together with services, where he further describe the quality perceived as subjective perception via measures which compares quality indirectly. Gronroos (1993) described services quality perception being a result of contrast between the customer’s real experience and expectation prior to consumption of the said services. He figure out three quality criteria to include functions, techniques and image of firm. (Parasuraman, A., Zeithaml, V. A., & Berry, L. L. 1985) adapted previous researches in which they came up with their service quality model which is basically on the perceived service quality concept. Ideally they describe the difficulty by consumer in evaluating service quality than quality of goods, also service quality perception was a result of contrast between expectation from consumer and the real service effectiveness, and to evaluate quality means to evaluate both outcome and processes involved in service delivery. In respect to these, Parasuraman et al. (1985) came up with 10 service quality dimensions, these are; understanding consumer, access, courtesy, reliability, communication, competence, creditability, tangibles, security and responsiveness. For simplicity, Brucks (2000) set forth other six quality dimensions to include performance, functionality, prestige for durable goods, Serviceability and durability

Extensive list of Service Quality Dimensions (SDQs) was proposed by Shahin (2007), for some international hotels and British Airways. In his research, two new dimensions were introduced in addition to the work of Parasuraman et al. (1985), these are price and flexibility. Security was substituted with creditability in Parasuraman et al. (1985). Shahin (2007) SQDs are categorized into two broader levels with the first level having 12 dimensions and the second level being the sub group of each of the first level dimensions having a total of 30 dimensions as shown in Table 2.1. Taking into account the Shahin (2007), compared to other quality studies, it shows that the 12 dimensions are relatively more comprehensive and extensive therefore, we adapt his work for our study.

Table 2.1: Twelve Service Quality Dimensions

First level	Second level
1. Reliability	• Performance • Accuracy and dependability • Consistency • Completeness
2. Responsiveness	• Willingness to help customer • Readiness, promptness • Comport
3. Security and confidentiality	• Physical security • Financial security • Safety
4. Access and approachability	• Ease of contact • Timely access
5. Communication	• Word-of-mouth communication • Giving information
6. Understanding the customer	• Comprehension • Individual attention
7. Credibility	• Trustworthiness and believability • Honesty • Reputation of service
8. Tangibles	• Appearance • Tools or equipment used to provide services • Availability of physical facilities
9. Courtesy	• Politeness, respect and consideratio • Empathy
10. Price	• Discountable for money • Valuable for money
11. Competence	• Skills • Knowledge and professionalism of personnel
12. Flexibility	• Specification and volume flexibility • Service delivery speed

Source: Shahin (2007)

2.2.2 Quality and Changing Customer Needs

Since the term 'quality' is dynamic and on-going entity, then it requires the understanding and changing habit of customer (firm in our case) demand and the supplier/vendors purposes, as described by Kotler (2003). As explained in the previous section, Parasuraman (2004), contended that relation between expectation and perceived service quality by the consumer determines the service quality. Petruzzellis (2006), argued that for consumer to be satisfied, there should be continuous checks between expectations and perception of services quality throughout their encounter so as to maintain their relationship. (Eagle and Brennan, 2007).

2.3 Supplier/Vendor Selection Methods

These are the ways or approaches employed to carry out the evaluation and selection of vendors (Li, C., Fun, Y., & Hung, J. 1997). Vendor selection process have a great significant effect on firms therefore methods employed by decision makers and analyst are of great importance, hence those concerned chooses a particular method or a combination different methods so that better results could be obtained. Varieties of selection methods have emerged from work of many scholars especially in the last two decades. Some methods have long been developed and are in use, which are the basics while some approaches are yet to be discovered, but most importantly is for firms to figure out their dimensions which they want to optimize for selecting the most appropriate ones, this will help the analyst in deciding a method or a combination of method that will suite the company's needs from vendors. Therefore, it is advantage cannot be overemphasized. There are several vendor selection methods in the literature, in this review we present the most used methods for supplier evaluation.

2.3.1 Data Envelopment Analysis (DEA)

Data Envelopment Analysis (DEA) is popularly adopted method in vendor evaluation and selection literature. (Ho, W., Xu, X., & Dey, P. K. 2010) shows that among the 78 papers published on vendor evaluation and selection between the years 2000 to 2008, DEA approach account for 17.95% of the papers studied indicating its extensive use.

DEA robustness accounted for its wide adaptability in many areas of application. Knowing that vendor selection problem comprises qualitative and quantitative measures, the traditional application of DEA to calculate the relative efficiency of regular DMUs basically on numerical data was further improved to contain qualitative data by quantifying the qualitative data, such as service quality, vendor reputation, and so on. (Saen, 2007).

DEA being non-parametric mathematical programming tool possesses the ability to assess the efficiency of relative DMUs and hence determine the efficiency frontier as a benchmark for inefficient DMUs to compare with the frontier based on criteria used, hence DEA has generally been applied to calculate vendors' performance in the existence of multiple inputs and outputs in the supplier selection problem because of its ability in handling such multiple dimensions. These multiple dimensions or criterions are used inform of numerical data which is fed into the DEA. The results obtained by DEA would further be used to reduce the number of vendors by considering the various relative efficiency scores of the evaluated vendors.

2.3.1.1 DEA background

The basic idea of DEA was dated back to Farrell's (1957) work on efficiency evaluation. His method was basically about production possibility set comprising of the convex structure of single input and output vectors. Twenty years after Farrell's work, Charnes, Cooper, and Rhodes (1978) (CCR) responds for the obligation to determine other acceptable method to evaluate the relative efficiencies od DMUs in multiple input and output situations, as a result, they present a powerful mathematical programming approach named Data Envelopment Analysis (DEA).

Charnes et al. (1978), chosen DEA as a programming model which can be applied on experimental data to obtain empirical estimates of relative extremity like the efficient production possibility. Typically, DEA is an approach directed to frontiers instead of central tendencies as other approached do. This unique feature of the DEA in discovering relations between two or more units differentiate it from other approaches.

The main idea behind the DEA approach was to evaluate comparable DMUs so that best performing DMUs would be identified easily. These efficient DMUs formed the efficiency frontier.

2.3.1.2 Production Possibility Set (PPS)

Production Possibility Set (PPS) is the set of inputs and outputs of a setup whereby the inputs produces the outputs. PPS is intersection of the many half spaces in which every half space corresponds to either of the defining hyperplane strong or weak described as facet. DEA forms efficient surfaces depending on inputs and outputs of the setup. A DMU is identified efficient if it lies on the surface, otherwise, it is inefficient.

Properties of PPS

Assume *n* DMUs having *m* number of inputs and *s* number of outputs. Each DMU*j*, $(j = 1,2,\cdots,n)$ produces *s* different outputs $y_{rj}(r = 1,2,\cdots,s)$, using m different inputs $x_{ij}(i = 1,2,\cdots,m)$, assuming all data to be nonnegative and a pair of semi positive input *x* and output *y* will be called *activity*. The PPS of the system can be showed as follows:

$$T = \{(x, y) \mid y \text{ produced by } x\}.$$

The properties below are presumed for PPS T

- Set of activities observed belongs to T that is,

  $(x_j, y_j) \in T \quad j = 1, 2, \cdots, n)$

- If an activity $(x, y) \in T$, then $(tx, ty) \in T$ for any positive scalar t. This is identified as the constant return to scale property.

- For any activity $(x, y) \in T$, any semi positive activity $(\bar{x}, \bar{y})$ with $(\bar{x} \geq x)$ and $(\bar{y} \leq y)$ elements of T.

- T is closed and convex

By considering $X = (x_j)$ and $Y = (y_j)$, for all $j = 1, 2, \cdots, n$ PPS T satisfying all the presumed properties can be defined as:

$$T_c = \{(x, y) \mid x \geq X\lambda,\ y \leq Y\lambda,\ \lambda \geq 0\}$$

Where $\lambda \in R_+^n$ is semi positive

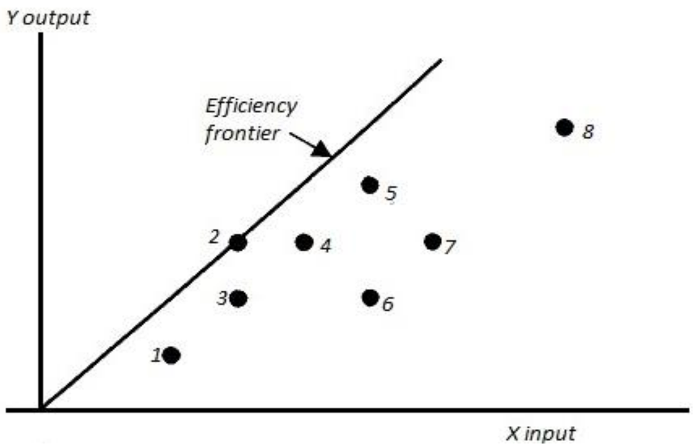

Figure 2.1: PPS for single input output space (CRS)

Figure 2.1 shows PPS for the single input and output space in two dimensions, in which m = 1 and s = 1 for the inputs and outputs respectively. In this figure (2.1), the

PPS is determined by point 2, the line which passes through 2 and the origin is the efficient frontier, and hence T_c is developed on CRS property assumption.

We know that some DMUs are not efficient in the previous models considered, so far they may become efficient if we assume variable return to scale (VRS) by relaxing the CRS property.

Charnes et al., (1978) (CCR), postulated that the PPS has the constant return to scale property, Banker et al., (1984) (BCC), further developed the work of Charnes et al., (1978) by introducing a convexity condition $\sum_{j=1}^{n} \lambda_j = 1$ in its constraints thus eliminating the constant return to scale property of the CCR model, which made the new efficiency to assume variable return to scale property. They introduces the BCC model whose production possibility set T_b is defined by:

$$T_b = \{(x, y) \mid x \geq X\lambda,\ y \leq Y\lambda,\ e\lambda = 1,\ \lambda \geq 0\}$$

Note that ($\sum_{j=1}^{n} \lambda_j = 1$) = ($e\lambda = 1$) where e is a row vector with all elements equal to one.

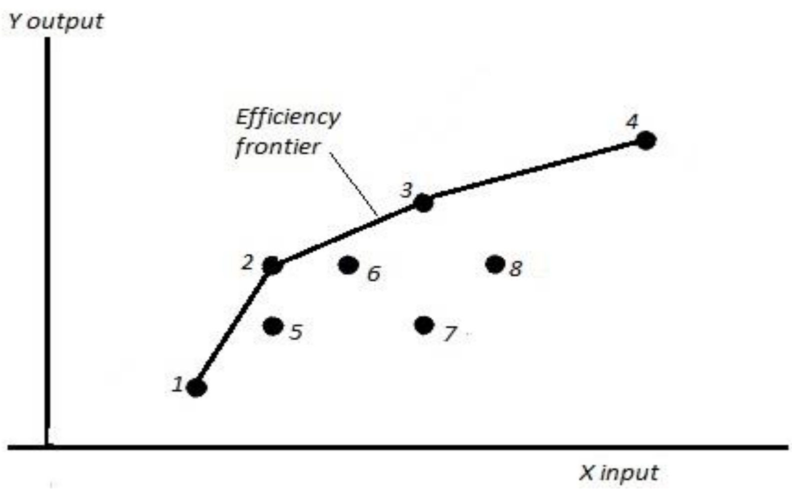

Figure 2.2: PPS for single input output space (VRS)

In Figure (2.2), part of the frontier starting at point 1 to point 2 (point 2 exclusive), shows increasing return to scale, point 2 undergo constant return to scale, while the remaining part of the frontier i.e. line segment from point 2 to point 4 experiences decreasing return to scale.

2.3.1.3 Basic DEA Models

DEA is a multi-criteria programming model for assessing the relative efficiencies of a set of Decision Making Units, where the efficiency value is defined as the ratio of weighted sum of the outputs to the weighted sum of inputs as shown in (2.1)

$$efficiency = \frac{weighted\ sum\ of\ outputs}{weighted\ sum\ of\ inputs} \qquad (2.1)$$

For a DMU to be evaluated, this ratio provides the measure of its efficiency which is a multiplier function. In a situation of unknown multipliers we cannot solve the above problem.

However in mathematical programming expression, this ratio which its aim is to maximize, forms the objective function for the DMU under evaluation, this will provide us with the efficiency measurement.

The CCR (Charnes Cooper and Rhodes) Model:

To evaluate the efficiency of $DMUo\ (o \in \{1,2,\cdots,n\})$ under the assumption of CRS, the CCR input-oriented linear programming (envelopment form) model (2.1) can be applied.

$$
\begin{aligned}
&\theta_o^* = \min \theta_o \\
&s.t. \\
&\quad \theta_o X_o - \sum_{j=1}^{n} X_j \lambda_j \geq 0 \\
&\quad \sum_{j=1}^{n} Y_j \lambda_j \geq Y_0 \\
&\quad \lambda_j \geq 0 \qquad j = 1, \ldots, n
\end{aligned}
\tag{2.1}
$$

The dual of model (2.1) is given below:

$$
\begin{aligned}
&z_o^* = \max UY_0 \\
&s.t. \\
&\quad UY_j - VX_j \leq 0 \\
&\quad VX_0 = 1 \\
&\quad U \geq 0 \\
&\quad V \geq 0
\end{aligned}
\tag{2.2}
$$

The optimal solution, θ_o* and z_o* gives an efficiency value of the *DMU* under evaluation. This procedure is repeated for all DMU_j j = (1,…,n). DMUs for which their optimal values are < 1 are inefficient, while *DMUs* for which their optimal values = 1 are boundary points.

From DEA literature, model (2.1) is said to comply with the assumption of strong disposal, hence it ignores the presence of nonzero slacks which may be present. It also evaluates radial (proportional) efficiency.

The BCC (Banker, Charnes and Cooper) model

The BCC model determines the efficiency of $DMUo\ (o \in \{1,2,\cdots,n\})$ under the assumption of VRS by solving the following linear program:

$$
\begin{aligned}
& b_o^* = \min b_o \\
& s.t \\
& \quad b_o X_0 - \sum_{j=1}^{n} X_j \lambda_j \geq 0 \\
& \quad \sum_{j=1}^{n} Y_j \lambda_j \geq Y_0 \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0 \qquad j = 1, \ldots, n
\end{aligned}
\tag{2.3}
$$

where b is a scalar

The dual of (2.3) is given below:

$$
\begin{aligned}
& z_o^* = \max UY_0 + u_0 \\
& s.t. \\
& \quad VX_0 = 1 \\
& \quad UY_j - VX_j + u_0 \leq 0 \qquad j = 1, \ldots, n \\
& \quad V \geq 0 \\
& \quad U \geq 0 \\
& \quad u_0 \ \ free
\end{aligned}
\tag{2.4}
$$

Solving either of the above equivalent problems gives the optimal performance score of DMU under evaluation. The dual programming problem is the model that we bring to bear most for this thesis.

When DEA is applied on the issue of vendor selection, the term DMU refers to the vendors which we are to select from. Inputs and outputs which are normally numerical values are the required data for the evaluation which can be applied on all the models we presented so far for evaluation. In most cases, inputs represents cost for firms while inputs represent gain, these models normally optimizes these processes by either minimizing cost or maximizing benefits.

2.3.1.4 Drawbacks of DEA based on Vendor Evaluation

DEA is a useful tool for benchmarking that can evaluate several inputs and outputs. This greatly helps in changing and improving management programs, R. Ramanathan (2003). With these advantages, DEA has some drawbacks. Firstly, DEA models shows poor discriminatory power. Secondly, the ability of DEA in making flexible weighting often results in determining some DMUs having absurd weight which is not its real weight. As such these DMUs may be efficient when their true efficiency score is not 1 because they have been favored by some other DMUs. Such DMUs are not effective generally hence the need to modify simple basic models may arise in order to do away with such limitation depending on the need of the analyst and managers.

Other limitations or drawbacks of DEA when adopted as a method for vendor selection may include confusion in selecting input and output criteria by the analyst and management. For instance, some researchers' regards price or cost as an output criterion Talluri et al. (2004), while others consider it an input (HO William et. al., 2010). As such, the vendor selection criteria need to be chosen based on the consumer and/or firms priorities. Other drawback is engagement of rating wen trying to convert certain qualitative dimension into numerical value. Researcher consider different rating methods such as the Fuzzy method. Saen (2006) establish some measuring scales for ranking priority of quality dimensions, but this may lead to variabilities in results due to personal perspective compared to when a different scale is chosen by other researcher. Another regard is the nature of DEA being a tool for measuring relatively the efficiencies of comparable DMUs, generally in most instances, vendors that generate more outputs and require less input are the efficient ones.

2.3.2 Other Vendor selection methods:

Here, we try to show other methods for the vendor selection problem with some of the authors who wrote about them.

The Analytical Hierarchic Process (AHP) developed by Saaty (1980). Analytical Network Process which is a broad procedure of AHP, it is also developed by Saaty, (1996). The Case Based Reasoning, developed by Schank and Abelson (1977) which normally deals with past experiences. The Decision Matrix Method proposed by Pugh in 1990. Also in several researches, several methods and modifications have been developed which are used in the evaluation process. They include Goal Programming (GP) approach and many others.

Many scholars also integrate different methods and applied them in their works i.e. they try to combine more than one approach in order to perfect their work depending on the type of problem they are handling. These integrated approach include AHP-DEA by Sevkli et al. (2007). Mendoza et al. (2008) proposed AHP-GP and several others.

In conclusion, according to William H O (2010), widely adapted approach is the DEA, followed by MP, AHP,CBR, ANP, Fuzzy set theory, SMART, lastly GA. This gave us more confidence in adapting the DEA in this work.

Chapter 3

BASIC DEA MODEL FOR VENDOR SELECTION

3.1 Introduction

In this section, we describe the model that is widely employed for measuring vendor's efficiency which is originated by Banker et. al. (1984), so as to facilitate the modification of the model.

Data envelopment analysis (DEA) applies LP to assess the relative efficiencies of decision making units (DMUs) with multiple inputs and output factors.

DEA identify such DMUs which yields higher amount of output by utilizing the least amount of inputs. DMU is regarded efficient when the ratio of weight sum of outputs to that of inputs is found to be the highest.

3.2 DEA model for Vendor Evaluation

Assume there are *n DMUs* where each DMU_j $(j = 1,2,\dots,n)$ utilizes *m* different inputs $x_{ij} \geq 0$ $(i = 1,2,\dots,m)$ to produce *s* different outputs $y_{rj} \geq 0$ $(r=1,2,\dots,s)$

The input-based (envelopment form) BBC model assess the efficiency of DMU_0 by solving the LP below:

$$
\begin{aligned}
& b_o{*} = \min b_o \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le b_o x_{i0} \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{r0} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \ge 0
\end{aligned}
\tag{3.1}
$$

Solving the above problem may often results in having non-zero slacks in our optimal solution, as such some boundary points may be "weakly efficient". This may be an issue because of this alternate optimal solutions with non-zero slacks. However, such can be avoided in cases like this by invoking these another stage to solving the LP problem.

In the next stage, we try to maximize sum of the input excesses and output shortages i.e. the slacks as follows:

$$
\begin{aligned}
& \max \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
& s.t. \\
& \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = b_o x_{i0} \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \ge 0 \\
& \quad s_i^- \ge 0 \\
& \quad s_r^+ \ge 0
\end{aligned}
\tag{3.2}
$$

In the development above, it can be noted that the slacks doesn't affect the optimal value ***b****, hence we define the relative efficiencies as follows.

Definition 3.1:

DMU_0 is considered *efficient* if and only if:

- $b^* = 1$
- $s_i^- = 0$ and $s_r^+ = 0$

Definition 3.2:

DMU_0 is considered *weakly efficient* if and only if:

- $b^* = 1$
- $s_i^- \neq 0$ and $s_r^+ \neq 0$ for some *i* or *r* in alternate optimal values

This two process in which we first find the efficiency score followed by maximization of slacks (Seiford and Cooper, 2007) .i.e. model (3.1) and (3.2) can be unified by joining them together in a single model as shown below:

$$\begin{aligned}
&\min b_o - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right) \\
&s.t \\
&\quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = b_o x_{i0} \\
&\quad \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{r0} \\
&\quad \sum_{j=1}^{n} \lambda_j = 1 \\
&\quad \lambda_j \geq 0 \\
&\quad s_i^- \geq 0 \\
&\quad s_r^+ \geq 0
\end{aligned} \tag{3.3}$$

Where $\varepsilon > 0$ is non-Archimedean number assumed to be very small number, the slack variables changes the equivalent inequalities to equations.

3.3 The Non –Archimedean number ε and the variable u_o

Since the introduction of non-Archimedean number ε in 1979 by Charnes et al. (1979), the number has extensively been used as the lower bound for factor weights in DEA models especially BCC model. These bound perturb the weak parts of frontier hence weakly efficient DMUs assume an efficiency value less than 1, but these values are not real efficiency values. Therefore using ε as the lower bound of factor weights is not suitable. Here the variations of u_o take the problem of feasibility and the above variation is the reason of inefficiency of some DMUs under consideration. In order to find the exact efficiencies of DMUs that lies to this weak frontier or DMUs which compare with these parts of efficiency frontier, a procedure is suggested to compute the value of ε and then it is used as the upper bound of u_o, which is the basis of our modification.

The dual of the above model (3.3) can also be represented as:

$$
\begin{aligned}
&\max\ UY_o + u_o \\
&s.t. \\
&\quad UY_j - VX_j + u_o \le 0 \qquad j = 1, \cdots, n \\
&\quad VX_o = 1 \qquad\qquad\qquad\qquad\qquad\qquad (3.4) \\
&\quad U \ge \varepsilon \\
&\quad V \ge \varepsilon \\
&\quad u_o \ \ free
\end{aligned}
$$

Alternatively, one can also consider the output side which will reorient the objective from maximization to minimization problem.

Note that in model (3.1), it is referred to assume "weak efficiency" because it does not take into account the possibility that non-zero slacks may be present in some optimal solutions, hence in economics term, this model is said to conform to the assumption of "strong disposal". However, by considering model (3.2), if we omit either of s_i^- or s_i^+ in their respective *m* and *s* constraints, then we have what is assumed as "weak disposal" in which inequalities are replaced with equalities directly hence there is no room for slacks in either case.

The Weak and strong disposal assumption gives the insight on the "free disposal" assumption which was initiated by TC Koopmans, (1951). This postulation means that there is no cost related with neglecting slacks in both the outputs and inputs, i.e. slacks in the objective are assumed zero coefficient.

Considering this "free disposal" idea, good decision maker DM cannot just ignore nonzero slacks by developing the assumption that multiplier that corresponds to these values would be zero. Therefore, applying the stage two optimization idea maximizes the slacks, as shown in the stage 2 process in order to bring out the maximum inefficiency value possible with regard to these nonzero excesses and shortfalls.

For evaluating the efficiency of vendors, it is also more accurate to neglect this assumption of "free disposal" and models (3.3) and (3.4) are widely considered.

3.4 Empirical Study in PVC Pipes Company (Darakar Co.) as an Example:

The data used in this thesis was deducted from empirical study performed in Darakar Company. This data was used in two different researches by Shirouyehzad H. et al.

(2009), and Shirouyehzad, H., Lotfi, F. H., (2011). Darakar Company is the biggest manufacturer of water pipes in Iran. Polyvinyl Chloride (PVC) is the major materials which accounts for more than 75% of the company's product. Hence, PVC remains the most utilized and critical material in this company. In the evaluation process, a set of 12 vendors are considered which have been doing business with Darakar Company for over two years.

The management of this company have gave thought to price which was on per unit basis for each item delivered, percentage of average late deliveries being number of times items are delivered late, percentage of rate of rejected parts representing expected incompatible deliveries, all for the more than two years relation with the company and service quality being the company's priority and perception from the vendors service quality provided. These are the four factors considered in evaluating the vendors.

Price, percentage of average late delivery and percentage rate of rejected items are used as the inputs criteria because they speak for the cost paid by the company. Service quality being utilized as output criterion because it is entitled to the advantage received by the company.

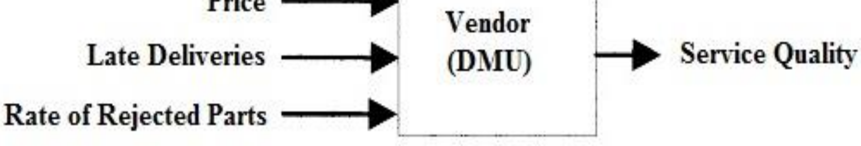

Figure 3.1: Vendor Evaluation Criteria

The data for the four criteria utilized for evaluating the relative efficiencies are represented in Table 3.1. The service quality data being a qualitative factor was obtained through a service quality checklist register gathered and filled by experts of company. These service quality values are measured through the company's perception from the vendors' services provided. Original PVC prices were altered as well, this is because of its sensitivity as the company's information. These gathered data representing the various inputs and output are as follows:

Table 3.1: Data for Empirical Study in Pipe Company

Criteria		Vendor											
		1	2	3	4	5	6	7	8	9	10	11	12
Inputs	Price (V_1)	290	240	300	255	295	250	245	285	270	270	285	275
	% Late Deliveries (V_2)	7	3	4	5	10	3	7	6	6	12	3	5
	% Rate of Rejects (V_3)	3	5	6	3	8	3	4	4	6	4	5	8
Output	Service Quality (U)	95	98	12	100	65	110	92	73	75	81	112	85

The analyst or decision maker may evaluate the efficiencies of the various vendors from the data given in Table 3.1 using Model 3.4 i.e. the Standard Model for measuring vendor efficiencies. The following Table shows us the results for the relative efficiencies of the various vendors using WinQSB software:

Table 3.2: Optimal weights and efficiency results of Basic Model for vendor selection

Vendor	V_1^*	V_2^*	V_3^*	U^*	u_o^*	**Efficiency**
1	0.0010	0.0010	0.2343	0.0010	0.8460	**0.9410**
2	0.0041	0.0010	0.0010	0.0033	0.6788	**1**
3	0.0031	0.0010	0.0096	0.0010	0.7037	**0.7157**

4	0.0010	0.0010	0.2467	0.0010	0.8830	**0.9830**
5	0.0033	0.0010	0.0010	0.0010	0.7089	**0.7739**
6	0.0040	0.0010	0.0010	0.0091	0	**1**
7	0.0038	0.0010	0.0132	0.0010	0.8920	**0.9840**
8	0.0033	0.0010	0.0107	0.0010	0.7595	**0.8325**
9	0.0037	0.0010	0.0010	0.0010	0.7882	**0.8632**
10	0.0035	0.0010	0.0114	0.0010	0.7998	**0.8808**
11	0.0010	0.2367	0.0010	0.0185	-1.0720	**1**
12	0.0036	0.0010	0.0010	0.0010	0.7714	**0.8564**

Considering the results in Table 3.2, we can see that using the standard model for vendor selection, we successfully find the efficiency score values of the vendors involved. It is evident that three vendors are found to be efficient with an efficiency score of 1 among the twelve vendors, the remaining vendors are rendered inefficient.

By optimizing the slacks and simultaneously finding the efficiencies of the vendors in the envelopment model, economically, we can say that the obtained results reflect the non-zero positive weights on the inputs and output variables, hence for each of the weights we have a lower bound of 0.001 which gives a better relative efficiencies of the vendors than when the slacks are neglected.

These scores help managements in vendor selection based on their relative performances. Those vendors having efficiency score of 1 shows that higher output was achieved by utilizing minimum amount of inputs. This will help the management in reflecting best vendors to employ so that they operate efficiently.

Considering various prices of the individual vendors, we can see that vendors with lowest prices are not substantially efficient. The price of vendor 7 is found to be 245 which is relatively low but this vendor is rendered inefficient, this is because of its relatively higher values in % of its late deliveries as well as rate of rejected items. When we look at vendor 11, we can see that it has relatively higher price but it is found to be efficient, this is because of its relatively higher service quality. So it is obvious that DEA is a good tool for evaluating efficiencies by considering several criteria.

Considering all these achievements, some of the evaluated vendors which are found to be efficient may not necessarily be efficient because they may lay in the weak part of the efficiency frontier hence their true value may be less than one, hence the need for the modification of the model arises.

Chapter 4

MODIFIED BCC MODEL

4.1 Introduction

The existence of multiple optimal solutions remain the big complication in classifying DMUs base on Return To Scale (RTS), that is to say, the classification may be a function of the particular solution selected.

It may be unreasonable to figure out all possible multiple optimal values in most real world applications, To provide a more definitive RTS, a number of modifications or extensions of the basic standard CCR and BCC approaches have been developed to deal with multiple optima problem for a given DMU..

This RTS classification has greatly been of interest by several researchers and authors, including Banker (1984) where he use the most productive scale size concept and letting the sum of lambda values suggests the RTS classification which come to be known as the BCC RTS method. Banker R.D. (1986) further report that a new free variable (u_o) estimates RTS by allowing variable returns to scale (VRS) for the CCR model, that is, the free variable u_o defines the RTS. Fare (1994) finally provide the scale efficiency index method for the determination of RTS using DEA. These 3 classifications are equivalent, but differ in way of presentation, Banker and Thrall (1992). Our focus here is the BBC RTS method.

4.2 Facet Analysis

In DEA aspect, the multi input-output efficiency frontier takes a polyhedral form in n- dimensional space *($n \geq 3$)*. The procedure for analyzing the defining hyperplanes of these polyhedral surfaces (Facets) of the efficiency frontier of DMUs is known as the Facet Analysis.

Facet analysis provides a correlation between geometric and algebraic ideas of DEA model as shown in several papers. Charnes et al (1978) and Banker et al (1984) characterizes the facet structures of their CCR and BCC models respectively. Thrall (1996) introduces the contrast between inner and outer facets. Also Daneshvar S. (2010) uses facet analysis to develop a modified standard BCC model.

When trying to evaluate the true efficiency of DMUs in multiple dimensions with multiple inputs and outputs, it is important that the polyhedral structured frontier is analyzed in facets, this allows the decision maker to find areas of improvement easily, either by reducing the amount of input to obtain the same amount of output or maintaining the same amount of input to achieve higher amount of output as we can see in Figure 4.1

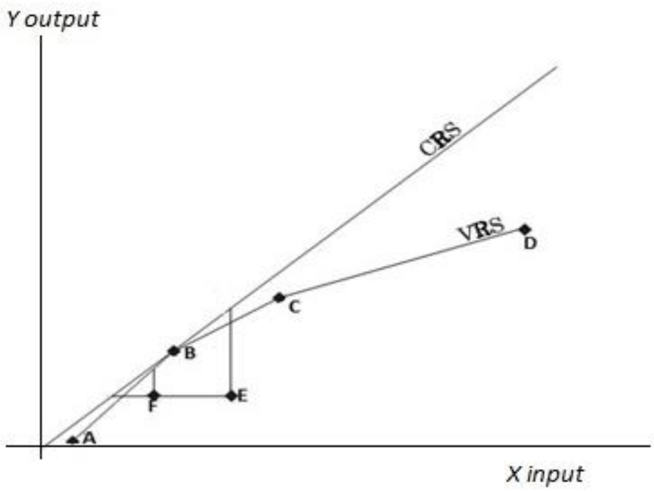

Figure 4.1: VRS and CRS frontiers in single input output space

We can see from Figure 4.1 that the input oriented as well as output oriented envelopment models will project F on to the Facet AB. Consider another DMU E which is inefficient, but it is projected onto different facets depending upon which orientation model is used. The input oriented model projects DMU E on the Facet AB, while the output oriented model projects it to the Facet BC, without the convexity constraint, the CCR envelopment model will project the DMU to the CRS frontier. Charnes et al. (1979).

4.2.1 Facet analysis on VRS frontier

The points in the efficiency frontier for the BCC model can be classified into 3 different categories:

- Strongly Efficient Points
- Efficient Points
- Weak Efficient Points

The Strong efficient points are the set of points located at the vertices of the frontier, i.e. the corner points, points as can be seen in Figure 4.1, points A,B and C are strong efficient points. Efficient points are the set of points considered to be efficient by the model. they lie on the line segments but not including the vertices, as we can see in Figure 4.1, all the points between line AB and BC with points A,B and C exclusive) , and the Weak Efficient Points are those set of points which are efficient in the input orientation and inefficient in the output orientation and vice versa. We can see it in figure 4.2 where all the points on H_1 and H_4 are rendered weak efficient. Charnes et al. (1991)

In evaluating the efficiency of *DMUo* (o ∈ {1, 2,…,*n*}), for the VRS frontier, we considered the following input oriented dual BBC model:

$$
\begin{aligned}
&\max \sum_{r=1}^{s} u_r y_{r0} + u_0 \\
&s.t. \\
&\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_0 \le 0 \\
&\quad \sum_{i=1}^{m} v_i x_{ij} = 1 \\
&\quad u_r \ge 0 \\
&\quad v_i \ge 0 \\
&\quad u_0 \ \ free
\end{aligned} \tag{4.1}
$$

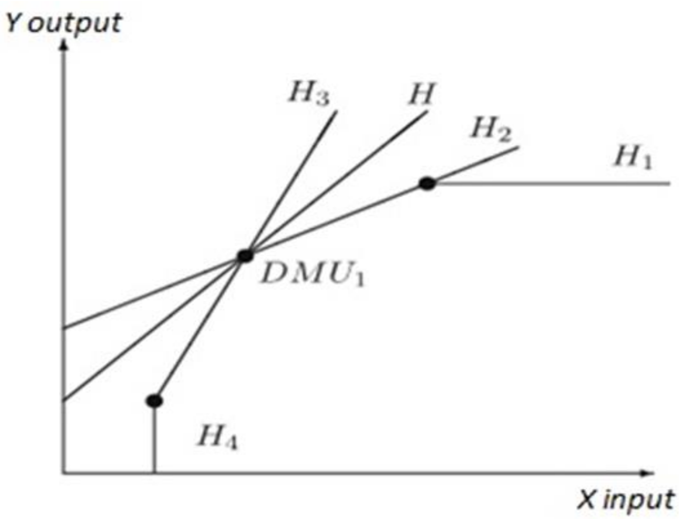

Figure 4.2: Hyperplanes in single input output space

Consider DMU_1 in Figure 4.2, in the evaluation of DMU_1 using model (4.1), we can see that several alternate optima which define an infinite number of supporting hyperplanes passing through DMU_1, of which two hyperplanes H_2 and H_3 are strong defining hyperplanes and H_1 and H_4 are weak defining hyperplanes. Cooper et al. (2000).

If (U^*, V^*, u_o^*) is an optimal solution of model (4.1), then $U^* y - V^* x + u_o^* = 0$ is equation of supporting hyperplane of the production possibility set (PPS)

Definition 4.1:

Hyperplane of PPS is assumed to be strong defining hyperplane if only if it is supporting at least $m + s$ strong efficient DMUs of PPS that lie on H considering figure 4.2 and all of components of its gradients are strictly positive.

Definition 4.2

Hyperplane of PPS is said to be weak defining hyperplane if and only if it is supporting at least $m + s$ strong efficient and weak efficient virtual DMUs of PPS that lie on H, that is, at least one components of its gradient is zero. Note that the hyperplanes H1 and H4 are weak defining hyperplanes (infinite edges; in the two dimensions space) of PPS

4.3 BCC Model Modification

Presence of multiple optima triggered the development of Banker and Thrall (1988) techniques which further generalize the BCC RTS (VRS) method on multiple outputs situation to empirically estimate Returns to Scale at the point that is radially technically efficient. A point is said to be radially technical efficient if it considered proportionate amount of input and output.

Assume that (X_o, Y_o) is a DMU on the frontier, which was evaluated. Our main focus is on the set of points of intersection between the production possibility set T_b and the plane P that pass through the radial technical efficient point.

As noted in Chapter 2, PPS T_b is given as:

$$T_b = \left\{ (X, Y) \mid X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j \right\} \qquad (4.2)$$

Hence P the set of points in the plane that cuts through the polyhedral figure illustrated in Figure (4.3) with its axes (α, β) is given as:

$$P = P(X_o, Y_o) = \{ (X, Y) \mid X = \alpha X_o, Y = \beta Y_o, \alpha, \beta \geq 0 \} \qquad (4.3)$$

See Figure 4.3

The intersection of (4.2) and (4.3) is given below:

$$P \cap T_b = \left\{ (X, Y) \mid X = \alpha X_o \geq \sum_{j=1}^{n} \lambda_j X_j, Y = \beta Y_o \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \ \forall j, \alpha, \beta \geq 0 \right\}$$

If this intersection plane is considered on the new axes α and β, the equivalent set will be defined as:

$$\overline{T}(X, Y) = \left\{ (\alpha X_o, \beta Y_o) \mid \alpha X_o \geq \sum_{j=1}^{n} \lambda_j X_j, \beta Y_o \leq \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \forall j, \alpha, \beta \geq 0 \right\}$$

See Figure 4.3

For optimal values U^*, V^*, and u_o^*, (X_o, Y_o) is radially technical efficient point, in the dual BCC formulation, that is, $b_o^* = 1$, hence $U^*Y_o + u_o^* = 1 = V^*X_o$

The supporting hyperplane $U^*Y_o - V^*X_o + u_o^* = 0$ in the multi input and output space passes through (X_o, Y_o), the points of intersection of this hyperplane and $\overline{T}$ is the line $\beta(U^*Y_o) - \alpha(V^*X_o) + u_o^* = 0$. This line will pass through $(\alpha, \beta) = (1, 1)$ for (X_o, Y_o)

As can be seen in Figure 4.3, many tangential (supporting hyperplanes) may result at (X_o, Y_o), hence u_o is not uniquely determined at this point.

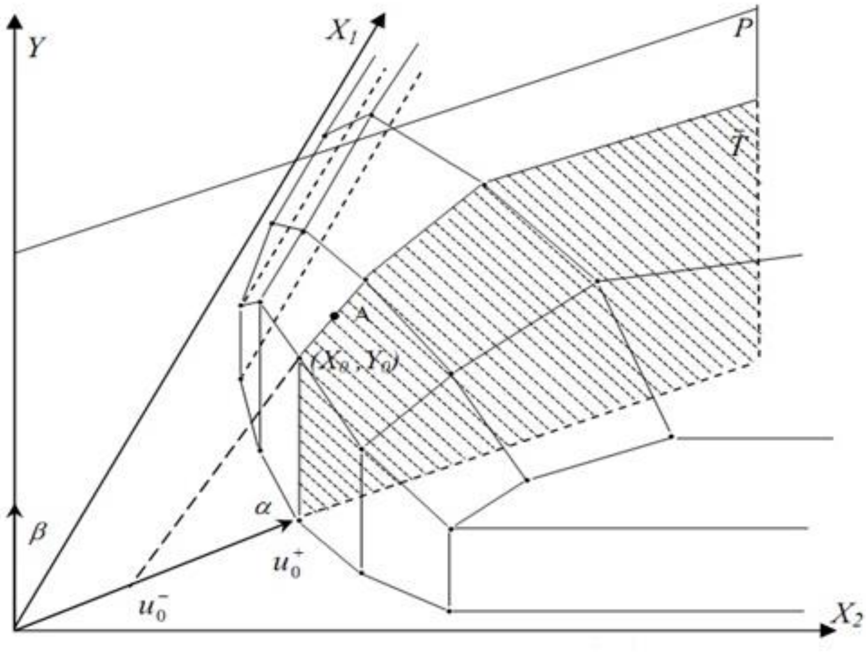

Figure 4.3: P and $\overline{T}$ in the two inputs one output space

Banker and Thrall (1988) proposed a modification on model (4.1) to deal with the multiple optima problem by computing u_o^+ as upper bound and u_o^- upper lower bound for the free variable of all the supporting hyperplanes which passes through such points. Daneshvar S. (2010). The model is as follows:

$$
\begin{aligned}
& \max u_o \\
& s.t. \\
& \quad UY_o + u_o = 1 \\
& \quad UY_j - VX_j + u_o \le 0 \quad for\ j = 1, \cdots, n \\
& \quad VX_o = 1 \\
& \quad U \ge 0 \\
& \quad V \ge 0 \\
& \quad u_o\ free
\end{aligned}
\tag{4.6}
$$

$$
\begin{aligned}
& \min u_o \\
& s.t. \\
& \quad UY_o + u_o = 1 \\
& \quad UY_j - VX_j + u_o \le 0 \quad for\ j = 1, \cdots, n \\
& \quad VX_o = 1 \\
& \quad U \ge 0 \\
& \quad V \ge 0 \\
& \quad u_o\ free
\end{aligned}
\tag{4.7}
$$

The optimal values of the above models are u_o^+ and u_o^- respectively, hence for any optimal value *U**, *V**, and u_o* alternative for model (4.1) we have $u_o^- \le u_o^* \le u_o^+$.

Note that u_o^- = -∞ may result considering model (4.7) but programming algorithm will detect this.

Definition 4.3:

Hyperplanes generated by u_o^* which passes through (X_o, Y_o) that is $U^*Y_o - V^*X_o + u_o^* = 0$ are known to be admissible supporting hyperplanes for set

T_b. Daneshvar S. (2010) DMUs that belong to this intersection of efficient and weak part of frontier have $u_o^+ = 1$ and $u_o^- < 1$ as seen in Figure 4.3

4.4 Empirical Study Example

Knowing that the basic DEA model for vendor evaluation is a BCC based model having same properties, with slack considered, same modification on the standard BCC model will also be applicable to the vendor evaluation model.

We first determine the efficiency of all the vendors using standard BCC model (2.4). Table 4.1 shows us the results.

Table 4.1: Optimal weights and efficiency results for standard BCC model

Vendor	V_1^*	V_2^*	V_3^*	U^*	u_o^*	Efficiency
1	0	0	0.3333	0	1	1
2	0.0042	0	0	0.0035	0.6597	1
3	0.0030	0	0.0512	0	0.8030	0.8036
4	0	0	0.3333	0	1	1
5	0.0034	0	0	0	0.8136	0.8136
6	0.0040	0	0	0.0091	0	1
7	0.0038	0	0.0189	0	1	1
8	0.0033	0	0.0164	0	0.8689	0.8689
9	0.0037	0	0	0	0.8889	0.8889
10	0.0034	0	0.0172	0	0.9138	0.9138
11	0.0005	0.2849	0	0.0089	0	1
12	0.0036	0	0	0	0.8727	0.8727

From Table (4.1), Vendors 1, 2, 4, 6 and 11 are found to be efficient. We then use these efficient vendors to calculate the optimal u_o^+ and u_o^- using models (4.6) and (4.7) respectively. The following results were obtained:

Table 4.2: Optimal values for u_o^+ and u_o^-

Vendor	u_o^+	u_o^-
1	1	1
2	1	**0.6597**
4	1	1
6	1	-35.6667
7	1	1
11	1	$-\infty$

Considering Table 4.2 above, from the results and definition of ϖ, the upper bound for the free variable u_o is found to be **0.6597** i.e. Vendor 2. Implementing this results will modify our basic vendor selection model.

Chapter 5

MODIFIED DEA MODEL FOR VENDOR SELECTION

5.1 Introduction

The model that is widely adapted by several researchers for evaluating the efficiencies of vendors is based on the BCC model. Depending on the decision maker, sometimes this standard model is assumed to dispose slacks (free disposal). This is a bad idea because some optima have nonzero slacks, hence in order to obtain better and more reliable results we must consider slacks. Hence model (3.4) earlier discussed in chapter 3 is widely employed.

Considering this widely adapted model (3.4), researchers mostly estimates the bound for the free variable in the model, but this did not address the issue of the weakly part of the efficiency frontier properly, as such more improvement need to be done in order to have more proper, accurate and reliable results for the evaluation process.

The proposed modified model attempts to modify the PPS frontier for vendor evaluation model (VRS frontier) by restricting the bound for the free variable. Similar modification process was achieved by Daneshvar S. (2010). The values of DMUs in this weakly part of the efficiency frontier and those that compare with them are changed due to this modification, hence reflecting on the true values of these DMUs thereby demonstrating great changes on the vendor evaluation process. This will have great impact on managerial and decision making issues particularly organizations that deals with vendor selection considering multiple criterions.

5.2 Problem Definition

When we are concerned with managerial responsibility of evaluating vendors for company supplies, managers mostly adopt DEA due to its vast advantages over other methods in evaluating the vendors as discussed in the literature. DEA find the most efficient vendors so that less vendor are employed for the company's supplies. Several researches were carried out and the DEA method was improved over its limitations.

Effect of weakly efficient DMUs in the DEA for vendor selection was not properly acknowledged, hence in this research we try to modify this weakly efficient frontier of the PPS so that DMUs which lies in this frontier shows their true efficiency values rather than enacting to be efficient when they are truly not. This will tremendously improve the vendor selection and evaluation problem which will help managers in finding only surely efficient vendors. Also for the DMUs that changes their efficiency after modification, managers can be able to easily identify areas of improvement for the vendors so they can suggest to them if they really want to continue with their partnership.

5.3 Proposed Modified Model Assumptions

For the modified model for vendor selection, we assume that:

- Non-zero slacks may exist for some optimal solutions hence we did not dispose them.

- Similar modification achieved on standard BCC model (Daneshvar et al. 2014) will also be applicable on the vendor selection model with a restriction that lower bounds for the weights of inputs and outputs should be strictly positive.
- When trying to find the appropriate optimal upper bound for the free variable u_o, there is no need to restrict the lower bounds for the weights of inputs and outputs. This is because we are trying to modify the weakly efficient frontier. We know that for this frontier, the upper bound for the free variable u_o should be $u_o^+ = 1$. If we assume weight restrictions for the optimal u_o, then we are not modifying the weak frontier hence our modification will be invalid because it does not in any way affects the weak efficiency frontier, this can be proved below;

$$
\begin{aligned}
& u_o^{*\varepsilon} = \max u_o \\
& s.t \\
& \quad UY_o + u_o = 1 \\
& \quad UY_j - VX_j + u_o \le 0 \qquad j = 1, \dots, n \\
& \quad VX_o = 1 \\
& \quad U \ge \varepsilon \\
& \quad V \ge \varepsilon \\
& \quad u_o \ \ free
\end{aligned}
\tag{5.1}
$$

Lemma 1: The optimal value of the above model (5.1) is less than one

Proof: Suppose that $(U^{*\varepsilon}, V^{*\varepsilon}, u_o^{*\varepsilon})$ is the optimal solution of model (5.1) for (X_o, Y_o) weakly efficient DMUs with $u_o^+ = 1$, then considering constraint 1 ($UY_o + u_o = 1$) we have $U^{*\varepsilon}Y_o = 0$. This is a contradiction when we consider the definition of *PPS* and constraint 4 ($U \ge \varepsilon$) in which we must have $U^{*\varepsilon}Y_o > 0$. Hence in finding the optimal bound of u_o, for the weakly efficient DMUs, inputs and outputs weights cannot be restricted to some positive values.

- The modified model does not change the efficiency of strong efficient DMUs. It changes only the efficiency of weakly efficient DMUs and the DMUs which compares with this frontier. Daneshvar S. (2010)

5.4 Proposed Modified Model

In this section we try to show the modified standard BBC model for vendor selection achieved by facet analysis and restricting the free variable.

Suppose that for the efficient DMUs obtained by using the Basic BBC Model in Chapter 4, u_o^- corresponds to supporting hyperplanes that passes through DMUs with minimum slope. If for weak parts of frontier u_o^+ cannot be equal to unity, then the frontier is modified by restricting u_o

For efficient DMUs satisfying the inequality $u_o^- \le u_o^* \le u_o^+$, ϖ is placed as upper bound for u_o of standard BBC model, hence ϖ is defined as:

$\varpi = Max\left\{ u_o^- \mid u_o^- \neq 1, \quad for\ efficient\ DMUs \right\}$ as seen in chapter 4.

Then the basic DEA model for Vendor Evaluation (4.1) is modified as follows:

$$
\begin{aligned}
& \max UY_o + u_o \\
& s.t. \\
& \quad UY_j - VX_j + u_o \le 0 \qquad j = 1, \cdots, n \\
& \quad VX_o = 1 \qquad\qquad\qquad\qquad\qquad (5.2) \\
& \quad U \ge \varepsilon \\
& \quad V \ge \varepsilon \\
& \quad u_o \le \varpi
\end{aligned}
$$

5.4.1 Summary of the Modification Process:

Implementing the following steps on DMUs modifies the basic model for vendor evaluation and selection:

- We first of all calculate the efficiency value of all the DMUs involved using standard BCC model (2.4) to find the efficient DMUs.
- For all efficient DMUs identified by model (2.4), we apply model (4.6) and (4.7) on those DMUs to find the optimal values for the bounds of our free variable which are u_o^+ and u_o^-. Hence the upper bound for the free variable will be defined by:

$$\varpi = Max\left\{ u_o^- \mid u_o^- \neq 1, \quad for\ efficient\ DMUs \right\}$$

- ϖ will then be used as the upper bound for the free variable u_o in model (3.4) for evaluating the vendor efficiencies.

Applying the above steps on the DMUs, we would obtain our new DMU values. These new values produces the modified VRS frontier for vendor efficiency evaluation problem with the weakly efficient frontier being modified thereby showing the real efficiency values of the DMUs in the frontier.

5.5 Empirical Study Example

In this section, we illustrate the modified model for vendor evaluation with an example by taking into account the empirical study in Chapter 3. WinQSB software was used for computing the values.

Applying the values $\varpi = 0.6597$ and $\varepsilon = 0.001$ to model (5.2) for each of the vendors with data in Table (3.1) we obtain the following results:

Table 5.1: Optimal weights and efficiency results for the modified model for vendor selection

Vendor	V_1^*	V_2^*	V_3^*	U^*	u_o^*	**Efficiency**
1	0.0010	0.0010	0.2343	0.0027	0.6597	**0.9156**
2	0.0041	0.0010	0.0010	0.0035	0.6597	**0.9979**
3	0.0032	0.0010	0.0072	0.0041	0.6597	**0.6770**
4	0.0010	0.0010	0.2467	0.0030	0.6597	**0.9627**
5	0.0033	0.0010	0.0010	0.0015	0.6597	**0.7573**
6	0.0040	0.0010	0.0010	0.0091	0	**1.0000**
7	0.0040	0.0010	0.0010	0.0032	0.6597	**0.9570**
8	0.0034	0.0010	0.0055	0.0019	0.6597	**0.8007**
9	0.0037	0.0010	0.0010	0.0023	0.6597	**0.8331**
10	0.0036	0.0010	0.0041	0.0023	0.6597	**0.8477**
11	0.0010	0.2367	0.0010	0.0088	0	**0.9805**
12	0.0036	0.0010	0.0010	0.0021	0.6597	**0.8416**

The above Table 5.1 shows the optimal weights of the various inputs and outputs, as well as efficiency values of the various vendors for the case study problem implemented in the modified model 5.2 for vendor evaluation.

From Table 3.4 we found 3 efficient vendors. As discussed in the literature, there exists some vendors which can be found to be weakly efficient. For these weakly efficient vendors, some of them are not showing their true efficiency score. This is a challenge for managers as decision makers, so we suggest some modifications to address this issue.

Using the standard model in Chapter 3, some of the vendors that are found to be efficient and those vendors that compare their performances with those efficient vendors tends to change their efficiency scores. This is because the benchmarking frontier for is modified hence their true efficiencies are revealed. This modified frontier will help decision maker in easily identifying areas of improvement for the weakly efficient vendors.

From the results obtained using the case study data on the modified model for vendor performance evaluation. We can see that vendors 2 and 11 are efficient when evaluated with standard model and they are inefficient when evaluated with the modified model. This shows that they are weakly efficient in the standard model. Considering vendor 6, this vendor does not change its efficiency score. This reaffirm that the modified model only changes the efficiency of the weakly efficient vendors.

Chapter 6

CONCLUSION AND FUTURE STUDY

6.1 Conclusion

The issue of efficiency with regards to vendors received several improvements in the purchasing literature which leads to several developments in multi criteria models for vendor evaluation for selection in order to ensure that best performing vendors are employed for better services. This is due to the great role they play which have a direct impact on firms. A well-functioning approach ensures a fair results for the efficiency evaluation and selection. Efficiency measurement systems as well as the criteria considered can be different in different firms or organizations. Therefore a suitable and extensive approach is required to enclose all the services provided.

This thesis was aimed mainly on proposing a modified DEA model for vendor evaluation based on the service quality they provide so that the overall best relatively performing vendors are selected. In the modified model, the output variable was the Service Quality and the input variables were price, late deliveries and rate of rejected items. This was demonstrated with an empirical study for a PVC pipe manufacturing firm.

Firstly, vendor selection problem was introduced together with DEA as method employed indicating areas of improving the method for selecting the best performing vendor. In chapter 2, comprehensive literature review of vendor selection, its methods and quality concepts of selecting best vendor was discussed as well as details of DEA. Basic DEA model for vendor evaluation was presented in chapter 3 which was explained with an empirical study example pointing out its limitations on the weak efficiency frontier. Facet analysis and its applications as well as the restricting the free variable of the basic vendor selection model was explained in chapter 4. The limitation of the basic model was properly tackled as shown in chapter 5 by modifying the basic model for vendor efficiency evaluation, which was achieved by applying the facet analysis and the free variable value obtained in chapter 4.

DEA is robust and has vast advantages over other multiple criteria decision making methods, because unlimited number of criteria can be handled, and the evaluation methodology DEA is relatively simple and easier to apply compared to other approaches. DEA approach gives an overall idea of how well vendors are performing relatively, compared to the traditional subjective vendor evaluation techniques, so long as data is available for evaluation which should be gathered for a while in order to have better picture of the services provided by the performing vendors.

This modified DEA approach allows the purchasing managers to evaluate effectively each vendor’s true performance score relative to the performance of other vendors in the cross-roads thereby giving far better results compared to the basic model employed. The results obtained from the Modified DEA model can be used in order to reduce the number of efficient vendors because some vendors tend to be efficient when evaluated with the standard DEA model, hence the modified model determine better benchmarks for comparing with inefficient vendors. This will tremendously help analysts and management based on the strategic purchasing objective and also to provide improvement targets for vendors.

6.2 Suggestions for Future Study

The sensitivity issue was in accordance with adjustments in the efficiency frontier. Some efficient DMUs become inefficient thus changes their position along the frontier, hence the sensitivity analysis of the efficient DMUs will be an interesting problem for further research.

Due to the large number of criteria a DEA methodology can hold, it is possible to consider the second level of Service Quality Dimensions as vendor's evaluation criteria in order to obtain more reliable results, though analyzing the model might be more complicated.

Finally, in this thesis work, the modification process was basically described in terms of input based DEA models. Similar developments can hold for the output based DEA models.

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APPENDICES

Appendix A: Optimal Coding Solutions of Basic Model for Vendor Selection Summarized in Table 3.2

Vendor 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	-40.0000	at bound	-M	40.0000
2	Late Deliveries %	0.0010	0	0	-4.0000	at bound	-M	4.0000
3	Rate of Rejected %	0.2343	0	0	0	basic	-0.4138	M
4	Service Quality	0.0010	95.0000	0.0950	-15.0000	at bound	-M	110.0000
5	Uo	0.8460	1.0000	0.8460	0	basic	0.8636	1.0000
	Objective	Function	(Max.) =	0.9410	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.3000	M
2	Vendor 1	-0.0590	<=	0	0.0590	0	-0.0590	M
3	Vendor 2	-0.4707	<=	0	0.4707	0	-0.4707	M
4	Vendor 3	-0.8520	<=	0	0.8520	0	-0.8520	M
5	Vendor 4	-0.0170	<=	0	0.0170	0	-0.0170	M
6	Vendor 5	-1.2687	<=	0	1.2687	0	-1.2687	M
7	Vendor 6	0	<=	0	0	1.0000	-0.8460	0.0170
8	Vendor 7	-0.2513	<=	0	0.2513	0	-0.2513	M
9	Vendor 8	-0.3093	<=	0	0.3093	0	-0.3093	M
10	Vendor 9	-0.7610	<=	0	0.7610	0	-0.7610	M
11	Vendor 10	-0.2923	<=	0	0.2923	0	-0.2923	M
12	Vendor 11	-0.5017	<=	0	0.5017	0	-0.5017	M
13	Vendor 12	-1.2237	<=	0	1.2237	0	-1.2237	M

Vendor 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0041	0	0	0	basic	0	M
2	Late Deliveries %	0.0010	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.0010	0	0	0	at bound	-M	0
4	Service Quality	0.0033	98.0000	0.3212	0	basic	98.0000	110.0000
5	Uo	0.6788	1.0000	0.6788	0	basic	0.8909	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.3440	M
2	Vendor 1	-0.2185	<=	0	0.2185	0	-0.2185	M
3	Vendor 2	0.0000	<=	0	0	1.0000	-0.0740	0.0273
4	Vendor 3	-0.5319	<=	0	0.5319	0	-0.5319	M
5	Vendor 4	-0.0554	<=	0	0.0554	0	-0.0554	M
6	Vendor 5	-0.3455	<=	0	0.3455	0	-0.3455	M
7	Vendor 6	0.0000	<=	0	0	0	-0.0273	0.0831
8	Vendor 7	-0.0433	<=	0	0.0433	0	-0.0433	M
9	Vendor 8	-0.2699	<=	0	0.2699	0	-0.2699	M
10	Vendor 9	-0.2034	<=	0	0.2034	0	-0.2034	M
11	Vendor 10	-0.1877	<=	0	0.1877	0	-0.1877	M
12	Vendor 11	-0.1401	<=	0	0.1401	0	-0.1401	M
13	Vendor 12	-0.1923	<=	0	0.1923	0	-0.1923	M

Vendor 3

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0031	0	0	0	basic	-17.5000	10.0000
2	Late Deliveries %	0.0010	0	0	-0.2121	at bound	-M	0.2121
3	Rate of Rejected %	0.0096	0	0	0	basic	-0.2000	2.0000
4	Service Quality	0.0010	12.0000	0.0120	-87.0909	at bound	-M	99.0909
5	Uo	0.7037	1.0000	0.7037	0	basic	0.1211	1.0000
	Objective	Function	(Max.) =	0.7157	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8030	0.4300	M
2	Vendor 1	-0.1441	<=	0	0.1441	0	-0.1441	M
3	Vendor 2	0.0000	<=	0	0	0.9091	-0.2340	0.0190
4	Vendor 3	-0.2843	<=	0	0.2843	0	-0.2843	M
5	Vendor 4	-0.0276	<=	0	0.0276	0	-0.0276	M
6	Vendor 5	-0.2409	<=	0	0.2409	0	-0.2409	M
7	Vendor 6	0.0000	<=	0	0	0.0909	-0.0190	0.0264
8	Vendor 7	-0.0160	<=	0	0.0160	0	-0.0160	M
9	Vendor 8	-0.1591	<=	0	0.1591	0	-0.1591	M
10	Vendor 9	-0.1295	<=	0	0.1295	0	-0.1295	M
11	Vendor 10	-0.1102	<=	0	0.1102	0	-0.1102	M
12	Vendor 11	-0.1267	<=	0	0.1267	0	-0.1267	M
13	Vendor 12	-0.1534	<=	0	0.1534	0	-0.1534	M

Vendor 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	-5.0000	at bound	-M	5.0000
2	Late Deliveries %	0.0010	0	0	-2.0000	at bound	-M	2.0000
3	Rate of Rejected %	0.2467	0	0	0	basic	-0.0588	M
4	Service Quality	0.0010	100.0000	0.1000	-10.0000	at bound	-M	110.0000
5	Uo	0.8830	1.0000	0.8830	0	basic	0.9091	1.0000
	Objective	Function	(Max.) =	0.9830	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.2630	M
2	Vendor 1	-0.0590	<=	0	0.0590	0	-0.0590	M
3	Vendor 2	-0.4953	<=	0	0.4953	0	-0.4953	M
4	Vendor 3	-0.8890	<=	0	0.8890	0	-0.8890	M
5	Vendor 4	-0.0170	<=	0	0.0170	0	-0.0170	M
6	Vendor 5	-1.3303	<=	0	1.3303	0	-1.3303	M
7	Vendor 6	0.0000	<=	0	0	1.0000	-0.8830	0.0170
8	Vendor 7	-0.2637	<=	0	0.2637	0	-0.2637	M
9	Vendor 8	-0.3217	<=	0	0.3217	0	-0.3217	M
10	Vendor 9	-0.7980	<=	0	0.7980	0	-0.7980	M
11	Vendor 10	-0.3047	<=	0	0.3047	0	-0.3047	M
12	Vendor 11	-0.5263	<=	0	0.5263	0	-0.5263	M
13	Vendor 12	-1.2853	<=	0	1.2853	0	-1.2853	M

Vendor 5

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0033	0	0	0	basic	-55.6250	M
2	Late Deliveries %	0.0010	0	0	-5.1356	at bound	-M	5.1356
3	Rate of Rejected %	0.0010	0	0	-1.5085	at bound	-M	1.5085
4	Service Quality	0.0010	65.0000	0.0650	-33.0000	at bound	-M	98.0000
5	Uo	0.7089	1.0000	0.7089	0	basic	0.6633	1.0000
	Objective	Function	(Max.) =	0.7739	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8136	0.4310	M
2	Vendor 1	-0.1714	<=	0	0.1714	0	-0.1714	M
3	Vendor 2	0.0000	<=	0	0	1.0000	-0.7089	0.0193
4	Vendor 3	-0.2877	<=	0	0.2877	0	-0.2877	M
5	Vendor 4	-0.0479	<=	0	0.0479	0	-0.0479	M
6	Vendor 5	-0.2261	<=	0	0.2261	0	-0.2261	M
7	Vendor 6	-0.0193	<=	0	0.0193	0	-0.0193	M
8	Vendor 7	-0.0256	<=	0	0.0256	0	-0.0256	M
9	Vendor 8	-0.1768	<=	0	0.1768	0	-0.1768	M
10	Vendor 9	-0.1269	<=	0	0.1269	0	-0.1269	M
11	Vendor 10	-0.1249	<=	0	0.1249	0	-0.1249	M
12	Vendor 11	-0.1358	<=	0	0.1358	0	-0.1358	M
13	Vendor 12	-0.1345	<=	0	0.1345	0	-0.1345	M

Vendor 6

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0040	0	0	0	basic	0	M
2	Late Deliveries %	0.0010	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.0010	0	0	0	at bound	-M	0
4	Service Quality	0.0091	110.0000	1.0000	0	basic	110.0000	110.0000
5	Uo	0	1.0000	0	0	at bound	-M	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.2560	M
2	Vendor 1	-0.2994	<=	0	0.2994	0	-0.2994	M
3	Vendor 2	-0.0713	<=	0	0.0713	0	-0.0713	M
4	Vendor 3	-1.0937	<=	0	1.0937	0	-1.0937	M
5	Vendor 4	-0.1128	<=	0	0.1128	0	-0.1128	M
6	Vendor 5	-0.6000	<=	0	0.6000	0	-0.6000	M
7	Vendor 6	0.0000	<=	0	0	1.0000	-0.8900	0.0801
8	Vendor 7	-0.1488	<=	0	0.1488	0	-0.1488	M
9	Vendor 8	-0.4795	<=	0	0.4795	0	-0.4795	M
10	Vendor 9	-0.4037	<=	0	0.4037	0	-0.4037	M
11	Vendor 10	-0.3532	<=	0	0.3532	0	-0.3532	M
12	Vendor 11	-0.1230	<=	0	0.1230	0	-0.1230	M
13	Vendor 12	-0.3337	<=	0	0.3337	0	-0.3337	M

Vendor 7

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	-66.2500	66.2500
2	Late Deliveries %	0.0010	0	0	-4.0000	at bound	-M	4.0000
3	Rate of Rejected %	0.0132	0	0	0	basic	-1.0816	1.0816
4	Service Quality	0.0010	92.0000	0.0920	-12.0000	at bound	-M	104.0000
5	Uo	0.8920	1.0000	0.8920	0	basic	0.8846	1.0000
	Objective	Function	(Max.) =	0.9840	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.3540	M
2	Vendor 1	-0.1725	<=	0	0.1725	0	-0.1725	M
3	Vendor 2	0.0000	<=	0	0	0.5000	-0.3760	0.0264
4	Vendor 3	-0.3305	<=	0	0.3305	0	-0.3305	M
5	Vendor 4	-0.0312	<=	0	0.0312	0	-0.0312	M
6	Vendor 5	-0.2906	<=	0	0.2906	0	-0.2906	M
7	Vendor 6	0	<=	0	0	0.5000	-0.0264	0.0301
8	Vendor 7	-0.0160	<=	0	0.0160	0	-0.0160	M
9	Vendor 8	-0.1875	<=	0	0.1875	0	-0.1875	M
10	Vendor 9	-0.1543	<=	0	0.1543	0	-0.1543	M
11	Vendor 10	-0.1279	<=	0	0.1279	0	-0.1279	M
12	Vendor 11	-0.1587	<=	0	0.1587	0	-0.1587	M
13	Vendor 12	-0.1889	<=	0	0.1889	0	-0.1889	M

Vendor 8

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0033	0	0	0	basic	-36.2500	116.2500
2	Late Deliveries %	0.0010	0	0	-2.2131	at bound	-M	2.2131
3	Rate of Rejected %	0.0107	0	0	0	basic	-1.6316	0.5088
4	Service Quality	0.0010	73.0000	0.0730	-34.1475	at bound	-M	107.1475
5	Uo	0.7595	1.0000	0.7595	0	basic	0.6813	1.0000
	Objective	Function	(Max.) =	0.8325	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8689	0.4090	M
2	Vendor 1	-0.1525	<=	0	0.1525	0	-0.1525	M
3	Vendor 2	0.0000	<=	0	0	0.2377	-0.3565	0.0207
4	Vendor 3	-0.2980	<=	0	0.2980	0	-0.2980	M
5	Vendor 4	-0.0287	<=	0	0.0287	0	-0.0287	M
6	Vendor 5	-0.2556	<=	0	0.2556	0	-0.2556	M
7	Vendor 6	0	<=	0	0	0.7623	-0.0207	0.0278
8	Vendor 7	-0.0160	<=	0	0.0160	0	-0.0160	M
9	Vendor 8	-0.1675	<=	0	0.1675	0	-0.1675	M
10	Vendor 9	-0.1368	<=	0	0.1368	0	-0.1368	M
11	Vendor 10	-0.1154	<=	0	0.1154	0	-0.1154	M
12	Vendor 11	-0.1362	<=	0	0.1362	0	-0.1362	M
13	Vendor 12	-0.1639	<=	0	0.1639	0	-0.1639	M

Vendor 9

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0037	0	0	0	basic	-15.0000	M
2	Late Deliveries %	0.0010	0	0	-2.3333	at bound	-M	2.3333
3	Rate of Rejected %	0.0010	0	0	-0.3333	at bound	-M	0.3333
4	Service Quality	0.0010	75.0000	0.0750	-23.0000	at bound	-M	98.0000
5	Uo	0.7882	1.0000	0.7882	0	basic	0.7653	1.0000
	Objective	Function	(Max.) =	0.8632	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8889	0.3900	M
2	Vendor 1	-0.1880	<=	0	0.1880	0	-0.1880	M
3	Vendor 2	0	<=	0	0	1.0000	-0.7882	0.0226
4	Vendor 3	-0.3076	<=	0	0.3076	0	-0.3076	M
5	Vendor 4	-0.0529	<=	0	0.0529	0	-0.0529	M
6	Vendor 5	-0.2443	<=	0	0.2443	0	-0.2443	M
7	Vendor 6	-0.0226	<=	0	0.0226	0	-0.0226	M
8	Vendor 7	-0.0273	<=	0	0.0273	0	-0.0273	M
9	Vendor 8	-0.1917	<=	0	0.1917	0	-0.1917	M
10	Vendor 9	-0.1368	<=	0	0.1368	0	-0.1368	M
11	Vendor 10	-0.1348	<=	0	0.1348	0	-0.1348	M
12	Vendor 11	-0.1507	<=	0	0.1507	0	-0.1507	M
13	Vendor 12	-0.1461	<=	0	0.1461	0	-0.1461	M

Vendor 10

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0035	0	0	0	basic	-47.5000	97.5000
2	Late Deliveries %	0.0010	0	0	-7.9655	at bound	-M	7.9655
3	Rate of Rejected %	0.0114	0	0	0	basic	-1.4444	0.7037
4	Service Quality	0.0010	81.0000	0.0810	-25.0690	at bound	-M	106.0690
5	Uo	0.7998	1.0000	0.7998	0	basic	0.7637	1.0000
	Objective	Function	(Max.) =	0.8808	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.9138	0.3940	M
2	Vendor 1	-0.1586	<=	0	0.1586	0	-0.1586	M
3	Vendor 2	0	<=	0	0	0.3276	-0.3610	0.0224
4	Vendor 3	-0.3078	<=	0	0.3078	0	-0.3078	M
5	Vendor 4	-0.0294	<=	0	0.0294	0	-0.0294	M
6	Vendor 5	-0.2663	<=	0	0.2663	0	-0.2663	M
7	Vendor 6	0	<=	0	0	0.6724	-0.0224	0.0285
8	Vendor 7	-0.0160	<=	0	0.0160	0	-0.0160	M
9	Vendor 8	-0.1736	<=	0	0.1736	0	-0.1736	M
10	Vendor 9	-0.1421	<=	0	0.1421	0	-0.1421	M
11	Vendor 10	-0.1192	<=	0	0.1192	0	-0.1192	M
12	Vendor 11	-0.1430	<=	0	0.1430	0	-0.1430	M
13	Vendor 12	-0.1715	<=	0	0.1715	0	-0.1715	M

Vendor 11

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	0	at bound	-M	0
2	Late Deliveries %	0.2367	0	0	0	basic	0	M
3	Rate of Rejected %	0.0010	0	0	0	at bound	-M	0
4	Service Quality	0.0185	112.0000	2.0720	0	basic	110.0000	112.0000
5	Uo	-1.0720	1.0000	-1.0720	0	at bound	-M	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.2930	2.0720
2	Vendor 1	-1.2642	<=	0	1.2642	0	-1.2642	M
3	Vendor 2	-0.2140	<=	0	0.2140	0	-0.2140	M
4	Vendor 3	-2.1027	<=	0	2.1027	0	-2.1027	M
5	Vendor 4	-0.6633	<=	0	0.6633	0	-0.6633	M
6	Vendor 5	-2.5392	<=	0	2.5392	0	-2.5392	M
7	Vendor 6	0.0000	<=	0	0	0	-M	0.0191
8	Vendor 7	-1.2757	<=	0	1.2757	0	-1.2757	M
9	Vendor 8	-1.4305	<=	0	1.4305	0	-1.4305	M
10	Vendor 9	-1.3805	<=	0	1.3805	0	-1.3805	M
11	Vendor 10	-2.6875	<=	0	2.6875	0	-2.6875	M
12	Vendor 11	0	<=	0	0	1.0000	-0.0195	M
13	Vendor 12	-0.9658	<=	0	0.9658	0	-0.9658	M

Vendor 12

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0036	0	0	0	basic	-68.1250	M
2	Late Deliveries %	0.0010	0	0	-1.3636	at bound	-M	1.3636
3	Rate of Rejected %	0.0010	0	0	-1.9818	at bound	-M	1.9818
4	Service Quality	0.0010	85.0000	0.0850	-13.0000	at bound	-M	98.0000
5	Uo	0.7714	1.0000	0.7714	0	basic	0.8673	1.0000
	Objective	Function	(Max.) =	0.8564	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8727	0.3980	M
2	Vendor 1	-0.1845	<=	0	0.1845	0	-0.1845	M
3	Vendor 2	0.0000	<=	0	0	1.0000	-0.7714	0.0219
4	Vendor 3	-0.3033	<=	0	0.3033	0	-0.3033	M
5	Vendor 4	-0.0518	<=	0	0.0518	0	-0.0518	M
6	Vendor 5	-0.2404	<=	0	0.2404	0	-0.2404	M
7	Vendor 6	-0.0219	<=	0	0.0219	0	-0.0219	M
8	Vendor 7	-0.0269	<=	0	0.0269	0	-0.0269	M
9	Vendor 8	-0.1885	<=	0	0.1885	0	-0.1885	M
10	Vendor 9	-0.1347	<=	0	0.1347	0	-0.1347	M
11	Vendor 10	-0.1327	<=	0	0.1327	0	-0.1327	M
12	Vendor 11	-0.1475	<=	0	0.1475	0	-0.1475	M
13	Vendor 12	-0.1436	<=	0	0.1436	0	-0.1436	M

Appendix B: Optimal Coding Solutions of Standard BCC Model summarized in Table 4.1

Vendor 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	0	at bound	-M	0
2	Late Deliveries %	0	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.3333	0	0	0	basic	0	M
4	Service Quality	0	95.0000	0	0	basic	95.0000	110.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0.8636	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	0	<=	0	0	1.0000	-0.1364	0
3	Vendor 2	-0.6667	<=	0	0.6667	0	-0.6667	M
4	Vendor 3	-1.0000	<=	0	1.0000	0	-1.0000	M
5	Vendor 4	0	<=	0	0	0	0	M
6	Vendor 5	-1.6667	<=	0	1.6667	0	-1.6667	M
7	Vendor 6	0	<=	0	0	0	0	0
8	Vendor 7	-0.3333	<=	0	0.3333	0	-0.3333	M
9	Vendor 8	-0.3333	<=	0	0.3333	0	-0.3333	M
10	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
11	Vendor 10	-0.3333	<=	0	0.3333	0	-0.3333	M
12	Vendor 11	-0.6667	<=	0	0.6667	0	-0.6667	M
13	Vendor 12	-1.6667	<=	0	1.6667	0	-1.6667	M

Vendor 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0042	0	0	0	basic	0	M
2	Late Deliveries %	0	0	0	0	at bound	-M	0
3	Rate of Rejected %	0	0	0	0	at bound	-M	0
4	Service Quality	0.0035	98.0000	0.3403	0	basic	98.0000	110.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.8909	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	-0.2188	<=	0	0.2188	0	-0.2188	M
3	Vendor 2	0	<=	0	0	1.0000	-0.0720	0.0278
4	Vendor 3	-0.5486	<=	0	0.5486	0	-0.5486	M
5	Vendor 4	-0.0556	<=	0	0.0556	0	-0.0556	M
6	Vendor 5	-0.3438	<=	0	0.3438	0	-0.3438	M
7	Vendor 6	0.0000	<=	0	0	0	-0.0417	0.0808
8	Vendor 7	-0.0417	<=	0	0.0417	0	-0.0417	M
9	Vendor 8	-0.2743	<=	0	0.2743	0	-0.2743	M
10	Vendor 9	-0.2049	<=	0	0.2049	0	-0.2049	M
11	Vendor 10	-0.1840	<=	0	0.1840	0	-0.1840	M
12	Vendor 11	-0.1389	<=	0	0.1389	0	-0.1389	M
13	Vendor 12	-0.1910	<=	0	0.1910	0	-0.1910	M

Vendor 3

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0030	0	0	0	basic	-17.5000	10.0000
2	Late Deliveries %	0	0	0	-0.2121	at bound	-M	0.2121
3	Rate of Rejected %	0.0152	0	0	0	basic	-0.2000	2.0000
4	Service Quality	0	12.0000	0	-87.0909	at bound	-M	99.0909
5	Uo	0.8030	1.0000	0.8030	0	basic	0.1211	1.0000
	Objective	Function	(Max.) =	0.8030	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8030	0	M
2	Vendor 1	-0.1212	<=	0	0.1212	0	-0.1212	M
3	Vendor 2	0.0000	<=	0	0	0.9091	-0.3333	0
4	Vendor 3	-0.1970	<=	0	0.1970	0	-0.1970	M
5	Vendor 4	-0.0152	<=	0	0.0152	0	-0.0152	M
6	Vendor 5	-0.2121	<=	0	0.2121	0	-0.2121	M
7	Vendor 6	0	<=	0	0	0.0909	-0.0333	0
8	Vendor 7	0.0000	<=	0	0	0	0	M
9	Vendor 8	-0.1212	<=	0	0.1212	0	-0.1212	M
10	Vendor 9	-0.1061	<=	0	0.1061	0	-0.1061	M
11	Vendor 10	-0.0758	<=	0	0.0758	0	-0.0758	M
12	Vendor 11	-0.1364	<=	0	0.1364	0	-0.1364	M
13	Vendor 12	-0.1515	<=	0	0.1515	0	-0.1515	M

Vendor 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	-5.0000	at bound	-M	5.0000
2	Late Deliveries %	0	0	0	-2.0000	at bound	-M	2.0000
3	Rate of Rejected %	0.3333	0	0	0	basic	-0.0588	M
4	Service Quality	0	100.0000	0	-10.0000	at bound	-M	110.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0.9091	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	0	<=	0	0	0	0	M
3	Vendor 2	-0.6667	<=	0	0.6667	0	-0.6667	M
4	Vendor 3	-1.0000	<=	0	1.0000	0	-1.0000	M
5	Vendor 4	0	<=	0	0	0	0	M
6	Vendor 5	-1.6667	<=	0	1.6667	0	-1.6667	M
7	Vendor 6	0	<=	0	0	1.0000	-1.0000	0
8	Vendor 7	-0.3333	<=	0	0.3333	0	-0.3333	M
9	Vendor 8	-0.3333	<=	0	0.3333	0	-0.3333	M
10	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
11	Vendor 10	-0.3333	<=	0	0.3333	0	-0.3333	M
12	Vendor 11	-0.6667	<=	0	0.6667	0	-0.6667	M
13	Vendor 12	-1.6667	<=	0	1.6667	0	-1.6667	M

Vendor 5

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0034	0	0	0	basic	-55.6250	M
2	Late Deliveries %	0	0	0	-5.1356	at bound	-M	5.1356
3	Rate of Rejected %	0	0	0	-1.5085	at bound	-M	1.5085
4	Service Quality	0	65.0000	0	-33.0000	at bound	-M	98.0000
5	Uo	0.8136	1.0000	0.8136	0	basic	0.6633	1.0000
	Objective	Function	(Max.) =	0.8136	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8136	0	M
2	Vendor 1	-0.1695	<=	0	0.1695	0	-0.1695	M
3	Vendor 2	0	<=	0	0	1.0000	-0.8136	0.0169
4	Vendor 3	-0.2034	<=	0	0.2034	0	-0.2034	M
5	Vendor 4	-0.0508	<=	0	0.0508	0	-0.0508	M
6	Vendor 5	-0.1864	<=	0	0.1864	0	-0.1864	M
7	Vendor 6	-0.0339	<=	0	0.0339	0	-0.0339	M
8	Vendor 7	-0.0169	<=	0	0.0169	0	-0.0169	M
9	Vendor 8	-0.1525	<=	0	0.1525	0	-0.1525	M
10	Vendor 9	-0.1017	<=	0	0.1017	0	-0.1017	M
11	Vendor 10	-0.1017	<=	0	0.1017	0	-0.1017	M
12	Vendor 11	-0.1525	<=	0	0.1525	0	-0.1525	M
13	Vendor 12	-0.1186	<=	0	0.1186	0	-0.1186	M

Vendor 6

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0040	0	0	0	basic	0	M
2	Late Deliveries %	0	0	0	0	at bound	-M	0
3	Rate of Rejected %	0	0	0	0	at bound	-M	0
4	Service Quality	0.0091	110.0000	1.0000	0	basic	110.0000	110.0000
5	Uo	0	1.0000	0	0	at bound	-M	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	-0.2964	<=	0	0.2964	0	-0.2964	M
3	Vendor 2	-0.0691	<=	0	0.0691	0	-0.0691	M
4	Vendor 3	-1.0909	<=	0	1.0909	0	-1.0909	M
5	Vendor 4	-0.1109	<=	0	0.1109	0	-0.1109	M
6	Vendor 5	-0.5891	<=	0	0.5891	0	-0.5891	M
7	Vendor 6	0.0000	<=	0	0	1.0000	-1.0000	0.0776
8	Vendor 7	-0.1436	<=	0	0.1436	0	-0.1436	M
9	Vendor 8	-0.4764	<=	0	0.4764	0	-0.4764	M
10	Vendor 9	-0.3982	<=	0	0.3982	0	-0.3982	M
11	Vendor 10	-0.3436	<=	0	0.3436	0	-0.3436	M
12	Vendor 11	-0.1218	<=	0	0.1218	0	-0.1218	M
13	Vendor 12	-0.3273	<=	0	0.3273	0	-0.3273	M

Vendor 7

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	-66.2500	66.2500
2	Late Deliveries %	0	0	0	-4.0000	at bound	-M	4.0000
3	Rate of Rejected %	0.0189	0	0	0	basic	-1.0816	1.0816
4	Service Quality	0	92.0000	0	-12.0000	at bound	-M	104.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0.8846	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	-0.1509	<=	0	0.1509	0	-0.1509	M
3	Vendor 2	0	<=	0	0	0.5000	-0.5000	0
4	Vendor 3	-0.2453	<=	0	0.2453	0	-0.2453	M
5	Vendor 4	-0.0189	<=	0	0.0189	0	-0.0189	M
6	Vendor 5	-0.2642	<=	0	0.2642	0	-0.2642	M
7	Vendor 6	0	<=	0	0	0.5000	-0.0408	0
8	Vendor 7	0	<=	0	0	0	0	M
9	Vendor 8	-0.1509	<=	0	0.1509	0	-0.1509	M
10	Vendor 9	-0.1321	<=	0	0.1321	0	-0.1321	M
11	Vendor 10	-0.0943	<=	0	0.0943	0	-0.0943	M
12	Vendor 11	-0.1698	<=	0	0.1698	0	-0.1698	M
13	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Vendor 8

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0033	0	0	0	basic	-36.2500	116.2500
2	Late Deliveries %	0	0	0	-2.2131	at bound	-M	2.2131
3	Rate of Rejected %	0.0164	0	0	0	basic	-1.6316	0.5088
4	Service Quality	0	73.0000	0	-34.1475	at bound	-M	107.1475
5	Uo	0.8689	1.0000	0.8689	0	basic	0.6813	1.0000
	Objective	Function	(Max.) =	0.8689	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8689	0	M
2	Vendor 1	-0.1311	<=	0	0.1311	0	-0.1311	M
3	Vendor 2	0.0000	<=	0	0	0.2377	-0.5000	0
4	Vendor 3	-0.2131	<=	0	0.2131	0	-0.2131	M
5	Vendor 4	-0.0164	<=	0	0.0164	0	-0.0164	M
6	Vendor 5	-0.2295	<=	0	0.2295	0	-0.2295	M
7	Vendor 6	0	<=	0	0	0.7623	-0.0351	0
8	Vendor 7	0	<=	0	0	0	0	M
9	Vendor 8	-0.1311	<=	0	0.1311	0	-0.1311	M
10	Vendor 9	-0.1148	<=	0	0.1148	0	-0.1148	M
11	Vendor 10	-0.0820	<=	0	0.0820	0	-0.0820	M
12	Vendor 11	-0.1475	<=	0	0.1475	0	-0.1475	M
13	Vendor 12	-0.1639	<=	0	0.1639	0	-0.1639	M

Vendor 9

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0037	0	0	0	basic	-15.0000	M
2	Late Deliveries %	0	0	0	-2.3333	at bound	-M	2.3333
3	Rate of Rejected %	0	0	0	-0.3333	at bound	-M	0.3333
4	Service Quality	0	75.0000	0	-23.0000	at bound	-M	98.0000
5	Uo	0.8889	1.0000	0.8889	0	basic	0.7653	1.0000
	Objective	Function	(Max.) =	0.8889	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8889	0	M
2	Vendor 1	-0.1852	<=	0	0.1852	0	-0.1852	M
3	Vendor 2	0	<=	0	0	1.0000	-0.8889	0.0185
4	Vendor 3	-0.2222	<=	0	0.2222	0	-0.2222	M
5	Vendor 4	-0.0556	<=	0	0.0556	0	-0.0556	M
6	Vendor 5	-0.2037	<=	0	0.2037	0	-0.2037	M
7	Vendor 6	-0.0370	<=	0	0.0370	0	-0.0370	M
8	Vendor 7	-0.0185	<=	0	0.0185	0	-0.0185	M
9	Vendor 8	-0.1667	<=	0	0.1667	0	-0.1667	M
10	Vendor 9	-0.1111	<=	0	0.1111	0	-0.1111	M
11	Vendor 10	-0.1111	<=	0	0.1111	0	-0.1111	M
12	Vendor 11	-0.1667	<=	0	0.1667	0	-0.1667	M
13	Vendor 12	-0.1296	<=	0	0.1296	0	-0.1296	M

Vendor 10

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0034	0	0	0	basic	-47.5000	97.5000
2	Late Deliveries %	0	0	0	-7.9655	at bound	-M	7.9655
3	Rate of Rejected %	0.0172	0	0	0	basic	-1.4444	0.7037
4	Service Quality	0	81.0000	0	-25.0690	at bound	-M	106.0690
5	Uo	0.9138	1.0000	0.9138	0	basic	0.7637	1.0000
	Objective	Function	(Max.) =	0.9138	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.9138	0	M
2	Vendor 1	-0.1379	<=	0	0.1379	0	-0.1379	M
3	Vendor 2	0	<=	0	0	0.3276	-0.5000	0
4	Vendor 3	-0.2241	<=	0	0.2241	0	-0.2241	M
5	Vendor 4	-0.0172	<=	0	0.0172	0	-0.0172	M
6	Vendor 5	-0.2414	<=	0	0.2414	0	-0.2414	M
7	Vendor 6	0	<=	0	0	0.6724	-0.0370	0
8	Vendor 7	0	<=	0	0	0	0	M
9	Vendor 8	-0.1379	<=	0	0.1379	0	-0.1379	M
10	Vendor 9	-0.1207	<=	0	0.1207	0	-0.1207	M
11	Vendor 10	-0.0862	<=	0	0.0862	0	-0.0862	M
12	Vendor 11	-0.1552	<=	0	0.1552	0	-0.1552	M
13	Vendor 12	-0.1724	<=	0	0.1724	0	-0.1724	M

Vendor 11

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0005	0	0	0	basic	0	0
2	Late Deliveries %	0.2849	0	0	0	basic	0	0
3	Rate of Rejected %	0	0	0	0	at bound	-M	0
4	Service Quality	0.0089	112.0000	1.0000	0	basic	112.0000	112.0000
5	Uo	0	1.0000	0	0	at bound	-M	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0	M
2	Vendor 1	-1.2938	<=	0	1.2938	0	-1.2938	M
3	Vendor 2	-0.1020	<=	0	0.1020	0	-0.1020	M
4	Vendor 3	-1.1854	<=	0	1.1854	0	-1.1854	M
5	Vendor 4	-0.6616	<=	0	0.6616	0	-0.6616	M
6	Vendor 5	-2.4188	<=	0	2.4188	0	-2.4188	M
7	Vendor 6	0.0000	<=	0	0	0	-0.0179	0.0794
8	Vendor 7	-1.2976	<=	0	1.2976	0	-1.2976	M
9	Vendor 8	-1.2028	<=	0	1.2028	0	-1.2028	M
10	Vendor 9	-1.1773	<=	0	1.1773	0	-1.1773	M
11	Vendor 10	-2.8329	<=	0	2.8329	0	-2.8329	M
12	Vendor 11	0.0000	<=	0	0	1.0000	-0.1069	0.0182
13	Vendor 12	-0.8057	<=	0	0.8057	0	-0.8057	M

Vendor 12

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0036	0	0	0	basic	-68.1250	M
2	Late Deliveries %	0	0	0	-1.3636	at bound	-M	1.3636
3	Rate of Rejected %	0	0	0	-1.9818	at bound	-M	1.9818
4	Service Quality	0	85.0000	0	-13.0000	at bound	-M	98.0000
5	Uo	0.8727	1.0000	0.8727	0	basic	0.8673	1.0000
	Objective	Function	(Max.) =	0.8727	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8727	0	M
2	Vendor 1	-0.1818	<=	0	0.1818	0	-0.1818	M
3	Vendor 2	0	<=	0	0	1.0000	-0.8727	0.0182
4	Vendor 3	-0.2182	<=	0	0.2182	0	-0.2182	M
5	Vendor 4	-0.0545	<=	0	0.0545	0	-0.0545	M
6	Vendor 5	-0.2000	<=	0	0.2000	0	-0.2000	M
7	Vendor 6	-0.0364	<=	0	0.0364	0	-0.0364	M
8	Vendor 7	-0.0182	<=	0	0.0182	0	-0.0182	M
9	Vendor 8	-0.1636	<=	0	0.1636	0	-0.1636	M
10	Vendor 9	-0.1091	<=	0	0.1091	0	-0.1091	M
11	Vendor 10	-0.1091	<=	0	0.1091	0	-0.1091	M
12	Vendor 11	-0.1636	<=	0	0.1636	0	-0.1636	M
13	Vendor 12	-0.1273	<=	0	0.1273	0	-0.1273	M

Appendix C: Optimal Coding Solutions of free variable for efficient vendors of standard BCC Model summarized in Table 4.2

Upper bound for Vendor 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	0	at bound	-M	0
2	Late Deliveries %	0	0	0	0	basic	0	M
3	Rate of Rejected %	0.3333	0	0	0	basic	0	0
4	Service Quality	0	0	0	-95.0000	at bound	-M	95.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	1.0000	1.0000	1.0000
2	VXo	1.0000	=	1.0000	0	0	1.0000	1.0000
3	Vendor 1	0	<=	0	0	0	0	M
4	Vendor 2	-0.6667	<=	0	0.6667	0	-0.6667	M
5	Vendor 3	-1.0000	<=	0	1.0000	0	-1.0000	M
6	Vendor 4	0	<=	0	0	0	0	0
7	Vendor 5	-1.6667	<=	0	1.6667	0	-1.6667	M
8	Vendor 6	0	<=	0	0	0	0	M
9	Vendor 7	-0.3333	<=	0	0.3333	0	-0.3333	M
10	Vendor 8	-0.3333	<=	0	0.3333	0	-0.3333	M
11	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
12	Vendor 10	-0.3333	<=	0	0.3333	0	-0.3333	M
13	Vendor 11	-0.6667	<=	0	0.6667	0	-0.6667	M
14	Vendor 12	-1.6667	<=	0	1.6667	0	-1.6667	M

Lower bound for Vendor 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	253.3333	at bound	-253.3333	M
2	Late Deliveries %	0	0	0	25.3333	at bound	-25.3333	M
3	Rate of Rejected %	0.3333	0	0	0	basic	-M	2.6207
4	Service Quality	0	0	0	0	basic	-M	95.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	1.0000	M
	Objective	Function	(Min.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	7.3333	0.8636	1.0000
2	VXo	1.0000	=	1.0000	0	-6.3333	1.0000	1.1579
3	Vendor 1	0	<=	0	0	0	0	M
4	Vendor 2	-0.6667	<=	0	0.6667	0	-0.6667	M
5	Vendor 3	-1.0000	<=	0	1.0000	0	-1.0000	M
6	Vendor 4	0	<=	0	0	0	0	M
7	Vendor 5	-1.6667	<=	0	1.6667	0	-1.6667	M
8	Vendor 6	0	<=	0	0	-6.3333	0	0
9	Vendor 7	-0.3333	<=	0	0.3333	0	-0.3333	M
10	Vendor 8	-0.3333	<=	0	0.3333	0	-0.3333	M
11	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
12	Vendor 10	-0.3333	<=	0	0.3333	0	-0.3333	M
13	Vendor 11	-0.6667	<=	0	0.6667	0	-0.6667	M
14	Vendor 12	-1.6667	<=	0	1.6667	0	-1.6667	M

Upper bound for Vendor 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	0	0
2	Late Deliveries %	0	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.0189	0	0	0	basic	0	M
4	Service Quality	0	0	0	-98.0000	at bound	-M	98.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	1.0000	0.6000	1.0000
2	VXo	1.0000	=	1.0000	0	0	1.0000	1.6667
3	Vendor 1	-0.1509	<=	0	0.1509	0	-0.1509	M
4	Vendor 2	0	<=	0	0	0	0	M
5	Vendor 3	-0.2453	<=	0	0.2453	0	-0.2453	M
6	Vendor 4	-0.0189	<=	0	0.0189	0	-0.0189	M
7	Vendor 5	-0.2642	<=	0	0.2642	0	-0.2642	M
8	Vendor 6	0	<=	0	0	0	-0.0417	0
9	Vendor 7	0	<=	0	0	0	0	M
10	Vendor 8	-0.1509	<=	0	0.1509	0	-0.1509	M
11	Vendor 9	-0.1321	<=	0	0.1321	0	-0.1321	M
12	Vendor 10	-0.0943	<=	0	0.0943	0	-0.0943	M
13	Vendor 11	-0.1698	<=	0	0.1698	0	-0.1698	M
14	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Lower bound for Vendor 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0042	0	0	0	basic	-M	81.6667
2	Late Deliveries %	0	0	0	1.0208	at bound	-1.0208	M
3	Rate of Rejected %	0	0	0	18.0347	at bound	-18.0347	M
4	Service Quality	0.0035	0	0	0	basic	-M	98.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	1.0000	M
	Objective	Function	(Min.) =	0.6597	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	9.1667	0.9280	1.0000
2	VXo	1.0000	=	1.0000	0	-8.5069	1.0000	1.0776
3	Vendor 1	-0.2188	<=	0	0.2188	0	-0.2188	M
4	Vendor 2	0	<=	0	0	0	0	M
5	Vendor 3	-0.5486	<=	0	0.5486	0	-0.5486	M
6	Vendor 4	-0.0556	<=	0	0.0556	0	-0.0556	M
7	Vendor 5	-0.3438	<=	0	0.3438	0	-0.3438	M
8	Vendor 6	0.0000	<=	0	0	-8.1667	-0.0417	0.0808
9	Vendor 7	-0.0417	<=	0	0.0417	0	-0.0417	M
10	Vendor 8	-0.2743	<=	0	0.2743	0	-0.2743	M
11	Vendor 9	-0.2049	<=	0	0.2049	0	-0.2049	M
12	Vendor 10	-0.1840	<=	0	0.1840	0	-0.1840	M
13	Vendor 11	-0.1389	<=	0	0.1389	0	-0.1389	M
14	Vendor 12	-0.1910	<=	0	0.1910	0	-0.1910	M

Upper bound for Vendor 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0042	0	0	0	basic	-M	81.6667
2	Late Deliveries %	0	0	0	1.0208	at bound	-1.0208	M
3	Rate of Rejected %	0	0	0	18.0347	at bound	-18.0347	M
4	Service Quality	0.0035	0	0	0	basic	-M	98.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	1.0000	M
	Objective	Function	(Min.) =	0.6597	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	9.1667	0.9280	1.0000
2	VXo	1.0000	=	1.0000	0	-8.5069	1.0000	1.0776
3	Vendor 1	-0.2188	<=	0	0.2188	0	-0.2188	M
4	Vendor 2	0	<=	0	0	0	0	M
5	Vendor 3	-0.5486	<=	0	0.5486	0	-0.5486	M
6	Vendor 4	-0.0556	<=	0	0.0556	0	-0.0556	M
7	Vendor 5	-0.3438	<=	0	0.3438	0	-0.3438	M
8	Vendor 6	0.0000	<=	0	0	-8.1667	-0.0417	0.0808
9	Vendor 7	-0.0417	<=	0	0.0417	0	-0.0417	M
10	Vendor 8	-0.2743	<=	0	0.2743	0	-0.2743	M
11	Vendor 9	-0.2049	<=	0	0.2049	0	-0.2049	M
12	Vendor 10	-0.1840	<=	0	0.1840	0	-0.1840	M
13	Vendor 11	-0.1389	<=	0	0.1389	0	-0.1389	M
14	Vendor 12	-0.1910	<=	0	0.1910	0	-0.1910	M

Lower bound for Vendor 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	50.0000	at bound	-50.0000	M
2	Late Deliveries %	0	0	0	20.0000	at bound	-20.0000	M
3	Rate of Rejected %	0.3333	0	0	0	basic	-M	0.5882
4	Service Quality	0	0	0	0	basic	-M	100.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	1.0000	M
	Objective	Function	(Min.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	11.0000	0.9091	1.0000
2	VXo	1.0000	=	1.0000	0	-10.0000	1.0000	1.1000
3	Vendor 1	0	<=	0	0	0	0	M
4	Vendor 2	-0.6667	<=	0	0.6667	0	-0.6667	M
5	Vendor 3	-1.0000	<=	0	1.0000	0	-1.0000	M
6	Vendor 4	0	<=	0	0	0	0	M
7	Vendor 5	-1.6667	<=	0	1.6667	0	-1.6667	M
8	Vendor 6	0	<=	0	0	-10.0000	0	0.1000
9	Vendor 7	-0.3333	<=	0	0.3333	0	-0.3333	M
10	Vendor 8	-0.3333	<=	0	0.3333	0	-0.3333	M
11	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
12	Vendor 10	-0.3333	<=	0	0.3333	0	-0.3333	M
13	Vendor 11	-0.6667	<=	0	0.6667	0	-0.6667	M
14	Vendor 12	-1.6667	<=	0	1.6667	0	-1.6667	M

Upper bound for Vendor 6

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	0	1,619.4450
2	Late Deliveries %	0	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.0189	0	0	0	basic	0	0
4	Service Quality	0	0	0	-110.0000	at bound	-M	110.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	1.0000	0.9600	1.0000
2	VXo	1.0000	=	1.0000	0	0	1.0000	1.0417
3	Vendor 1	-0.1509	<=	0	0.1509	0	-0.1509	M
4	Vendor 2	0	<=	0	0	0	-0.6667	0
5	Vendor 3	-0.2453	<=	0	0.2453	0	-0.2453	M
6	Vendor 4	-0.0189	<=	0	0.0189	0	-0.0189	M
7	Vendor 5	-0.2642	<=	0	0.2642	0	-0.2642	M
8	Vendor 6	0	<=	0	0	0	0	M
9	Vendor 7	0	<=	0	0	0	0	M
10	Vendor 8	-0.1509	<=	0	0.1509	0	-0.1509	M
11	Vendor 9	-0.1321	<=	0	0.1321	0	-0.1321	M
12	Vendor 10	-0.0943	<=	0	0.0943	0	-0.0943	M
13	Vendor 11	-0.1698	<=	0	0.1698	0	-0.1698	M
14	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Lower bound for Vendor 6

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	7,241.6670	at bound	-7,241.6670	M
2	Late Deliveries %	0	0	0	110.0000	at bound	-110.0000	M
3	Rate of Rejected %	0.3333	0	0	0	basic	-M	86.9000
4	Service Quality	0.3333	0	0	0	basic	-M	110.0000
5	Uo	-35.6667	1.0000	-35.6667	0	at bound	1.0000	M
	Objective	Function	(Min.) =	-35.6667	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	56.0000	-M	1.0000
2	VXo	1.0000	=	1.0000	0	-91.6667	1.0000	M
3	Vendor 1	-5.0000	<=	0	5.0000	0	-5.0000	M
4	Vendor 2	-4.6667	<=	0	4.6667	0	-4.6667	M
5	Vendor 3	-33.6667	<=	0	33.6667	0	-33.6667	M
6	Vendor 4	-3.3333	<=	0	3.3333	0	-3.3333	M
7	Vendor 5	-16.6667	<=	0	16.6667	0	-16.6667	M
8	Vendor 6	0	<=	0	0	0	0	M
9	Vendor 7	-6.3333	<=	0	6.3333	0	-6.3333	M
10	Vendor 8	-12.6667	<=	0	12.6667	0	-12.6667	M
11	Vendor 9	-12.6667	<=	0	12.6667	0	-12.6667	M
12	Vendor 10	-10.0000	<=	0	10.0000	0	-10.0000	M
13	Vendor 11	0	<=	0	0	-55.0000	-0.6485	M
14	Vendor 12	-10.0000	<=	0	10.0000	0	-10.0000	M

Upper bound for Vendor 7

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	0	0
2	Late Deliveries %	0	0	0	0	basic	0	M
3	Rate of Rejected %	0.0189	0	0	0	basic	0	0
4	Service Quality	0	0	0	-92.0000	at bound	-M	92.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	1.0000	0.4286	1.0000
2	VXo	1.0000	=	1.0000	0	0	1.0000	2.3333
3	Vendor 1	-0.1509	<=	0	0.1509	0	-0.1509	M
4	Vendor 2	0	<=	0	0	0	0	0.0394
5	Vendor 3	-0.2453	<=	0	0.2453	0	-0.2453	M
6	Vendor 4	-0.0189	<=	0	0.0189	0	-0.0189	M
7	Vendor 5	-0.2642	<=	0	0.2642	0	-0.2642	M
8	Vendor 6	0	<=	0	0	0	0	0.0235
9	Vendor 7	0	<=	0	0	0	0	M
10	Vendor 8	-0.1509	<=	0	0.1509	0	-0.1509	M
11	Vendor 9	-0.1321	<=	0	0.1321	0	-0.1321	M
12	Vendor 10	-0.0943	<=	0	0.0943	0	-0.0943	M
13	Vendor 11	-0.1698	<=	0	0.1698	0	-0.1698	M
14	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Lower bound for Vendor 7

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0038	0	0	0	basic	-1,015.8330	338.6111
2	Late Deliveries %	0	0	0	30.6667	at bound	-30.6667	M
3	Rate of Rejected %	0.0189	0	0	0	basic	-5.5283	16.5850
4	Service Quality	0	0	0	0	basic	-M	92.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	1.0000	M
	Objective	Function	(Min.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	8.6667	0.9592	1.0000
2	VXo	1.0000	=	1.0000	0	-7.6667	1.0000	1.0426
3	Vendor 1	-0.1509	<=	0	0.1509	0	-0.1509	M
4	Vendor 2	0	<=	0	0	-3.8333	0	0.0272
5	Vendor 3	-0.2453	<=	0	0.2453	0	-0.2453	M
6	Vendor 4	-0.0189	<=	0	0.0189	0	-0.0189	M
7	Vendor 5	-0.2642	<=	0	0.2642	0	-0.2642	M
8	Vendor 6	0	<=	0	0	-3.8333	0	0.0313
9	Vendor 7	0	<=	0	0	0	0	M
10	Vendor 8	-0.1509	<=	0	0.1509	0	-0.1509	M
11	Vendor 9	-0.1321	<=	0	0.1321	0	-0.1321	M
12	Vendor 10	-0.0943	<=	0	0.0943	0	-0.0943	M
13	Vendor 11	-0.1698	<=	0	0.1698	0	-0.1698	M
14	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Upper bound for Vendor 11

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0	0	0	-1,960.0000	at bound	-M	1,960.0000
2	Late Deliveries %	0.3333	0	0	0	basic	-20.6316	M
3	Rate of Rejected %	0	0	0	-112.0000	at bound	-M	112.0000
4	Service Quality	0	0	0	0	basic	-M	112.0000
5	Uo	1.0000	1.0000	1.0000	0	basic	0	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	UYo+uo	1.0000	=	1.0000	0	-55.0000	1.0000	1.0000
2	VXo	1.0000	=	1.0000	0	56.0000	1.0000	1.0000
3	Vendor 1	-1.3333	<=	0	1.3333	0	-1.3333	M
4	Vendor 2	0	<=	0	0	0	0	M
5	Vendor 3	-0.3333	<=	0	0.3333	0	-0.3333	M
6	Vendor 4	-0.6667	<=	0	0.6667	0	-0.6667	M
7	Vendor 5	-2.3333	<=	0	2.3333	0	-2.3333	M
8	Vendor 6	0	<=	0	0	56.0000	-0.0179	0
9	Vendor 7	-1.3333	<=	0	1.3333	0	-1.3333	M
10	Vendor 8	-1.0000	<=	0	1.0000	0	-1.0000	M
11	Vendor 9	-1.0000	<=	0	1.0000	0	-1.0000	M
12	Vendor 10	-3.0000	<=	0	3.0000	0	-3.0000	M
13	Vendor 11	0	<=	0	0	0	0	M
14	Vendor 12	-0.6667	<=	0	0.6667	0	-0.6667	M

Lower bound for Vendor 11

Unbounded	solution!!!	Make any of	the following	changes and	solve it again.
12-30-2016 05:01:49	Constraint	Decision Variable	Coefficient A(i,j)	Subtract More Than This From A(i,j)	Or Add More Than This To A(i,j)
	Change	the direction	of constraint	Vendor 6	

Appendix D: Optimal Coding Solutions of Modified Model for Vendor Selection summarized in Table 5.1

Vendor 1

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	-34.5455	at bound	-M	34.5455
2	Late Deliveries %	0.0010	0	0	-3.4545	at bound	-M	3.4545
3	Rate of Rejected %	0.2343	0	0	0	basic	-0.3574	M
4	Service Quality	0.0027	95.0000	0.2559	0	basic	0	110.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.8636	M
	Objective	Function	(Max.) =	0.9156				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8636	0.8137	M
2	Vendor 1	-0.0844	<=	0	0.0844	0	-0.0844	M
3	Vendor 2	-0.4910	<=	0	0.4910	0	-0.4910	M
4	Vendor 3	-1.0180	<=	0	1.0180	0	-1.0180	M
5	Vendor 4	-0.0339	<=	0	0.0339	0	-0.0339	M
6	Vendor 5	-1.3449	<=	0	1.3449	0	-1.3449	M
7	Vendor 6	0	<=	0	0	0.8636	-0.1863	0.0373
8	Vendor 7	-0.2818	<=	0	0.2818	0	-0.2818	M
9	Vendor 8	-0.3720	<=	0	0.3720	0	-0.3720	M
10	Vendor 9	-0.8203	<=	0	0.8203	0	-0.8203	M
11	Vendor 10	-0.3414	<=	0	0.3414	0	-0.3414	M
12	Vendor 11	-0.4983	<=	0	0.4983	0	-0.4983	M
13	Vendor 12	-1.2660	<=	0	1.2660	0	-1.2660	M

Vendor 2

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0041	0	0	0	basic	-8.9091	M
2	Late Deliveries %	0.0010	0	0	-0.1114	at bound	-M	0.1114
3	Rate of Rejected %	0.0010	0	0	-1.9674	at bound	-M	1.9674
4	Service Quality	0.0035	98.0000	0.3382	0	basic	0	110.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.8909	M
	Objective	Function	(Max.) =	0.9979				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.9280	0.9711	M
2	Vendor 1	-0.2211	<=	0	0.2211	0	-0.2211	M
3	Vendor 2	-0.0021	<=	0	0.0021	0	-0.0021	M
4	Vendor 3	-0.5489	<=	0	0.5489	0	-0.5489	M
5	Vendor 4	-0.0572	<=	0	0.0572	0	-0.0572	M
6	Vendor 5	-0.3533	<=	0	0.3533	0	-0.3533	M
7	Vendor 6	0.0000	<=	0	0	0.8909	-0.2696	0.0023
8	Vendor 7	-0.0465	<=	0	0.0465	0	-0.0465	M
9	Vendor 8	-0.2764	<=	0	0.2764	0	-0.2764	M
10	Vendor 9	-0.2095	<=	0	0.2095	0	-0.2095	M
11	Vendor 10	-0.1928	<=	0	0.1928	0	-0.1928	M
12	Vendor 11	-0.1398	<=	0	0.1398	0	-0.1398	M
13	Vendor 12	-0.1966	<=	0	0.1966	0	-0.1966	M

Vendor 3

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0032	0	0	0	basic	-2.0792	1.2245
2	Late Deliveries %	0.0010	0	0	-0.0257	at bound	-M	0.0257
3	Rate of Rejected %	0.0072	0	0	0	basic	-0.0245	0.2182
4	Service Quality	0.0014	12.0000	0.0173	0	basic	0	99.0909
5	Uo	0.6597	1.0000	0.6597	0	basic	0.1211	M
	Objective	Function	(Max.) =	0.6770				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.0972	0.9452	1.2139
2	Vendor 1	-0.1527	<=	0	0.1527	0	-0.1527	M
3	Vendor 2	0	<=	0	0	0.1101	-0.0484	0.0123
4	Vendor 3	-0.3230	<=	0	0.3230	0	-0.3230	M
5	Vendor 4	-0.0323	<=	0	0.0323	0	-0.0323	M
6	Vendor 5	-0.2510	<=	0	0.2510	0	-0.2510	M
7	Vendor 6	0.0000	<=	0	0	0.0110	-0.0138	0.0312
8	Vendor 7	-0.0213	<=	0	0.0213	0	-0.0213	M
9	Vendor 8	-0.1748	<=	0	0.1748	0	-0.1748	M
10	Vendor 9	-0.1387	<=	0	0.1387	0	-0.1387	M
11	Vendor 10	-0.1216	<=	0	0.1216	0	-0.1216	M
12	Vendor 11	-0.1227	<=	0	0.1227	0	-0.1227	M
13	Vendor 12	-0.1536	<=	0	0.1536	0	-0.1536	M

Vendor 4

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	-4.5455	at bound	-M	4.5455
2	Late Deliveries %	0.0010	0	0	-1.8182	at bound	-M	1.8182
3	Rate of Rejected %	0.2467	0	0	0	basic	-0.0535	M
4	Service Quality	0.0030	100.0000	0.3030	0	basic	0	110.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.9091	M
	Objective	Function	(Max.) =	0.9627				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.9091	0.7767	M
2	Vendor 1	-0.0895	<=	0	0.0895	0	-0.0895	M
3	Vendor 2	-0.5197	<=	0	0.5197	0	-0.5197	M
4	Vendor 3	-1.0879	<=	0	1.0879	0	-1.0879	M
5	Vendor 4	-0.0373	<=	0	0.0373	0	-0.0373	M
6	Vendor 5	-1.4217	<=	0	1.4217	0	-1.4217	M
7	Vendor 6	0.0000	<=	0	0	0.9091	-0.2233	0.0410
8	Vendor 7	-0.3002	<=	0	0.3002	0	-0.3002	M
9	Vendor 8	-0.3968	<=	0	0.3968	0	-0.3968	M
10	Vendor 9	-0.8691	<=	0	0.8691	0	-0.8691	M
11	Vendor 10	-0.3635	<=	0	0.3635	0	-0.3635	M
12	Vendor 11	-0.5223	<=	0	0.5223	0	-0.5223	M
13	Vendor 12	-1.3361	<=	0	1.3361	0	-1.3361	M

Vendor 5

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0033	0	0	0	basic	-36.8941	M
2	Late Deliveries %	0.0010	0	0	-3.4063	at bound	-M	3.4063
3	Rate of Rejected %	0.0010	0	0	-1.0005	at bound	-M	1.0005
4	Service Quality	0.0015	65.0000	0.0976	0	basic	0	98.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.6633	M
	Objective	Function	(Max.) =	0.7573				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.5396	0.9395	1.2018
2	Vendor 1	-0.1729	<=	0	0.1729	0	-0.1729	M
3	Vendor 2	0.0000	<=	0	0	0.6633	-0.0492	0.0118
4	Vendor 3	-0.3309	<=	0	0.3309	0	-0.3309	M
5	Vendor 4	-0.0469	<=	0	0.0469	0	-0.0469	M
6	Vendor 5	-0.2427	<=	0	0.2427	0	-0.2427	M
7	Vendor 6	-0.0133	<=	0	0.0133	0	-0.0133	M
8	Vendor 7	-0.0287	<=	0	0.0287	0	-0.0287	M
9	Vendor 8	-0.1894	<=	0	0.1894	0	-0.1894	M
10	Vendor 9	-0.1384	<=	0	0.1384	0	-0.1384	M
11	Vendor 10	-0.1334	<=	0	0.1334	0	-0.1334	M
12	Vendor 11	-0.1288	<=	0	0.1288	0	-0.1288	M
13	Vendor 12	-0.1410	<=	0	0.1410	0	-0.1410	M

Vendor 6

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0040	0	0	0	basic	0	M
2	Late Deliveries %	0.0010	0	0	0	at bound	-M	0
3	Rate of Rejected %	0.0010	0	0	0	at bound	-M	0
4	Service Quality	0.0091	110.0000	1.0000	0	basic	110.0000	M
5	Uo	0	1.0000	0	0	at bound	-M	1.0000
	Objective	Function	(Max.) =	1.0000	(Note:	Alternate	Solution	Exists!!)
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0000	0.2560	M
2	Vendor 1	-0.2994	<=	0	0.2994	0	-0.2994	M
3	Vendor 2	-0.0713	<=	0	0.0713	0	-0.0713	M
4	Vendor 3	-1.0937	<=	0	1.0937	0	-1.0937	M
5	Vendor 4	-0.1128	<=	0	0.1128	0	-0.1128	M
6	Vendor 5	-0.6000	<=	0	0.6000	0	-0.6000	M
7	Vendor 6	0.0000	<=	0	0	1.0000	-0.8900	0.0801
8	Vendor 7	-0.1488	<=	0	0.1488	0	-0.1488	M
9	Vendor 8	-0.4795	<=	0	0.4795	0	-0.4795	M
10	Vendor 9	-0.4037	<=	0	0.4037	0	-0.4037	M
11	Vendor 10	-0.3532	<=	0	0.3532	0	-0.3532	M
12	Vendor 11	-0.1230	<=	0	0.1230	0	-0.1230	M
13	Vendor 12	-0.3337	<=	0	0.3337	0	-0.3337	M

Vendor 7

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0040	0	0	0	basic	-55.4091	M
2	Late Deliveries %	0.0010	0	0	-3.4649	at bound	-M	3.4649
3	Rate of Rejected %	0.0010	0	0	-0.9046	at bound	-M	0.9046
4	Service Quality	0.0032	92.0000	0.2973	0	basic	0	110.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.8364	M
	Objective	Function	(Max.) =	0.9570				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.8534	0.9941	M
2	Vendor 1	-0.2139	<=	0	0.2139	0	-0.2139	M
3	Vendor 2	-0.0004	<=	0	0.0004	0	-0.0004	M
4	Vendor 3	-0.5225	<=	0	0.5225	0	-0.5225	M
5	Vendor 4	-0.0545	<=	0	0.0545	0	-0.0545	M
6	Vendor 5	-0.3391	<=	0	0.3391	0	-0.3391	M
7	Vendor 6	0.0000	<=	0	0	0.8364	-0.2455	0.0005
8	Vendor 7	-0.0430	<=	0	0.0430	0	-0.0430	M
9	Vendor 8	-0.2649	<=	0	0.2649	0	-0.2649	M
10	Vendor 9	-0.1998	<=	0	0.1998	0	-0.1998	M
11	Vendor 10	-0.1845	<=	0	0.1845	0	-0.1845	M
12	Vendor 11	-0.1368	<=	0	0.1368	0	-0.1368	M
13	Vendor 12	-0.1887	<=	0	0.1887	0	-0.1887	M

Vendor 8

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0034	0	0	0	basic	-24.0568	86.5944
2	Late Deliveries %	0.0010	0	0	-1.5078	at bound	-M	1.5078
3	Rate of Rejected %	0.0055	0	0	0	basic	-1.2154	0.3376
4	Service Quality	0.0019	73.0000	0.1410	0	basic	0	107.1475
5	Uo	0.6597	1.0000	0.6597	0	basic	0.6813	M
	Objective	Function	(Max.) =	0.8007				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.5920	0.8851	1.1537
2	Vendor 1	-0.1694	<=	0	0.1694	0	-0.1694	M
3	Vendor 2	0.0000	<=	0	0	0.1619	-0.3581	0.0093
4	Vendor 3	-0.3772	<=	0	0.3772	0	-0.3772	M
5	Vendor 4	-0.0384	<=	0	0.0384	0	-0.0384	M
6	Vendor 5	-0.2747	<=	0	0.2747	0	-0.2747	M
7	Vendor 6	0.0000	<=	0	0	0.5194	-0.0105	0.0400
8	Vendor 7	-0.0272	<=	0	0.0272	0	-0.0272	M
9	Vendor 8	-0.1993	<=	0	0.1993	0	-0.1993	M
10	Vendor 9	-0.1552	<=	0	0.1552	0	-0.1552	M
11	Vendor 10	-0.1387	<=	0	0.1387	0	-0.1387	M
12	Vendor 11	-0.1265	<=	0	0.1265	0	-0.1265	M
13	Vendor 12	-0.1629	<=	0	0.1629	0	-0.1629	M

Vendor 9

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0037	0	0	0	basic	-11.4796	M
2	Late Deliveries %	0.0010	0	0	-1.7857	at bound	-M	1.7857
3	Rate of Rejected %	0.0010	0	0	-0.2551	at bound	-M	0.2551
4	Service Quality	0.0023	75.0000	0.1734	0	basic	0	98.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.7653	M
	Objective	Function	(Max.) =	0.8331				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.6803	0.8554	1.0955
2	Vendor 1	-0.1919	<=	0	0.1919	0	-0.1919	M
3	Vendor 2	0.0000	<=	0	0	0.7653	-0.1285	0.0061
4	Vendor 3	-0.4203	<=	0	0.4203	0	-0.4203	M
5	Vendor 4	-0.0503	<=	0	0.0503	0	-0.0503	M
6	Vendor 5	-0.2875	<=	0	0.2875	0	-0.2875	M
7	Vendor 6	-0.0069	<=	0	0.0069	0	-0.0069	M
8	Vendor 7	-0.0352	<=	0	0.0352	0	-0.0352	M
9	Vendor 8	-0.2245	<=	0	0.2245	0	-0.2245	M
10	Vendor 9	-0.1669	<=	0	0.1669	0	-0.1669	M
11	Vendor 10	-0.1571	<=	0	0.1571	0	-0.1571	M
12	Vendor 11	-0.1323	<=	0	0.1323	0	-0.1323	M
13	Vendor 12	-0.1631	<=	0	0.1631	0	-0.1631	M

Vendor 10

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0036	0	0	0	basic	-34.9773	80.5867
2	Late Deliveries %	0.0010	0	0	-6.0829	at bound	-M	6.0829
3	Rate of Rejected %	0.0041	0	0	0	basic	-1.1939	0.5182
4	Service Quality	0.0023	81.0000	0.1880	0	basic	0	106.0690
5	Uo	0.6597	1.0000	0.6597	0	basic	0.7637	M
	Objective	Function	(Max.) =	0.8477				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.6978	0.8467	1.0995
2	Vendor 1	-0.1828	<=	0	0.1828	0	-0.1828	M
3	Vendor 2	0	<=	0	0	0.2502	-0.3634	0.0064
4	Vendor 3	-0.4206	<=	0	0.4206	0	-0.4206	M
5	Vendor 4	-0.0432	<=	0	0.0432	0	-0.0432	M
6	Vendor 5	-0.2937	<=	0	0.2937	0	-0.2937	M
7	Vendor 6	0	<=	0	0	0.5135	-0.0071	0.0446
8	Vendor 7	-0.0318	<=	0	0.0318	0	-0.0318	M
9	Vendor 8	-0.2189	<=	0	0.2189	0	-0.2189	M
10	Vendor 9	-0.1684	<=	0	0.1684	0	-0.1684	M
11	Vendor 10	-0.1523	<=	0	0.1523	0	-0.1523	M
12	Vendor 11	-0.1295	<=	0	0.1295	0	-0.1295	M
13	Vendor 12	-0.1703	<=	0	0.1703	0	-0.1703	M

Vendor 11

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0010	0	0	-35.6364	at bound	-M	35.6364
2	Late Deliveries %	0.2367	0	0	0	basic	-0.3751	M
3	Rate of Rejected %	0.0010	0	0	-2.0364	at bound	-M	2.0364
4	Service Quality	0.0088	112.0000	0.9805	0	basic	110.0000	M
5	Uo	0	1.0000	0	-0.0182	at bound	-M	1.0182
	Objective	Function	(Max.) =	0.9805				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	1.0182	0.2930	2.0720
2	Vendor 1	-1.1180	<=	0	1.1180	0	-1.1180	M
3	Vendor 2	-0.0971	<=	0	0.0971	0	-0.0971	M
4	Vendor 3	-1.1476	<=	0	1.1476	0	-1.1476	M
5	Vendor 4	-0.5659	<=	0	0.5659	0	-0.5659	M
6	Vendor 5	-2.1006	<=	0	2.1006	0	-2.1006	M
7	Vendor 6	0.0000	<=	0	0	1.0182	-0.8530	0.0191
8	Vendor 7	-1.1002	<=	0	1.1002	0	-1.1002	M
9	Vendor 8	-1.0699	<=	0	1.0699	0	-1.0699	M
10	Vendor 9	-1.0394	<=	0	1.0394	0	-1.0394	M
11	Vendor 10	-2.4049	<=	0	2.4049	0	-2.4049	M
12	Vendor 11	-0.0195	<=	0	0.0195	0	-0.0195	M
13	Vendor 12	-0.7222	<=	0	0.7222	0	-0.7222	M

Vendor 12

	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	Price	0.0036	0	0	0	basic	-59.0880	M
2	Late Deliveries %	0.0010	0	0	-1.1827	at bound	-M	1.1827
3	Rate of Rejected %	0.0010	0	0	-1.7189	at bound	-M	1.7189
4	Service Quality	0.0021	85.0000	0.1819	0	basic	0	98.0000
5	Uo	0.6597	1.0000	0.6597	0	basic	0.8673	M
	Objective	Function	(Max.) =	0.8416				
	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	VXo	1.0000	=	1.0000	0	0.7570	0.8720	1.1165
2	Vendor 1	-0.1879	<=	0	0.1879	0	-0.1879	M
3	Vendor 2	0.0000	<=	0	0	0.8673	-0.1117	0.0073
4	Vendor 3	-0.4014	<=	0	0.4014	0	-0.4014	M
5	Vendor 4	-0.0496	<=	0	0.0496	0	-0.0496	M
6	Vendor 5	-0.2780	<=	0	0.2780	0	-0.2780	M
7	Vendor 6	-0.0082	<=	0	0.0082	0	-0.0082	M
8	Vendor 7	-0.0338	<=	0	0.0338	0	-0.0338	M
9	Vendor 8	-0.2170	<=	0	0.2170	0	-0.2170	M
10	Vendor 9	-0.1609	<=	0	0.1609	0	-0.1609	M
11	Vendor 10	-0.1520	<=	0	0.1520	0	-0.1520	M
12	Vendor 11	-0.1316	<=	0	0.1316	0	-0.1316	M
13	Vendor 12	-0.1584	<=	0	0.1584	0	-0.1584	M