Efficiency Improvement with Target Setting Models in Data Envelopment Analysis: Theory and Applications

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ABSTRACT

Data Envelopment Analysis (DEA) evaluates efficiency of homogeneous units using a frontier as an approximation for production function, to identify the efficient and inefficient units. Target setting offers strategic efficiency improvement for inefficient units, thus providing ex-ante efficiency improvement strategy. To that effect, two approaches for target setting are proposed. First approach uses the most productive scale size (MPSS) hyperplane vector to guide an inefficient unit to the efficiency frontier, consequently incorporating feasible productivity improvement and enhancing efficiency. The second approach has two folds which are based on decision makers' desire. One is based on predefined inputs, which uses decision makers' input capabilities to propose efficient output targets. The other is based on predefined outputs targets by the decision maker, where desired output are presented, and the required efficient inputs are proposed. Empirical analysis with real life applications are used to validate the proposed models.

Keywords: Data Envelopment Analysis, Efficiency improvement, Target setting, Most Productive Scale Size, Predefined inputs, Predefined outputs.

ÖZ

Veri Zarflama Analizi (VZA), etkin ve verimsiz birimleri tanımlamak için bir sınır kullanarak homojen birimlerin verimliliğini değerlendirir. Hedef belirleme, etkin olmayan birimler için stratejik verimlilik artışı sağlar, böylece ön stratejis sağlar. Bu amaçla, hedef belirleme için iki yaklaşım önerilmiştir. İlk yaklaşım, verimsiz bir üniteyi verimlilik sınırına yönlendirmek için en üretken ölçek boyutu (EÜÖB) hiperdüzlem vektörünü kullanır ve sonuç olarak uygun verimlilik iyileştirmesini ve verimliliği artırır. İkinci yaklaşım, karar vericilerin isteğine dayanan iki katlı yaklaşımdır. Biri, verimli çıktı hedefleri önermek için karar vericilerin girdi yeteneklerini kullanan önceden tanımlanmış girdilere dayanmaktadır. Diğeri, karar vericinin önceden tanımlanmış çıktı hedeflerine dayanır, istenen çıktılar sunulur ve gerekli verimli girdiler önerilir. Gerçek yaşam uygulamaları ile deneysel analizler, önerilen modelleri doğrulamak için kullanılır.

Anahtar Kelimeler: Veri Zarflama Analizi, Verimlilik Artışı, Hedef Belirleme, En Üretken Ölçek Büyüklüğü, Ön tanımlı girişler, Ön tanımlı çıkışlar.

DEDICATION

To my late Dad

(Ibrahim Abubakar Dalahs)

ACKNOWLEDGMENT

First of all, I would like to thank Allah for making it possible for me to achieve this milestone. Also, I would like to express my deepest appreciation to my mum, my brothers, Dauda and Mohammed, and my sister Zainab for their support and encouragement. A special gratitude I give to my supervisor, Asst. Prof. Dr. Sahand Daneshvar for his guidance throughout my studies. I have to appreciate the invaluable support and contribution given to me by Asst. Prof. Dr. Mevhibe Hocaoglu, which led to the completion of my program. Furthermore, I would also like to acknowledge with much appreciation the crucial role of my monitoring committee members, Prof. Dr. Bela Vizvari and Asst. Prof. Dr. Hüseyin Güden for their vital contributions. Special gratitude goes to the staffs of the Department of Industrial Engineering. Last but not least, I would like to appreciate the external jury members whose comments improved the thesis.

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Chapter 1

INTRODUCTION

To better understand production theory, (Farrell, 1957) demonstrated how to measure cost inefficiency. Based on the work of Farrell, Data Envelopment Analysis (DEA) was developed by (Charnes, Cooper, & Rhodes, 1978) CCR to measure efficiency of homogeneous entities known as Decision Making Units (DMUs) operating under similar conditions. To accommodate the wide application of DEA, the term DMU generic. It represents any entity that can convert inputs to outputs.

Numerous models have been proposed for the advancement of DEA. The notable of which are (Banker, Charnes, & Cooper, 1984) BCC, which modified the constant return to scale (CRS) model of (Charnes et al., 1978) to introduce variable return to scale (VRS) in the production function, and (Chambers, Chung, & Färe, 1998; Färe & Grosskopf, 2000) by introducing the Directional Distance Function (DDF) to estimate efficiency.

DEA uses the homogeneous DMUs operating under similar conditions to form a production possibility set (PPS), where the relative efficiency of each DMU is analyzed. The PPS is an assumption of all possible combination of inputs/outputs. A frontier is projected for the efficient DMUs from the PPS, DMUs that are on the frontier are the efficient DMUs, and the inefficient DMUs are enveloped by the frontier. An efficiency score of 1 (100%) is attributed to the efficient DMUs, and less than 1(<100%) for the inefficient DMUs.

1.1 Efficiency evaluation in DEA

Relative efficiency: A DMU is considered to be fully (100%) DEA efficient if, and only if, it cannot be demonstrated through the performance of other DMUs that some of their inputs or outputs could be improved without negatively affecting others (Cooper & Tone, 1997).

To illustrate the above mentioned definition, assume there are n DMUs to be evaluated $DMU_j \left(j = 1, \ldots, n\right)$ uses m amount of inputs $x_{ij}, i = 1, \ldots, m$ to produce s amount of outputs $y_{rj}, r = 1, \ldots, s$. To evaluate a DMU_0 with input and output $\left(x_0, y_0\right)$, model 1.1 illustrates the input oriented VRS model as introduced by (Banker et al., 1984) and model 1.2 describes the output oriented VRS model. Input oriented model evaluates efficiency by keeping the output constant while the input is minimized. While output oriented model hold the input constant and the outputs are increased.

$$
\begin{aligned}
&\theta^* = \min \theta \\
&\textit{subject to} \\
&\sum\nolimits_{j=1}^{n} \lambda x_{ij} \leq \theta x_{io} \qquad i = 1, \ldots, m, \\
&\sum\nolimits_{j=1}^{n} \lambda y_{rj} \geq y_{ro} \qquad r = 1, \ldots, s, \\
&\sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \geq 0 \qquad\qquad j = 1, \ldots, n.
\end{aligned}
\tag{1.1}
$$

$$
\begin{aligned}
&\theta^* = \max \theta \\
&\textit{subject to} \\
&\sum\nolimits_{j=1}^{n} \lambda x_{ij} \le x_{io} \qquad i = 1,\ldots,m, \\
&\sum\nolimits_{j=1}^{n} \lambda y_{rj} \ge \theta y_{ro} \quad r = 1,\ldots,s, \\
&\sum\nolimits_{j=1}^{n} \lambda_j = 1 \\
&\lambda_j \ge 0 \qquad\qquad\quad j = 1,\ldots,n.
\end{aligned} \tag{1.2}
$$

By removing the third constraint of model 1.1 and model 1.2, the models becomes the CRS input oriented and output oriented model of (Charnes et al., 1978).

The direction distance function introduced by (Chambers et al., 1998) estimate the amount that can be translated as input or output vector radially to the production frontier, reference to a pre-assigned direction. The direction distance function is a general case of the (Shephard, 1970) and McFadden's gauge function (Fuss & McFadden, 2014). The direction distance function is illustrated as follows:

Let $x \in \Re_+^N$ represent a vector of inputs and $y \in \Re_+^m$ denote a vector of outputs. The technology is given by:

$T = \{(x, y)\ \textit{such that x can produce y}\}.$

The following assumptions are made for the technology.

- B1. T is closed
- B2. The inputs and outputs are freely disposed, meaning if $(x, y) \in T$ and $(x', y') \ge (x, -y)\,\textit{then}\,(x', y') \in T$
- B3. There is no free lunch: i.e. if $(x, y) \in T$ *and* $x = 0,$ *then* $y = 0.$
- B4. $(0,0) \in T$ is feasible
- B5. T is convex.

Let $g=(-g_x, g_y)$ be a "directional" vector. The directional distance function is defined as follows:

$$\vec{D}_T\left(-x, y; g_x, g_y\right) = \sup\left\{\beta : \left(-x+\beta g_x, y+\beta g_y\right) \in T\right\}.$$

Efficiency evaluation using DDF simultaneously contracts inputs and expands outputs as illustrated in Figure1.1 $g=\left(-g_x, g_y\right)$ represents the directional vector in which the input-output vector (x, y) is projected onto the frontier of T at $\left(x-\vec{D}_T g_x, y+\vec{D}_T g_y\right)$, where $\vec{D}_T$ is a step of length for projecting (x, y) on the frontier of T in $g=\left(-g_x, g_y\right)$ direction.

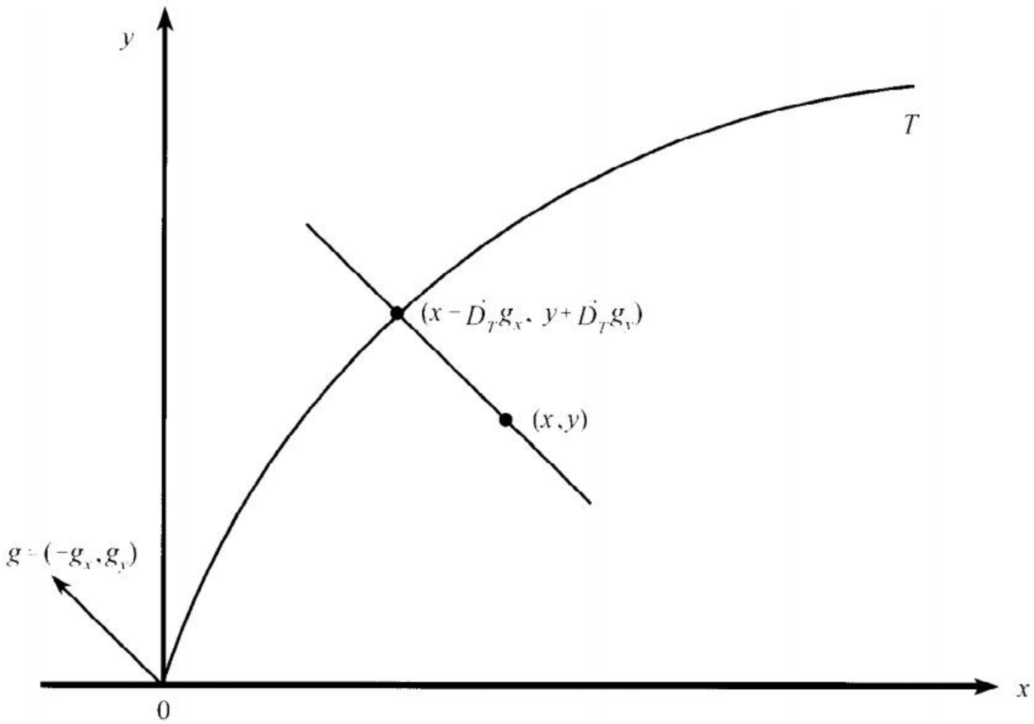

Figure 1.1: The Directional Distance Function.

1.2 Application of Data Envelopment Analysis

DEA has established itself to be salient technique in Management science/Operations Research. Its application as a performance evaluation tool extends to both the private and public sector, such as, natural resources (water, energy land, and food), hospitals, education, banks, insurance, policies, information technology (IT), transportation, supply chain, and tourism, among others. For example, the efficiency of water-energy-food nexus of China was evaluated using DEA by (Li, Huang, & Li, 2016). Efficiency of public vs private hospitals was evaluated by (Guerrini, Romano, Campedelli, Moggi, & Leardini, 2018). A detail description of DEA in education was presented by (Thanassoulis et al., 2016). The efficiency of higher education institution in Europe and United States was estimated by (Wolszczak-Derlacz, 2017). The efficiency of Turkish industry was evaluated by (Bal & Gölcükcü, 2002). Also, the United Kingdom's commercial banks performance was analyzed using DEA by (Ouenniche & Carrales, 2018). (Nourani, Chandran, Kweh, & Lu, 2018) used DEA to measure the human, physical and structural capital efficiency of insurance companies. Efficiency of cross-border healthcare policy was evaluated by (Ibrahim, Hocaoglu, Numan, & Daneshvar, 2018). The sustainability of Airport and its efficiency was evaluated by (Carlucci, Cirà, & Coccorese, 2018). (Emrouznejad, Cabanda, & Gholami, 2010) introduced a method for measuring information communication and technology (ICT)-Opportunity index for countries using DEA. (Ibrahim & Daneshvar, 2017) evaluated the supply chain efficiency of pharmaceutical companies in India by applying DEA. The regional tourism efficiency of China was measured by (Chaabouni, 2018).

The above mentioned applications of DEA are some of the performance assessment capabilities that makes it a robust management tool, and gives it added advantage over other performance evaluation methods.

An extensional advantage of DEA is its ability to suggest efficiency improvement strategies or targets post efficiency evaluation. (Färe & Grosskopf, 2012) mentioned that DEA can be utilized as an ex-ante management tool, as oppose to ex-post or wait and see method. To achieve that, targets needs to be set for the inefficient DMUs on the efficiency frontier.

1.3 Motivation of the study

Conventional efficiency improvement in standard DEA models is carried out in two forms, input minimization and output maximization. The input minimization requires holding the output constant while the input is minimized until it reaches the efficiency frontier (See DMU E^{*} Figure 1.2), while output maximization entails holding the input constant while the output it maximized until it reaches the efficiency frontier (see DMU E^{**} Figure 1.2). Both approaches are highly impractical and likely to be infeasible for the decision maker.

Another standard technique used is the radial projection approach. The radial projection approach improves the inputs/outputs of the inefficient DMU proportionally until it reaches the frontier of the PPS. However, this sometimes moves the DMU to the weak part of the frontier, thus proposing a weak efficiency improvement for the inefficient DMU, which is not necessarily efficient (Korhonen, Dehnokhalaji, & Nasrabadi, 2018).

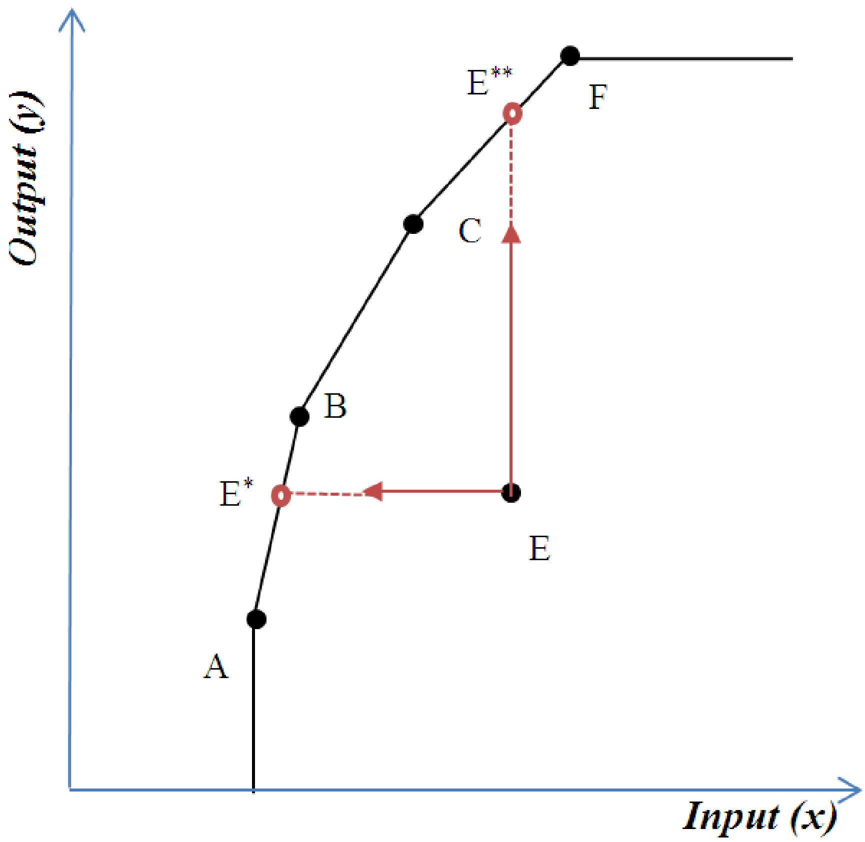

Figure 1.2: Conventional efficiency improvement methods

In this thesis, we aim to offer alternative efficiency improvement options that require less drastic modifications and are feasible for the decision maker. In addition, other performance improvement characteristics such as, productivity consideration in efficiency improvement and decision makers' desires in the form of predefined targets are explored.

1.4 Conceptual framework

The aim of this research is to explore other improvement direction to the efficiency frontier for inefficient DMUs. The efficiency frontier has multiple characteristics, therefore, projecting an inefficient DMU to a part of the efficiency frontier gives the inefficient DMU the characteristics of that section of the frontier.

An important section of the frontier is the hyperplane where the most productive DMUs are identified, known as the Most Productive Scale Size (MPSS) hyperplane. These are the efficient DMUs that produce maximum outputs from the consumed inputs. Using the properties of these DMUs to set targets for the inefficient DMUs should also produce better outputs for the inefficient DMUs. Therefore, we propose using the MPSS hyperplane vectors to guide the inefficient DMUs to the frontier as an alternative for efficiency improvement.

Based on efficiency improvement literature, decision makers' capabilities/capacity or desires have not gained much research focus. Most studies impose performance improvement without the decision makers' information/input. However, these are important factors that should be incorporated into target setting for efficiency improvement. Therefore, we propose models that accommodate predefined inputs/output targets of the decision maker.

To achieve the aims of the study, the remainder of the thesis is organized as follows: Chapter 2 presents a literature review of studies that have proposed models with the aim of setting target or efficiency improvement. Chapter 3 discusses the proposed method of target setting using MPSS vectors with empirical examples on Nigeria's electricity industry and production lines of a beverage producing company. Chapter 4 then presents target setting models that accommodate predefined inputs/outputs of decision makers', supported with numerical example and case studies on water, energy, land, and food nexus (WELF-Nexus), cross-border healthcare in Europe and a poultry chain. Conclusions of the thesis with future study are presented in chapter 5.

Chapter 2

LITERATURE REVIEW

2.1 Advances in DEA

Over the past couple of decades, there have been significant progress in DEA, both in methodology and application. Given the wide range of DEA utilization, theoretical development to mitigate new huddles and enhance DEA have emerged. Most of the advances are based on the standard CCR model of (Charnes et al., 1978), BCC model of (Banker et al., 1984) or DDF model of (Chambers et al., 1998).

Some prominent contributions to DEA literature are as follows. Weight restrictions proposed to optimize efficiency evaluation contributes enormously to managerial application, given that DEA was developed based on the assumption of common weights for all variables before evaluation, thus, allowing the model to allocate the weights, which sometimes gives zero weights to some variables. Contributions such as (Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997; Dyson & Thanassoulis, 1988) help improve efficiency evaluation discrimination by weight allocation in efficiency. To increase discrimination in DEA, Podinovski & Thanassoulis (2007) suggest weight restriction as a practical solution.

The inputs and outputs used in DEA are often considered to be neutral in terms signs (-/+). Production systems such as healthcare and industrial systems have shown to produce negative outputs, which was a problem in DEA models. (Emrouznejad, Anouze, & Thanassoulis, 2010; Portela, Thanassoulis, & Simpson, 2004) proposed models to accommodate negative data in DEA. In addition, ranking of DMUs is an interesting aspect of DEA. The contribution of research such as (Andersen & Petersen, 1993) made ranking DMUs easy. To mitigate the drawbacks of Anderson and Peterson, ranking models such as (Johnson & McGinnis, 2009) have contributed significantly.

Composite indicators which are often index/ratio variables are important in efficiency analysis, as they cover a wide range of factors that provides a robust definition for efficiency. It was shown that ratio/index variables are problematic in DEA, as it does not conform to the convexity assumption. (Emrouznejad & Amin, 2009) introduced models to accommodate ratio/index variables for efficiency evaluation. However, their model requires the numerators and denominators of the variables to be known, which may not often be the case. In situation were the numerators and denominators are unknown, (Podinovski, Olesen, & Sarrico, 2017) developed a model to account that.

The existence of multiple efficiency improvement options on the frontier prompted researches to develop methodological contribution for efficiency improvement. The integration of DEA and multi-objective linear programming (MOLP) which was inspired by (Golany, 1988) when trying to find efficient solution to map out interactive ways for efficiency improvement created a new theme for performance improvement in DEA. In-depth literature on advances in DEA are presented in (Emrouznejad & Yang, 2018; Fukuyama & Weber, 2017; Tone, 2017).

2.2 Overview of efficiency improvement models

After efficiency evaluation, the efficient and inefficient DMUs are distinguished by their efficiency scores. The efficient DMUs are on the frontier, and the inefficient DMUs are below the frontier. The inefficient DMUs are required to be projected onto the frontier for them to be considered efficient. It is widely acknowledged that the conventional efficiency improvement methods are sometimes impractical, hence the necessity for alternative efficiency improvement options. Numerous studies have proposed methods for improving inefficient DMUs or target setting, however, this area of DEA has not received appropriate research attention.

When a DMU is inefficient, a target on the frontier is projected for the DMU. This may be an existing or virtual target. The radial projection is a standard technique for finding a target by proportionally improving the inputs/outputs of the DMU until it reaches the boundary of the PPS frontier. The radial projection target sometimes proposes a weak efficient target which is not necessarily efficient (Cooper, Seiford, & Zhu, 2011). Unfortunately, there is no distinction between the efficiency score of the weak efficient and efficient target points. To guarantee an efficient target, the sum of slack variables of the inputs-outputs variables is inserted into the objective function, to lexicographically guarantee efficiency of the solution which is no longer a target point on the frontier but a radial projection (Korhonen et al., 2018).

The radial projection is a special case of the general DDF of (Chambers et al., 1998) which proposes a target on the boundary for an inefficient DMU. (Chambers et al., 1998) introduced an improvement direction which reflects real input usage or output production. However, the DDF suffer similar drawbacks of projecting a weak efficient target like the radial projection. (Asmild & Pastor, 2010) proposed an extension to account for technical inefficiency and overcome the problem of projecting weak efficient DMUs.

A more straight forward approach for finding an efficient target based on radial projection was introduced by (Korhonen et al., 2018). They employed a lexicographic method which finds a final solution using a stepwise procedure. The first step projects the DMU onto the boundary using radial projection, and if it is not efficient, it is further projected using the sub-vector of the original projection vector. This is repeated until the final projection is a coordinate on the efficiency frontier.

To support planning and management control, an assessment that integrates past performance and future planning targets with decision makers' desire is vital (Cook & Green, 2004). Incorporating decision makers' preference in efficiency improvement is imperative for implementing strategic performance improvement. Some DEA models have been proposed to actively involve decision makers' in setting targets. (Golany, 1988) introduced an interactive model incorporating both DEA and multi-objective linear programming (MOLP) model to allow a DMU allocate inputs and choose the most suitable set of outputs from a set of multiple points on the frontier.

(J.-B. Yang, Wong, Xu, & Stewart, 2009) developed a MOLP made up of three equivalent models (super ideal point model, the ideal point model and the shortest distance model) to assess performance and set target with MOLP. The super ideal point model was shown to be identical to the dual of output oriented DEA model (Cooper, Seiford, & Zhu, 2004). The method of (J.-B. Yang et al., 2009) is a radial model which solves *n* linear programming problem and projects the DMUs onto the frontier.

(Malekmohammadi, Lotfi, & Jaafar, 2011) proposed a non-radial super ideal method, identical to a target model that simultaneously reduce total inputs and increase total outputs. The method solves only one mathematical programming problem as oppose to *n* linear programming problem by (J.-B. Yang et al., 2009).

Another commonly used approach that include decision makers' preference in efficiency improvement is weight restriction (Halme, Joro, Korhonen, Salo, & Wallenius, 1999). (Thanassoulis & Dyson, 1992) considered decision makers' preference by proposing a weight-based general preference structure model, in which specific weights are set for a selected subset of preferred inputs and outputs by the decision maker. In the model, higher priority targets are satisfied before the low priority target levels.

A dynamic interactive DEA (IDEA) three step model was proposed by (Post & Spronk, 1999). They stated that pre-emptive preferences can be integrated in sequence (i.e. lexicographic) optimization method. However, no model was presented to support the notion.

(Lins, Angulo-Meza, & Da Silva, 2004) proposed a multi-objective ratio optimization and multi-objective target optimization (MOTO) model that suggest efficient coordinates on the frontier for the decision maker to choose based on preference.

(Sebastián Lozano & Villa, 2009) presented two multi-objective target setting approach. One allows for articulation of decision makers' preferences, and the other is a lexicographic technique that solve sequential models using weights to improve inputs and outputs. Both methods are incorporated with analytical hierarchy method (AHP) to facilitate decision makers' preferences.

The stepwise preference information for efficiency improvement was originally proposed by (Seiford & Zhu, 2003). It is a "context-dependent" DEA model that improves efficiency by successive levels. The method has extensive merit because it can identify ambiguous decision makers.

(Suzuki, Nijkamp, Rietveld, & Pels, 2010) proposed a unique distance function minimization (DFM) model for generating (non-radial) performance improvement projection. Their approach does not utilize a value judgement of the DMU, but allows the data to select its direction. The approach is based on a generalized distance function using Euclidean distance metric in weighted space. Overall, they presented multi-objective quadratic programming (MOQP) model to improve efficiency.

Alternative consideration for efficiency improvement is finding the closest target for the inefficient DMU. This has made considerable contribution to DEA literature. The argument here is that, the closest target requires less effort for improving efficiency (Ramón, Ruiz, & Sirvent, 2018).

(S Lozano & Villa, 2005) developed a sequential targets within close proximity of an inefficient DMU. A unique mixed-integer zero-one model was developed by

(Aparicio, Ruiz, & Sirvent, 2007) that directly suggest a close target in a single step. Least-norm projection was used by (Frei & Harker, 1999) to move a DMU to the frontier. (Baek & Lee, 2009) used the work of (Frei & Harker, 1999) to obtain the shortest target to the strong part of the frontier. (Jesus T Pastor & Aparicio, 2010) demonstrated that the work of (Baek & Lee, 2009) lacks monotonicity. A linear bi-level programming approach was developed by (Jahanshahloo, Vakili, & Zarepisheh, 2012) to find the closest target and shorted distance to the strong efficiency frontier by different norms. The use of DDF was proposed as a direction for improvement by (Zofio, Pastor, & Aparicio, 2013).

A problem in using the closest target as an efficiency improvement option for an inefficient DMU is finding a well-defined target among the closest target. To mitigate this drawback, (Aparicio & Pastor, 2013) utilized the full dimensional efficient facets (FDEFs) to satisfy the requirement of a well-defined target.

A centralized model was developed by (Fang, 2015) to set target for an inefficient DMU. Using the DDF as a base model, the direction of marginal productivity was proposed as a direction for efficiency improvement and solution to the capacity adjustment problem by (Lee, 2016).

Similar to the method of (Jesús T Pastor & Ruiz, 2007), (Diabat, Shetty, & Pakkala, 2015) introduced a mixed-integer linear programming problem to minimize the distance between a similar efficient DMU and an inefficient DMU. The use of ideal points to determine the range at which an inefficient DMU can be improved was introduced by (He, Xu, Chen, & Zhu, 2016). When the closest targets are unattainable, (Ramón et al., 2018) proposed a two-step benchmarking method using the context of dependent DEA.

Efficiency improvement based on DDF has been explored by previous studies such as (Chung, Färe, & Grosskopf, 1997; Fare, Grosskopf, & Kokkelenberg, 1989; Färe, Grosskopf, Lovell, & Pasurka, 1989). The use of output oriented measures, input oriented measures and hyperbolic measures to improve efficiency was illustrated by (Färe, Grosskopf, & Zaim, 2002; Kuosmanen, 2005).

The simultaneous improvement of inputs and outputs using hyperbolic efficiency measure was introduced by (Johnson & McGinnis, 2009). The use of allocative efficiency benchmark with DDF was proposed by (Zofío & Prieto, 2006). Based on the work of (Zofío & Prieto, 2006), (Lee, 2014) proposed moving in the direction of marginal profit maximization which focuses more on future planning as a strategy for improving efficiency.

The above mentioned studies are some of the major breakthroughs in DEA development and target setting studies. However, other studies may exist that offer similar contribution to DEA literature

Chapter 3

TARGET SETTING USING MOST PRODUCTIVE SCALE SIZE (MPSS) VECTORS

In this chapter, target setting for efficiency improvement using MPSS vectors is discussed. The proposed method in this chapter emphasizes on efficiency improvement with possibility of optimal feasible productivity. The premise of this concept is as follows:

- MPSS hyperplane is the part of the frontier that holds the most productive DMUS. Therefore, target setting of inefficient DMUs guided by the MPSS hyperplane offers the possibility of parallel productivity improvement in addition to efficiency improvement. Figure 3.1 shows the MPSS hyperplane on the efficiency frontier.
- In cases where multiple MPSS hyperplane are observed, multiple efficient targets can be proposed, which will enable versatility in efficiency improvement.
- Less drastic modification of the inefficient DMUs are possible using the MPSS hyperplane vectors.

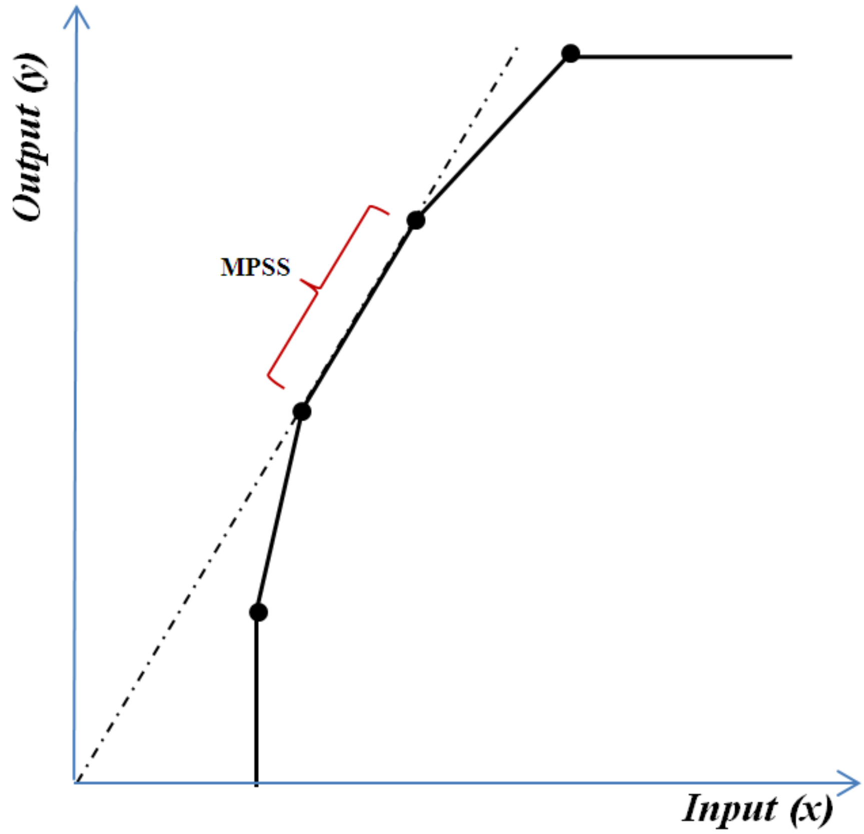

Figure 3.1: MPSS region of the frontier

3.1 Most Productive Scale Size

The MPSS DMU is an optimal size DMU in the PPS. MPSS is directly related to the return to scale (RTS) concept where expansion or contraction of DMUs with increasing or decreasing return to scale is determined. RTS in DEA has three subscales: increasing return to scale (IRTS), constant return to scale (CRS) and decreasing return to scale (DRTS). The MPSS concept is based on the comparison of average productiveness, where maximizing the average productivity of an operating unit is the objective, by increasing the scale size if increasing return to scale (IRTS) is favourable, maintaining the scale size if CRS favourable, or decreasing the scale size if decreasing return to scale (DRTS) is prevailing. (Banker, 1984) introduced the MPSS concept using the CCR linear programming function to identify the MPSS pattern. Subsequently, (Cooper, Thompson, & Thrall, 1996b) developed the BCC based method with fractional objective function to identify the MPSS DMUs. In a situation of stochastic data, (Khodabakhshi, 2009) introduced a method for identifying MPSS patterns. From a pessimistic point of view, (Y.-M. Wang & Lan, 2013) measured MPSS. To solve capacity dilemma in production systems (Lee, 2016) used MPSS to simultaneously address both demand fulfillment and Economic scale size problem.

To identify MPSS pattern in a PPS, consider n DMUs. Where each DMU_j uses m inputs x_{ij} $(i=1,\ldots,m)$ to produce s outputs y_{rj} $(r=1,\ldots,s)$. The general PPS is defined as follows:

$$T=\left\{(x,y)\in \mathbb{R}_{+}^{m+s} \,\middle|\, y \text{ can be produced from } x\right\}. \tag{3.1}$$

(Charnes et al., 1978) introduced the CRS PPS as follows:

$$T_c=\left\{(x,y)\in \mathbb{R}_{+}^{m+s} \,\middle|\, X\lambda\le x, Y\lambda\ge y, \lambda\ge 0\right\}. \tag{3.2}$$

From the foregoing PPS, the input oriented CCR envelopment model to evaluate efficiency of DMU_0 is written as:

$$\theta^{CCR}=\min\left\{\theta \,\middle|\, X\lambda+s^{-}=\theta x_0, Y\lambda-s^{+}=y_0, \lambda\ge 0\right\}. \tag{3.3}$$

(Banker et al., 1984) introduced VRS PPS by eliminating the ray unboundedness postulate from the CCR postulates and presented the following PPS:

$$T_v=\left\{(x,y)\in \mathbb{R}_{+}^{m+s} \,\middle|\, X\lambda\le x, Y\lambda\ge y, e\lambda=1, \lambda\ge 0\right\}. \tag{3.4}$$

The vector is the sum of all components equal to one. The BCC input oriented model to evaluate efficiency of DMU_0 under the PPS T_v is as follows:

$$\theta^{BCC}=\min\left\{\theta \,\middle|\, X\lambda+s^{-}=\theta x_0, Y\lambda-s^{+}=y_0, e\lambda=1, \lambda\ge 0\right\}. \tag{3.5}$$

(Banker, 1984) define MPSS as follows: (x_0, y_0) is MPSS if

$$\forall \beta, \alpha \geq 0, (\beta x_0, \alpha y_0) \in T \Rightarrow \beta \geq \alpha. \tag{3.6}$$

From the above definition, if inputs are proportionally changed by β for a specific MPSS, then the output can be proportionally changed by β at most.

A model for identifying MPSS DMUs was introduced by (Cooper, Thompson, & Thrall, 1996a) based on (Banker, 1984). They provided a necessary condition for DMU_0 with input X_0 and output Y_0 to be MPSS as $Max\ \beta/\alpha = 1$, in which case it will be constant return to scale.

$$\begin{aligned}
&Max \quad \beta/\alpha \\
&subject\ to \\
&\qquad \beta y_0 \leq \sum_{j=1}^{n} y_j \lambda_j, \\
&\qquad \alpha x_0 \leq \sum_{j=1}^{n} x_j \lambda_j, \\
&\qquad\quad 1 \leq \sum_{j=1}^{n} \lambda_j, \\
&\qquad 0 \leq \beta, \alpha \quad \lambda_j,\ j = 1, \ldots n.
\end{aligned} \tag{3.7}$$

3.2 Development of MPSS target setting model

The optimal weight (v^*, u^*) of inputs and outputs are produced using the multiplier side of DEA models. These weights can be considered as unit price vectors of the variables. Figure 3.2 illustrates the weights vectors of MPSS DMUs C and B. These weights are used to construct the vectors $(-t_x, t_y)$ which will be called MPSS vectors.

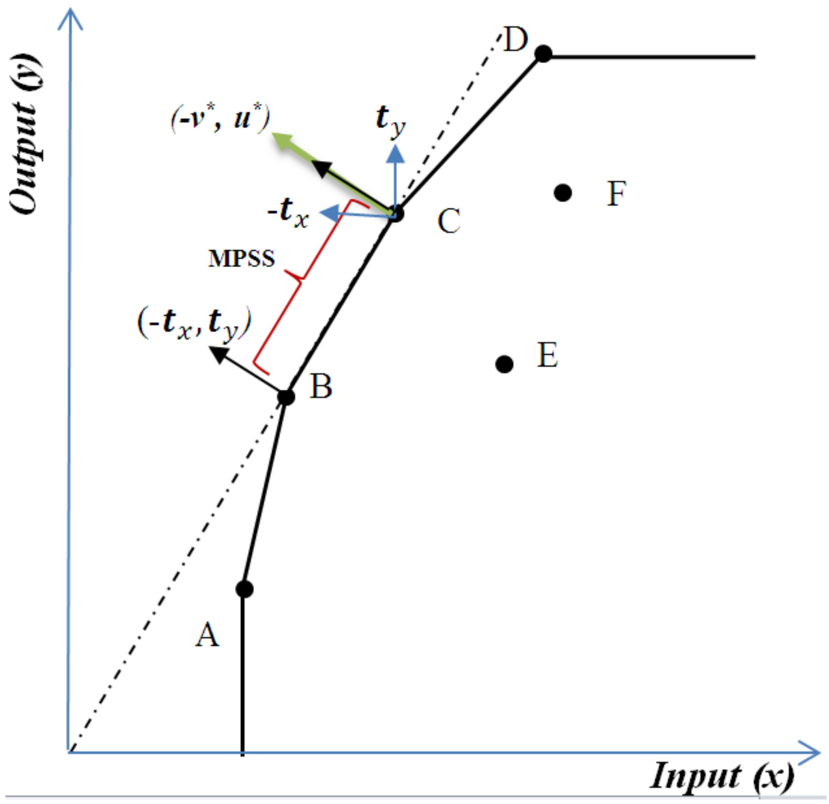

Figure 3.2: MPSS region and MPSS vectors of a production frontier.

In general, $(-t_x, t_y)$ is in the same direction as the vector $(-v^*, u^*)$, where v^* and u^* are the optimal solution from the multiplier side of input or output oriented CCR or BCC when a MPSS DMU (x_0, y_0) is under evaluation. This is because the objective function value of the model is $u^* y_0 = 1$ (in BCC model u_0 is equal to zero) and from the constraints $v^* x_0 = 1$, then $u^* y_0 = v^* x_0$ and $-v^* x_0 + u^* y_0 = 0$. The hyperplane $-v^* x_0 + u^* y_0 = 0$ or $(-v^*, u^*)(x, y)^t = 0$ is the supporting hyperplane of PPS at point (x_0, y_0) and $(-v^*, u^*)$ is its normal vector (Daneshvar, Izbirak, & Javadi, 2014).

Figure 3.3 demonstrates the proposed method of setting targets for inefficient DMUs E and F guided by the MPSS hyperplane vectors $\left(-t_x, t_y\right)$ from the MPSS DMUs B and C. Note that, the vectors of MPSS DMUs B and C are equal because they are both on the same MPSS hyperplane. In cases of multiple inputs and outputs. More than one MPSS hyperlane may be observed.

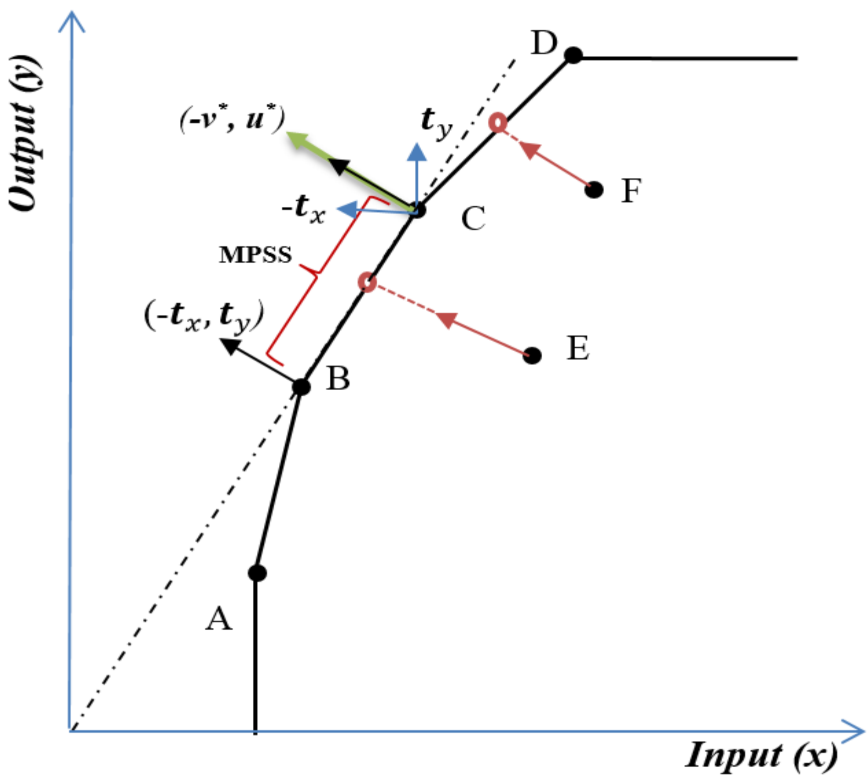

Figure 3.3: Target setting of inefficient DMUs using MPSS vectors.

The proposition of this study is using the MPSS hyperplane vectors $\left(-t_x, t_y\right)$ to guide an inefficient DMU to a target point on the frontier. For that, the weights of the MPSS DMUs are transformed to $\left(-t_x, t_y\right)$. This notion is already DEA validated. In addition, it is supported by economic viewpoint. To justify the assumption, the work of (Weitzman, 2000) is referenced. Where he posed and answered the question “is there a *(m+s)*-dimensional price vector $p = \left(-t_x, t_y\right)$ that supports a solution of a decentralized profit maximization problem, the ‘input’ and ‘output’ combination (x, y) which is a point on the efficiency frontier?” where *m* is represents the inputs and *s* is the number of outputs. Using economic intuition, he stated that-with a convex production structure, the augmented *(m+s)*-dimensional price vector that should competitively support (x, y) is $p = (-t_x, t_y)$

In addition, the supporting hyperplane theorem of a PPS is equivalent to a decentralized solution of profit maximization problem. Also, a price vector sustaining an input combination on the supporting hyperplane is considered to be a set of prices that would guarantee that particular input combination to be demanded by a cost minimizing producer. Therefore, the use of vectors as a replacement for optimal weight is justified using this economic explanation.

3.2.1 Target setting model using MPSS vectors

The propose model is as follows:

Assume a target setting value (θ) for an inefficient DMU.

The MPSS vectors to guide the inefficient DMU to the frontier are

$$(-t_x, t_y) = \left(-t_{x_1}, \ldots, -t_{x_i}, \ldots, -t_{x_m}, t_{y_1}, \ldots, t_{y_r}, \ldots, t_{y_s}\right)$$

Where $t_{x_i} = v_i / Max(v_i, u_r)$ and $t_{y_r} = u_r / Max(v_i, u_r)$

A set of n DMUs $(j = 1, \ldots, n)$, each DMU$_j$ produce s output $y_{rj}, r = 1, \ldots, s$ using m input $x_{ij}, \imath = 1, \ldots, m$. For any DMU (x_0, y_0), we want to move as much as possible in direction $(-t_x, t_y)$ in a manner that $(x_0, y_0) + \theta(-t_x, t_y)$ belongs to the PPS. Therefore, the propose model is as:

$$
\begin{aligned}
& Max\ \theta \\
& subject\ to \\
& \quad (x_0 - \theta t_x, y_0 + \theta t_y) \in PPS
\end{aligned}
\tag{3.8}
$$

The model to find the target setting value using MPSS vectors is presented in model (3.9) and the dual form in (3.10)

$$
\begin{aligned}
& Max\ \theta \\
& subject\ to \\
& \quad \sum\nolimits_{j=1}^{n} \lambda_j y_{rj} - \theta t_{y_r} \geq y_{r0}, \qquad r = 1,\ldots,s \\
& \quad \sum\nolimits_{j=1}^{n} \lambda_j x_{ij} + \theta t_{x_i} \leq x_{i0}, \qquad i = 1,\ldots,m \\
& \quad \sum\nolimits_{j=1}^{n} \lambda_j = 1, \\
& \quad \lambda_j \geq 0, \qquad j = 1,\ldots,n.
\end{aligned}
\tag{3.9}
$$

$$
\begin{aligned}
& Min \quad \sum\nolimits_{r=1}^{s} u_r y_{r0} - \sum\nolimits_{i=1}^{m} v_i x_{i0} + u_0, \\
& Subject\ to, \\
& \quad \sum\nolimits_{j=1}^{n} u_r y_{rj} - \sum\nolimits_{j=1}^{n} v_i x_{ij} + u_0 \geq 0, \\
& \quad -\sum\nolimits_{r=1}^{s} u_r t_{y_r} - \sum\nolimits_{i=1}^{m} v_i t_{x_i} = 1, \\
& \quad u_r, v_i \geq 0, \qquad u_0\ \ free.
\end{aligned}
\tag{3.10}
$$

After finding the target value (θ) from model (3.9), the efficient target (x_0^*, y_0^*) for an inefficient DMU (x_0, y_0) is proposed by equations (3.11), where $(-t_x, t_y)$ are the inputs-outputs vectors of the inefficient DMU.

$$
\begin{aligned}
x_0^* &= x_0 - \theta t_x \\
y_0^* &= y_0 + \theta t_y
\end{aligned}
\tag{3.11}
$$

Applying model (3.9) on efficient DMUs present an improvement value of zero, because improvement is not needed based on the PPS. However, a non-zero value is suggested for inefficient DMUs.

Since the efficient target is dependent on the MPSS hyperplane vectors, in cases of more than one MPSS hyperplane, multiple efficient target value θ_k are proposed, where k represents the MPSS vector used to guide the inefficient DMU. This provides alternatives for efficiency improvement.

3.2.2 Numerical Examples

This section presents two numerical examples to illustrate the application of the proposed MPSS target setting model. Example 1 is a single input single output system used to present a visual description of the target points. Example 2 is a two inputs two outputs system to demonstrate the application of the model in multiple netput system.

Example 1: One input one output data set

Data set: A= (1, 1), B= (1.5, 2), C= (3, 4), D= (4, 5), E= (3, 2.5), F= (4, 4.5)

Table 3.1 presents the summary of the procedure. Using the BCC model (1.1), the efficiency of the DMUs are evaluated as shown in column 4. DMUs E and F are identified as the inefficient DMUs with an efficiency score of 0.625 and 0.875 respectively. Therefore, they need to find a target on the efficiency frontier for them to be considered efficient. The weights of the output (u_r) and input (v_i) are in column 5 and 6. Applying the MPSS model of (Cooper et al., 1996b) model (3.7), the MPSS DMUS are identified as B and C. The weights are converted to vectors as shown in columns 8 and 9. DMUs B and C are on the same MPSS hyperplane, therefore they have equal vectors.

The MPSS hyperplane vectors (0.75, 1) representing $(-t_x, t_y)$ are used in model (3.9) to find the target value. It can be observed that the target values for all efficient DMUs A, B, C and D are zero, as shown in the last column of Table 3.1, while that of DMUs E and F are 0.72 and 0.286 respectively. Using the target values in equations 3.11, the efficient targets are proposed (see Table 3.2). Figure 3.4 illustrates the targets proposed for the inefficient DMUs.

Table 3.1: Example 1 data and Model procedure

DMUs	x	y	BCC	u_r	v_i	MPSS	t_y	t_x	θ
A	1	1	1	0.5	1		0.5	1	0
B	1.5	2	1	0.5	0.67	***	0.75	1	0
C	3	4	1	0.25	0.33	***	0.75	1	0
D	4	5	1	0.25	0.25		1	1	0
E	3	2.5	0.625	0.25	0.33		0.75	1	0.72
F	4	4.5	0.875	0.25	0.25		1	1	0.286

Table 3.2: Proposed targets

	x_0^*	y_0^*
E'	2.28	3.04
F'	3.71	4.79

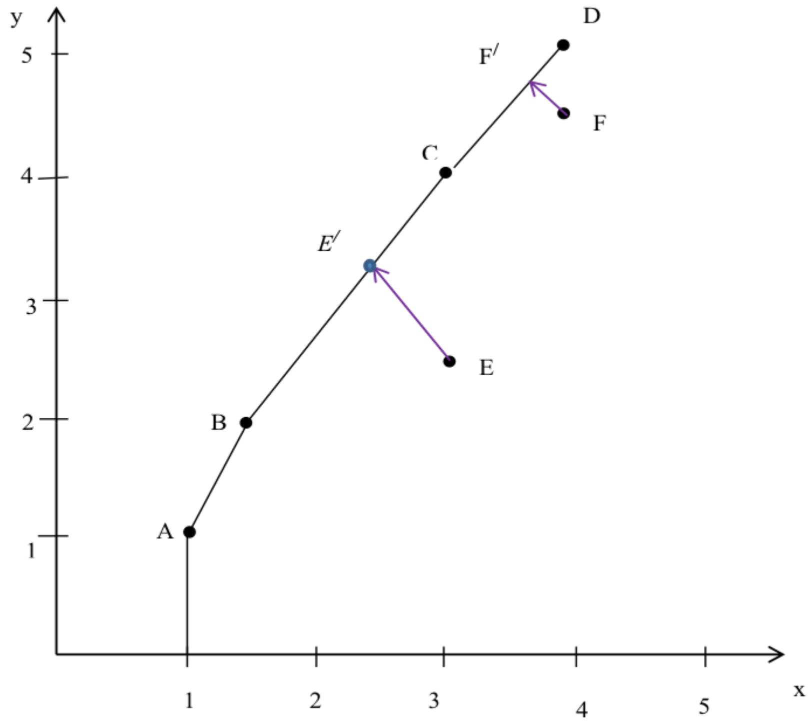

Figure 3.4: MPSS Target setting for Example 1

To test the effectiveness of the proposed model, the efficient targets from the MPSS model is compared to that of the input orientation and output orientation targets. The output orientation proposes the points (3, 4) and (4, 5) on the frontier for DMUs E and F, while input orientation proposes (2, 2.5) and (3.6, 4.5) on the frontier for DMUs E and F respectively. Comparing the proposed targets of both orientations with that of the MPSS model, shows that the modification suggested by the MPSS vector model is less drastic in comparison with the output and input orientation targets.

Furthermore, the output per unit input ratio, which is a form of productivity measurement, shows that, that of the propose target from the MPSS vector model is better compared to input and output orientation targets. Output per unit input for input orientation for DMU E is 1.33 and output orientation is 1.25. That of the MPSS vector is 1.33 which is equal to the ratio of input orientation. However, the modification suggested by the MPSS vector model is more feasible because it requires less drastic measures. For DMU F, the ratio for input and output orientation is 1.25, while that of the MPSS vector model is 1.29. It is therefore obvious that, the targets proposed by the MPSS vector model is a better option compared to that of the input/output orientation.

Example 2: Two inputs two outputs data set

Table 3.3 presents the data set for the two inputs two outputs sample and summarizes the procedure for setting efficient targets for the inefficient DMUs.

DMUs D11, D12 and D13 are identified as inefficient (see column 6), DMUs D1, D2, D3, D4 and D5 are identified as MPSS DMUs (see column 11). Four MPSS hyperplanes are identified. DMUs D1 and D4 are on the same MPSS hyperplane, while DMUs D2, D3 and D5 are on separate hyperplanes (see column 12). Column 13 shows improvement value of zero for all efficient DMUs, and non-zero for inefficient DMUs. Table 3.4 presents the efficient targets for the inefficient DMUs proposed by the MPSS vector model.

Table 3.3: Example 2 data set and procedure

DMUs	x_1	x_2	y_1	y_2	BCC	u_1^*	u_2^*	v_1^*	v_2^*	MPSS	t_{x1}^*	t_{x2}^*	t_{y1}^*	t_{y2}^*	θ
D1	2	2	1	1	1	1	0	0.37	0.133	***	0.37	0.13	1	0	0
D2	3	1	1	0.95	1	1	0	0.25	0.25	***	0.25	0.25	1	0	0
D3	1	4	0.9	0.9	1	0	0	0.33	0.167	***	1	0.5	0	0	0
D4	3	3	1.5	1.5	1	0.667	0	0.24	0.089	***	0.37	0.13	1	0	0
D5	4.5	1.5	1.45	1.45	1	0.741	0	0.16	0.185	***	0.22	0.25	1	0	0
D6	2	8	1.4	1.4	1	1	0	0.32	0.044		0.32	0.04	1	0	0
D7	4	4	1.8	1.8	1	0.833	0	0.22	0.028		0.26	0.03	1	0	0
D8	14	5	1.65	1.85	1	0	4	0	0.2		0	0.05	0	1	0
D9	3.5	13.5	1.75	1.65	1	1.219	0	0.26	0.007		0.21	0	1	0	0
D10	2	2	0.8	0.8	1	0	0	0.33	0.167		1	0.5	0	0	0
D11	5	6	1.2	1.3	0.49	0	0.38	0.14	0.051		0.37	0.13	0	1	0.6
D12	4	3	0.5	0.4	0.57	0	0	0.14	0.143		1	1	0	0	2.0
D13	2.5	2.5	1.15	1.15	0.92	0.8	0	0.29	0.108		0.37	0.13	1	0	0.0868

Table 3.4: Proposed Coordinate's (Example 2)

	x'_{10}	x'_{20}	y'_{10}	y'_{20}
D11	4.78	5.92	1.2	1.9
D12	2	1	0.5	0.4
D13	2.47	2.47	1.24	1.15

3.3 Empirical Studies: Efficiency improvement using MPSS hyperplane vectors

Two case studies are presented to further elaborate the real life application of the proposed MPSS vector target setting model. The first case study is on Nigeria's electricity industry. Annual data of the electricity industry is considered as DMUs, and efficiency is evaluated. Consequently, targets are set for future reference to ensure efficiency of the electricity industry. The second case study is on a beverage producing company, where the production lines over a time period are considered as DMUs, and their efficiencies are evaluated. Subsequently, targets are set for the inefficient production lines to ensure efficiency and profitability of the company.

3.3.1 Case 1: Nigeria's electricity industry (efficiency of policy reforms)

One of the driving forces behind economic growth, technical innovations and an improved overall standard of living of a nation, lies in the presence of a reliable electricity industry. Nigeria, in such regards, has made attempts to ensure she can boast of the presence of a reliable electricity industry and rip the benefits thereof. Major attempts of the Nigerian government were the institution of the Electricity Power Sector Reform (EPSR) and the National Integrated Power Project (NIPP) in 2005 and 2004 respectively, the purpose is to improve the performance of the sector. However, the persistent frequent blackouts and low access to electricity (% of the population) experienced in most parts of the country casts huge doubts over the efficiency of these policies in improving the overall efficiency of the electricity industry in Nigeria. Nigeria has one of the lowest world's electricity generation per capita (WorldBank, 2017b), and only 57.6% of the population have access to electricity in 2014 (WorldBank, 2017a). Electricity generation falls short of demand, resulting to frequent blackouts, load shedding and a reliance on private generators for electricity (EIA, 2016). Recently, Nigeria recorded 0MW electricity supply for several hours on March 31st 2016 (Okechukwu, 2016).

The EPSR and the NIPP can be seen as strong statements of intents by the government to tackle the prevailing electricity conditions in Nigeria. The NIPP highlights the large investment of the Nigerian government in the formation of power plants and electrical infrastructures. The EPSR delineates the government's shift to semi-privatization of the electricity sector. Both efforts aimed at providing lasting solutions to the electricity quandaries facing the nation. From Nigeria Power Baseline Report 2015, Figure 3.5 shows the journey of Nigeria's electricity industry (David & Pedro, 2016).

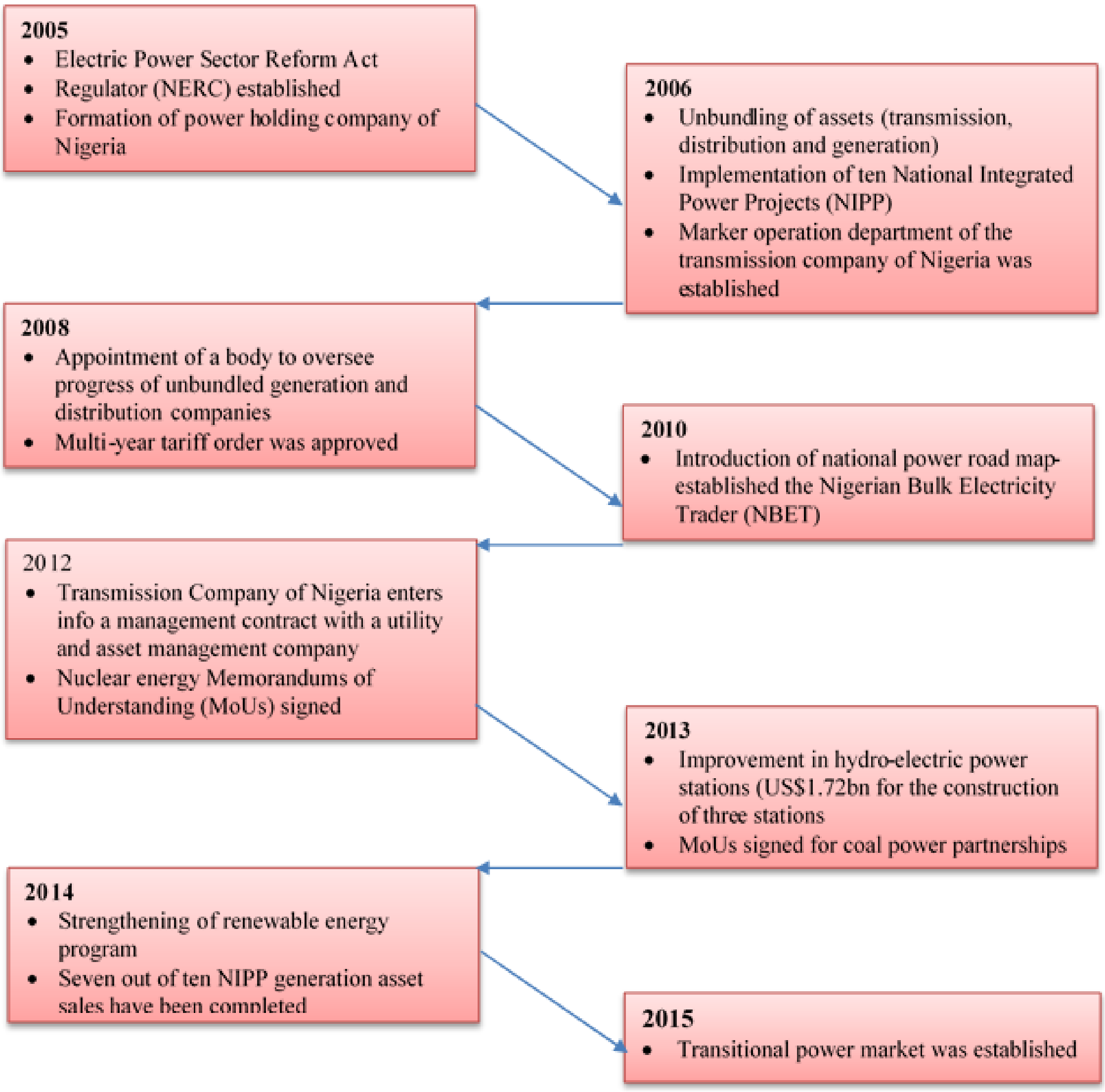

Figure 3.5: Nigeria Electricity industry transition timeline

An essential path of Nigeria's desire to join the world leading economies is access to electricity (vision 20:2020)(Nigeria, 2010). A declaration of the Nigerian government to attain an ambitious installed capacity of 32.712 GW by increasing the present capacity by 20,000MW fossil-fuel generation and 5,960MW hydroelectricity generation (EIA, 2016) with the aim of fixing the long electricity problems in the country. In this thesis, we adopt a performance model aimed at analyzing the efficiency of the electricity industry. Two benchmark scenarios are set to compare the efficiency so far. The probable scenario (PS) represents the most likely performance level of the country at 2020, and the desired scenario (DS) represents the 2020 vision of the country's electricity industry.

The question we attempt to answer is whether these policies have boded any significant impact on the efficiency of Nigeria's electricity industry. And if not, what are the efficient target that should be set to ensure efficiency.

Data is extracted for the period of 1980 through 2014, using technical-economic indicators and environmental indicators that are measurable and publicly available Factors that are relevant and could not be found in a publicly available resource were exempted such as economic losses from energy disruption and operating expenses.

Achieving reliable electric power supply necessitates a comprehensive combination of processes which include, generation, transmission, distribution and retailing- coupled with strong labor force, capital and financial resources (Morey, 2001). The capital and financial resources are by the NIPP, the labor force and infrastructural maintenance is covered by the EPSR. In addition, security of the power system is its ability to withstand exigencies such as reduction in water level in hydroelectric power generation and changes in generator availability for thermal generation. Adequacy of the system refers to the competency of the system's capacity at hand to maintain aggregate power supply under all but most critical circumstances (Morey, 2001).

DEA is one of the most frequently used method for evaluating productive efficiency of an electricity industry (Cook, Du, & Zhu, 2015; Cook & Zhu, 2007; Fallahi, Ebrahimi, & Ghaderi, 2011; Lo, Chien, & Lin, 2001; Vaninsky, 2006). DEA has been used to evaluate efficiency of electric power industry. For example, (H. Yang & Pollitt, 2010), estimate the operational efficiency of China's 221 and 582 coal fired power planed in 2002 using DEA. (Bi, Song, Zhou, & Liang, 2014), estimated the efficiency of China's thermal generation system in each province from 2007 to 2009. The electricity distribution utilities in India was evaluated by (Bobde & Tanaka, 2018).

To evaluate the efficiency of electricity industry, the selection of inputs and outputs should cover the whole electric power supply chain including power generation, transmission, distribution, and utilization (Xiong, Li, Wu, Li, & Liu, 2013). We applied two inputs (Installed capacity and electric power losses), and two output variables (Capacity utilization and % of population with access to electricity) for the efficiency estimation which are similar to studies evaluating efficiency of electricity industries (Vaninsky, 2006; K. Wang, Lee, Zhang, & Wei, 2016). Installed capacity covers generation aspect and energy losses covers transmission and distribution, capacity utilization which is a function of net generation and installed capacity covers the utilization aspect (see equation 3.12), and % of population with access to electricity signifies the overall product of the electricity industry. Table 3.5 shows the descriptive statistics of the data set from 1980 through 2014.

$$CU = \frac{Net\ generation}{Installed\ Capacity\ \times\ 8760} \tag{3.12}$$

Table 3.5: Descriptive statistics of Nigeria's electricity industry

Indicator	Installed Capacity, (Million kw)	Energy loss, share of output, (%)	Net Generation (Billion KWh)	Capacity utilization (%)	Access to electricity (%)
Role	Input 1	Input 2		Output 1	Output 2
Max	9.951	49.27	28.83	49.93	57.65
Min	2.507	5.865	6.867	23.27	38.82
Mean	6.02	29.47	16.25	30.61	44.84

Table 3.6 shows the correlation matrix of the variables, it shows a 90.8% correlation between installed capacity and net generation, and 93.33% correlation between net generation and access to electricity. To evaluate the efficiency of PS and DS, we forecast the missing data for Net generation of DS, and access to electricity of both PS and DS. Forecasting net generation for DS with installed capacity as the independent variable gives 93.17 Billion KWh, and the forecasts for access to electricity with net generation as the independent variable are 83.46% and 100% for PS and DS respectively.

Table 3.6: Correlation matrix

	Installed Capacity	Energy Losses	Net Generation	Access to electricity
Installed Capacity	1			
Energy Losses	-0.638	1		
Net Generation	0.908	-0.602	1	
Access to electricity	0.9164	-0.6335	0.9333	1

To evaluate efficiency, the modified DEA model of (Daneshvar et al., 2014) is utilized. This is chosen to increase the discrimination in efficiency. The complete data set of the inputs and outputs are presented in Appendix 1. The efficiency scores and MPSS DMUs are displayed in Table 3.7.

Table 3.7: Efficiency of Nigeria's Electricity Industry

Years	DMUs	Efficiency	MPSS
1980	1	1	***
1981	2	0.947	
1982	3	1	***
1983	4	0.979	
1984	5	0.719	
1985	6	0.737	
1986	7	0.795	
1987	8	0.7556	
1988	9	0.7326	
1989	10	0.7519	
1990	11	0.6064	
1991	12	0.6186	
1992	13	0.6202	
1993	14	0.7393	
1994	15	0.6698	
1995	16	0.6432	
1996	17	0.6115	
1997	18	0.6101	
1998	19	0.6273	
1999	20	0.6076	
2000	21	0.65593	
2001	22	0.6599	
2002	23	0.7358	
2003	24	0.7484	
2004	25	0.918	
2005	26	1	
2006	27	0.6742	
2007	28	0.9654	
2008	29	0.9377	
2009	30	1	***
2010	31	0.8196	
2011	32	1	***
2012	33	1	***
2013	34	0.885	
2014	35	0.991	
2020-PS	36	0.813	
2020-DS	37	1	***

It can be seen that Nigeria's electricity industry operates inefficiently for the thirty five years considered. Only six years i.e. 1980, 1982, 2005, 2009, 2011 and 2012 operate relatively efficiently. The 2020-PS is also expected to operate inefficiently. Therefore, an efficient target needs to be set for efficient realistic expectation. Five periods are also identified to be MPSS. Using the vectors of the identified MPSS DMUs. Efficient targets are proposed.

To demonstrate the application of the proposed MPSS vector target setting model. The most recent inefficient years i.e. 2010, 2013, 2014 and the 2020-PS are considered. These periods are chosen because there is no need improving previous years. Furthermore, the most important period is the 2020-PS. Since it will ensure efficiency in the future. Table 3.8 presents the weights and transformed vectors of the MPSS and inefficient DMUs. Out of the six MPSS DMUs. Five MPSS hyperplane are observed, because MPSS DMUS 3 and 30 are on the same hyperplane.

Appling model 3.9 on the inefficient DMUs, and using the MPSS vectors in columns 6-9 from Table 3.8, the improvement values are evaluated and presented in Table 3.9. The first row corresponds to the MPSS DMU vectors used. It can be observed that the target values of all inefficient DMUs using vectors of MPSS DMUs 3 and 30 are equal, since they are on the same MPSS hyperplane.

Table 3.8: Weights and Vectors of MPSS and inefficient DMUs

MPSS	u_1	u_2	v_1	v_2	t_{y1}	t_{y2}	t_{x1}	t_{x2}
1	0	0	6.64	0.84	0	0	1	0.12
3	0	0	3.1	1.47	0	0	1	0.47
30	0	0	3.1	1.47	0	0	1	0.47
32	0	1.57	2.98	0.98	0	0.53	1	0.33
33	0.51	0	2.67	1.49	0.19	0	1	0.56
37	0	1.62	1	0	0	1	0.62	0
Inefficient Units	u_1	u_2	v_1	v_2	t_{y1}	t_{y2}	t_{x1}	t_{x2}
31	0.75	0	2.25	1.2	0.33	0	1	0.53
34	0	3.82	3.29	0	0	1	0.86	0
35	0.79	5.35	3.29	0	0.15	1	0.62	0
2020-PS	0	1.56	0.79	1.98	0	0.79	0.4	1

Table 3.9: Target values for inefficient DMUs

DMUs	θ_1	θ_3	θ_{30}	θ_{32}	θ_{33}	θ_{37}
31	0.0705	0.0639	0.0639	0.0461	0.0589	0.0421
34	0.0351	0.0351	0.0351	0.0196	0.0351	0.0162
35	0.0027	0.0027	0.0027	0.0014	0.0025	0.0012
2020-PS	0.1409	0.1122	0.1122	0.0753	0.1022	0.0621

Using equations (3.11) and the target values in Table 3.9. Efficient targets are set for the inefficient DMUs as presented in Table 3.10. DMUs 31 and 34 has four proposed efficient Targets. DMU 35 has two efficient targets, while the 2020-PS has five proposed targets.

Table 3.10: Efficient targets for Inefficient DMUs

DMUs	x1	x2	y1	y2
31	6.121	15.36	34.748	48
	6.337	15.54	34.649	48
	9.935	18.43	34.383	48
	6.500	15.67	34.574	48
34	8.962	15.12	31.458	59.11
	10.500	15.12	31.458	57.56
	8.962	15.12	31.458	59.11
	9.494	15.12	31.458	57.22
35	9.979	16.10	33.087	57.79
	9.927	16.11	33.086	57.77
2020-PS	19.55	5.058	30.256	81.66
	19.93	6.472	30.256	79.39
	22.39	15.70	30.256	76.46
	20.06	6.965	30.256	78.59
	20.59	8.941	30.256	75.42

As can be observed, the method is capable of proposing multiple efficient targets. The efficient target for the 2020-PS from the first proposed target points shows that, for the electricity industry to operate efficiently, it has to achieves a 19.56 thousand MW installed capacity with about 5% energy losses and operate at the current capacity utilization then 81% of the population should have electricity.

3.3.2 Case 2: Beverage company production line efficiency

The market of carbonated soft drinks is having a negative growth, therefore, producers have to efficiency of investment in manufacturing carbonated soft drinks. To have profit, the companies must have operational efficiency which is an indicator of success.

As the market is getting narrower, companies need competitive edge to maintain or increase profit. From a managerial perspective, the production lines that perform efficiently are those that should be maintained. To identify the efficient production lines, factors used in the production must be analyzed. The factors include: operational design process, stock keeping units, production cost and product pricing (Sharma & Choudhary, 2010).

Most companies are involved in producing multiple soft drink products known as production lines. The reduction in revenue as a result of market shrinkage presents a problem for managers to priorities production lines and eliminate any waste or inefficient production lines. Addressing inefficiency from a controllable source ensures efficiency and profitability.

In this thesis, we present a case study of a beverage producing company by evaluating efficiency of the production lines over six year period with the aim of identifying the inefficient production lines. Furthermore, we set efficient targets using the MPSS target setting model for the inefficient production lines in order to be considered efficient.

Efficiency analysis

The factory has five production lines (Pet-6, Pet-2, Can, Glass bottle and Premix lines).

Annual data are collected for each production line from the year 2010-2015. The annual data for each production line is pooled to get a total of 30 DMUs. Over the six years period, the efficiency of the production lines are evaluated. The factors considers are operational and quality factors that are used in production using the management description.

Using the discussion with the factory manager, management and literature on production efficiency. The inputs and outputs are decided. To evaluate efficiency, four inputs and two outputs are considered. They comprise of quality and operational factors. The operational factors contributes to operational efficiency while the quality factors contribute to quality efficiency. The operational factors include labor and quality factors satisfy the quality standard of the food and beverage production. Efficiency evaluation of the production lines satisfies the DEA homogeneity because the production lines use the same resources to produce different product to serve the same purpose.

The inputs and outputs are as follows:

- **Input variables**

(x1) Electricity consumption (operational factors) - KWh

(x2) Labor wages (direct and indirect labor wages) (quality + operational factors) - TL

(x3) Number of labor involved directly in the production lines (operational factors) - Numeral

(x4) Number of defected products removed by the quality assurance (QA) department -Numeral

- **Output Variables**

(y1) Production SKU (stock keeping unit) quality. Number of approved products by the quality assurance department (Quality + Operational factor) -SKU

(y2) Income contribution of each production line (Operational factor) -TL

The procedure for collecting the data is as follows: Input 1 is from the energy consumption report. With guidance from the production supervisor, all the production lines are carefully examined.

To collect data for input 2, labor is considered into two categories. Direct labor and indirect labor. The direct labor operate directly with the production lines, while indirect labor operate indirectly with the production such as the general labor workers. Using the Analytical hierarchy process (AHP), the total labor cost is calculated for each production line.

Input 3 is from counting the total number of direct labor involved in each production line, and input 4 is collected from the defected raw materials and finished product from the QA department annual report. Output 1 is the addition of all products certified by the quality assurance (QA) department annual report. Output 2 is from the company's sales department, by multiplying the prices of the SKU and the quantity. Appendix 2 shows the complete data set for the five production lines from 2010-2015. Table 3.11 shows the efficiency score and DMUs identified as MPSS. Ten DMUs are identified as inefficient DMUs, and thirteen DMUs are identifies as MPSS DMUs. Table 3.12 shows the weights and vectors of the MPSS and inefficient DMUs. The improvement values are presented in Table 3.13 and multiple efficient targets as a result of the multiple MPSS hyperplanes are presented in Table 3.14.

Table 3.11: Efficiencies of the Production Lines and MPSS DMUs

Production Line	DMUs	Efficiency	MPSS
Pet-6	D1	1	
	D6	0.96	
	D11	1	***
	D16	0.97	
	D21	1	***
	D26	1	***
Pet-2	D2	0.6	
	D7	0.61	
	D12	0.98	
	D17	1	***
	D22	0.93	
	D27	0.82	
Can	D3	1	***
	D8	1	***
	D13	1	***
	D18	1	***
	D23	1	***
	D28	1	***
Glass Bottle	D4	1	
	D9	1	
	D14	1	
	D19	1	
	D24	0.89	
	D29	0.89	
Premix	D5	0.95	
	D10	1	
	D15	1	***
	D20	1	***
	D25	1	***
	D30	1	

Table 3.12: Weights and Vectors of MPSS and inefficient DMUs

MPSS units	u_1	u_2	v_1	v_2	v_3	v_4	t_{y1}	t_{y2}	t_{x1}	t_{x2}	t_{x3}	t_{x4}
D3	1.48	0.00	0.48	0.48	0.62	0.41	1.00	0.00	0.32	0.32	0.42	0.28
D8	1.71	0.00	0.57	0.86	0.00	0.61	1.00	0.00	0.33	0.50	0.00	0.36
D11	1.05	0.00	0.41	0.27	0.00	0.96	1.00	0.00	0.39	0.26	0.00	0.91
D13	0.00	1.18	0.95	1.62	0.00	0.00	0.00	0.73	0.59	1.00	0.00	0.00
D17	0.00	0.00	4.71	0.00	1.31	3.24	0.00	0.00	1.00	0.00	0.28	0.69
D18	0.00	1.17	9.19	0.05	0.00	0.00	0.00	0.13	1.00	0.01	0.00	0.00
D20	0.00	0.00	0.00	0.00	2.15	2.06	0.00	0.00	0.00	0.00	1.00	0.96
D21	1.10	0.00	0.71	0.07	0.00	0.85	1.00	0.00	0.65	0.06	0.00	0.77
D23	0.35	0.74	0.00	1.23	0.00	0.23	0.28	0.60	0.00	1.00	0.00	0.19
D25	0.00	2.64	9.27	0.00	0.45	3.82	0.00	0.69	2.43	0.00	0.12	1.00
D26	0.41	0.88	0.00	0.15	0.07	2.10	0.20	0.42	0.00	0.07	0.03	1.00
D28	0.00	0.87	2.05	0.00	0.15	0.96	0.00	0.42	1.00	0.00	0.07	0.47
D30	0.00	7.70	0.00	0.00	1.58	15.72	0.00	0.49	0.00	0.00	0.10	1.00
Inefficient units	u_1	u_2	v_1	v_2	v_3	v_4	t_{y1}	t_{y2}	t_{x1}	t_{x2}	t_{x3}	t_{x4}
D2	0.00	0.00	0.00	0.00	2.20	0.00	0.00	0.00	0.00	0.00	1.00	0.00
D5	1.48	0.00	6.35	1.06	0.00	3.79	0.23	0.00	1.00	0.17	0.00	0.60
D6	1.24	0.00	0.00	1.20	0.00	0.00	1.00	0.00	0.00	0.97	0.00	0.00
D7	0.00	0.00	9.03	0.00	1.20	0.47	0.00	0.00	1.00	0.00	0.13	0.05
D10	4.86	1.61	0.00	3.82	0.00	0.00	1.00	0.33	0.00	0.79	0.00	0.00
D12	0.00	0.00	5.06	1.67	0.00	3.98	0.00	0.00	1.00	0.33	0.00	0.79
D16	1.52	1.51	0.00	1.12	0.00	0.00	1.00	0.99	0.00	0.74	0.00	0.00
D22	2.65	0.00	0.00	3.07	0.00	0.00	0.86	0.00	0.00	1.00	0.00	0.00
D27	1.36	0.00	0.00	0.00	2.75	0.00	0.49	0.00	0.00	0.00	1.00	0.00
D29	0.00	0.00	5.88	0.00	1.45	1.88	0.00	0.00	1.00	0.00	0.25	0.32

Table 3.13: Target values for inefficient DMUs

	θ_3	θ_8	θ_{11}	θ_{13}	θ_{17}	θ_{18}	θ_{20}	θ_{21}	θ_{23}	θ_{25}	θ_{26}	θ_{28}	θ_{30}
D2	0.068	0.061	0.059	0.050	0.029	0.029	0.167	0.039	0.130	0.012	0.222	0.029	0.224
D5	0.011	0.010	0.007	0.022	0.009	0.014	0.008	0.007	0.003	0.004	0.008	0.010	0.008
D6	0.021	0.019	0.021	0.032	0.135	0.134	0.145	0.002	0.024	0.056	0.103	0.135	0.468
D7	0.061	0.060	0.053	0.040	0.027	0.027	0.165	0.035	0.113	0.011	0.214	0.027	0.207
D10	0.011	0.010	0.006	0.011	0.006	0.085	0.006	0.007	0.016	0.003	0.005	0.006	0.005
D12	0.004	0.004	0.003	0.036	0.021	0.003	0.004	0.002	0.007	0.001	0.004	0.002	0.004
D16	0.012	0.011	0.013	0.011	0.077	0.117	0.055	0.015	0.009	0.017	0.019	0.030	0.021
D22	0.017	0.014	0.018	0.020	0.266	0.046	0.074	0.021	0.016	0.179	0.082	0.291	0.228
D27	0.064	0.071	0.087	0.076	0.231	0.360	0.064	0.097	0.008	0.146	0.240	0.330	0.244
D29	0.021	0.021	0.019	0.011	0.007	0.007	0.033	0.011	0.053	0.003	0.045	0.007	0.044

Table 3.14: Efficient Target for Inefficient production lines

	x1	x2	x3	x4	y1	y2
DMU2	33180.06	208134.5	2.54	46330.01	93896.88	471103.38
DMU5	62998.63	273992.48	6	1626.77	87588.57	1600429.8
	63667.58	274094.83	6	1690.68	87443.23	1600429.8
	53716.91	272572.33	6	740.1	89605.17	1600429.8
	63583.96	274082.04	6	1682.69	87461.4	1600429.8
DMU6	812786.01	622105.18	10	118233.4	732366.07	10337667
	812786.01	623143.58	10	118233.4	731274.79	10337667
	812786.01	613946.3	10	118233.4	740940.42	10337667
	812786.01	596070.94	10	118233.4	759726.03	10337667
DMU7	5008.58	194254.27	4.96	45412.63	66069.23	450235.01
DMU10	69554.37	198407.84	6	2549.58	145678.14	2078114.3
	69554.37	239734.42	6	2549.58	92205.4	1682910.7
DMU12	18153.71	203597.81	5	13756.71	47868.24	526404.55
	18571.8	203724.29	5	13809.34	47868.24	526404.55
	19491.61	204002.54	5	13925.14	47868.24	526404.55
	4189.32	199373.46	5	11998.69	47868.24	526404.55
	18989.9	203850.77	5	13861.98	47868.24	526404.55
	18070.09	203572.52	5	13746.18	47868.24	526404.55
	19742.47	204078.43	5	13956.72	47868.24	526404.55
	15979.61	202940.13	5	13483	47868.24	526404.55
	19742.47	204078.43	5	13956.72	47868.24	526404.55
	18237.33	203623.11	5	13767.23	47868.24	526404.55
DMU16	713321.09	680159.62	9	68319.26	701662.12	16071930
	713321.09	680837.31	9	68319.26	700726.73	16051199
	713321.09	679877.24	9	68319.26	702051.86	16080568
	713321.09	681176.16	9	68319.26	700259.04	16040834
	713321.09	643564.11	9	68319.26	752172.81	17191409
	713321.09	655819.09	9	68319.26	735257.96	16816522
	713321.09	678860.7	9	68319.26	703454.94	16111665
	713321.09	681797.38	9	68319.26	699401.61	16021830
	713321.09	677448.84	9	68319.26	705403.65	16154855
	713321.09	697949.1	9	68319.26	707040.57	16191134
	713321.09	675076.91	9	68319.26	708677.49	16227414

Table 3.14 *continuation*: Efficient Target for Inefficient production lines

	x1	x2	x3	x4	y1	y2
DMU22	436374.64	236929.66	5	85902.64	323103.95	5592722.3
	436374.64	238615.83	5	85902.64	321623.69	5592722.3
	436374.64	236163.22	5	85902.64	323776.8	5592722.3
	436374.64	234553.7	5	85902.64	325189.78	5592722.3
	436374.64	214564.94	5	85902.64	342737.61	5592722.3
	436374.64	193089.28	5	85902.64	361590.76	5592722.3
	436374.64	233327.39	5	85902.64	326266.33	5592722.3
	436374.64	237542.81	5	85902.64	322565.68	5592722.3
	436374.64	112536.42	5	85902.64	432306.9	5592722.3
DMU27	329300.38	221202.31	3.3	64644.13	233168.78	3719960.3
	329300.38	221202.31	3.22	64644.13	235944.32	3719960.3
	329300.38	221202.31	3.05	64644.13	241957.98	3719960.3
	329300.38	221202.31	3.17	64644.13	237756.13	3719960.3
	329300.38	221202.31	3.29	64644.13	233245.88	3719960.3
	329300.38	221202.31	2.94	64644.13	245735.8	3719960.3
	329300.38	221202.31	6.63	64644.13	300861.05	3719960.3
DMU29	5385.07	170456.3	3.98	25016.47	50697.77	1291404.1
	5385.07	170456.3	3.97	25936.48	50697.77	1291404.1
	8478.98	170456.3	3.98	26094.81	50697.77	1291404.1
	5050.59	170456.3	3.97	25919.36	50697.77	1291404.1

The model proposes multiple targets for some inefficient units, either option provides an efficient option. It can also be observed that the modifications proposed by the model are not aggressive. It presents practical and realistic options for decision makers

Chapter 4

TARGET SETTING WITH PREDEFINED INPUTS-OUTPUTS

Estimating required inputs for predefined output targets or efficient production possibilities for available inputs is imperative in implementing improvement strategies for inefficient firms. In practice, decision makers often express future desires of their production system. Furthermore, the tradeoffs between multiple inputs adjustment produces an expectable difference in the degree of outputs. This plays a role in management’s desire for efficiency improvement models with predefined targets. In this chapter, models with the ability to estimate efficient input/output for an inefficient DMU when a predefined input/output is presented by the decision maker are proposed.

From an engineering perspective, the efficient estimation of required resources (inputs) to achieve a predefined target or production possibilities (output) contributes to various applications such as, resource allocation and capacity planning problems. In cases of frequent demand fluctuation which requires short run planning, models capable of explicitly defining the required capacity adjustment with the ability to adapt to the required output are limited. In addition, predefined target setting models can priorities inputs or outputs during efficiency improvement.

One of the major challenges in determining improvement options for inefficient firms is incorporating the firm's desires and capabilities when proposing improvement strategies. This study proposes models capable of estimating efficient output as a result of simultaneous input expansion or reduction, and estimating the required inputs in cases of predefined output targets. The models are also capable of accommodating the microeconomic theory were some inputs are considered to be nondiscretionary inputs and are fixed constant (Banker & Morey, 1986), while other inputs are discretionary inputs. Also addressing cases were some predefined outputs (such as negative outputs) are kept constant and other discretionary inputs and outputs are efficiently estimated. Moreover, the models also contributes to the implications of the second welfare micro economic theorem where firms should be allowed to generate efficient outcome given the new endowment (Sheldon, 2017).

Production system response to efficiency improvement in DEA is related to the point on the efficiency frontier, i.e. increasing return to scale (IRS), constant return to scale (CRS) and decreasing return to scale (DRS). These measures remain outside our development, because we are focus on proposing efficient production possibilities and required inputs based on the decision maker's desires. For details on the differential characteristics of efficiency frontier and their elasticity measures see (Podinovski & Førsund, 2010).

4.1 Target setting models with predefined inputs or outputs

This section describes the models for target setting of inefficient DMUs. The models show the required efficient inputs with predefined outputs target, and the efficient production possibilities with predefined available input.

Base on the DDF of Färe & Grosskopf (2000), the target setting model is developed. Assume DMU$_0$ consumes inputs x_{i0} to produce outputs y_{r0}, the target setting model of an inefficient DMU in the production possibility set T is shown in model (4.1). Using the weights, the direction vectors are produced. Weights represents the unit tradeoffs between inputs and outputs (Lee, 2017), for the results to be independent of the units, we eliminate the units of each factor: let $\left(g_i = \frac{1}{\sqrt{m}}\right)$ and $\left(w_r = \frac{1}{\sqrt{s}}\right)$, and propose the general model (4.2).

$$
\begin{aligned}
&Max\ \phi \\
&subject\ to \\
&(x_{i0} - \phi g_i, y_{r0} + \phi w_r) \in T
\end{aligned}
\tag{4.1}
$$

$$
\begin{aligned}
&Max\ \phi \\
&s.t \quad \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} + \phi g_i \le x_{i0} && i = 1, \ldots, m \\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} - \phi w_r \ge y_{r0} && r = 1, \ldots, s \\
&\qquad \textstyle\sum_{j=1}^{n} \lambda_j = 1 && j = 1, \ldots, n \\
&\qquad \lambda_j \ge 0 \\
&\qquad \phi\ free
\end{aligned}
\tag{4.2}
$$

4.1.1 Case 1: Target setting model with predefined inputs

In this case, the decision maker presents the amount of resources (input) available x_{i0}^*, and seeks to identify efficient production possibilities y_{r0}^* with the predefined inputs that allows it to be efficient. Model (4.3) provides the target value θ for addition/reduction to the existing inefficient output $y_{r0}, r = 1, \ldots, s$ as shown in equation (4.4). R denote a subset of efficient DMUs.

$$
\begin{aligned}
&\theta = Max \sum_{j \in R} \eta_j + \phi \\
&s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} \qquad\qquad\qquad\qquad \le x_{i0}^* \qquad i = 1, \dots, m \\
&\qquad \sum_{j=1}^{n} \lambda_j y_{rj} - \sum_{j \in R} \eta_j y_{rj} - \phi w_r \ \ge y_{r0} \qquad r = 1, \dots, s \\
&\qquad \sum_{j=1}^{n} \lambda_j + \sum_{j \in R} \eta_j = 1 \\
&\qquad \lambda_j \ge 0 \\
&\qquad \eta_{j \in R} \ge 0 \\
&\qquad \phi \ \ free
\end{aligned}
\tag{4.3}
$$

$$
y_{r0}^* = y_{r0} + \theta w_r \quad r = 1, \dots, s \tag{4.4}
$$

Proposition 1: Given the above models, if the DMU is efficient, the objective function value θ will be zero and the proposed outputs $y_{r0}^*, r = 1, \dots, s$ will remain the unchanged. However, if the DMU is inefficient or weak efficient, the objective function value is nonzero and equation 4.4 estimates the efficient outputs for the available input of the inefficient DMU.

Proof of model 4.3

$x_{i0}^*, i = 1, \dots, m$ in model 4.3 is the available predefined input set by the decision maker of the inefficient DMU with the aim of operating efficiently.

The dual of model (4.3) is as follows:

$$
\begin{aligned}
&Min \sum_{i=1}^{m} v_i x_{i0}^* - \sum_{r=1}^{s} u_r y_{r0} + u_0 \\
&s.t \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_0 \ge 0 \qquad j = 1, \dots, n \\
&\qquad\qquad\qquad \sum_{r=1}^{s} u_r w_r \qquad = 1 \qquad r = 1, \dots, s \\
&\qquad v_i \ge 0 \quad i = 1, \dots, m \\
&\qquad u_r \ge 0 \quad r = 1, \dots, s \\
&\qquad u_0 \quad free
\end{aligned}
\tag{4.3.1}
$$

To ensure that the original PPS is maintained, and the desired predefined input is feasible, the original efficiency frontier must be preserved. To do that, we add the third constraint to model (4.3.1) for the set indices of efficient DMUs $(j \in R)$ to maintain the original efficiency frontier as follows:

$$
\begin{aligned}
& Min \sum_{i=1}^{m} x_{i0}^{*} - \sum_{r=1}^{s} u_r y_{r0} + u_0 \\
& s.t \quad \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} + u_0 \geq 0 \qquad j = 1, \ldots, n \\
& \qquad\qquad \sum_{r=1}^{s} u_r w_r \qquad = 1 \qquad r = 1, \ldots, s \\
& \qquad\qquad \sum_{r=1}^{s} u_r y_{rj} \ + u_0 = 1 \qquad j \in R \\
& \quad v_i \geq 0 \quad i = 1, \ldots, m \\
& \quad u_r \geq 0 \quad r = 1, \ldots, s \\
& \quad u_0 \quad free
\end{aligned}
\tag{4.3.2}
$$

The dual of model (4.3.2) gives the target setting model with predefined input as presented earlier in model 4.3.

4.1.2 Case 2: Target setting model with predefined outputs

In this case, the decision maker sets an output target $y_{r0}^{*}, r = 1, \ldots, s$ for the inefficient DMU, and seeks to identify the amount of inputs required to efficiently achieve the predefined output. Model (4.5) and equation (4.6) presents the required input for the predefined output.

$$
\begin{aligned}
& \theta = Max \sum_{j \in R} \eta_j + \phi \\
& s.t \quad \sum_{j=1}^{n} \lambda_j x_{ij} + \sum_{j \in R} \eta_j x_{ij} + \phi g_i \leq x_{i0} \qquad i = 1, \ldots, m \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} \qquad\qquad\qquad \geq y_{r0}^{*} \qquad r = 1, \ldots, s \\
& \quad \sum_{j=1}^{n} \lambda_j + \sum_{j \in R} \eta_j = 1 \\
& \quad \lambda_j \geq 0 \\
& \quad \eta_{j \in R} \geq 0 \\
& \quad \phi \ free
\end{aligned}
\tag{4.5}
$$

$$
x_{i0}^{*} = x_{i0} - \theta g_i, \quad i = 1, \ldots, m \tag{4.6}
$$

Proposition 2: Given the above models, if the DMU is efficient, the objective function value θ will be zero, and the estimated required inputs $x_{i0}^*, i=1,\ldots,m$ will remain the unchanged. However, if the DMU is inefficient or weak efficient, the objective function value is nonzero and equation (4.6) proposes the new efficient required inputs for the predefined output target.

Proof of model 4.5

$y_{r0}^*, r=1,\ldots,s$ in model 4.5 is the predefined output target set by the decision maker for the inefficient DMU with the aim of improving efficiency.

The dual of model 4.5 is as follows:

$$
\begin{aligned}
&Min \ \sum_{i=1}^{m} v_i x_{i0} + \sum_{r=1}^{s} u_r y_{r0}^* + u_0 \\
&s.t \quad \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 \ge 0 \\
&\qquad \sum_{i=1}^{m} v_i g_i = 1 \\
&\qquad v_i \ge 0 \quad i=1,\ldots,m \\
&\qquad u_r \ge 0 \quad r=1,\ldots,s \\
&\qquad u_0 \quad free
\end{aligned}
\tag{4.5.1}
$$

To ensure that the original PPS is maintained, and the improvement desire of the decision maker is feasible, the original efficiency frontier must be preserved. To do that, we add the third constraint to model (4.5.1) for the set indices of efficient DMUs $(j \in R)$ as follows:

$$
\begin{aligned}
& Min \sum_{i=1}^{m} v_i x_{i0} + \sum_{r=1}^{s} u_r y_{r0}^{*} + u_0 \\
& s.t \quad \sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + u_0 \geq 0 \\
& \qquad \sum_{i=1}^{m} v_i g_i \qquad = 1 \\
& \qquad \sum_{i=1}^{m} v_i x_{ij} + u_0 = 1 \qquad\qquad j \in R \\
& \qquad v_i \geq 0 \quad i = 1, \ldots, m \\
& \qquad u_r \geq 0 \quad r = 1, \ldots, s \\
& \qquad u_0 \quad free
\end{aligned}
\tag{4.5.2}
$$

The dual of model (4.5.2) gives the target setting value for predefined output presented in model (4.5).

To maintain feasibility of the predefined input/output target in the models and the original production possibility set, the DMU adjustment must be within the production possibility set. A limited range of adjustment is recommended to ensure the DMU remains in the production possibility set due to the law of diminishing marginal returns (Lee & Johnson, 2014).

From a microeconomic perspective, the proposed models are particularly relevant for second welfare theorem, where redistribution of income from endowment of labor and ownership shares of firms is practically difficult to implement, especially in production systems of pure exchange economy where every Pareto efficient allocation is a possible competitive equilibrium and, consumers' and firm's exhibit convex preferences (Varian, 2010).

The above models assume the proposed efficient targets are equally important. However, this might not be the case. When decision makers' express preference or priories on some targets, weights can be used to emphasis preference. The use of

Analytical hierarchy process (AHP) (Millet & Harker, 1990) can be used to distribute the vectors according to the decision makers' preference.

4.2 Numerical examples for models with predefined inputs-outputs

This section includes two constructed examples to illustrate the proposed models. Using WinQSB linear and integer programming software, we apply models 4.3 and 4.4 for the case of predefined inputs, and models 4.5 and 4.6 for the case of predefined outputs. An example of single input and output is used to give a graphical illustration of the models estimates, followed by a multiple input and output example.

Example 1: single input and output

Table 4.1 shows a single input (x)single output (y) system with six DMUs labeled A to F. The BCC model identifies DMUs E and F to be inefficient, with an efficiency score of 0.778 and 0.769 respectively.

Table 4.1: Single input and output system

DMUs	x	y	BCC efficiency
A	2	4	1
B	4	8	1
C	6	12	1
D	10	16	1
E	4.5	7	0.778
F	6.5	10	0.769

Assuming decision makers present a case of predefined input ***(Case 1),*** by decreasing the input to 2.5 for DMU E and 5.5 for DMU F with $w_r = 1$ (Table 4.2), model (4.3) gives an objective function value of -2 and 1 for E and F, and equation 4.4 estimates an efficient output of 5 and 11 for E and F respectively.

Table 4.3 presents the case of predefined outputs ***(Case 2),*** by increasing the output to 9.5 and 13 for DMUs E and F, and applying the proposed models (4.5 and 4.6) with $g_i = 1$, an efficient required input is estimated in Table 4.3 which are 4.75 and 7. Figure 4.1 illustrates the proposed coordinates for efficiency of all cases.

Table 4.2: Predefined input

DMUs	x_{i0}^{*}	θ	θw_r	y_{r0}^{*}
E^{*}	**2.5**	-2	-2	**5**
F^{*}	**5.5**	1	1	**11**

Table 4.3: Predefined output

DMUs	y_{r0}^{*}	θ	θg_i	x_{i0}^{*}
E^{**}	**9.5**	-0.25	-0.25	**4.75**
F^{**}	**13**	-0.5	-0.5	**7**

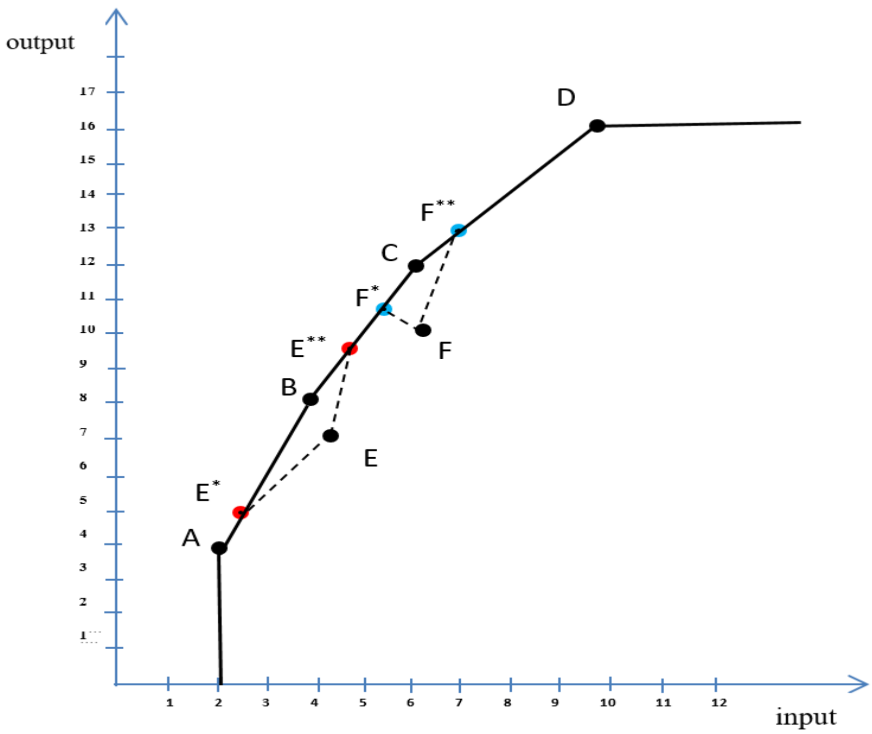

Figure 4.1: DEA frontier and target coordinates

Example 2: Two inputs two outputs

This is an example of two inputs (x_1, x_2), two outputs (y_1, y_2) with twelve observations $(D1\ldots D12)$. Table 4.4 shows the data set and efficiency scores of the twelve observations. The BCC model shows DMUs D11 and D12 to be inefficient with efficiency score of 0.49 and 0.57 respectively. The cases of predefined inputs (Table4.5), nondiscretionary predefined inputs (Table 4.6), predefined outputs (Table 4.7) and nondiscretionary predefined outputs (Table 4.8) as improvement alternatives for the inefficient DMUs are analyzed.

The case of predefined inputs assumes input reduction for the inefficient DMUs, and models (4.3 and 4.4) estimates the efficient production possibilities $\left(y_{10}^*, y_{20}^*\right)$ of the predefined inputs with $w_r = 0.7071$. The case of nondiscretionary predefined inputs assumes that input 1 $\left(x_{10}^*\right)$ for DMUs 11 and 12 are to remain constant, assuming they are not at the discretion of the decision maker.

For predefined output, increase in outputs is assumed for both DMUs 11 and 12. The case of nondiscretionary predefined outputs assumes that output 1 $\left(y_{10}^*\right)$ for DMU11 and DMU12 are to remain constant while other variables can be improved, and models (4.5 and 4.6) with $g_i = 0.7071$ estimates the required inputs to efficiently achieve the predefined output targets.

Table 4.4: Two input two output system

DMUs	x_1	x_2	y_1	y_2	BCC
D1	2	2	1	1	1
D2	3	1	1	0.95	1
D3	1	4	0.9	0.9	1
D4	3	3	1.5	1.5	1
D5	4.5	1.5	1.45	1.45	1
D6	2	8	1.4	1.4	1
D7	4	4	1.8	1.8	1
D8	14	5	1.65	1.85	1
D9	3.5	13.5	1.75	1.65	1
D10	2	2	0.8	0.8	1
D11	5	6	1.2	1.3	0.49
D12	4	3	0.5	0.4	0.57

Table 4.5: Predefined inputs

DMUs	x_{10}^*	x_{20}^*	θ	θw_r	y_{10}^*	y_{20}^*
D11	**2.5**	**3.5**	0.0864	0.0611	**1.2611**	**1.3611**
D12	**2.6**	**1.5**	0.7425	0.5250	**1.0250**	**0.9250**

Table 4.6: Nondiscretionary predefined inputs

DMUs	x_{10}^*	x_{20}^*	θ	θw_r	y_{10}^*	y_{20}^*
D11	**5**	**3**	0.5091	0.3600	**1.5600**	**1.6600**
D12	**4**	**2**	1.367	0.9667	**1.4667**	**1.3667**

Table 4.7: Predefined output

DMUs	y_{10}^*	y_{20}^*	θ	θg_i	x_{10}^*	x_{20}^*
D11	**1.8**	**1.7**	1.4142	1.0000	**2.2000**	**2.3000**
D12	**1.5**	**1.4**	0.6285	0.4444	**0.9444**	**0.8444**

Table 4.8: Nondiscretionary predefined outputs

DMUs	y_{10}^*	y_{20}^*	θ	θg_i	x_{10}^*	x_{20}^*
D11	**1.2**	**1.7**	2.0428	1.4317	**2.6317**	**2.7317**
D12	**0.5**	**1.4**	0.9428	0.6667	**1.1667**	**1.0667**

4.3 Empirical studies using predefined inputs-outputs target setting models

This section we present empirical studies to validate the proposed target setting models. The empirical examples considers different viewpoint of decision makers, where desired inputs are known, thus predefined inputs are set for the inefficient DMUs, or output targets are known, hence predefined outputs are set.

4.3.1 Transnational resource generativity: Efficiency analysis and target setting of Water, Energy, Land, and Food Nexus for OECD countries.

Generativity in resource management requires quantification and analysis of water-energy-land-food (WELF) nexus. Transnational comparisons of countries such as those under the Economic Co-Operation and Development (OECD) provide a more informative conclusion to causes of inefficiency of WELF-Nexus. Utilizing the input-output index system, the efficiency of WELF-Nexus for OECD countries is performed in intervals (2007, 20012, and 2016) and, to ensure resource generativity, a target setting model that accommodates predefined input was utilized.

Water, energy, and land resources are significant contributors to food security and sustainability of the ecosystem. The growing natural resource scarcity and environmental impact of resource consumption has made the interconnection between these resources apparent. The Economic Co-operation and Development (OECD 1998) defines eco-efficiency as the use of ecological resources to satisfy human needs.

The theory of resource nexus stems from the understanding that natural resources are getting scarce, affecting both the economic growth and human well-being (Hoff, 2011). The growing pressure on the resources could hamper social-economic development and subsequently lead to irreparable environmental damage. A strategy for ensuring human well-being and environmental sustainability on both the short and long run is to identify the connections between key natural resources that guarantee efficiency (Ringler, Bhaduri, & Lawford, 2013). The benefits of Nexus thinking include improving the resource efficiency in terms of usage and evading the adverse effects of single resource development strategies. Several studies have documented the importance of quantifying the interconnection between Water, Energy, Land, and Food (WELF-Nexus) to ensure its efficient using (Liu, Wang, Li, & Zhang, 2010; Picazo-Tadeo, Gómez-Limón, & Reig-Martínez, 2011; Ringler et al., 2013).

As resources are interconnected with human well-being and environmental outcomes for both present and future generations, a rigorous theoretical framework is needed to even the tradeoffs and identify the synergies across the resources (Ringler et al., 2013). The relative performance of OECD countries in terms of resource utilization and eco-efficiency has been studied at some level in the literature. The convergence in eco-efficiency of 22 OECD countries was studied by (Camarero, Castillo, Picazo-Tadeo, & Tamarit, 2013) using Gross Domestic product (GDP) to cover economic value of goods and services, and three air pollutants namely, carbon dioxide (CO_2), nitrogen oxides (NO_X) and sulfur oxides (SO_X) to account for ecological performance. As a case study for measuring eco-efficiency, (Rashidi & Saen, 2015) evaluated 19 OECD countries based on greenhouse gases. The inputs are labor, average precipitation, and energy use, and the outputs are GDP per unit of energy use and CO_2 emission. More recently, studies on resource nexus efficiency focus on water, energy, and food nexus. (Li et al., 2016) evaluated the input-output efficiency of China's water, energy, and food nexus using population as a fundamental component of the production process. They applied an index system to account for the inputs and outputs of the nexus using total waste gas, waste water, and solid waste to create an environmental index, and GDP per capita as an economic index. The authors did not consider the role of land in the ecosystem.

The objective of this section is twofold. (i) Evaluate efficiency of WELF-Nexus; of OECD countries; and (ii) Utilize the target setting model to ensure future WELF-Nexus efficiency and resource generativity.

The WELF-Nexus efficiency of 31 OECD countries are evaluated according to the method described by (Li et al., 2016), instead of using only water, energy, and food, we included land in the input for the nexus, and social welfare in the output index. The inclusion of land is justified, as it is an integral part of the ecosystem and possesses several interconnections with other sectors of the nexus that cannot be ignored. Land is not only important in food production, but also as a source of both energy (biofuel or shale gas) and water supply (underground water). It is also a recipient of the environmental effects of production processes and resource consumption such as fertilizer residue from agriculture, waste disposal, and environmental impact from energy extraction. Social welfare is used as an output index because it is a fundamental part of the OECD mission. Further contributions to previous empirical studies include utilizing indicators for social-economic and environmental indexes for input-output efficiency of the nexus. Providing broader analysis of policies aimed at understanding the evolution and trends of WELF-Nexus efficiency among countries.

Application of DEA in WELF-Nexus

In the WELF-Nexus framework (see Figure 4.2), several factors and drivers are interconnected, including WELF resources, population, social-economic development, and environmental effects (Hoff, 2011). The WELF- Nexus system consists of multiple inputs and outputs. The system is much more like a “Black box”, for it is difficult to quantify explicit relationships between each resource unit in the nexus (Li et al., 2016). DEA has the advantage of addressing systems with black box characteristics because it relaxes the complex relation between individual factors in a system and evaluate it as a unit by using the resources consumed and outcomes as evaluating criteria and which has been applied in various areas.

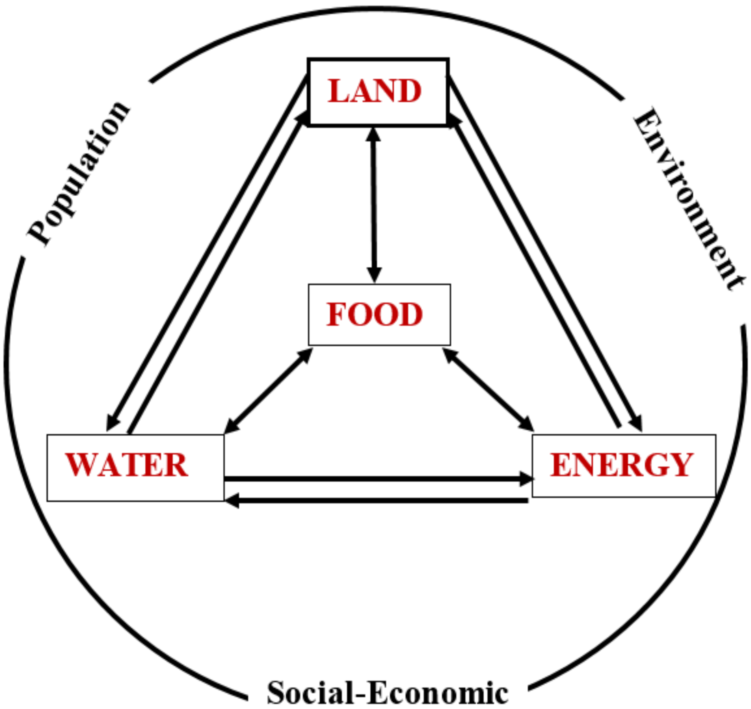

Figure 4.2: Water, Energy, Land and Food Nexus framework

The WELF- Nexus efficiency of a country refers to the amount of social-economic services per unit consumption or resources or per unit discharged pollution during resource metabolism (Liu et al., 2010). The environmental consequences of resource consumption not only lie on the quantity and quality consumed by the country, but also on that of the waste discharged. In this study, the indicators for quantifying WELF-Nexus efficiency were developed based on literature and characteristics of the nexus. There are essentially two classes of indicators that could be disintegrated further: They are the resources consumed considered as inputs, and social-economic outcomes, environmental waste and emission as outputs.

Developing an input-output index system is contingent on the target evaluation of the input-output efficiency (Li et al., 2016). In the WELF- Nexus system, high input-output efficiency indicates a greater output benefits with less WELF consumption and minimal environmental cost. The indicators of WELF-Nexus have a wide contentious interpretation depending on the viewpoint selected. However, it has become customary to define it as an integrated process of resource-environment activities and social-economic outcomes in which WELF are put in, and social-economic developments are provided, while waste and emissions are unavoidable component of the process. In sustainable development and resource generativity, the production system is not only focused on resources. The effects human factor has on the system contributes directly or indirectly because a principle component of sustainable development continuum is the population in the production and consumption process.(Zeng & Gu, 2000). The interconnection between WELF with human, social-economic and environmental sustainability (Liu et al., 2010; Ringler et al., 2013), shows evidence of strong interdependence between WELF- Nexus and the economic system, environmental system and population (see Figure 4.2).

To develop the indicators in this study, the most practical and effective input-output measures used in the literature are selected, in combination with intrinsic and comprehensive variables. Finally, four inputs and four outputs were defined. Figure 4.3 shows the origin of the variables used in the WELF-nexus framework, and Table 1 shows the input-output indexes implemented.

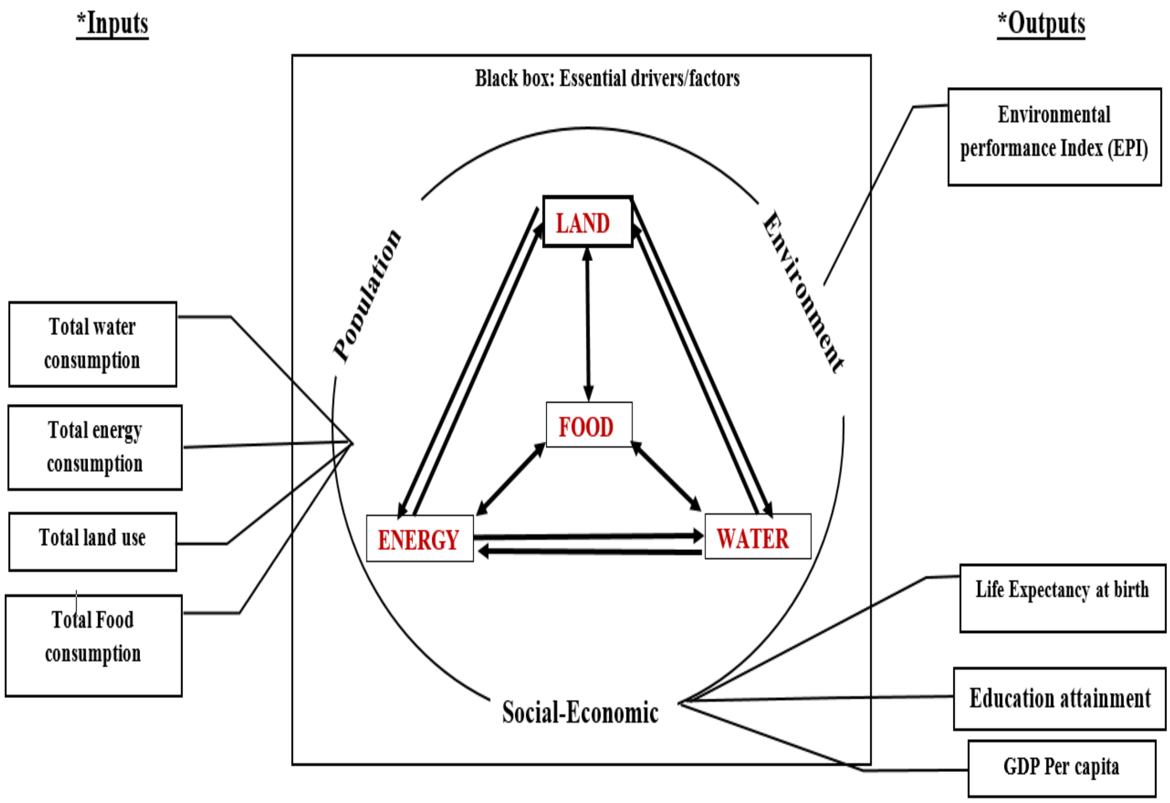

Figure 4.3: Origin of inputs and outputs in the WELF-nexus framework

Table 4.9: Input-output indexes for OECD countries

Indexes		OECD case
Input	Total water consumption	Annual freshwater withdrawals, total (billion cubic meters)
	Total energy consumption	Total final energy consumption (TFEC) (TJ)
	Total land use	Land area (Square kilometers)
	Total food consumption	Expenditure per capita on food (US$ per person)
Output	Environment	Environmental Performance Index (EPI)
	Social welfare	Life expectancy at birth (yrs.)
	Social welfare	Education attainment (% same age population)
	Economic	GDP per capita PPP (constant 2011 international $)

Input indexes: Direct inputs are considered for the WELF-Nexus analysis. They are the total amount of WELF consumed by the countries: total water consumption, total energy consumption, total land use and total food consumption.

Annual freshwater withdrawal refers to the entire water extraction from water basins including desalination plants in countries where they are a significant source, but excludes evaporation from sources. They include water consumption for agriculture, industries, domestic, municipal and commercial establishments. Total final energy consumption refers to all forms of energy consumed excluding non-energy use. Land area here refers to total land excluding under inland water bodies. Expenditure per capita on food represents expenditure on food per person.

Output indexes: The output indexes include the environment and social-economic consequence of WELF-Nexus. A holistic viewpoint is implemented in evaluating the efficiency of WELF consumption. The outputs used are environmental performance index (EPI), life expectancy at birth, education attainment and GDP per capita.

EPI is an aggregate environmental performance measure that centers around two broad and comprehensive environmental protection objectives (environmental public health and ecosystem vitality) using nineteen indicators tracked by nine policy categories. EPI was developed and maintained by Yale Center for Environmental Law and Policy (YCELP) and Center for International Earth Science Information Network (CIESIN). The EPI incorporates a comprehensive picture of high-priority environmental issues including resources consumption (WELF), depletion of environmental resources, species loss and other important environmental properties.

EPI was selected as an output measure because of its intrinsic environmental structure and its strength as an expert consensus-based framework that determines critical environmental issues and calculate scientifically rigorous metrics on a common and comparable scale. Using the EPI enables comparison of countries environmental progress and efficacy of policies implemented.

Social welfare, part of the social-economic dynamics of WELF consumption includes health indicators and social variables that affect the human population. A common health indicator is life expectancy at birth. Defined as how long, on average, a newborn is expected to live assuming common health rates do not change. Gains in life expectancy can be attributed to numerous factors including greater access to quality health service.

Another common social variable that affects human population is education. We use population with tertiary education as a proxy measure for population education. It is the percentage of population that have completed the highest level of education by age group.

The economic return of WELF consumption is to enhance regional economic development, best represented by (GDP). However, the goal of resource consumption is improvement in living standards of the population, which is an integral part of the WELF-Nexus. Therefore, economic outcomes are best represented by GDP *per capita* that measures output of a country per person, show economic growth, and reflects productivity of a country.

Public and readily available data were extracted for the variables. Annual freshwater withdrawals and Total final energy consumption was extracted from (Word development indicators). Total land use was from (OECD Stats) while Expenditure *per capita* on food was extracted from (knoema.com). Environmental Performance Index was obtained from (epi.yale.edu), Education attainment and Life expectancy were both extracted from (OECD STATS) and GDP *per capita* from Word development indicators.

Analysis and Results

The 34 OECD member states were initially considered, but to achieve a balanced panel data and eliminating outliers, 31 member states were finally analyzed. The descriptive statistics of the inputs and outputs used for the efficiency analysis are presented in Table 4.10 and the complete data set are depicted in Appendix 3.

Table 4.10: Descriptive statistics of inputs/outputs of WELF-Nexus

		Total water consumption	Total energy consumption	Total land use	Total food consumption	EPI	LE	Education Attainment	GDP per capita
2007	Mean	36.36	4553524.01	1156455.48	2204.04	61.77	78.59	33.82	36228.92
	Median	9.15	1338654.90	267710.00	2087.10	62.99	79.53	36.20	37772.08
	Std. Dev.	100.11	10591871.18	2693628.32	922.19	7.04	3.09	11.85	11625.19
	Min.	0.22	203566.19	20270.00	524.80	43.87	68.41	14.20	16044.25
	Max.	562.40	59567475.18	9984670.00	4163.60	77.99	82.51	55.70	65083.26
2012	Mean	33.89	4390954.55	1162830.52	2554.98	62.19	79.78	38.97	35798.74
	Median	9.15	1303347.07	267710.00	2508.21	63.36	80.63	40.20	36367.58
	Std. Dev.	86.77	10065189.21	2714721.58	839.82	6.65	2.87	10.88	10964.15
	Min.	0.26	204431.69	20270.00	1226.14	44.80	70.21	20.00	16324.43
	Max.	485.60	56628804.85	9984670.00	4614.17	76.69	83.10	65.70	63003.41
2016	Mean	34.15	4409210.73	1162830.52	2301.16	84.05	80.56	42.51	38032.74
	Median	9.15	1240301.81	267710.00	2272.71	85.42	81.50	43.40	38058.87
	Std. Dev.	86.73	10612790.60	2714721.58	685.35	5.61	2.80	10.39	11641.00
	Min.	0.24	183793.32	20270.00	1181.05	67.68	71.24	21.80	16832.46
	Max.	485.60	59772457.32	9984670.00	3631.25	90.68	83.84	70.00	64179.04

Given the convexity assumed by DEA, the use of index/ratio variables (i.e. EPI) variable cannot be directly used by the previous model (Emrouznejad & Amin, 2009). However, as claimed by (Olesen, Petersen, & Podinovski, 2015), this problem can be surpassed if we disregard convexity and take nonconvexity, instead, as assumption. This is achieved by simply imposing that coefficients λ_j are Boolean, i.e., $\lambda_j \in \{0,1\}$, which means that each DMU has one and only one possible benchmark. The linear model becomes a mixed linear programming model, which can be solved using computational programming tools or a simplification resulting from the asymptotic properties of the so-called partial frontiers. Using the $order-\alpha$ (Aragon, Daouia, & Thomas-Agnan, 2005) DEA approach. The efficiency of WELF-Nexus for OECD countries is evaluated.

Figure 4.4 depicts the efficiencies of the 31 countries evaluated for the three periods (2007, 2012 and 2016). The WELF-Nexus efficiency improved over the evaluated period, with an average efficiency of 91%, 92% and 97% for 2007, 2012 and 2016 respectively. However, only Chile and New Zealand were efficient in 2007, while Canada, Czech Rep., Hungary, Ireland, Italy, Japan, Poland, Slovenia, Spain, Sweden and Switzerland were efficient in 2016. No country was estimated to be efficient in 2012. Over the evaluated periods, United states, Turkey and Korea Rep. were estimated to be the least effcient countries consecutively.

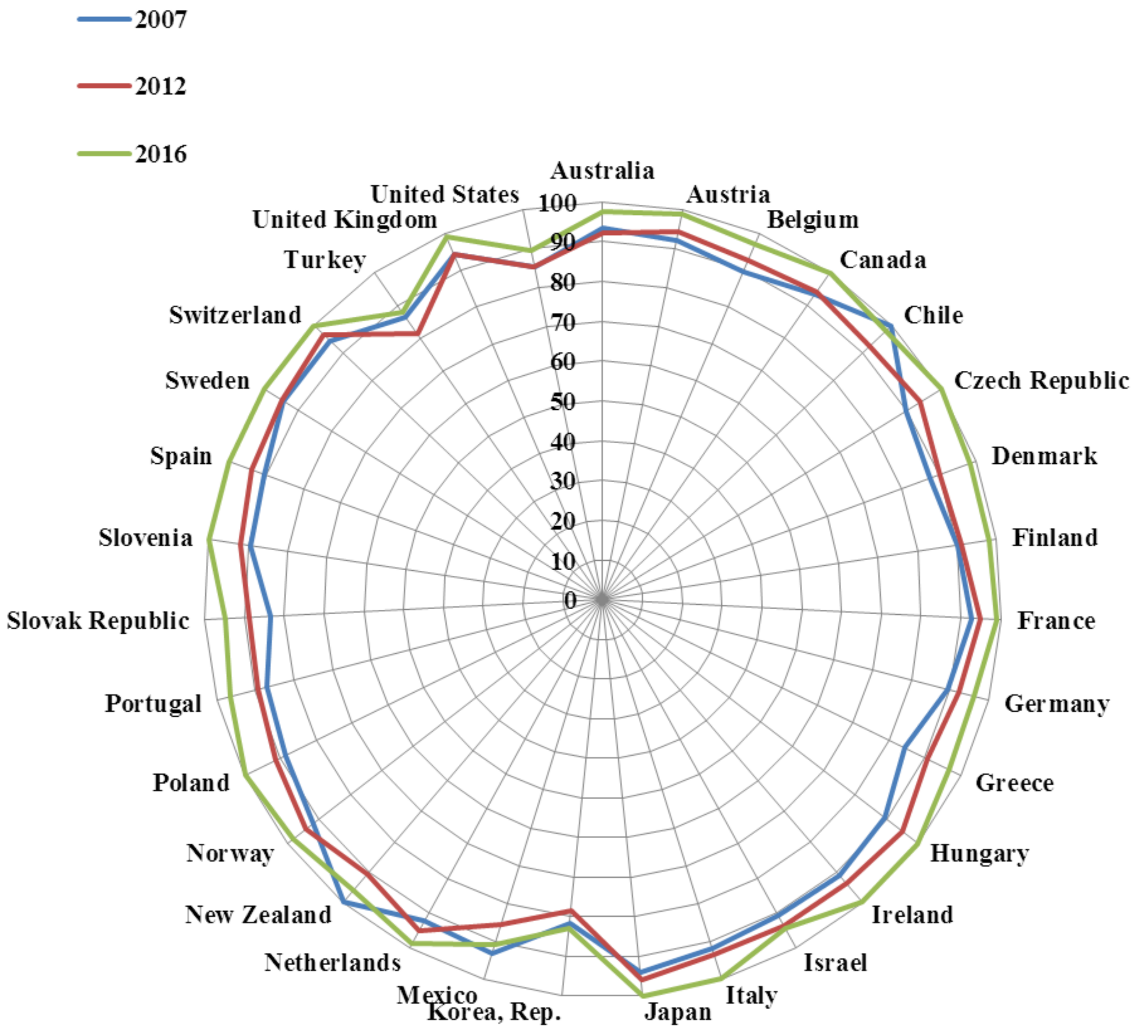

Figure 4.4: Efficiency scores of WELF-Nexus in OECD countries

Resource generativity prevents stagnation in existing inefficient WELF-Nexus. As an attempt to prevent future inefficiency, outputs modifications are recommended. These modifications are based on predefined inputs. The data for 2016 was used as the predefined inputs with the assumption that significant changes in WELF consumption are not expected in the near future. By applying the target setting model with predefined inputs (see model 4.3), percentage output improvements are recommended using model 4.4, for the weak efficient and inefficient countries. Figure 4.5 presents the estimated output percentage improvement for the countries to perform efficiently in the future using the 2016 data.

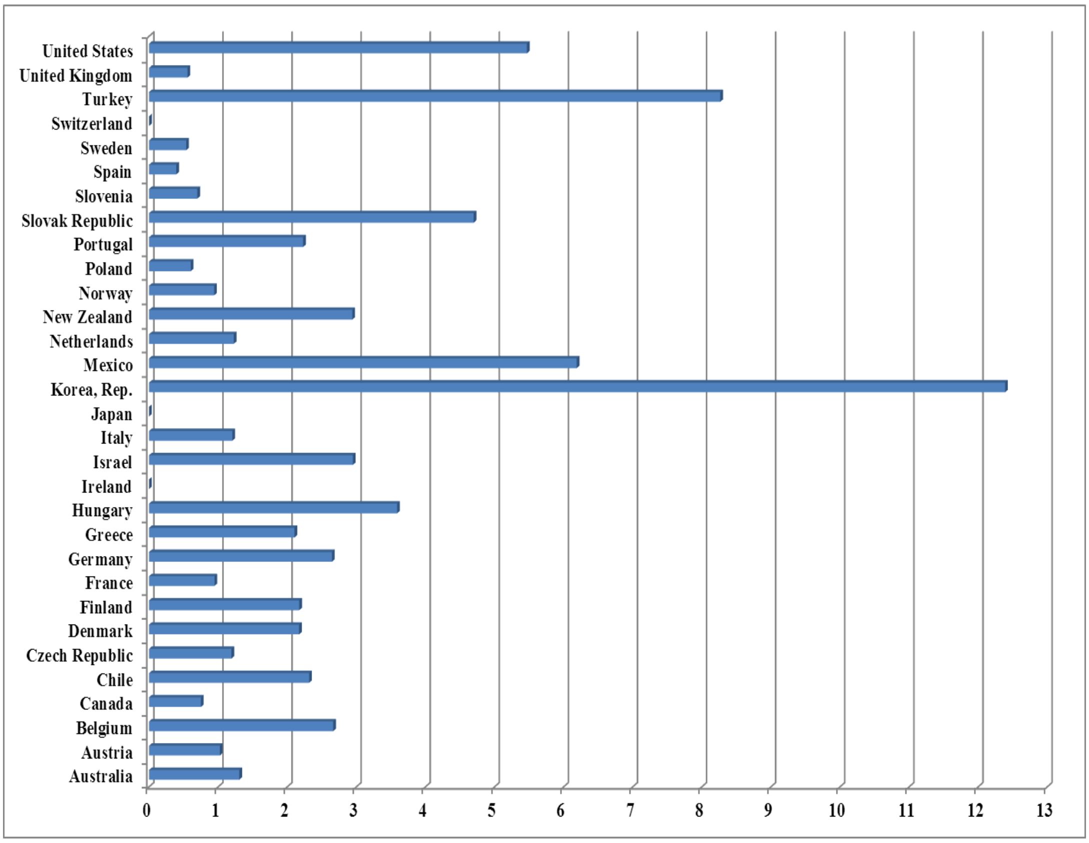

Figure 4.5: Recommended percentage output improvement

Evidence presented in Figure 4.5 shows no output improvement recommendation for Ireland, Japan and Switzerland, since they are performing relatively better than others in 2016. An output improvement of 0.75%, 0.95%, 0.94%, 0.70%, 0.39%, 0.54% and 0.56% are recommended for Canada, France, Norway, Poland, Slovenia, Spain, Sweden and United Kingdom respectively. The relatively low percentage improvement recommendations for those countries are a result of their performance in 2016. However, Korea Rep., Turkey, Mexico, United States and Slovak Rep., had percentage improvement recommendation of 12.5%, 8.3%, 6.2%, 5.5% and 4.7% respectively. This is due to their inefficiency in 2016.

Discussion and Conclusion

Results of the performed study shows that Ireland, Japan, and Switzerland are the most WELF efficicient countries among the OECD members. It was interesting to find that none of the high perfoming countries had minimum WELF consumption in 2016 or previous years. Inferinig that, WELF-Nexus efficiency has less to do with minimum consumption of WELF resources, and more to do with adequate utilization of the consumed resources. Morever, Switzerland had the highest EPI while Japan had the highest life expectancy in 2016.

No common denominator among the WELF component was identified among the low performing countries i.e. Korea Rep. Turkey, Mexico, United state, Slovak Rep., and Hungary. Therefore, the cause of WELF inefficiency in each member state might be different. However,some conclusions can be made individually as to the major cause of inefficiency among the nexus component, by comparing the worst and best performance. United states (US) for exampple, consumes 155 times the amount of energy consumed by Ireland, 5.5 times compared to Japan and 80 times compared to Switzerland. However, the outputs such GDP *per capita* does not compensate for the enormous difference in input. The US have about 15% lesser GDP *per capita* compared to Ireland, 7% lesser compared to Switzerland but 39% more than Japan. Justification can be made that population size is the reason for the great energy concumption gap, however, GDP *per capita*, an economic outcome which is heavily dependent on energy, already factor in population. Therefore it is fair to conclude that energy inefficient among the WELF component is the cause for inefficiency in the US. For Korea Rep., the relatively low EPI and life expectancy might be the cause of inefficiency.

As a step towards recource generativity and WELF efficiency, the recommended percentage improvement in output while consuming relatively the same input in 2016, ensures efficient consumption of WELF in the future. Environmental performance, education attainment and GDP per capita for Turukey is relatively low compared to other member states given the amount of WELF it consumes. Therefore, 8.3% improvement in outputs (EPI and GDP per capita) may improve efficiency for Turkey. Similar output modification with the recommended percentage improvement may gurantee efficiency.

From this analysis, we conclude the following: (1) The WELF efficiency of OECD countries improved significantly during the study period. (2) Reduction in WELF consumption might not neccesarily lead to improvement in WELF efficiency, but proper utilization of the available resource could significantly boost efficiency. (3) The recommeded percentage output improvement are within feasible range, therefore efficient performance is expected.

4.3.2 Estimating Efficiency of Directive 2011/24/EU Cross-border Healthcare in Member States

Improving the health and promoting people's wellbeing are among the key ambitions of the European Union (EU) and as Article 35 of the EU Charter of Fundamental Human Rights declares: *"A high level of human health protection shall be ensured in the definition and implementation of all Union policies and activities"* Health in the EU is conceptualized as people's right and safeguarded in all of the policies of the Union. Legislations in the form of Directives, Regulations and Rulings of the Court of Justice of the European Union are sources of policies in the EU.

Directive 2011/24/EU on patients' rights in cross-border healthcare defines the context under which patients can go to another EU state and receive healthcare, and get reimbursement for the healthcare costs including prescriptions and delivery of medications and medical devices. The Directive prescribes creation of national contact points that provide accurate information to citizens, defines minimum required elements that should be included in medical prescriptions taken from a different EU country to another and promotes the collaboration of medical expertise, health technology assessments and e-health tools (EU, 2011). Directive 2011/24/EU is a critical step towards harmonizing principles in all EU health systems (Azzopardi-Muscat et al., 2018) and a wider cooperation in European healthcare. Protection of health across the EU member states can not only be attained with common health and health-related policies but also with similar commitment to their implementation.

In this thesis, we assess the uptake of the Directive by evaluating efficiency of each member state in implementing it from operational, financial and quality perspective.

Materials and Methods

Data Sources

To evaluate the efficiency of the Directive, this study used data from the Special Eurobarometer 411 on Patient Safety and Quality of Care administered in 2013 and Special Eurobarometer 425 on Patients' Rights in the EU administered in 2014. Items from the Eurobarometer surveys were used as quality and operational success indicators respectively. The Eurobarometer is a series of public opinion surveys that are administered face-to-face in the appropriate national language to about 1000 persons per member state with residents who are 15 years and older. Number of interviews in Malta, Luxemburg and Republic of Cyprus are approximately 500.

The Eurobarometer surveys use stratified random sampling technique in line with population density. The published reports contain no breakdown by gender, age or other characteristics by country. However, technical specifications of the surveys report that a multistage random sampling design has been used and stratified according to sex, age and region (European Commission. Patient Safety and Quality of Care. Special Eurobarometer 411.2014 and European Commission. Patient's Rights in the EU. Special Eurobarometer 425.2015).

The efficiency model utilized here, aims to minimize factors that discourages and maximize factors that promotes implementation of the Directive. Table 4.11 describes the selected inputs and outputs used in efficiency analysis.

The quality indicators used in the efficiency evaluation are participant responses to QC6a and QC6b which are considered as inputs (x1 and x2), and QC7 as output (y1) from the Eurobarometer 411 (Commission, June 2014) The operational indicators used in the efficiency evaluation are participant responses to QD7 (x3) which is considered as an input and QD6 (y2) which is considered as an output from the Eurobarometer 425 (Commission, May 2015)Health expenditure per capita has also been used as an input (x4).

Table 4.11: Inputs and Outputs Used in Evaluating Efficiency

Variables	Role	Indicator	Definition	Source
Harmed by hospital	Input (x1)	Quality	% of people that answered yes they think patients are most likely to be harmed by hospital care	QC6a (EB-411). How likely do you think patients could be harmed by hospital care in our country?
Harmed by non-hospital	Input (x2)	Quality	% of people that answered yes think patients are most likely to be harmed non-hospital health care	QC6b (EB-411). How likely do you think patients could be harmed by non-hospital care in our country?
Re-imbursement problem	Input (x3)	Operational	Number of people that answered yes they encounter problems getting reimbursement from their national health service or health insurer	QD7 (EB-425). Thinking about the last you received treatment in another EU country, did you encounter any problems getting reimbursement from your national health service or health insurer?
Health expenditure	Input (x4)	Operational	Health Expenditure per Capita	Word bank
No. of adverse events	Output (y1)	Quality	Number of people that answered no they did not experience any adverse events when receiving treatment in another EU country	QC7 (EB-411). Have you or your family member ever experienced adverse event when receiving healthcare in another EU country.
Medical treatment in another EU country	Output (y2)	Operational	Number of people that answered yes they have received any medical treatment in another EU country	QD6 (EB-425). Have you received treatment in another EU country in the last 12 months.

To estimate efficiency, the hyperbolic distance function model of (Färe, Margaritis, Rouse, & Roshdi, 2016) is used. Furthermore, the HDF model is adapted to the hyperbolic super-efficiency ranking model of (Johnson & McGinnis, 2009).

Appendix 4 presents the data set used for the cross-border healthcare efficiency analysis. Table 4.12 depicts the efficiency scores and rank of the countries considered.

Table 4.12: Efficiency Scores and Ranks

Country	Efficiency (%)	Rank
Austria	100	2
Belgium	83.5	22
Bulgaria	100	7
Croatia	100	8
Cyprus	73.3	25
Czech Republic	96.5	15
Denmark	87.2	20
Estonia	100	10
Finland	100	9
France	74.2	24
Germany	100	11
Greece	90.9	18
Hungary	100	4
Ireland	98.1	14
Italy	100	6
Latvia	100	11
Netherlands	78.7	23
Poland	100	5
Portugal	99.1	13
Romania	100	3
Slovakia	94.7	16
Slovenia	85.4	21
Spain	94.6	17
Sweden	90	19
United Kingdom	100	1
Average	**93**	

An average efficiency score of 0.93 (93%) is observed, suggesting overall inefficiency of the Directive in the EU countries. Only twelve (48%) of the member states had an efficient uptake of the directive. United Kingdom, Austria and Romania are the top three ranked countries, while Netherlands, France and Republic of Cyprus ranked the lowest.

The results of efficiency improvement using the predefined output target setting model (i.e. model 4.5 and 4.6), and the outputs of UK as the benchmark for inefficient DMUs since it is the best performing country is shown in Figure 4.6. The result indicate that Sweden is able to reach efficiency by making an improvement of 17.9% in quality and operation indicators by reducing hospital and non-hospital harm and number of patient encountering re-imbursement problems. While Cyprus and Ireland need an improvement of 55.9% and 50.4% respectively in the quality and operational indicators to attain efficiency.

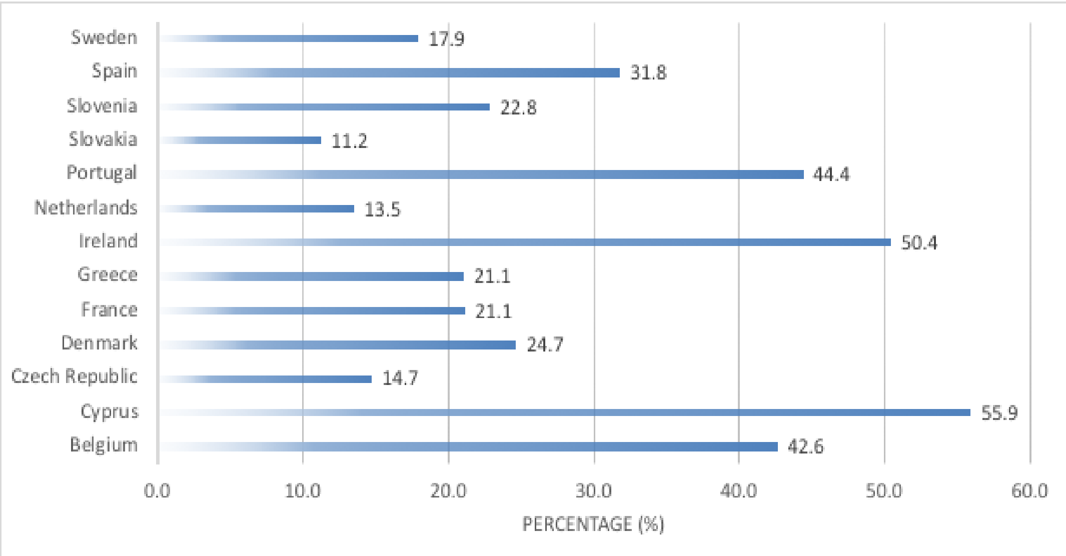

Figure 4.6: Required Percentage Improvement in Selected Inputs for Attaining Efficiency of the Directive

This section evaluates the efficiency of Cross-border Healthcare policy. The study indicates that more than half of the EU member states included in the study attained efficiency of the Directive however variations in the uptake of the Directive is also evident. Countries with similar welfare traditions and close geographical proximity are expected to have well established cross-border collaborations therefore display similar trends in the uptake of the Directive.

As expected, countries with the lowest efficiency scores, such as Cyprus, Ireland, Portugal and Belgium requires the highest percentage improvement, while countries such as Slovakia, Netherlands and Czech Republic needs less percentage improvement.

4.3.3 Green Hen poultry chain

This empirical study is on the poultry chain of 'Green Hen poultry' in Guilan Province, Iran presented by (Homayounfar, Amirteimoori, & Toloie-Eshlaghy, 2014). The central decision making team supervises all thirteen poultries which are considered as DMUs. The central decision making team manages and makes future plans for each unit. The production system produces desirable outputs, produced meat ($y1$) and feed conversion ratio ($y2$), and also undesirable output (mortality and condemn stock $y3$). 'Produced meat' refers to the gross weight of matured chickens, 'Feed conversion ratio' is the amount of body weight gained for every Kilogram of feed consumed, while 'Mortality and condemn stock' refers to the amount of dead or discarded chicks along the production season. The inputs considered are, New born chicks $x1$, Feed cost $x2$ and operational expenses $x3$. 'New born chicks' refers to the newly hatched chicks and 'Feed cost' refers to the dietary cost of the chicks and chickens, while 'operational cost' refers to the gross expenses including hygiene, safety, rent and energy (gas, power, gasoline and water). Table 4.13 shows the inputs and outputs for the thirteen DMUs and their respective efficiency scores.

Table 4.13: Data set for Poultry chain

DMUs	*x1*	*x2*	*x3*	*y1*	*y2*	*y3*	Efficiency
TI	12700	587000	155290	28582.2	1.98	640	0.9788
T2	14670	663500	174060	32387.2	1.93	710	0.9904
T3	13300	590340	169370	28506.3	2	1569	0.9705
T4	15000	701440	193240	34075	1.95	500	1
T5	12000	562620	157730	26256.5	1.98	1014	0.938
T6	14000	614790	177340	29828	1.97	1361	0.9785
T7	13000	637380	172570	30158.7	2.03	790	1
T8	14900	707620	190780	33414.6	2.04	1035	1
T9	13500	650320	169070	30439	1.94	764	0.9766
T10	12800	577220	166170	28223.5	2.03	790	1
T11	19800	921770	225390	44581.2	2.01	1378	1
T12	11000	511640	138220	25683.4	2	474	1
T13	12600	589600	159630	28405.3	1.88	665	0.9729

Table 4.13 shows DMUs T1, T2, T3, T5, T6, T9 and T13 to be inefficient. Using the proposed efficiency improvement models with $(w_r \text{ and } g_i) = 0.5774$. Table 4.14 shows the efficient production possibilities in an assumption of predefined inputs and nondiscretionary inputs (implementing models 4.3 and 4.4). Input *x1* is assumed to be nondiscretionary in DMUs T5, T9 and T13, and x3 is assumed to be nondiscretionary in DMUs T1, T9 and T13, while other inputs are modified. The last three columns estimate the efficient production possibilities for the available predefined inputs.

Table 4.15 shows the estimates of the efficient required inputs in a case of predefined output target (applying models 4.5 and 4.6). Output 3 which is a negative output, and it is assumed to be nondiscretionary in DMU T2, and decreased in DMUs T3 and T6. The last three columns present the required inputs to achieve the predefined output target. The results presented in these examples validate the proposed models.

Table 4.14: Predefined inputs of Poultry chain and target outputs

DMUs	x_{10}^{*}	x_{20}^{*}	x_{30}^{*}	θ	θw_r	y_{10}^{*}	y_{20}^{*}	y_{30}^{*}
T1	**13500**	**645000**	**155290**	0.0207	0.01195	**29115.04**	**2.00438**	**640**
T5	**12000**	**595000**	**145000**	0.0205	0.01184	**26784.19**	**2.00414**	**1014**
T9	**13500**	**625000**	**169070**	0.0182	0.01051	**30907.49**	**1.96144**	**764**
T13	**12600**	**600000**	**159630**	0.0304	0.01755	**29187.83**	**1.91581**	**665**

Table 4.15: Predefined outputs of Poultry chain with target input

DMUs	y_{10}^{*}	y_{20}^{*}	y_{30}^{*}	θ	θg_i	x_{10}^{*}	x_{20}^{*}	x_{30}^{*}
T2	**27500**	**2**	**710**	0.2111	0.121889	**12256.6**	**551146.2**	**146587.4**
T3	**27000**	**1.98**	**850**	0.0942	0.054391	**12223.06**	**540203.9**	**157110.8**
T6	**30000**	**2**	**800**	0.0178	0.010278	**13796.5**	**605316.3**	**175023.5**

Tables 4.14 and 4.15 shows the efficient for the inefficient poultries after applying the proposed predefined input-output target setting models

Chapter 5

CONCLUSION

This thesis discussed models for efficiency improvement of inefficient DMUs by proposing targets for the inefficient DMUs on the frontier. The models proposed presents direct efficient coordinates as oppose to the stepwise approach by most models. In the thesis, three models are proposed under two categories. The first model under the first category is the target setting model using MPSS vectors as a guide, described in chapter 3. The second and third models are under the second category, which is target setting with predefined inputs or outputs illustrated in chapter 4.

The MPSS vector target setting model presented in chapter 3 emphasizes on finding a target on the efficiency frontier with the principal focus on optimal feasible productivity rather than pure efficiency improvement. The idea behind this approach is that MPSS is an appropriate benchmark for inefficient units, thus improvement on the premise of MPSS direction provides improvements in efficiency and productivity for units in the PPS. Most studies suggest points that exclude the properties of economic efficiency, where improvement should follow optimal scale size (i.e. MPSS). This is an important drawback because; optimal scale size is a significant characteristic that should be incorporated while improving performance of an inefficient unit. The approach suggested in this thesis overcomes this drawback by unifying improvement in efficiency and possible optimal productivity, also providing managers with a decision supporting tool for efficiency and productivity improvement. The proposed approach also overcomes the possibility of projecting a target point to the weak efficient frontier like the radial projection approach. The proposed model evades this because it is guided by the MPSS hyperplane. Further advantages of the proposed method include recommendation of multiple target points, providing versatility in improvement options for managers. An empirical study undertaken on Nigeria's electricity industry from 1980-2014, and the analysis of five production lines of a beverage producing company from 2010-2015, validates the proposed model. The model successful proposed multiple targets for Nigeria's electricity industry to attain efficiency in 2020. Application of the model in the beverage producing company also presents multiple efficient targets for the inefficient production lines.

The results of the MPSS vector target setting model proposes a better target compared to the conventional (input /output orientation) efficiency improvement options, and presents less drastic and feasible modifications to the inefficient units.

The second category of the models proposed in this thesis is aimed at incorporating decision makers' desire in efficiency improvement. The models are, target setting using predefined inputs and target setting using predefined outputs. The predefined input model considers decision makers input and propose the efficient output targets, and the predefined output model uses the decision makers' output target and proposes the efficient required inputs. Three empirical studies on WELF-Nexus efficiency of OECD countries, efficiency of cross-border healthcare in EU member states, and Green Hen poultry chain validate the predefined target setting models.

Application of the predefined input model on WELF-Nexus efficiency of OECD countries successfully estimates outputs that will make WELF consumption efficient in the future. Furthermore, application of the predefined output target setting model on cross-border healthcare in EU member states illustrates the required input modification that will enable the inefficient countries undertake the policy efficiently. Lastly, the predefined input and predefined output applied on the Green Hen poultry chain shows the robust nature of the model. Nondiscretionary inputs and outputs were assumed to test the robustness of the model. Results show that the model is capable of accommodating decision makers' desires. The managerial implications of the proposed models enhance decision makers' ability to implement effective and inexorable efficiency improvement strategies.

The methodological contributions of the models proposed in this thesis are evident. The target setting models proposed caters to different needs of decision makers' and efficiency improvement with special intrinsic performance enhancement of DMUs.

As for directions of future studies, modification of the target setting models to directly accommodate negative outputs without converting them to its inverse form, should present and interesting facet of the proposed models. In addition, applying the target setting approach in parametric method of efficiency analysis such as stochastic frontier analysis (SFA) will present a novelty in efficiency improvement. Further modification of the model to handle ratio data is also an interesting direction.

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APPENDICES

Appendix A: Nigeria's Electricity Input/Output Data set

Years	DMUs	Input 1	Input 2	Output 1	Output 2
1980	1	2.5	29.07	31.36	38.82
1981	2	2.8	49.27	31.96	39.12
1982	3	2.9	24.29	32.04	39.42
1983	4	3.2	24.03	29.55	39.72
1984	5	3.6	42.23	27.18	40.03
1985	6	4.2	32.85	26.99	40.33
1986	7	4.6	27.60	28.28	40.63
1987	8	4.6	30.20	26.69	40.93
1988	9	4.6	32.41	27.68	41.23
1989	10	4.9	29.70	28.54	41.54
1990	11	5.9	38.42	23.27	41.84
1991	12	5.9	37.58	26.34	42.14
1992	13	5.8	38.57	28.04	42.45
1993	14	5.8	27.85	27.38	42.76
1994	15	5.8	34.32	29.28	43.06
1995	16	5.8	37.72	27.34	43.37
1996	17	5.9	41.47	27.80	43.67
1997	18	5.9	42.27	28.44	43.98
1998	19	5.9	40.75	28.50	44.29
1999	20	5.9	43.84	29.86	44.59
2000	21	5.9	38.15	27.34	44.9
2001	22	5.9	38.72	28.89	45.21
2002	23	5.9	37.53	39.98	45.52
2003	24	5.9	33.39	37.43	45.83
2004	25	5.9	31.08	44.93	46.14
2005	26	5.90	23.71	43.58	46.45
2006	27	7.44	31.07	33.80	46.76
2007	28	7.96	11.53	31.41	47.07
2008	29	8.47	9.42	27.13	47.38
2009	30	8.70	5.87	24.68	47.69
2010	31	8.43	17.22	33.69	48
2011	32	8.91	9.55	32.93	55.9
2012	33	9.03	8.66	34.47	55.45
2013	34	9.95	15.12	31.46	55.6
2014	35	9.95	16.11	33.08	57.65
2020-PS	36	21.4	12	30.26	70.5
2020-DS	37	32.71	12	35.73	100

Appendix B: Production Lines Input/Output Data set

	Input 1	Input 2	Input 3	Input 4	Output 1	Output 2
DMU1	836191	729215.22	11	115235	779486	13846881
DMU2	33182	208132.51	5	46330	93896	471150
DMU3	108773	436009.43	6	106928	518787	13073963
DMU4	14422	118049.16	3	37039	77643	1072267.3
DMU5	72280	275413.67	6	2513	85574	1600515.8
DMU6	812782	637309.7	10	118233	716385	10337609
DMU7	27507	194255.38	5	45598	66071	450192.95
DMU8	109864	398676.46	6	105982	505381	13013014
DMU9	13999	106691.63	3	37040	81866	1156515.1
DMU10	69551	249610.83	6	2549	79419	1588380
DMU11	787492	626690.89	9	55986	729653	14271344
DMU12	21749	204686.23	5	14209	47865	526322.4
DMU13	95105	422429.78	6	106595	487924	13789412
DMU14	17266	111016.33	3	26925	166373	2839733.9
DMU15	66760	237368.76	5	2082	85310	1838951.4
DMU16	713322	687102.15	9	68319	692072	15859469
DMU17	466	223696.97	5	16589	26421	205064
DMU18	88014	459321.28	6	81282	478374	14813418
DMU19	17529	120731.37	3	133836	113214	1557216.3
DMU20	64905	261676.24	5	1584	88118	2089638.4
DMU21	633045	756110.18	7	61829	738233	16244660
DMU22	436378	249654.64	5	85903	311934	5592703.2
DMU23	249452	522519	6	95332	513083	17390306
DMU24	17031	163061.18	4	17664	55085	981231.78
DMU25	63670	322098.22	6	1697	94787	2289852.4
DMU26	560201	766440.21	7	50855	556809	14758001
DMU27	329300	221205.43	4	64644	208534	3719996.1
DMU28	207499	530946.61	6	57565	410623	17272191
DMU29	14585	170453.07	4	26407	50700	1291422.4
DMU30	80034	329798.57	6	1179	91991	2306254.9

Appendix C: WELF data set for OECD countries

Appendix C.1: 2007 WELF data set for OECD countries

Country Name	x1	x2	x3	x4	y1	y2	y3	y4
Australia	18.76	2931287.79	7741220.00	1782.90	56.88	81.29	40.70	40649.66
Austria	3.58	1055872.48	83879.00	2914.80	69.01	80.18	31.10	43927.13
Belgium	6.22	1369002.50	30530.00	3020.00	63.28	79.78	41.30	41623.44
Canada	41.32	7438147.04	9984670.00	1768.30	57.22	80.54	55.70	41647.39
Chile	35.43	887436.77	756096.00	524.80	54.15	77.92	19.50	18572.57
Czech Republic	1.97	1024109.49	78870.00	1492.60	64.23	76.72	15.50	28844.07
Denmark	0.57	620661.24	43090.00	2923.10	62.93	78.20	36.20	46373.52
Finland	6.56	1060748.68	338420.00	2903.20	63.66	79.26	39.30	42467.26
France	31.41	6103504.65	549087.00	3187.50	68.62	81.11	41.40	37772.08
Germany	32.30	8247962.05	357100.00	2429.70	66.64	79.53	22.60	40473.53
Greece	9.63	877509.61	131960.00	4163.60	58.92	79.44	28.10	32073.96
Hungary	5.58	688718.72	93030.00	1126.00	56.76	73.15	22.00	23491.75
Ireland	0.73	509078.75	70280.00	2522.00	58.27	79.64	44.10	48937.48
Israel	1.95	477026.49	22070.00	2196.50	55.65	80.50	41.50	28744.36
Italy	82.98	5431812.64	301340.00	2991.90	69.36	81.43	18.90	38612.01
Japan	29.20	11890617.93	377930.00	3315.70	62.99	82.51	53.70	36697.31
Korea, Rep.	0.22	4710012.30	99720.00	810.70	56.54	68.41	55.50	28013.70
Mexico	9.15	4489156.88	1964380.00	873.80	47.93	75.59	16.30	16044.25
Netherlands	0.69	2015022.90	41540.00	2285.80	65.33	80.10	36.70	46527.62
New Zealand	0.93	507011.89	267710.00	1285.50	66.86	80.15	34.00	33193.63
Norway	35.57	770084.15	385178.00	4060.40	70.16	80.40	42.70	65083.26
Poland	2.63	2508333.23	312680.00	1262.30	63.25	75.24	30.00	19563.30
Portugal	2.26	756338.22	92090.00	2026.00	54.90	78.32	21.40	27575.24
Slovak Republic	40.10	425869.87	49030.00	1752.30	66.12	74.21	17.50	24049.98
Slovenia	8.51	203566.19	20270.00	2062.40	61.48	78.56	30.10	30190.72
Spain	562.40	3925604.35	505370.00	2172.90	59.27	80.87	40.00	34329.65
Sweden	49.58	1338654.90	450300.00	2087.10	68.20	80.90	40.00	44051.47
Switzerland	78.95	799649.49	41290.00	3351.70	77.99	81.74	35.00	56269.16
Turkey	10.95	2957885.52	785350.00	1193.90	43.87	73.18	14.20	17901.48
United Kingdom	4.91	5571082.52	243610.00	1912.10	68.35	79.45	43.10	38384.25
United States	12.03	59567475.18	9632030.00	1925.60	55.92	77.99	40.40	51011.43

Appendix C.2: 2012 WELF data set for OECD countries

Country Name	x1	x2	x3	x4	y1	y2	Y3	y4
Australia	16.02	3122808.85	7741220.00	3781.50	56.61	82.05	47.20	42561.12
Austria	3.49	1043785.74	83879.00	2586.57	68.92	80.94	36.10	44580.52
Belgium	6.01	1365537.17	30530.00	2856.00	63.02	80.39	43.00	41046.48
Canada	38.80	7180462.86	9984670.00	2633.82	58.41	81.58	57.10	41794.54
Chile	35.43	1014598.63	756096.00	1524.17	55.34	78.61	27.00	21330.24
Czech Republic	1.84	945861.47	78870.00	1544.48	64.79	78.08	27.80	28527.14
Denmark	0.65	560788.95	42890.00	3128.94	63.61	80.05	40.20	44336.81
Finland	6.56	1010030.92	338420.00	3052.57	64.44	80.63	39.70	39912.94
France	30.23	5864272.03	549087.00	3036.50	69.00	81.97	42.90	37344.54
Germany	33.04	8300447.23	357170.00	2317.90	66.91	80.54	29.00	42822.10
Greece	9.63	684435.71	131960.00	2649.47	60.04	80.63	35.00	24364.27
Hungary	5.05	604057.73	93030.00	1226.14	57.12	75.06	30.40	22582.07
Ireland	0.76	415836.66	70280.00	2212.75	58.69	80.85	49.20	45097.07
Israel	1.95	552683.68	22070.00	2821.08	54.64	81.70	44.50	30582.15
Italy	81.45	4920561.24	301340.00	3089.06	68.90	82.24	22.30	35227.62
Japan	29.20	11038188.17	377960.00	3909.83	63.36	83.10	58.60	36367.58
Korea, Rep.	0.26	5120552.91	100150.00	1579.65	57.20	70.21	65.70	31776.90
Mexico	9.15	4763988.56	1964380.00	1549.38	49.11	76.35	20.00	16324.43
Netherlands	0.67	1965271.53	41540.00	2508.21	65.65	81.10	43.00	45411.35
New Zealand	0.93	499717.57	267710.00	3369.50	66.05	81.16	37.00	33286.76
Norway	37.35	763007.04	385178.00	4614.17	69.92	81.45	45.00	63003.41
Poland	2.69	2634756.46	312680.00	1421.93	63.47	76.75	40.80	23218.11
Portugal	2.01	630972.36	92210.00	2403.60	57.64	80.37	28.30	25805.85
Slovak Republic	42.01	387977.48	49036.00	1688.14	66.62	76.11	27.00	26218.47
Slovenia	8.21	204431.69	20270.00	2054.94	62.25	80.12	35.30	27976.92
Spain	485.60	3294173.74	505940.00	2224.88	60.31	82.43	40.40	31109.19
Sweden	53.75	1303347.07	447420.00	3193.44	68.82	81.70	43.50	43308.21
Switzerland	80.30	810175.57	41290.00	3948.32	76.69	82.70	39.30	56149.67
Turkey	10.72	3359232.09	785350.00	1686.07	44.80	74.64	21.00	20282.03
United Kingdom	5.20	5128825.06	243610.00	2278.08	68.82	80.90	47.90	36892.85
United States	11.48	56628804.85	9831510.00	2313.44	56.59	78.74	44.00	50519.53

Appendix C.3: 2016 WELF data set for OECD countries

Country Name	x1	x2	x3	x4	y1	y2	y3	y4
Australia	24.35	3260565.46	7741220.00	2926.56	87.22	82.45	49.30	44414.03
Austria	3.49	1023803.48	83879.00	2272.71	86.64	81.84	39.70	44358.01
Belgium	6.01	1289531.02	30530.00	2706.26	80.15	81.29	44.30	42058.66
Canada	38.80	7545587.25	9984670.00	2167.66	85.06	82.13	60.60	43087.76
Chile	35.43	1053181.04	756096.00	1402.35	77.67	79.16	31.00	22706.72
Czech Republic	1.48	892412.49	78870.00	1510.46	84.67	79.47	32.60	31352.82
Denmark	0.65	503380.92	42890.00	2820.74	89.21	81.10	45.90	45966.28
Finland	6.56	965635.13	338420.00	2757.10	90.68	81.39	41.10	39523.05
France	30.23	5413181.13	549087.00	2727.14	88.20	82.67	44.00	38058.87
Germany	33.04	7996976.24	357170.00	2245.70	84.26	81.09	30.50	44260.36
Greece	9.63	555552.13	131960.00	2245.01	85.81	81.59	41.00	24277.60
Hungary	5.05	615484.68	93030.00	1196.35	84.60	75.96	30.40	25653.85
Ireland	0.76	388005.65	70280.00	1973.14	86.60	81.50	43.30	62991.91
Israel	1.95	533754.44	22070.00	3320.46	78.14	82.05	47.40	32684.20
Italy	81.45	4328283.40	301340.00	2664.08	84.48	83.49	25.60	34715.27
Japan	29.20	10774100.80	377960.00	3019.72	80.59	83.84	60.10	38252.30
Korea, Rep.	0.24	5379569.22	100150.00	1683.27	70.61	71.24	70.00	34985.85
Mexico	9.15	4704662.38	1964380.00	1331.25	73.59	76.88	21.80	16832.46
Netherlands	0.47	1703792.15	41540.00	2340.56	82.03	81.71	45.20	47302.70
New Zealand	1.44	540726.94	267710.00	3148.76	88.00	81.46	43.40	35271.40
Norway	37.35	733619.56	385178.00	3524.06	86.90	82.10	48.60	64179.04
Poland	2.69	2499897.08	312680.00	1181.05	81.26	78.20	43.50	26050.98
Portugal	2.01	584348.83	92210.00	2310.63	88.63	81.52	35.00	27104.87
Slovak Republic	42.01	352357.42	49036.00	1581.46	85.42	77.21	33.40	29223.65
Slovenia	8.21	183793.32	20270.00	1820.52	88.98	81.08	43.00	29932.89
Spain	485.60	2927451.14	505940.00	2137.71	88.91	83.38	41.00	33349.14
Sweden	53.75	1240301.81	447420.00	2819.78	90.43	82.55	47.20	46662.05
Switzerland	80.30	738968.39	41290.00	3631.25	86.93	83.20	48.80	57430.05
Turkey	10.72	3541700.64	785350.00	1470.70	67.68	75.41	30.50	23756.48
United Kingdom	5.20	4642451.09	243610.00	1991.50	87.38	81.60	52.00	39229.85
United States	11.48	59772457.32	9831510.00	2408.10	84.72	78.74	47.50	53341.82

Appendix D: Cross-border healthcare data set for EU Member states

Country	(X1)	(X2)	(X3)	(X4)	(Y1)	Y2
AT Austria	21	33	19	5580.493943	87	5
BE Belgium	49	47	27	4884.066261	73	4
BG Bulgaria	58	62	10	661.8465273	88	1
CY Republic of Cyprus	82	75	20	1819.112996	64	4
CZ Czech Republic	53	55	15	1378.521202	80	7
DE Germany	37	34	11	5410.634639	66	2
DK Denmark	62	59	5	6463.243218	50	4
EE Estonia	37	47	28	1248.279632	61	2
EL Greece	78	71	27	1743.037533	80	2
ES Spain	54	50	13	2658.270424	77	3
FI Finland	34	34	10	4612.290456	61	4
FR France	63	53	26	4958.989226	64	3
HR Croatia	47	47	11	1050.334391	74	5
HU Hungary	41	41	20	1036.623859	82	10
IE Ireland	54	54	8	4239.154489	73	6
IT Italy	57	57	14	3257.75341	84	12
LV Latvia	71	69	8	920.7037654	59	3
NL The Netherlands	47	48	13	5693.859921	53	3
PL Poland	73	70	8	910.2837803	80	7
PT Portugal	75	71	34	2096.823176	85	7
RO Romania	67	61	36	556.8096847	80	8
SE Sweden	40	43	10	6807.717778	46	3
SI Slovenia	45	51	23	2160.746701	68	5
SK Slovakia	51	53	13	1454.81013	75	5
UK The United Kingdom	49	43	1	3934.823561	61	2