Decision Making Under Fuzzy Environment

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ABSTRACT

High competition between universities has been increasing over the years, and stimulates higher education institutions to attain higher positions in the ranking list. Ranking is an important performance indicator of university status evaluation, and therefore plays an essential role in students' university selection. In this thesis, analytic hierarchy process (AHP) and fuzzy analytic hierarchy process (FAHP) are applied for the comparison and ranking of performances of five UK universities, according to four criteria. The criteria used for the evaluation of universities' performances are teaching, research, citations, and international outlook. The comparison matrix is used to compare criteria as well as alternatives with respect to each criterion. Consistency index and consistency ratio are calculated by using eigenvalues to check the consistency of comparison matrix and by obtaining the final priorities, ranking is possible with AHP. Eigenvalues are also calculated to check the consistency for FAHP as well as eigenvectors are calculated for ranking process. Besides these, by the calculation of coefficient of variation for all alternatives, it becomes possible to rank the universities in prioritized order with FAHP.

Keywords: Ranking, Analytic Hierarchy Process (AHP); Fuzzy Analytic Hierarchy Process (FAHP); ranking; consistency; eigenvalue; eigenvector; coefficient of variation.

ÖZ

Üniversiteler arasındaki yüksek rekabet yıllar geçtikçe artmakta ve yüksek öğretim kurumlarını sıralama listesinde daha üst sıralara ulaşmaya teşvik etmektedir. Sıralama, üniversite durum değerlendirmesinin önemli bir performans göstergesidir ve bu nedenle öğrencilerin üniversite seçiminde önemli bir rol oynar. Bu tezde, Birleşik Krallık'taki beş üniversitenin performanslarının dört kritere göre karşılaştırılması ve sıralanması için analitik hiyerarşi süreci (AHP) ve bulanık analitik hiyerarşi süreci (FAHP) uygulanmıştır. Üniversitelerin performanslarının değerlendirilmesi için kullanılan kriterler öğretim, araştırma, alıntılar ve uluslararası görünümdür. Karşılaştırma matrisi, kriterleri ve her bir kritere göre alternatifleri karşılaştırmak için kullanılır. Karşılaştırma matrisinin tutarlılığını kontrol etmek için özdeğerler kullanılarak tutarlılık indeksi ve tutarlılık oranı hesaplanır ve nihai öncelikler elde edilerek AHP ile sıralama yapılabilir. Özdeğerler ayrıca FAHP için tutarlılığı kontrol etmek için hesaplanır ve ayrıca sıralama işlemi için özvektörler hesaplanır. Bunların yanı sıra tüm alternatifler için varyasyon katsayısı hesaplanarak FAHP ile üniversiteleri öncelik sırasına göre sıralamak mümkün hale geliyor.

Anahtar Kelimeler: Sıralama, Analitik Hiyerarşi Süreci (AHP); Bulanık Analitik Hiyerarşi Süreci (FAHP); sıralama; tutarlılık; özdeğer; özvektör; varyasyon katsayısı.

DEDICATION

To My Family

ACKNOWLEDGMENTS

I would like to give special thanks to my supervisor Prof. Dr. Rashad Aliyev for his valuable contribution and for supporting me all the time.

I would like to give many thanks to my family for providing me all kinds of support both material and spiritual all the time.

I would like to give many thanks to my wife for her support and giving me morale in all circumstances.

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Chapter 1

INTRODUCTION

In the real world, one of the characteristics of being human is having the ability to make appropriate decisions in various situations. These decisions offer benefits and conveniences in every aspect of life. The important elements of decision making process are alternatives and criteria. Decision making is a process of choosing and identifying the best alternative among available options to achieve the desired purpose in many areas of human activity.

The ranking of universities has been carried out using different techniques. Main goal of decision processes in real-life problems is to deal with the symmetry or asymmetry of different types of information. We consider that multi-criteria decision making (MCDM) is well applicable to symmetric information modelling.

MCDM has been a very fast-growing field in recent years. It has taken its role in different application areas due to being useful and attractive for solving complex real-world problems. MCDM includes a finite set of alternatives, so that a decision maker can rank them, and there is a finite set of criteria which are weighted by decision maker, with respect to the importance of criteria. So, MCDM is an advanced discipline which can be applied to complex decision making problems by providing an effective solution for ranking of alternatives. The desired result can be achieved, adhering to the appropriate criteria of the related decision making problem.

Analytic hierarchy process (AHP) is a frequently used and a well-known technique of MCDM discipline, which is based on pairwise comparisons of criteria/alternatives for alternatives' evaluation. AHP was firstly introduced by Thomas Saaty in 1980 [1]. AHP uses criteria and sub-criteria initial weights as well as hierarchical dependencies between criteria [2]. AHP is a powerful structured technique used to organize and analyze complex decisions when the alternatives in a problem are hardly quantified and compared. AHP provides appropriate results if the uncertainty in comparative judgment is not taken into consideration. On the other hand, preferences or judgments of decision makers sometimes include ambiguous expressions and inconsistencies. In such cases, it is preferable to use the extension of AHP which is fuzzy analytic hierarchy process (FAHP).

In this thesis, five United Kingdom universities is ranked according to AHP as well as fuzzy AHP (FAHP). The criteria to be considered for ranking is taken from the Times Higher Education data provider.

1.1 Thesis Outline

The chapters of this thesis are organized as follows. Chapter one includes the introduction part of the thesis. Chapter two consists of literature review, preliminaries and statement of the problem. Chapter three explains AHP model and demonstrates the ranking of five UK universities by using AHP technique. Chapter four explains fuzzy AHP model and demonstrates the ranking of five UK universities by using fuzzy AHP technique. Finally, the conclusion part is given in chapter five.

Chapter 2

LITERATURE REVIEW AND PRELIMINARIES

2.1 General Review

After the AHP method was introduced by Thomas Saaty, it has been discussed in many studies. Choice, ranking, prioritization, resouce allocation, benchmarking, quality management, conflict resolution are some of the decision cases that can be obtained using AHP method. The application areas of this method are very wide, some of these areas are tourism, education, engineering, and economy. In this thesis, due to the fact that a study is carried out on the field of education, the application of both AHP and Fuzzy AHP (FAHP) methods to education process are discussed in literature review.

AHP technique provides a significant result to solve group decision making processes for a large number of academic evaluation cases [3]. Research papers have been evaluated by using AHP method in Villanova University.

In [4], AHP approach is used to develop an effective academic staff promotion system in University of Kuala Lumpur. Some important criteria for staff evaluation are used to obtain the best alternative. AHP method is used to select the most convenient staff having enough qualifications for academic promotion.

Teaching evaluation is essential to improve the quality of teaching at the university. This may be possible by assessing the quality parameters of teachers. In [5], AHP method is used in to evaluate the quality of university teaching.

In [6], fuzzy AHP method is applied on three Dutch universities to demonstrate the performance of the model, and the universities are ranked according to the importance of such criteria as networking and knowledge exchanging ability, general attractiveness, research ability and commercialization ability, and this method can provide obtaining the best university in an uncertain situation.

In [7], fuzzy AHP is used for evaluation of management quality at private higher education. As a case study, STMIC Pringsewu college located in Lampung's Province is used, that needs an improvement in graduates' quality. Six main criteria and 30 sub-criteria are used to make an evaluation of management quality. For this reason, the weights for each criterion are calculated, and ranking of criteria is performed according to the optimal weight.

The knowledge management, leadership effectiveness and organizational culture are the factors used to evaluate the university organizational performance in Taiwan [8]. The results show that the weights of these factors are 55.6%, 28.1% and 16.3%, respectively. So, the knowledge management is defined as being the most significant factor.

An intellectual capital (IC) evaluation model is developed to measure its contribution for the university performances in Taiwan [9]. IC measurement indicators are formulated by using AHP method to develop the IC evaluation model. The integration of AHP method and fuzzy approach allows one to eliminate the vagueness on decision makers' judgments, and to develop a hierarchy structure to prioritize the IC measurement indicators for better understanding.

In [10], the selection of university academic staff involving uncertainty is made up by applying fuzzy AHP method. It is mentioned that the AHP model is unable to cope with imprecise judgment in pairwise comparisons, and this disadvantage can be overcome by using fuzzy AHP. Three alternatives based on three criteria are considered in numerical example, where the first criterion is work experience, the second criterion is academic background, and the third criterion is individual skill. The results show that the candidate with the highest normalized weight is determined as the best alternative for employment.

Examining fuzzy AHP for the optimal academic staff selection to be suitable for the required post is studied in [11]. Five candidates are evaluated and ranked according to ten distinct sub-criteria. Since AHP is insufficient to overcome the impreciseness and subjectivity in pairwise comparisons, the fuzzy AHP method becomes an important tool in terms of using fuzzy numbers and linguistic variables to achieve accuracy and consistency. In addition, triangular fuzzy numbers are used to set the fuzzy rating and fuzzy weights.

The combination of fuzzy AHP and fuzzy comprehensive evaluation approach to conduct a teaching performance evaluation is proposed in [12]. The weights of factors and sub-factors are estimated by analysis of fuzzy AHP method, which significantly reduces uncertainty in group decision making. The proposed framework is a useful tool to improve the education quality level in higher education institutions.

In [13], the proposed method evaluates the students' performance in e-learning systems. Six main criteria and 24 sub-criteria are contained in a framework intended for selection of a best student involved, and the criteria weights are calculated by using fuzzy AHP method.

In [14], an e-learning system performance is evaluated on the base of FAHP and critical success factors (CSFs). The evaluation is realized according to seven main CSFs. This performance evaluation helps in the development of strategic planning of e-learning plan.

Fuzzy AHP method is used to assess the ranks of priorities of multiple factors affecting e-learning success in higher education [15]. The study defines the five most influential factors from lecturers' points of view and five most influential factors from students' point of view, to successfully implement the e-learning at Sebelas Maret University.

2.2 Statement of the Problem

The decision making problem should be stipulated by wide range of alternatives and criteria. In this research, five alternatives and four main criteria are addressed for the university ranking problem. These criteria are teaching, research, citations and international outlook [16].

There are five factors affecting the teaching criterion, which are reputation survey, staff to student ratio, doctorate to bachelor's ratio, doctorates awarded to academic staff ratio and institutional income. Reputation survey, research income and research productivity are the factors affecting the research criterion. Field weighted citation impact is the factor affecting the citations criterion. There are three factors affecting the international outlook criterion which are the proportion of international students, proportion of international staff and international collaboration [16].

The purpose of this thesis is an application of AHP and fuzzy AHP techniques to select the optimal alternative from the given five alternatives (universities) A, B, C, D and E.

2.3 Preliminaries

Definition 2.3.1 (Fuzzy Set). Let X be the universe set of the set of objects and x be the elements of the universe set X. Let y be the subset of X which is used for the membership, and the characteristic function μ_y from X to {0,1} can be described as follows

$$\mu_y(x) = \begin{cases} 1 \; if \; and \; only \; if \; x \in y, \\ 0 \; if \; and \; only \; if \; x \notin y \end{cases}$$

{0,1} is the value set where 1 shows the membership and 0 shows non-membership. If the value set is in the interval [0,1], then A is a fuzzy set. Moreover, μ_y (x) is the degree of membership of elements x in the fuzzy set y, $\mu_y : X \to [0,1]$. As much as the value of membership μ_y (x) for the element x is closer to 1, then so much element x belongs to the fuzzy set y. This fuzzy set y is described as

$$y = \{(x, \mu_y(x)), x \in X\}$$

where x is an element in the universe set, μ_y (x) is the degree of membership of x and X is the universe set [17-19].

Definition 2.3.2 (Fuzzy Number and Triangular Fuzzy Number). Let y be a fuzzy subset of the universe set. Fuzzy number y is a fuzzy subset of real numbers that has important characteristics:

- The membership function μ_y (x) is continuous from R to [0,1].
- The membership function μ_y (x) is normal, that is, there exists the number x_0 so that $\mu_y(x_0) = 1$.
- If all of the level sets are convex in classical sense for a fuzzy set y, that means that this fuzzy set y is convex.

A triangular fuzzy number y can be represented as (y_l, y_m, y_u). Then, the membership function of the triangular fuzzy number μ_y (x) can be expressed in the following form [17,18]:

$$\mu_y(x) = \begin{cases} \dfrac{x - y_l}{y_m - y_l}, & If \quad y_l \le x \le y_m \\ \dfrac{y_u - x}{y_u - y_m}, & If \quad y_m \le x \le y_u \\ 0, & otherwise \end{cases}$$

Definition 2.3.3 (Fuzzy Decision Making). Fuzzy decision making is used to choose the best alternative among several ones in the presence of uncertainty. A set of alternatives $A_1, A_2, \ldots, A_n$ depends on some criteria $H_1, H_2, \ldots, H_m$. So, the best alternative is one that fulfills all criteria [17,18].

Definition 2.3.4 (Fuzzy Preferences). Fuzzy preferences are actually based on fuzzy logic and fuzzy sets. In MCDM, fuzzy or uncertain preferences can be written as fuzzy utilities or weighted sums. These fuzzy utilities and fuzzy weighted sums are fuzzy numbers. A fuzzy preference is a significant type for fuzzy binary relation, and is used to generate the degree of preference between two alternatives when there are certainty and uncertainty preferences.

Let A represent a set of alternatives $A_1, A_2, \ldots, A_n$ and $n > 1$. A fuzzy preference for the set of alternatives A is a fuzzy relation on A denoted by $R = (r_{ij})_{nxn}$ which has a membership function denoted by $\mu_R : A \times A \rightarrow [0,1]$. Here, $\mu_R (A_i, A_j) = r_{ij}$ represents the degree of preference for alternative A_i over A_j.

Chapter 3

ANALYTIC HIERARCHY PROCESS (AHP) FOR RANKING OF UNIVERSITIES

3.1 Comparison Matrix

In AHP method, the comparison matrix (n x n) should be created for criteria as well as for alternatives with respect to each criterion. The form of the comparison matrix is as follows:

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{n1} & a_{n2} & \ldots & a_{nn} \end{bmatrix}$$

The diagonal entries from a_{11} to a_{nn} take the value 1 ($[a_{ii} \ldots a_{nn}] = 1$). The rest entries take the value between 1-9 describing numerical values of comparison matrix [20]. If a_{12} takes the value 3, then a_{21} becomes $1/a_{12}$ that is 1/3 ($[a_{ji} = 1/a_{ij}]$).

3.2 Methodology of AHP

There are some steps to be followed to reach results using AHP method [20]. The principle of working with AHP method is stepwise explained below.

Step 1. The model is developed: the decision problem is broken down into a hierarchy structure, and the first, second, and third levels of the hierarchy consist of goal, criteria, and alternatives, respectively. Hierarchy structure for ranking of universities is shown in Figure 1.

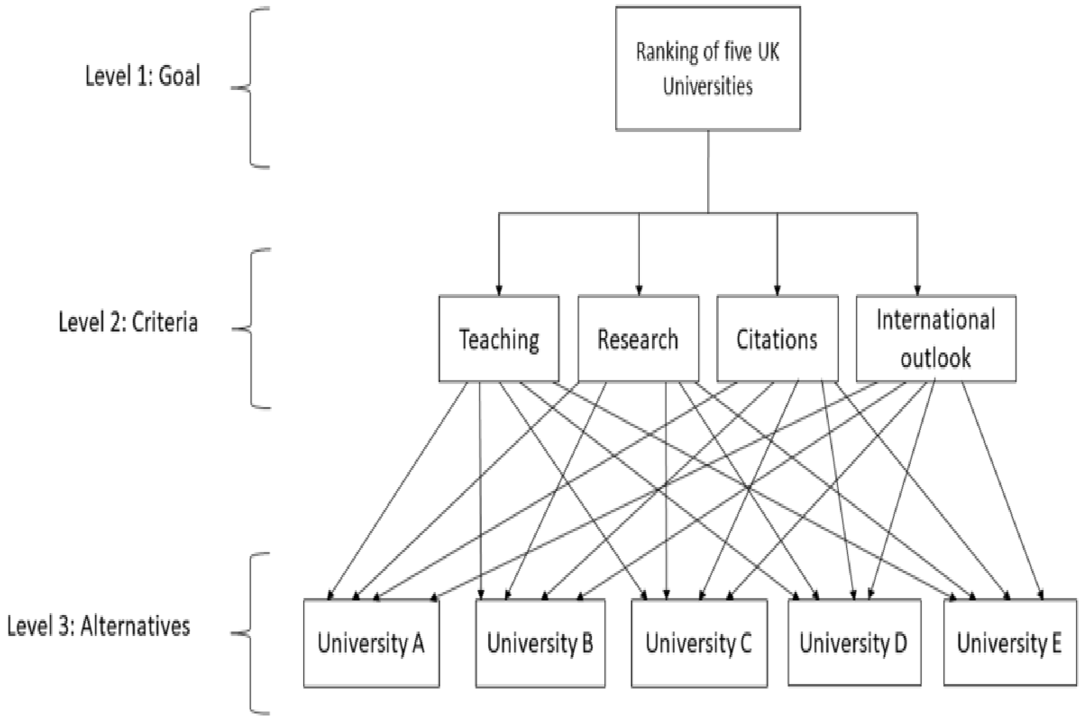

Figure 1: Hierarchy Structure for Ranking of Universities

Step 2. Priorities (weights) for the criteria are obtained: for this reason, pairwise comparisons are executed between criteria. Then the consistency of judgments is checked to be sure about the proportionality and transitivity. The pairwise comparisons between the criteria should be realized to obtain the priorities by considering a scale developed by Saaty [20]. The maximum numerical value 9 shows the extremely important property whereas the minimum numerical value 1 describes equally important property. The numerical values 2 and 3 describe moderately more important property; numerical values 4 and 5 describe strongly more important property. After defining the comparison matrix of criteria, it is required to calculate the normalized matrix which is carried out in the following order:

- Sum the values in each column of the comparison matrix.
- Divide each value by sum of related column to calculate the normalized matrix.
- Obtain the priorities for the criteria by taking the average of each row from the normalized matrix.

The consistency of comparison matrix is checked as follows:

- By using the comparison matrix, multiply each value in the first column with the first criterion priority, then multiply each value in the second column with the second criterion priority and continue this process for all columns.
- Sum the values in each row to obtain the values that are called weighted sum.
- Divide the values of weighted sum by related priority of each criterion. After division, take the average of the values to calculate λ_{max}.
- Calculate the consistency index (CI) and consistency ratio (CR) by using the following formulas (n is a number of compared elements):

$$CI = \frac{\lambda_{max} - n}{n - 1} \qquad CR = \frac{CI}{RI}$$

where RI (Random Index) gets available values from a randomly generated comparison matrix. The value of CR should be ≤ 0.1 to claim that it is consistent. For n=4 the value of RI is 0.9; for n=5 the value of RI is 1.12; for n=6 the value of RI is 1.24 [20].

Step 3. Priorities for alternatives are obtained: the pairwise comparison is done between the alternatives with respect to each criterion. Then the consistency of the pairwise comparisons is checked:

- The comparison matrix is done with respect to each criterion.
- Calculate the normalized matrix and take the average of rows to obtain the priorities for each alternative.
- Steps for consistency checking are same.

Step 4. Final (overall) priorities are obtained: the priorities of the alternatives with respect to each criterion are combined as weighted sum by considering the weight of each criterion to determine the final priorities. The alternative with the highest final priority is the best choice [20].

The priorities of alternatives for each criterion and priorities of criteria are considered. Final priorities of alternatives are calculated by multiplying each of alternative priorities (with respect to each criterion) with corresponding criteria weights, then by taking the summation of each row. So the best alternative is obtained.

3.3 Numerical Example of AHP

In this section, AHP model is applied for ranking five UK universities A, B, C, D and E.

3.3.1 Criteria Priorities

The values of the comparison matrix for criteria are shown in Table 1. Table 2 shows the normalized matrix for Table 1. In order to have the criteria priorities, we use the average of each row from the normalized matrix (Table 2).

Table 1: Comparison matrix for criteria

Criterion	Teaching	Research	Citations	International outlook
Teaching	1	1	1	4
Research	1	1	1	4
Citations	1	1	1	4
International outlook	0.25	0.25	0.25	1
Sum	3.25	3.25	3.25	13

Table 2: Normalized matrix and criteria priorities

Criterion	Teaching	Research	Citations	International outlook	Priority
Teaching	0.308	0.308	0.308	0.308	0.308
Research	0.308	0.308	0.308	0.308	0.308
Citations	0.308	0.308	0.308	0.308	0.308
International outlook	0.076	0.076	0.076	0.076	0.076

The consistency of comparison matrix should be checked. The comparison matrix is multiplied with the criteria priorities, and the weighted sum is calculated in Table 3.

Table 3: Results of weighted sum for criteria

Criterion	**Teaching**	**Research**	**Citations**	**International outlook**	**Weighted sum**
Criteria Priorities	0.308	0.308	0.308	0.076	
Teaching	1	1	1	4	1.228
Research	1	1	1	4	1.228
Citations	1	1	1	4	1.228
International outlook	0.25	0.25	0.25	1	0.307

The weighted sum for each criterion is divided by respective priority to find eigenvalues λ`s, then average is calculated to find the maximum eigenvalue λ_{max} shown in Table 4.

Table 4: Result for maximum eigenvalue λ_{max}

Weighted sum	**Priority**	**λ**	**λ_{max}**
1.228	0.308	3.987	
1.228	0.308	3.987	
1.228	0.308	3.987	
0.307	0.076	4.039	16/4 = 4
		Sum: 16	

The consistency index $\text{CI} = \frac{\lambda_{\text{max}} - n}{\text{n} - 1} = \frac{4-4}{4-1} = 0$, and the consistency ratio $\text{CR} = \frac{\text{CI}}{\text{RI}} = \frac{0}{0.9} = 0 < 0.1$. So, the comparison matrix is perfectly consistent. Figure 2 shows the computer simulation results for criteria priorities.

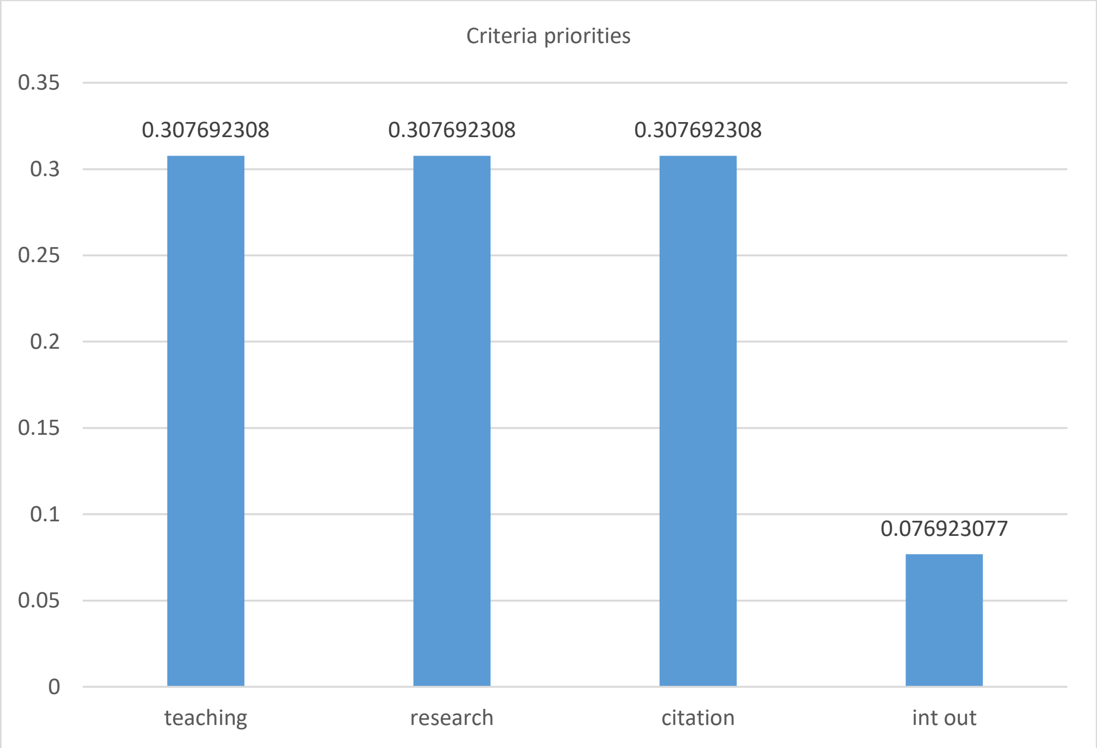

Figure 2: Computer Simulation Results for Criteria Priorities

The same steps are done for each criterion by considering all alternatives. The weights for alternatives with respect to each criterion are taken from [21].

3.3.2 Teaching Criterion Priorities

For teaching criterion priorities, comparison matrix is shown in Table 5, and the computer simulation results for teaching criterion priorities are shown in Figure 3. $\lambda_{max} = 5.0192, \mathrm{CI} = 0.0048, \mathrm{CR} = 0.0042 < 0.1$

Table 5: Comparison matrix for teaching criterion

Teaching	**A**	**B**	**C**	**D**	**E**
A	1	1	2	3	1
B	1	1	2	2	1
C	0.5	0.5	1	1	0.5
D	0.333	0.5	1	1	0.5
E	1	1	2	2	1

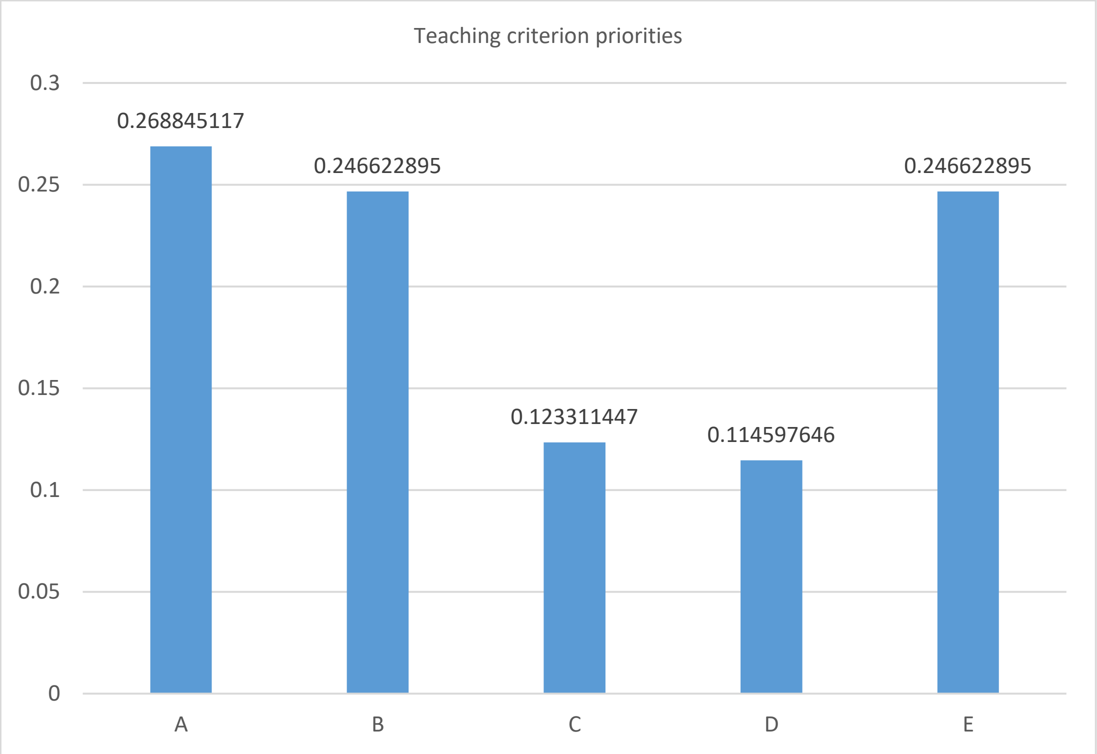

Figure 3: Computer Simulation Results for Teaching Criterion Priorities

3.3.3 Research Criterion Priorities

For research criterion priorities, comparison matrix is shown in Table 6, and the computer simulation results for research criterion priorities are shown in Figure 4. $\lambda_{max} = 5.093, \mathrm{CI} = 0.02325, \mathrm{CR} = 0.0207 < 0.1$

Table 6: Comparison matrix for research criterion

Research	**A**	**B**	**C**	**D**	**E**
A	1	1	2	3	2
B	1	1	1	2	1
C	0.5	1	1	1	1
D	0.333	0.5	1	1	0.5
E	0.5	1	1	2	1

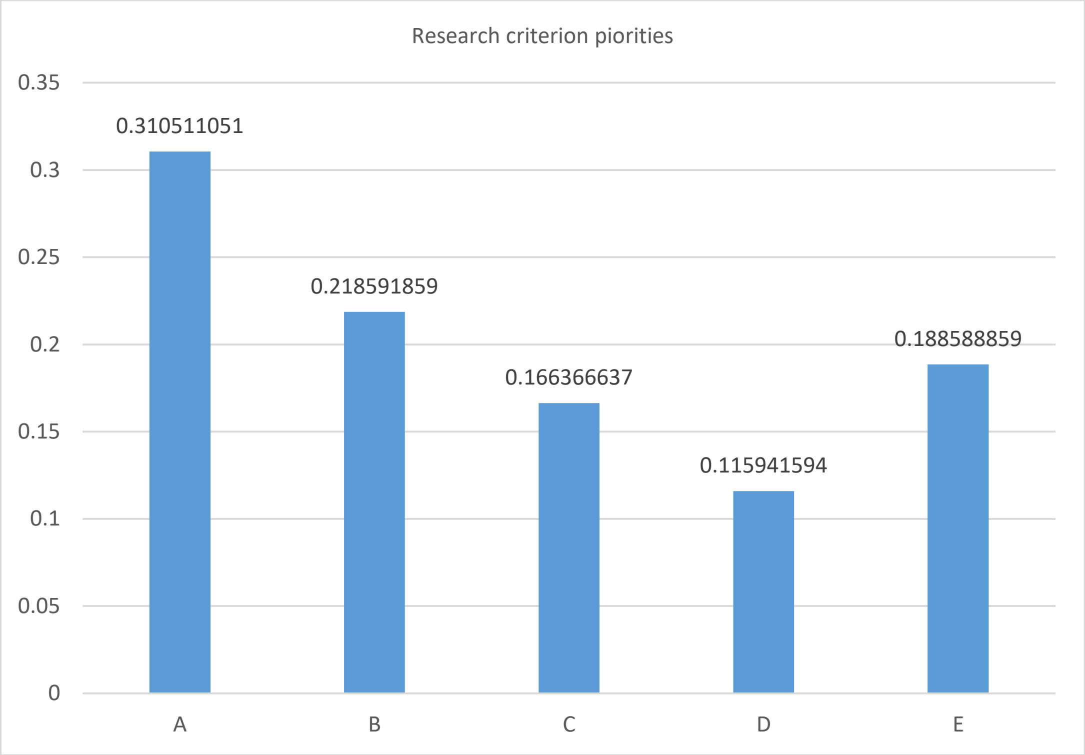

Figure 4: Computer Simulation Results for Research Criterion Priorities

3.3.4 Citations Criterion Priorities

For citations criterion priorities, the comparison matrix is shown in Table 7, and the computer simulation results for citations criterion priorities are represented in Figure 5. $\lambda_{max} = 5.0172, CI = 0.0043, CR = 0.0038 < 0.1$

Table 7: Comparison matrix for citations criterion

Citations	**A**	**B**	**C**	**D**	**E**
A	1	1	1	1	3
B	1	1	1	1	3
C	1	1	1	1	3
D	1	1	1	1	2
E	0.333	0.333	0.333	0.5	1

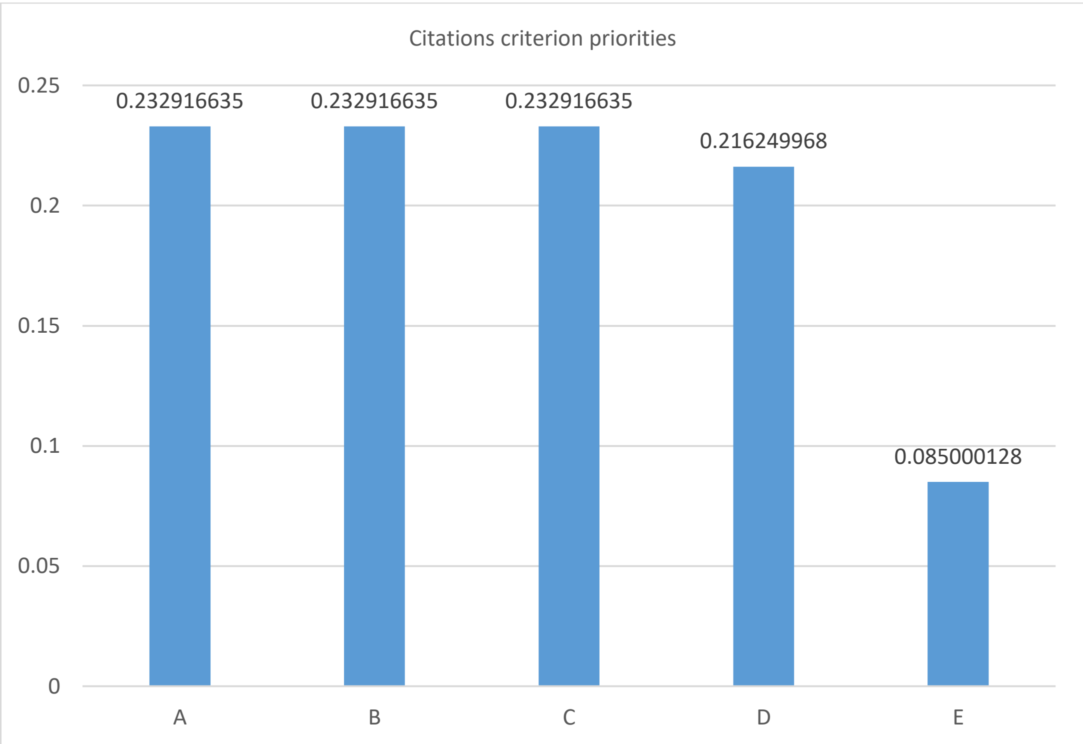

Figure 5: Computer Simulation Results for Citations Criterion Priorities

3.3.5 International Outlook Criterion Priorities

For international outlook criterion priorities, the comparison matrix is shown in Table 8. Figure 6 shows the computer simulation results for international outlook priorities. Being very close to each other, the value 1 for weights of alternatives is assigned. $\lambda_{\max} = 5, \mathrm{CI} = 0, \mathrm{CR} = 0 < 0.1$

Table 8: Comparison matrix for international outlook criterion

International outlook	**A**	**B**	**C**	**D**	**E**
A	1	1	1	1	1
B	1	1	1	1	1
C	1	1	1	1	1
D	1	1	1	1	1
E	1	1	1	1	1

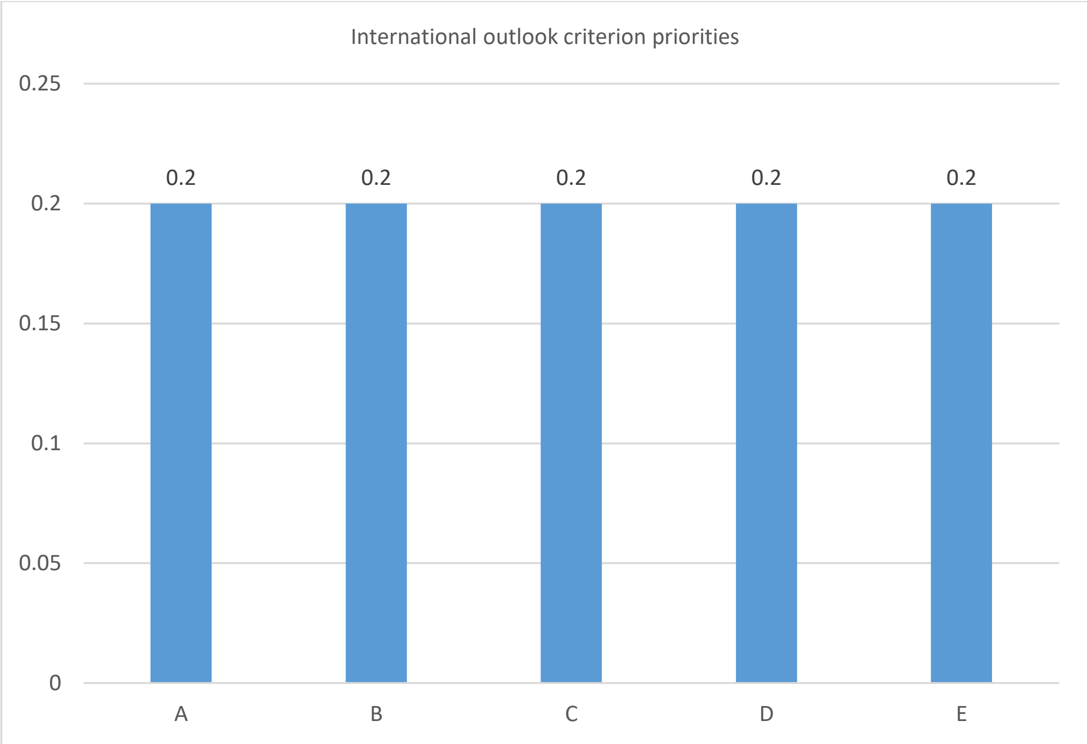

Figure 6: Computer Simulation Results for International Outlook Criterion Priorities

3.3.6 Final Priorities

The priority of alternatives for each criterion is multiplied by criteria priorities and then each row is summed up to find final priorities of each alternative as shown in Table 9.

Table 9: Results for final priorities of each alternative

	Teaching	**Research**	**Citations**	**International outlook**	**Final Priorities**
Criteria Priorities	0.308	0.308	0.308	0.076	
A	0.269	0.310	0.233	0.2	0.2653
B	0.247	0.219	0.233	0.2	0.2305
C	0.123	0.166	0.233	0.2	0.1760
D	0.114	0.116	0.216	0.2	0.1526
E	0.247	0.189	0.085	0.2	0.1756

The computer simulation results for final priorities of each alternative are shown in Figure 7, and according final priorities results, the universities are ranked as A > B > C > E > D. So, the best alternative is obtained to be the university A [22].

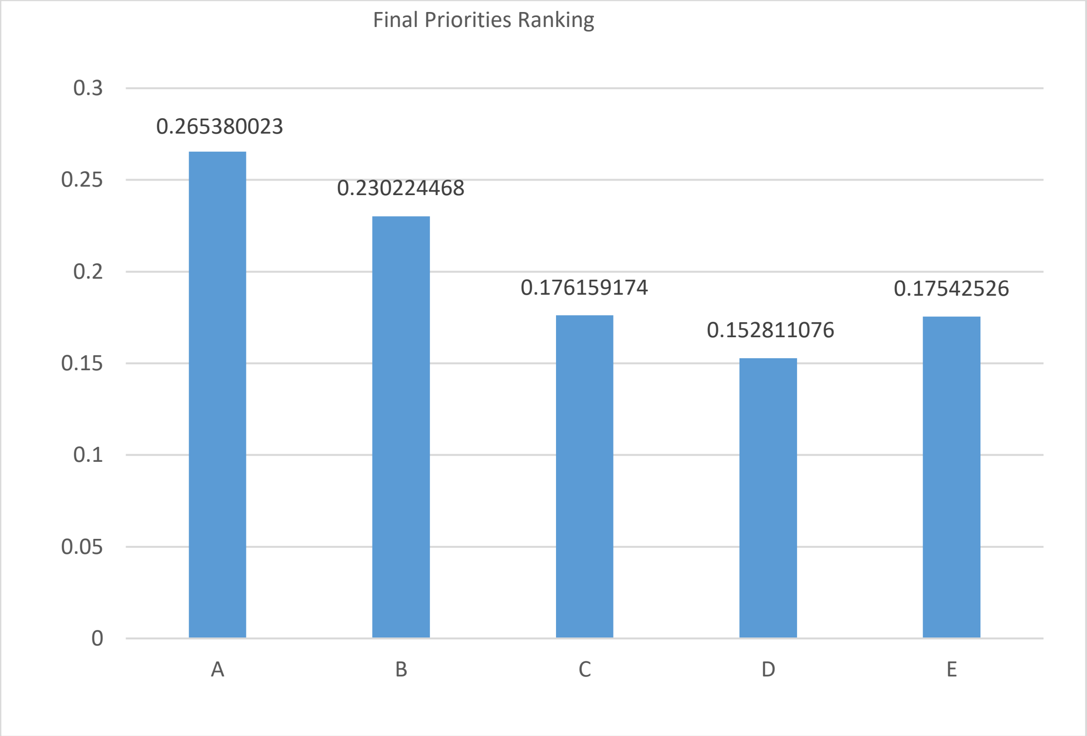

Figure 7: Computer Simulation Results for Final Priorities

Chapter 4

FUZZY ANALYTIC HIERARCHY PROCESS (FAHP) FOR RANKING OF UNIVERSITIES

Uncertainty is a common phenomenon in the real world. Human decisions which include preferences are mostly vague or uncertain. In other words, the selection of alternatives is performed under the environment filled by complex and imprecise information. As the system complexity increases, uncertainty of problems and in human's thoughts increases consequently. Therefore, there is a need for a system that provides a reliable and precise solution, while dealing with incomplete and uncertain information.

The classical decision making approaches such as AHP are applied in the presence of certain and complete information. Unfortunately, such approaches are not always capable of providing an exact solution of complex problems, and are insufficient to work under many circumstances of real-life situations. For this reason, in order to provide an optimal solution of the problem, fuzzy decision making concept becomes extremely important in the presence of vague information, and this information can be assessed by applying fuzzy set theory, which was firstly proposed by Zadeh in 1965 [23].

AHP method can cause some problems while dealing with rank reversal problem that means the alternatives' preferences can be reversed in case of adding or deleting any alternative(s), priorities derivations method, the comparison scale [24-31] as well as decision maker's preference aggregation from pairwise comparison matrices in the environment of inaccurate evaluations for determining an appropriate solution for decision making problem [32]. Therefore, AHP is not considered as a convenient method for decision making under uncertainty. Hence, it is necessary to reach better outcomes by using the extension of AHP which is fuzzy analytic hierarchy process (FAHP). Despite the fact that application of fuzzy approach in decision making process using AHP can sometimes lead to disorientation of the eigenvector of the matrix of pairwise comparisons while perturbing the entries of the matrix [33], fuzzy sets can be included in the pairwise comparison to cope with the uncertainty and vagueness in problems. The FAHP model allows decision makers to better specify their preferences in fuzzy environments.

Fuzzy AHP has the capability and the power to represent the uncertain situations. This method investigates the pairwise comparisons of alternatives and criteria in terms of importance and dominance of these alternatives and criteria. Fuzzy AHP is frequently used in the solution of complex decision making problems, as it takes into account both qualitative and quantitative factors [34]. Fuzzy comparison matrices are used to tolerate uncertainty. When comparing alternatives, it is required to use uncertainty by the decision maker. For this reason, in such situations, the use of fuzzy numbers is preferred instead of using crisp numbers. In addition, consistency checking is an important factor in the fuzzy AHP method. Consistency checking proves that the constructed fuzzy pairwise matrices are acceptable and consistent [35,36].

4.1 Methodology of Fuzzy AHP (FAHP)

In addition to many decision-making problems in the field of education, ranking among universities is also considered as a decision-making problem. Main goal in this MCDM problem is to rank the universities from the best to the worst by using multiple criteria. In this section, the university ranking problem will be modelled by using the fuzzy AHP method according to eight steps. The steps of fuzzy AHP method are described below:

Step 1: Construct the fuzzy matrix $\tilde{C}$ and then decompose it into three matrices called C_l, C_m, C_u [37]. Fuzzy matrix is a matrix with entries as triangular fuzzy numbers. Such a matrix shows the pairwise comparisons of the criteria (mxm matrix) or pairwise comparison of the alternatives with respect to each other (nxn matrix). After constructing the fuzzy triangular matrix, it is divided into three matrices as C_l, C_m, C_u which mean matrices of lower, medium and upper values of triangular fuzzy numbers-based entries, respectively.

Step 2: The three matrices obtained in step 1 will be used in the next step to calculate the system of fuzzy linear homogeneous equations [37].

$$\overline{C}_l w_l + \overline{C}_m w_m + \overline{C}_u w_u - \overline{\lambda}_l w_l - \overline{\lambda}_m w_m - \overline{\lambda}_u w_u = 0$$

$$\overline{C}_l = 2C_l + C_m, \qquad \overline{C}_m = C_l + 4C_m + C_u, \qquad \overline{C}_u = C_m + 2C_u$$

Step 3: Calculate the eigenvalues $\overline{\lambda}_l$, $\overline{\lambda}_m$, $\overline{\lambda}_u$ of matrices $\overline{C}_l$, $\overline{C}_m$, $\overline{C}_u$ that were determined in step 2. After that, calculate λ_l, λ_m, λ_u by using the following equations [37]:

$$\overline{\lambda}_l = 2\lambda_l + \lambda_m, \qquad \overline{\lambda}_m = \lambda_l + 4\lambda_m + \lambda_u, \qquad \overline{\lambda}_u = \lambda_m + 2\lambda_u$$

Step 4: Calculate the eigenvectors w_l, w_m, w_u of matrices $\overline{C}_l$, $\overline{C}_\mathrm{m}$, $\overline{C}_\mathrm{u}$. Next, calculate $\overline{w}_l$, $\overline{w}_\mathrm{m}$, $\overline{w}_\mathrm{u}$ by using the following formulas [37]:

$$\overline{w}_l = \frac{w_l \, \lambda_l}{s_l \, \lambda_m}, \qquad \overline{w}_m = \frac{w_m}{s_m}, \qquad \overline{w}_u = \frac{w_u \, \lambda_u}{s_u \, \lambda_m}$$

$$s_l = \sum_{i=1}^{n} w_{i,l}, \qquad s_m = \sum_{i=1}^{n} w_{i,m}\,, \qquad s_u = \sum_{i=1}^{n} \mathrm{w}_{\mathrm{i,u}}$$

Step 5: Calculate the consistency index (CI) and consistency ratio (CR) of the matrix C_m by using the following formulas. CR should be ≤ 0.1 to claim that the comparison matrix is consistent and RI is the random index [37]. RI is used for random consistency which depends on the size of the matrix. The values of random index are recommended by Saaty in [1].

$$\mathrm{CI} = \frac{\lambda_{\max} - \mathrm{n}}{\mathrm{n} - 1}, \qquad \mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}$$

Step 6: Set the priority fuzzy matrices $\overline{P}_l, \overline{P}_m, \overline{P}_u$ that contain normalized eigenvectors $\overline{w}_l, \overline{w}_\mathrm{m}, \overline{w}_\mathrm{u}$ of the alternatives with respect to each criterion (use $\overline{w}_l^{\,T}, \overline{w}_\mathrm{m}^{\,T}, \overline{w}_\mathrm{u}^{\,T}$) [38].

Step 7: Vectors of global priorities g_l, g_m, g_u are calculated according to the following formulas (where $\overline{w}_l, \overline{w}_\mathrm{m}, \overline{w}_\mathrm{u}$ are the eigenvectors of criteria) [38]:

$$\overline{\mathrm{w}}_\mathrm{l}^{\,\mathrm{T}} = \left[\overline{\mathrm{w}}_{1,\mathrm{l}} \quad \overline{\mathrm{w}}_{2,\mathrm{l}} \;\cdots\; \overline{\mathrm{w}}_{\mathrm{n,l}}\right]^\mathrm{T}$$

$$\overline{\mathrm{w}}_\mathrm{m}^{\,\mathrm{T}} = \left[\overline{\mathrm{w}}_{1,\mathrm{m}} \quad \overline{\mathrm{w}}_{2,\mathrm{m}} \;\cdots\; \overline{\mathrm{w}}_{\mathrm{n,m}}\right]^\mathrm{T}$$

$$\overline{\mathrm{w}}_\mathrm{u}^{\,\mathrm{T}} = \left[\overline{\mathrm{w}}_{1,\mathrm{u}} \quad \overline{\mathrm{w}}_{2,\mathrm{u}} \;\cdots\; \overline{\mathrm{w}}_{\mathrm{n,u}}\right]^\mathrm{T}$$

$$\mathrm{g}_\mathrm{l} = \overline{\mathrm{P}}_\mathrm{l}\overline{\mathrm{w}}_\mathrm{l} = \left[\mathrm{g}_{1,\mathrm{l}}\ \mathrm{g}_{2,\mathrm{l}} \cdots \mathrm{g}_{\mathrm{m,l}}\right]^\mathrm{T}$$

$$\mathrm{g}_\mathrm{m} = \overline{\mathrm{P}}_\mathrm{m}\overline{\mathrm{w}}_\mathrm{m} = \left[\mathrm{g}_{1,\mathrm{m}}\ \mathrm{g}_{2,\mathrm{m}} \cdots \mathrm{g}_{\mathrm{m,m}}\right]^\mathrm{T}$$

$$\mathrm{g}_\mathrm{u} = \overline{\mathrm{P}}_\mathrm{u}\overline{\mathrm{w}}_\mathrm{u} = \left[\mathrm{g}_{1,\mathrm{u}}\ \mathrm{g}_{2,\mathrm{u}} \cdots \mathrm{g}_{\mathrm{m,u}}\right]^\mathrm{T}$$

Step 8: Calculate the expected value (fuzzy mean) and standard deviation (fuzzy spread) by using the following formulas [38]:

$$g_{i,e} = \frac{g_{i,l} + 2g_{i,m} + g_{i,u}}{4}$$

$$\sigma_i = \left(\frac{1}{80}\left(3{g_{i,l}}^2 + 4{g_{i,m}}^2 + 3{g_{i,u}}^2 - 4g_{i,l}g_{i,m} - 2g_{i,l}g_{i,u} - 4g_{i,m}g_{i,u}\right)\right)^{1/2}$$

4.2 Numerical Example of FAHP

An implementation of fuzzy AHP is discussed in this chapter. The expert's decision about the collected information is mainly up to the amount and characteristics of information affecting its certainty degree. So, error rate of left and right deviations of comparison matrices can be 5%, 8%, 10% and so on. Left and right deviations of crisp comparison matrices formed in AHP example are represented with 5% error rate, as triangular fuzzy number in this example. The eigenvalues, eigenvectors, and consistency are calculated for fuzzy AHP.

4.2.1 Fuzzy Pairwise Matrix of Criteria

Table 10 shows the fuzzy pairwise matrix of criteria for Table 1. Fuzzy pairwise matrix of criteria is decomposed into three matrices C_l, C_m, C_u as shown in Table 11.

Table 10: Fuzzy pairwise matrix of criteria

Criterion	**Teaching**	**Research**	**Citations**	**International Outlook**
Teaching	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(3.8,4,4.2)
Research	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(3.8,4,4.2)
Citations	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(3.8,4,4.2)
International Outlook	(0.2375,0.25,0.2625)	(0.2375,0.25,0.2625)	(0.2375,0.25,0.2625)	(0.95,1,1.05)

Table 11: Decomposing of fuzzy pairwise matrix of criteria into three matrices C_l, C_m, C_u

Matrix C_l					**Matrix C_m**					**Matrix C_u**			
0.95	0.95	0.95	3.8		1	1	1	4		1.05	1.05	1.05	4.2
0.95	0.95	0.95	3.8		1	1	1	4		1.05	1.05	1.05	4.2
0.95	0.95	0.95	3.8		1	1	1	4		1.05	1.05	1.05	4.2
0.2375	0.2375	0.2375	0.95		0.25	0.25	0.25	1		0.2625	0.2625	0.2625	1.05

Then matrices $\overline{C}_l, \overline{C}_m, \overline{C}_u$ of criteria become as in Table 12.

Table 12: Matrices $\overline{C}_l, \overline{C}_m, \overline{C}_u$ of criteria

Matrix $\overline{C}_l$					**Matrix $\overline{C}_m$**					**Matrix $\overline{C}_u$**			
2.9	2.9	2.9	11.6		6	6	6	24		3.1	3.1	3.1	12.4
2.9	2.9	2.9	11.6		6	6	6	24		3.1	3.1	3.1	12.4
2.9	2.9	2.9	11.6		6	6	6	24		3.1	3.1	3.1	12.4
0.725	0.725	0.725	2.9		1.5	1.5	1.5	6		0.775	0.775	0.775	3.1

In the next step the eigenvalues of matrices $\overline{C}_l$, $\overline{C}_m$ and $\overline{C}_u$ of criteria are calculated. The eigenvalues of these matrices are $\overline{\lambda}_l$ = 11.6, $\overline{\lambda}_m$ = 24, $\overline{\lambda}_u$ = 12.4 respectively, and it is obtained that λ_l = 3.8, λ_m = 4, λ_u = 4.2.

The eigenvectors of matrices $\overline{C}_l$, $\overline{C}_m$, $\overline{C}_u$ of criteria are

w_l= [0.5714 0.5714 0.5714 0.1429]

w_m= [0.5714 0.5714 0.5714 0.1429]

w_u= [0.5714 0.5714 0.5714 0.1429]

Then eigenvectors $\overline{w}_l, \overline{w}_\mathrm{m}, \overline{w}_\mathrm{u}$ of criteria become as

$\overline{w}_l$ = [0.2923 0.2923 0.2923 0.0731]

$\overline{w}_\mathrm{m}$ = [0.3077 0.3077 0.3077 0.0769]

$\overline{w}_\mathrm{u}$ = [0.3230 0.3230 0.3230 0.0807]

The consistency index and consistency ratio for criteria are calculated as

$$CI = (4 - 4)/(4 - 1) = 0$$

$$CR = 0/0.9 = 0 \leq 0.10$$

So, the comparison matrix is completely consistent for criteria.

4.2.2 Fuzzy Pairwise Matrix of Teaching Criterion

Table 13 shows the fuzzy pairwise matrix of teaching criterion for Table 5.

Table 13: Fuzzy pairwise matrix of teaching criterion

Teaching	**A**	**B**	**C**	**D**	**E**
A	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(2.85,3,3.15)	(0.95,1,1.05)
B	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(1.9,2,2.1)	(0.95,1,1.05)
C	(0.475,0.5,0.525)	(0.475,0.5,0.525)	(0.95,1,1.05)	(0.95,1,1.05)	(0.475,0.5,0.525)
D	(0.31635,0.333,0.34965)	(0.475,0.5,0.525)	(0.95,1,1.05)	(0.95,1,1.05)	(0.475,0.5,0.525)
E	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(1.9,2,2.1)	(0.95,1,1.05)

Fuzzy pairwise matrix of teaching criterion is decomposed into three matrices C_l, C_m, C_u as shown in Table 14. Then matrices $\overline{C}_l, \overline{C}_{\mathrm{m}}, \overline{C}_u$ of teaching criterion become as in Table 15.

The eigenvalues of these matrices are $\overline{\lambda}_l$ = 14.5571, $\overline{\lambda}_{\mathrm{m}}$ = 30.1181, $\overline{\lambda}_{\mathrm{u}}$ = 15.5610.

From $\overline{\lambda}_l$, $\overline{\lambda}_{\mathrm{m}}$, $\overline{\lambda}_{\mathrm{u}}$, it is obtained that λ_l = 4.768708, λ_m= 5.019683, λ_{u} = 5.270658.

The eigenvectors of matrices $\overline{C}_l$, $\overline{C}_{\mathrm{m}}$, $\overline{C}_{\mathrm{u}}$ of teaching criterion are

w_l = [0.5711 0.5228 0.2614 0.2424 0.5228]

w_m = [0.5711 0.5228 0.2614 0.2424 0.5228]

w_u = [0.5711 0.5228 0.2614 0.2424 0.5228]

Then eigenvectors $\overline{w}_l$, $\overline{w}_{\mathrm{m}}$, $\overline{w}_{\mathrm{u}}$ of teaching criterion become as

$\overline{w}_l$ = [0.2558 0.2342 0.1171 0.1085 0.2342]

$\overline{w}_{\mathrm{m}}$ = [0.2693 0.2465 0.1232 0.1143 0.2465]

$\overline{w}_{\mathrm{u}}$ = [0.2827 0.2588 0.1294 0.1200 0.2588]

The consistency index and consistency ratio for teaching criterion are calculated as

$$CI = (5.019683 - 5)/(5 - 1) = 0.004920$$

$$CR = 0.004920/1.12 = 0.0043 \leq 0.10$$

So, the comparison matrix is consistent for teaching criterion.

Table 14: Decomposing of fuzzy pairwise matrix of teaching criterion into three matrices C_l, C_m, C_u

Matrix C_l						**Matrix C_m**						**Matrix C_u**				
0.95	0.95	1.9	2.85	0.95		1	1	2	3	1		1.05	1.05	2.1	3.15	1.05
0.95	0.95	1.9	1.9	0.95		1	1	2	2	1		1.05	1.05	2.1	2.1	1.05
0.475	0.475	0.95	0.95	0.475		0.5	0.5	1	1	0.5		0.525	0.525	1.05	1.05	0.525
0.31635	0.475	0.95	0.95	0.475		0.333	0.5	1	1	0.5		0.34965	0.525	1.05	1.05	0.525
0.95	0.95	1.9	1.9	0.95		1	1	2	2	1		1.05	1.05	2.1	2.1	1.05

Table 15: Matrices $\overline{C}_l, \overline{C}_{\mathrm{m}}, \overline{C}_u$ of teaching criterion

Matrix $\overline{C}_l$						**Matrix $\overline{C}_{\mathrm{m}}$**						**Matrix $\overline{C}_{\mathrm{u}}$**				
2.9	2.9	5.8	8.7	2.9		6	6	12	18	6		3.1	3.1	6.2	9.3	3.1
2.9	2.9	5.8	5.8	2.9		6	6	12	12	6		3.1	3.1	6.2	6.2	3.1
1.45	1.45	2.9	2.9	1.45		3	3	6	6	3		1.55	1.55	3.1	3.1	1.55
0.9657	1.45	2.9	2.9	1.45		1.998	3	6	6	3		1.0323	1.55	3.1	3.1	1.55
2.9	2.9	5.8	5.8	2.9		6	6	12	12	6		3.1	3.1	6.2	6.2	3.1

4.2.3 Fuzzy Pairwise Matrix of Research Criterion

Table 16 shows the fuzzy pairwise matrix of research criterion for Table 6. Fuzzy pairwise matrix of research criterion is decomposed into three matrices C_l, C_m, C_u as shown in Table 17.

Then matrices $\overline{C}_l$, $\overline{C}_\mathrm{m}$, $\overline{C}_\mathrm{u}$ of research criterion become as in Table 18.

The eigenvalues of these matrices are $\overline{\lambda}_l$ = 14.7725, $\overline{\lambda}_\mathrm{m}$ = 30.5637, $\overline{\lambda}_\mathrm{u}$ = 15.7912 and λ_l = 4.839275, λ_m = 5.09395, λ_u = 5.348625.

The eigenvectors of matrices $\overline{C}_l$, $\overline{C}_\mathrm{m}$, $\overline{C}_\mathrm{u}$ of research criterion are

w_l= [0.6618 0.4657 0.3525 0.2457 0.4007]

w_m= [0.6618 0.4657 0.3525 0.2457 0.4007]

w_u= [0.6618 0.4657 0.3525 0.2457 0.4007]

Then eigenvectors $\overline{w}_l$, $\overline{w}_\mathrm{m}$, $\overline{w}_\mathrm{u}$ of research criterion become as

$\overline{w}_l$ = [0.2956 0.2080 0.1574 0.1097 0.1790]

$\overline{w}_\mathrm{m}$ = [0.3112 0.2190 0.1657 0.1155 0.1884]

$\overline{w}_\mathrm{u}$ = [0.3267 0.2299 0.1740 0.1213 0.1978]

The consistency index and consistency ratio for research criterion are

$$CI = (5.09395 - 5)/(5 - 1) = 0.0234875$$

$$CR = 0.0234875/1.12 = 0.0209709821 \leq 0.10$$

So, the comparison matrix is consistent for research criterion.

Table 16: Fuzzy pairwise matrix of research criterion

Research	**A**	**B**	**C**	**D**	**E**
A	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(2.85,3,3.15)	(1.9,2,2.1)
B	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(0.95,1,1.05)
C	(0.475,0.5,0.525)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)
D	(0.31635,0.333,0.34965)	(0.475,0.5,0.525)	(0.95,1,1.05)	(0.95,1,1.05)	(0.475,0.5,0.525)
E	(0.475,0.5,0.525)	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)	(0.95,1,1.05)

Table 17: Decomposing of fuzzy pairwise matrix of research criterion into three matrices C_l, C_m, C_u

Matrix C_l						**Matrix C_m**						**Matrix C_u**				
0.95	0.95	1.9	2.85	1.9		1	1	2	3	2		1.05	1.05	2.1	3.15	2.1
0.95	0.95	0.95	1.9	0.95		1	1	1	2	1		1.05	1.05	1.05	2.1	1.05
0.475	0.95	0.95	0.95	0.95		0.5	1	1	1	1		0.525	1.05	1.05	1.05	1.05
0.31635	0.475	0.95	0.95	0.475		0.333	0.5	1	1	0.5		0.34965	0.525	1.05	1.05	0.525
0.475	0.95	0.95	1.9	0.95		0.5	1	1	2	1		0.525	1.05	1.05	2.1	1.05

Table 18: Matrices $\overline{C}_l, \overline{C}_m, \overline{C}_u$ of research criterion

Matrix $\overline{C}_l$						**Matrix $\overline{C}_m$**						**Matrix $\overline{C}_u$**				
2.9	2.9	5.8	8.7	5.8		6	6	12	18	12		3.1	3.1	6.2	9.3	6.2
2.9	2.9	2.9	5.8	2.9		6	6	6	12	6		3.1	3.1	3.1	6.2	3.1
1.45	2.9	2.9	2.9	2.9		3	6	6	6	6		1.55	3.1	3.1	3.1	3.1
0.9657	1.45	2.9	2.9	1.45		1.998	3	6	6	3		1.0323	1.55	3.1	3.1	1.55
1.45	2.9	2.9	5.8	2.9		3	6	6	12	6		1.55	3.1	3.1	6.2	3.1

4.2.4 Fuzzy Pairwise Matrix of Citations Criterion

Table 19 shows the fuzzy pairwise matrix of citations criterion for Table 7. Fuzzy pairwise matrix of citations criterion is decomposed into three matrices C_l, C_m, C_u as shown in Table 20.

Then matrices $\overline{C}_l$, $\overline{C}_{\mathrm{m}}$, $\overline{C}_{\mathrm{u}}$ of citations criterion become as in Table 21.

The eigenvalues of these matrices are $\overline{\lambda}_l$ = 14.5559, $\overline{\lambda}_{\mathrm{m}}$ = 30.1157, $\overline{\lambda}_{\mathrm{u}}$ = 15.5598 and λ_l = 4.768308, λ_m = 5.019283, λ_u = 5.270258.

The eigenvectors of matrices $\overline{C}_l$, $\overline{C}_{\mathrm{m}}$, $\overline{C}_{\mathrm{u}}$ of citations criterion are

w_l = [0.5005 0.5005 0.5005 0.4642 0.1821]

w_m = [0.5005 0.5005 0.5005 0.4642 0.1821]

w_u = [0.5005 0.5005 0.5005 0.4642 0.1821]

Then eigenvectors $\overline{w}_l$, $\overline{w}_{\mathrm{m}}$, $\overline{w}_{\mathrm{u}}$ of citations criterion become as

$\overline{w}_l$ = [0.2213 0.2213 0.2213 0.2053 0.0805]

$\overline{w}_{\mathrm{m}}$ = [0.2330 0.2330 0.2330 0.2161 0.0847]

$\overline{w}_{\mathrm{u}}$ = [0.2446 0.2446 0.2446 0.2269 0.0890]

The consistency index and consistency ratio for citations criterion are

$$CI = (5.019283 - 5)/(5 - 1) = 0.00482075$$

$$CR = 0.00482075/1.12 = 0.0043042411 \leq 0.10$$

So, the comparison matrix is consistent for citations criterion.

Table 19: Fuzzy pairwise matrix of citations criterion

Citations	**A**	**B**	**C**	**D**	**E**
A	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(2.85,3,3.15)
B	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(2.85,3,3.15)
C	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(2.85,3,3.15)
D	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(1.9,2,2.1)
E	(0.31635,0.333,0.34965)	(0.31635,0.333,0.34965)	(0.31635,0.333,0.34965)	(0.475,0.5,0.525)	(0.95,1,1.05)

Table 20: Decomposing of fuzzy pairwise matrix of citations criterion into three matrices C_l, C_m, C_u

Matrix C_l						**Matrix C_m**						**Matrix C_u**				
0.95	0.95	0.95	0.95	2.85		1	1	1	1	3		1.05	1.05	1.05	1.05	3.15
0.95	0.95	0.95	0.95	2.85		1	1	1	1	3		1.05	1.05	1.05	1.05	3.15
0.95	0.95	0.95	0.95	2.85		1	1	1	1	3		1.05	1.05	1.05	1.05	3.15
0.95	0.95	0.95	0.95	1.9		1	1	1	1	2		1.05	1.05	1.05	1.05	2.1
0.31635	0.31635	0.31635	0.475	0.95		0.333	0.333	0.333	0.5	1		0.34965	0.34965	0.34965	0.525	1.05

Table 21: Matrices $\overline{C}_l, \overline{C}_m, \overline{C}_u$ of citations criterion

Matrix $\overline{C}_l$					**Matrix $\overline{C}_m$**					**Matrix $\overline{C}_u$**				
2.9	2.9	2.9	2.9	8.7	6	6	6	6	18	3.1	3.1	3.1	3.1	9.3
2.9	2.9	2.9	2.9	8.7	6	6	6	6	18	3.1	3.1	3.1	3.1	9.3
2.9	2.9	2.9	2.9	8.7	6	6	6	6	18	3.1	3.1	3.1	3.1	9.3
2.9	2.9	2.9	2.9	5.8	6	6	6	6	12	3.1	3.1	3.1	3.1	6.2
0.9657	0.9657	0.9657	1.45	2.9	1.998	1.998	1.998	3	6	1.0323	1.0323	1.0323	1.55	3.1

4.2.5 Fuzzy Pairwise Matrix of International Outlook Criterion

Table 22 shows the fuzzy pairwise matrix of international outlook criterion for Table 8.

Table 22: Fuzzy pairwise matrix of international outlook criterion

Inter. outlook	**A**	**B**	**C**	**D**	**E**
A	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)
B	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)
C	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)
D	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)
E	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)	(0.95,1,1.05)

Fuzzy pairwise matrix of international outlook criterion is decomposed into three matrices C_l, C_m, C_u as shown in Table 23.

Table 23: Decomposing of fuzzy pairwise matrix of international outlook criterion into three matrices C_l, C_m, C_u

Matrix C_l						**Matrix C_m**						**Matrix C_u**				
0.95	0.95	0.95	0.95	0.95		1	1	1	1	1		1.05	1.05	1.05	1.05	1.05
0.95	0.95	0.95	0.95	0.95		1	1	1	1	1		1.05	1.05	1.05	1.05	1.05
0.95	0.95	0.95	0.95	0.95		1	1	1	1	1		1.05	1.05	1.05	1.05	1.05
0.95	0.95	0.95	0.95	0.95		1	1	1	1	1		1.05	1.05	1.05	1.05	1.05
0.95	0.95	0.95	0.95	0.95		1	1	1	1	1		1.05	1.05	1.05	1.05	1.05

Then matrices $\overline{C}_l, \overline{C}_{\mathrm{m}}, \overline{C}_{\mathrm{u}}$ of international outlook criterion become as in Table 24.

Table 24: Matrices $\overline{C}_l, \overline{C}_m, \overline{C}_u$ of international outlook criterion

Matrix $\overline{C}_l$						**Matrix $\overline{C}_m$**						**Matrix $\overline{C}_u$**				
2.9	2.9	2.9	2.9	2.9		6	6	6	6	6		3.1	3.1	3.1	3.1	3.1
2.9	2.9	2.9	2.9	2.9		6	6	6	6	6		3.1	3.1	3.1	3.1	3.1
2.9	2.9	2.9	2.9	2.9		6	6	6	6	6		3.1	3.1	3.1	3.1	3.1
2.9	2.9	2.9	2.9	2.9		6	6	6	6	6		3.1	3.1	3.1	3.1	3.1
2.9	2.9	2.9	2.9	2.9		6	6	6	6	6		3.1	3.1	3.1	3.1	3.1

The eigenvalues of these matrices are $\overline{\lambda}_l$ = 14.5, $\overline{\lambda}_m$ = 30, $\overline{\lambda}_u$ = 15.5 and λ_l = 4.75, λ_m = 5, λ_u= 5.25.

The eigenvectors of matrices $\overline{C}_l$, $\overline{C}_m$, $\overline{C}_u$ of international outlook criterion are

w_l = [0.4472 0.4472 0.4472 0.4472 0.4472]

w_m = [0.4472 0.4472 0.4472 0.4472 0.4472]

w_u = [0.4472 0.4472 0.4472 0.4472 0.4472]

Then eigenvectors $\overline{w}_l$, $\overline{w}_m$, $\overline{w}_u$ of international outlook criterion become as

$\overline{w}_l$ = [0.19 0.19 0.19 0.19 0.19]

$\overline{w}_m$ = [0.2 0.2 0.2 0.2 0.2]

$\overline{w}_u$ = [0.21 0.21 0.21 0.21 0.21]

The consistency index and consistency ratio for international outlook criterion are

$$CI = (5-5)/(5-1) = 0$$

$$CR = 0/1.12 = 0 \leq 0.10$$

So, the comparison matrix is completely consistent for international outlook criterion.

4.2.6 Ranking Process

Priority fuzzy matrices $\overline{P}_l, \overline{P}_m, \overline{P}_u$ that contain normalized eigenvectors $\overline{w}_l, \overline{w}_\mathrm{m}, \overline{w}_\mathrm{u}$ are given below:

$$\overline{P}_l = \begin{bmatrix} 0.2558 & 0.2956 & 0.2213 & 0.19 \\ 0.2342 & 0.2080 & 0.2213 & 0.19 \\ 0.1171 & 0.1574 & 0.2213 & 0.19 \\ 0.1085 & 0.1097 & 0.2053 & 0.19 \\ 0.2342 & 0.1790 & 0.0805 & 0.19 \end{bmatrix}$$

$$\overline{P}_m = \begin{bmatrix} 0.2693 & 0.3112 & 0.2330 & 0.2 \\ 0.2465 & 0.2190 & 0.2330 & 0.2 \\ 0.1232 & 0.1657 & 0.2330 & 0.2 \\ 0.1143 & 0.1155 & 0.2161 & 0.2 \\ 0.2465 & 0.1884 & 0.0847 & 0.2 \end{bmatrix}$$

$$\overline{P}_u = \begin{bmatrix} 0.2827 & 0.3267 & 0.2446 & 0.21 \\ 0.2588 & 0.2299 & 0.2446 & 0.21 \\ 0.1294 & 0.1740 & 0.2446 & 0.21 \\ 0.1200 & 0.1213 & 0.2269 & 0.21 \\ 0.2588 & 0.1978 & 0.0890 & 0.21 \end{bmatrix}$$

Eigenvectors of criteria are:

$$\overline{w}_l = [0.2923\ 0.2923\ 0.2923\ 0.0731]$$

$$\overline{w}_\mathrm{m} = [0.3077\ 0.3077\ 0.3077\ 0.0769]$$

$$\overline{w}_\mathrm{u} = [0.3230\ 0.3230\ 0.3230\ 0.0807]$$

By multiplying priority fuzzy matrices $\overline{P}_l, \overline{P}_m, \overline{P}_u$ by $\overline{w}_l, \overline{w}_{\mathrm{m}}, \overline{w}_{\mathrm{u}}$, the global priorities are obtained. Table 25 represents global priorities, expected value, standard deviation and coefficient of variation.

Table 25: Global priorities, expected value, standard deviation and coefficient of variation

Alternative	**Vector g_l**	**Vector g_m**	**Vector g_u**	**Exp. Val. $g_{i,e}$**	**Stand. Dev. (%)**	**CV_i**
A	0.2397	0.2657	0.2928	0.2660	0.8397	3.1568
B	0.2078	0.2303	0.2538	0.2306	0.7274	3.1544
C	0.1588	0.1760	0.1940	0.1762	0.5566	3.1589
D	0.1377	0.1526	0.1682	0.1528	0.4823	3.1564
E	0.1582	0.1753	0.1932	0.1755	0.5535	3.1538

According to [38], the alternative with higher expected value and lower standard deviation should be considered as the best one. Apart from the obtained results for these measures in Table 25, there is no possibility to rank the universities under such circumstances. For this reason, ranking can be only possible according to the values of coefficient of variation ($\mathrm{CV_i} = \sigma_i / g_{i,e}$) which are also represented in Table 25. An alternative with a smaller $\mathrm{CV_i}$ is taken as the best one [38]. So, the universities E and C are respectively best and worst alternatives according to the results of coefficient of variation (CV_i). Ranking of the universities has the prioritized order $E > B > D > A > C$ [39].

Chapter 5

CONCLUSION

The quality of a university is stipulated by its ranking in the international higher education sector; in other words, ranking is a tool which is used to measure the university performance. Selecting a prestigious university to study is a process that requires strategic decision making.

Ranking of five UK universities by applying AHP and FAHP approaches is studied in this paper. The criteria and the alternatives are mutually compared and the comparison matrices are formed by using the comparison scale. Consistency ratio is 0 for criteria comparisons and the consistency ratios for alternatives comparison for teaching, research, citations, and international outlook criteria are 0.0042, 0.0207, 0.0038, and 0, respectively for AHP. Since all the consistency ratios of comparison matrices are less than 0.1, it is decided that all comparison matrices are consistent; so the ranking of universities is achieved by using AHP method.

Nowadays, most decisions are made in complex and uncertain environments. When the complexity of a decision making problem increases, it becomes difficult to reach the perfect solution. In such cases, a better decision making method and more precise results are required. Fuzzy AHP method can be a very useful tool for optimal problem solving. Since fuzzy AHP provides precise results in cases of uncertainty, it is one of the most widely used approaches among MCDM methodologies. In this thesis, consistency ratios are acceptable for FAHP as well. After checking the consistency of all criteria and alternatives' pairwise matrices, their eigenvectors are calculated. By using these eigenvectors, a ranking of five universities is carried out by using FAHP method.

According to AHP, ranking of the universities is as A > B > C > E > D, and the best alternative is defined to be the University A. On the other hand, after applying fuzzy AHP which gives more reliable and precise results, the prioritized order for universities becomes in the form of E > B > D > A > C, and it means that the best alternative is defined to be the University E.

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