

# **Comparison of the Behaviour of Curved and Straight Types of Steel Shell Roof Structures**

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Submitted to the  
Institute of Graduate Student and Research  
in partial fulfillment of the requirements for the Degree of

Master of Science  
in  
Civil Engineering

Eastern Mediterranean University  
February 2010  
Gazimağusa, North Cyprus

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## **ABSTRACT**

In this research, the straight and curved models of the steel shell roof with different plates were analysed, designed and the results were compared with one another. Through this exercise it is aimed at achieving an ideal shell roof structure which could cover a larger surface. Therefore, three types of shell roofs were considered duopitch, cylindrical and dome and the main objective was to compare the straight and curved model of the shells. According to the findings of the literature review, this is the first time for such comparison to be carried out among the basic types (duopitch, cylindrical, dome) of shell roof structure.

In addition to the advantage of covering large openings the shell roof structures also use the least materials to do this. Most of its resistance against the forces imposed on it is due to its curved surface. It is this particular characteristic of the shell structures that has been attracting architects and civil engineers more in the recent years. This research shows how effective the curved surfaced shell roof can be with different angles in helping the structure to resistance its self weight and applied loads.

Moreover, it also indicates under which conditions a particular structure can be more reliable. This kind of shell roof is not common in all shapes and each model needs special characteristics. When all the comparisons were made, an optimal structure with the largest span of shell roof and resistance to loads was obtained. In this research, three different angles were used in three shell roof models in simple and general conditions and the results were compared in order that a structure with ideal

condition can be obtained in future while basic conditions also need to be understood.

For each shell roof type, three plates were considered for analyses and designed. A metal box with either 5cm or 7cm I-section edges, each box has 0.5m length and breadth and I-section edges with either 0.05m or 0.07m height and 0.003m thickness. A metal box with L-section edges, each box has 0.5m length and breadth and L section edge with 0.05m height and 0.02m edge and 0.03m thicknesses. The comparison between these plates indicates the load carrying capacity and span capability of the structures. When all different shell roof types are able to cover the same opening, then it is these plates which determine the weight and the amount of stresses in the structure.

The research showed that it is necessary and important to consider the maximum possible loads these structures could carry and also to find out the maximum span and the loads that these types of structures can tolerate.

This kind of design is very economic in shell structures and has the capability of replacement and frequent usage in different areas while the other structures do not have this ability.

**Key words:** Shell, Metal box, Steel structure, Curved model, Straight model.

## ÖZ

Bu arařtırmada, farklı levhalar kullanılarak yapılmıř elik kabuk atı dz ve kavisli modelleri, analiz ve tasarımı yapıldı ve sonuları karřılařtırıldı. Bu egzersiz sayesinde daha byk bir alanı kaplayabilecek ideal kabuk atısı elde edilmeye alıřıldı. Bu nedenle,  tip kabuk atı duopitch, silindir Őeklinde ve kubbe kullanılarak dz ve kavisli model kabukların karřılařtırmalarının yapılması amalandı. Literatr inceleme sonucunda elde edilen bulgulara gre, kabuk atı yapı temel trleri arasında (duopitch, silindirik, kubbe) ilk kez byle bir karřılařtırma yapıldı.

Kabuk atı yapıların en nemli avantajı olan byk aıklıkları kaplayabilmeye ek olarak bunu yapmak iin de en az malzeme kullanmasıdır. Yklere karřı dayanabilmesinin en nemli nedeni eđri bir yzeyi oluřundandır. Son yıllarda mimar ve inřaat mhendislerinin bu tr yapılara olan ilgilerinin en nemli nedenlerinden birisi de kabuk yapıların bu zelliđidir. Bu arařtırma eđri yzeyli ve farklı aıları olan kabuk atıların kendi ađırlıklarına ve dıřtan uygulanan yklere karřı ne denli etkin direnc sađlayabildiklerini gstermiřtir.. Bundan te, belli yapıların hangi kořullar altında daha gvenilir olabileceđine dikkat ekmiřtir. Bu tr kabuk atı tm Őekillerde yaygın deđildir ve her modelin zel niteliklere ihtiyaı vardır. Tm karřılařtırmalar yapıldıđında, yklere en ok diren gsteren ve en byk aıklıkları geebilen ideal bir kabuk atı yapısı elde edildi. Bu arařtırmada,  farklı aı basit ve genel kořullarda  farklı kabuk atı modelinde kullanıldı ve sonuları karřılařtırılarak gelecekte elde edilebilecek ideal yapının ne olabileceđi anlařılmaya alıřıldı.

Her kabuk çatı tipinin analiz ve tasarımı yapılırken üç farklı çelik levha kullanıldı. 5cm veya 7cm I-şeklinde kenarları olan metal bir kutu, her kutunun 0.5m uzunluk ve genişliği ve I-şeklindeki kenarının ise 0.05m veya 0.07m yüksekliği ve 0.003m kalınlığı vardır. L şeklinde kenarı olan metal bir kutu, kutunun 0.05m yüksekliği, 0.02m kenarı ve 0.003m kalınlıkları ile 0.5m uzunluk ve genişliği vardır. Bu levhaları kullanan çelik kabukların davranışları arasında yapılan karşılaştırma bu tür yapıların yük aşırıya çıkma ve büyük açıklıkları geçebilme kapasitelerini gösterir. Tüm farklı kabuk çatı türleri aynı açıklığı kaplayabildiğinde, yapıların ağırlık ve gerilme miktarlarını bu plakalar belirler.

Araştırma bu tür yapılarda en çok taşınabilecek yük miktarının ve en büyük geçilebilecek açıklıkların anlaşılmasının çok gerekli ve önemli olduğunu göstermiştir.

Bu tür tasarımın kabuk yapılarda çok ekonomik olduğu ve diğer yapılarda olmayan bir özelliğe, yapıların sık kullanım ve farklı alanlarda değiştirilebilme özelliğine, sahip olduğu anlaşılmıştır.

**Anahtar Kelimeler:** Kabuk, Metal kutu, Çelik yapı, Eğimli model, Düz model.

## ACKNOWLEDGEMENTS

This thesis could not have been written without great moral advices and suggestions of Assist. Prof. Dr. Murude Celikag, who was not only served as my supervisor but also encouraged me throughout my academic and social life. She supported me from the moment I stated my graduate studies and let me benefit from her precious engineering knowledge.

I would like to express my deepest appreciation to my lovely family, particularly my father who support me for doing a masters degree at EMU. My family were always there when I needed and they indebted me with their endless patience and love.

My sincere respect goes to Prof. Dr. Nasrullah Dianat who introduced me to the fundamentals of shell structures and always available for my queries.

I am so grateful to my friends Anoshe Iravaniyan, Amin Abrishambaf, Fatih Parlak, Saameh Golzadeh, Reza Nastaranpoor, Dr. Amir Hedayat, Golnaz Dianat, Mediya Behnam, Majid Yazadani.

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## LIST OF SYMBOLS

$S$	Snow load
$W_e$	Wind pressure acting on the external surfaces
$q_{\text{ref}}$	Reference mean wind velocity pressure
$W_i$	Wind pressure acting on the internal surfaces
$P_0$	Load-displacement
$\Delta P$	Incremental load-displacement
$[K_e]$	Elastic stiffness matrix
$[K\sigma(\sigma)]$	Initial stress matrix evaluated at the stress $\{\sigma\}$
$\lambda$	Load factor
$\Delta \lambda$	Incremental load factor
$U$	Displacement
$\Delta u$	Incremental displacement
$\{F^a\}$	Load vector

# Chapter 1

## INTRODUCTION

### 1.1 Spatial Structure

Architects and civil engineers are continuously looking for new and efficient methods to cover large spaces using the least materials and minimum number of columns. Modern and industrial world need large spaces to gather people and to keep consumable material. These spaces, such as airport terminals, leisure center, railway stations, aircraft hangars, and spatial structures require coverage with economical and new design. If they are designed correctly, undoubtedly the structures will be economic in terms of construction material.

Spatial structures are those kinds of structures using the least construction material to cover large space which are defined by Space Structures Research Centre (SSRC) at the University of Surrey as follows:

Spatial structure refers to a structural system in which the load transfers mechanism involves three dimensions. This is in contrast to a 'linear structure' and 'plane structure', such as a beam and a plane truss, where the load transfer mechanism involves no more than two dimensions. (Candela. F. (1970)).

They include structures such as reinforced concrete shells, tensile membranes and cable nets, single and double layer space grids, and deployable structures.

In fact, workshops and construction of this kind of structures had been limited in developed countries since 1970s. Nevertheless, using this kind of structures are still

common found in Asia and other parts where the construction materials are expensive and the demands for structures with high efficiency exist.

In recent years there is a tendency towards this kind of structures. Of course, this tendency is not restricted to the present time and it has contributed to the progress in structural designing and construction. At present, there are numerous spatial structures in the process of construction. Two of them are 2010 World Expo Shanghai and 2010 Asia Games Guangzhou.

## **1.2 Shells Roof**

### **1.2.1 History of Shells**

Shell construction began in the 1920s. The shell emerged as a major long-span concrete structure after World War II. Thin parabolic shell vaults stiffened with ribs have been built with spans up to about 90 m. More complex forms of concrete shells have been constructed, including hyperbolic paraboloids or saddle shapes and intersecting parabolic vaults less than 1.25cm thick. Revolutionary thin-shell designers include Felix Candela and Pier Luigi Nervi.

### **1.2.3 Steel Shell Structures and Space Grid Structures Roof**

In the past few decades, the proliferation of space grid structures all over the world has taken much of the potential market for steel shells. The wider availability of structural steel and high-strength materials permit the construction of space grid structures of longer spans and more complex shapes. In addition, the wider use of powerful computers and the development of computer programs enable the analysis and design of space grids to be accurate and confident. The history of development and the recent world-wide achievements of space grid structures can be found in the

open literature, e.g. Makowski, Z.S. (1981, 1985), Nooshin, H. (1991). Some of the general benefits gained from the use of space grid structures are as follows:

- a) The high redundancy of space grid structures means that, in general, failure of one or a limited number of elements does not necessarily lead to the overall collapse of the structure.
- b) The naturally high modularity of space grid structures accelerates the fabrication and the assembly of members on site, resulting in savings in erection time.
- c) The great freedom of choice of support locations leads to ease in space planning beneath the grid.

Naturally, there are also some disadvantages with the use of space grid structures. The regular nature of the geometry makes the design appear very 'busy' to some eyes, whilst if it is necessary the fire protection is difficult to achieve without excessive cost penalty.

Although steel shells and space grids are at opposite ends of the structural spectrum, they can usually be compared, due to the similarity of their structural behavior. Although space grids consist of a large number of members dominantly in tension and compression, on the whole they behave much like thin continuum shells. In general, any applied load is distributed throughout the structure and to all the supports, with all elements contributing to the load carrying capacity. A space grid with an overall geometric shape following that of a similar steel shell, equally supported and loaded, would clearly exhibit similar distributions of internal forces. The axial forces in the members of its top and bottom layers of a space grid would indicate the same distribution of tension and compression present within a shell.

Steel shells and space grid structures are modern, efficient, light, versatile, and capable of covering a large column-free area.

### **1.3 The Steel Shell Folded Plate Roof System**

This kind of structures are formed of plates joined together to cover a surface easily. Therefore, they can cover small surfaces more conveniently than curved surfaces. When the plates are put next to one another, they can form a modern structure compared to ordinary and more economical structures. This structure can behave like beam. It can cover a span or several spans with different lengths while it is fixed from one end to cantilever supports without the need to be fixed at the other end.

### **1.4 Advantages over Other Shell Structures**

All spatial structures are designed for a specific one place, but if screws and bolts are used in the design of Steel shell folded plate, then after being using in one place the structure can be dismantled and moved to another place, such as the temporary structures used in the army. This property of the structure can make it very economical. It's other property is the speed of construction; after fabricating the structure , it is very fast to erect it on site. It has another important property, it does not need trained people for its fabrication and erection.

### **1.5 Objectives of the Study**

This study aims to carry out a comparison between the straight model and curve model of the steel shell roof. This will be done through using different roof slopes, roof shapes and steel plate element with different spans. Steel plate elements are similar to those used for composite shell roof elements which are used to form the steel shell roof in this study. The details geometry and design are given in chapter 3. This particular plate element is suitable for models that are subject to structural

analysis. Furthermore, comparison of the results of the stress and buckling analysis for different roof shell models are also carried out.

## **1.6 Reasons for the Objectives**

All steel shell roof structures are designed and constructed require spatial structure system. Fashion and performance and economy are the three parameters influencing the type of structural systems to be used, especially spatial structural systems. By comparing these three parameters, this research can form the basis for new methods of evaluation for steel shell structural systems.

On the other hand, accurate information about nonlinear behavior of different structural systems leads to higher quality in their design.

## **1.7 Guides to the Thesis**

This study is presented in six chapters.

Chapter two includes literature review, being divided into seven sections. The first section (section 2.1) is devoted to the introduction of different types of Shell that considered in this study. The historical development of theory and analytical solution methods were given for shell. Section 2.2 introduces Eurocode for steel shell. These standard is possibility of structures and have principles and constraints. In section 2.3, is devoted to the introduction standard for strength and stability of shells. Then, consider to possibility and limit states that exist for the shells. Sections 2.4, is devoted to the introduction stability of the shells and consist history of thin shell buckling, general theories for shells and general theories of stiffened shells. Section 2.5 and 2.6 are devoted to review of past research on the technique for the shell buckling. They review the research being carried out on method of buckling. Section

2.7, is devoted to the introduction folded plate shell. Then, consider to the history and character of the structure.

Chapter three is devoted to design of the model frames. The methodology of design of the structures is first introduced in section 3.1. Then, the results of design of the shells including plate sections and code design are given in section 3.2.

Methodology of buckling analysis, evaluation of the actual nonlinear buckling, idealizing the response nonlinear curves are given in chapter four.

Chapter five includes results and discussion. This chapter is divided into five sections. Actual introduction design used of the shells is given in section 5.1. kind of material and base palate design that used in this study are in sections 5.2 and 5.3. Idealized stress analyses for each model are given in section 5.4. At the next step, checking the stability and stiffness of models in buckling analysis and compare them together in sections 5.5.

Chapter six includes summary, conclusion and recommendation. A summary of what has been done its consequential findings and discussions of findings are given in sections 6.1. The conclusion of the thesis can be found in section 6.2 and the brief recommendations for future studies are in section 6.3.

## **Chapter 2**

### **LITERATURE REVIEW**

#### **2.1 Introduction**

This chapter presents a review of existing knowledge pertinent to the duopitch, cylindrical, dome shell roof system. In addition, experimental and numerical techniques to determine shell buckling loads are discussed. The chapter begins with discussing about the code used for shells and the historical development and recent advances in folded plates. Moreover, details of relevant research on the shell action and local buckling behavior of thin steel plates in construction are summarized and presented in this thesis. The historical development of theory and analytical solution methods for shells, with emphasis being given to the buckling behavior of shell structures, are then described. In addition, experimental and numerical techniques for shell buckling research are surveyed.

#### **2.2 Eurocode for Steel Shells**

##### **2.2.1 Introduction**

Shell structures are some of the most complicated structures that a structural engineer may have to deal with at some point in their career. Conditions seen in other structural forms are almost definitely exist somewhere in shell structure. The behavior of a variety of shell structure responses need to be obtained through methodical approach as per the requirements of the relevant standards.



Standards are possibly of two types: “applications” standards, pertinent to particular structural type, such as tower or tank, and “genetic” standards, anticipated to be applicable to the whole class of structures with a common form, such as shell or box. There are numerous genetic standards which can be regarded as appropriate to metal shell. The best generic standards for metal shells are Australian Standard (AS 2327.1 2003), the European code (Eurocode 3 1994), the British Standard (BS 5950-4 1994), and the ASCE code (ANSI/ASCE 3-91 1991). Metal shells have special characteristics which make them less capable of carrying external or internal or internal pressure when compared to shells with other material.

### **2.2.2 Principles and Constraints**

1. Consistency of regulations for different requests: The thought that all shell structure forms should be designed according to the same rules. Moreover, the principal properties of the shell materials need to be in compliance with the design.
2. Maintaining the existing rules for various structures: Shell structures, like many other successful structures, should be constructed as per the approved standards.
3. Variations between regulations for dissimilar use: different types of structure require different rules. The general rules of design code requirements specified for other structures must be adequate for shell structures.

### **2.2.3 Structure of the Considered Standards**

There are two common standards to cover the following structural form:

- 1- Strength and stability of shell structure.
- 2- Strength and stability of plated box structure.

## **2.3 Scope of Using Standards for the Strength and Stability of Shells**

### **2.3.1 Introduction**

Transverse shear and normal stresses are important for the thickness distributions from one side to the other side. Shell stress induced failures occur through three mechanisms. Three dimensional equilibriums are used to determine the transverse and transverse normal stress distribution all the way through the thickness of the shell. If the in-plane stress gets too large, fiber breakage or material yield occurs. Transverse normal stress may develop on the structure which is strong enough to tolerate de bonding failure where two layers pull away from each other. It is critical to understand how to calculate transverse shear and normal stress distribution taking into account the thickness of the shell. Traditionally, the estimates of the transverse shear stress obtained via the equilibrium equation have relied on numerous restrictive assumptions: large deformation effects, initial curvature, directional bending and membrane forces which were all neglected. These assumptions lead to the usual parabolic distribution of transverse shear stress, and the transverse normal stress vanishes.

“The behavior of shell against the applied dynamic load is modeled by conventional formulations of shell. Considered the dynamic behavior of shell of revolution and included the effect of transverse shear.” (Tene, 1978).

### **2.3.2 Possible Methods of Checking Strength and Stability**

Generic shell standard are only concerned about the strength and stability rules. For this study buckling is considered for the sake of checking the strength and stability which are very dominant in shell structures. Eurocode is one of the standards that generally deal with ordinary structural engineering problems. However, it has the

capacity of dealing with more complicated structural engineering problems too. Therefore, Eurocode can be used to solve shell roof structures.

### **2.3.3 Limit States**

The design standard defines four limit states as follows:

1. plastic collapse and crack
2. cyclic plasticity
3. buckling
4. weakness

For each of them, the types of analysis which may be used failure criteria are defined. The limit state of cyclic plasticity limits the extent to which local stresses can follow the local plastic deformations which leads to low cycle weakness failure.

### **2.3.4 Different logical approaches**

The design standard recognizes that the structure can be analyzed by membrane theory, linear shell bending theory, linear eigenvalue analysis, geometrically nonlinear analysis or geometrically non linear analysis including modeling of imperfection. Each of these analyses is used by a significant group of engineers, so the conditions under which each may be used, and a codification of the assumptions is necessary. The criteria of failure must be defined and qualified to each analysis type, since all analyses require an additional factor on the outcome before they can be used in design.

## **2.4 Stability of Shells**

### **2.4.1 Introduction**

Structure with the stable form is the structure that achieves its strength in accordance with the shell form and it is this form that should have the tolerance to additional

loads. If membranes reverse without changing forms and if it is set under the loads for which the certain form has been designed, structure have stable form that act under pressure. Once the shell roof under load changes from elastic behavior then the structure becomes inefficiency and will not tolerate. Compressive stresses. Thin shells have lot of advantages in terms of their usage, however, their design and construction is difficult since they require skilled labour. The advantages in their usage often overcome the difficulties faced during design and construction. Thin shells are structures with stable form with enough thickness so that no bending stresses will appear on them. They are sufficiently thick enough to have the ability to tolerate the tension and shear loads.

The basis of modern application of shell structure has been formed according to the benefits having high strength to weight ratio combined with its natural stiffness. As it may have been seen in more details in the historical review of Sechler (1974), thin shell structures have been widely used in many fields of engineering and have been studied scientifically for more than one hundred years. Great variety of shell roofs have been designed and constructed in many part of the world. Using shells for roofs leads to considerable materials saving.

A significant value in evaluating the good geometrical arrangement of thin shells is the ratio between the thickness and the span of the roof structure. Failure by buckling, therefore, is often the controlling design criterion. It is essential that the buckling behavior of these shells is correctly understood so that appropriate design methods can be recognized.

Many memorable researchers as well as a large number of papers and textbooks are concerned with this subject. Only those aspects related to the design of duopitch, arch and dome stiffened metal shells roofs are examined, as they are directly relevant to the stiffened steel base shell.

#### **2.4.2 Concise Historical Outline of Thin Shell Buckling Research**

To check stability a local load is applied to the surface which results in a global response distributing the load throughout the surface. This behavior allows shell structures to lose portion of the structure and still remain stable and standing. Regarding buckling problem, it is very important to define the concept relating to the loss of stability and the method to determine the collapse load. The shell roofing element should be analysed as a 3 Dimensional problem. Progress in the analysis of the buckling of shells illustrates the close relationship between theory and experiment required to improve the corresponding design approaches.

The first theoretical solution to the shell buckling problem was provided by Lorenz (1908), Timoshenko (1910) and Southwell (1914) who determined the linear bifurcation stress for a cylindrical shell under axial compression with simply supported ends. The book 'Buckling of Shell Structures' was the first attempt to summarize in a unified manner, at the level of the knowledge (1970) in this field. As well, even when great care was taken with the testing include in the book, the test strengths exhibited a wide scatter for apparently identical specimens. The unacceptable inconsistency between predictions and experimental results led to a re-valuation of the linear shell buckling theory as well as simplifying the assumptions adopted in the early theoretical investigations. Some very important studies in behavior of reticulated shell field were performed by Wright (1965), Sumec (1986), Heki (1986). First attempts were made by Flügge (1932) and Donnel (1934) who

studied plasticity induced by end restraint in the cylinders. For sensible aspects referring to the factors influencing the buckling and post buckling behavior, some very important studies were performed by See (1983), Kani (1981), McConnel (1978), Hatzeis (1987), Fathelbab (1989), Gioncu and Balut (1985), Lenza (1988), Suzuki (1989), Kato (1988), Heki (1993), and others. Through the hard work put into many subsequent investigations, it is now quite well accepted that the large difference between the experimental results and theoretical predictions and the wide scatter of experimental results is qualified to four factors:

- 1) buckling deformations and their related changes in stress (e.g. Fischer 1965, Almroth 1966, Gorman and Evan-Iwanowski 1970, Yamaki and Kodama 1972, Yamaki 1984)
- 2) boundary conditions (e.g. Ohira 1961, Hoff and Soong 1965, Almroth 1966, Yamaki 1984)
- 3) non uniformities and eccentricities in the applied load or support (e.g. Simitse et al. 1985, Calladine 1983)
- 4) associated residual stresses and geometric imperfections (e.g. Kármán and Tsien 1941, Donnell and Wan 1950, Rotter and Teng 1989; Guggenberger 1996, Holst et al. 1999).

The situation of the art surveys on the effects of these factors has appeared at various times. An exhaustive compilation of research on concrete shell buckling is available in Popov and Medwadowski (1981). More recently, a comprehensive review of research on thin metal shell buckling has been given by Teng and Rotter (2004).

The enormous advances of numerical techniques especially the finite element method and remarkable improvements in computer power have had a far reaching

effect on shell buckling research, during the past two decades. For example, the development of the arc length method (Wempner 1979, Riks 1979, Crisfield 1981, and Ramm 1981), which was a notable step forward in the path of the following methods, has enabled the solution of shell buckling with high nonlinearities to be possible. More than a few common purpose computer packages such as ABAQUS, ANSYS and MARC are now commercially available. Numerical approaches are now on an equal footing with logical and experimental methods for exploring shell buckling, and it is believed that it will play a much more important function in the research and design process of shells in future. Good examples of numerical studies of shell buckling behavior include those of Zhao (2001), Song (2002), Pircher and Bridge (2001), Lin (2004), Rotter and Teng (1989), and Teng and Rotter (1992).

### **2.4.3 General Theories on shell Roof Structure**

The theory of shell structures is a vast subject and is set firmly in the field of structure mechanics, for which the main ideas have been well recognized for many years. It has existed as a well defined branch of structural technicalities, and the literature is not only extensive but also rapidly growing. In the theory of shell structures, the interaction between bending and stretching effects is crucial. It is obviously important for engineers to understand clearly in physical terms how the bending and stretching effects combine to carry the loads which are applied to shell structure. The character of any book depends of course mainly on the author's conception of it is subject matter. Most authors of books and paper on the theory of shell structures would agree that the subject exists for the benefit of engineers who are responsible for the design and manufacture of shell structure. It is important for engineers to understand how shell structures act, and how to be able to express this understanding in the physical language rather than purely mathematical ideas.

#### **2.4.3.1 General Theories of Duopitch Shells**

The attributed feature of the duopitch roof is the ease in forming plane surfaces. Consequently, they are more adaptable areas than curved surfaces. A number of advantages might be gained by increasing the thickness of the slab just at the valleys. Therefore it will act as an arched beam and as an I section plate girder. When a duopitch shell has been constructed as a roof structure, the main loading on the structure which consists of self weight (Dead Load), snow loading, and wind loading should be considered. The first construction was made of concrete shells and developed by Beltman and spit (1962), Cement (1961), Shelling (1959), Caspers (1962), Brekelmans and Meischke (1964) and Seyna and Hofman (1964).

#### **2.4.3.2 General Theories of Cylindrical Shells**

A surface of transference shells, which is usually perpendicular under the first arch, has the ability of finding tension transferred across the flat arch and the tension caused by the flat arch shakes transferred to another flat arch. A cylindrical form would show the result of transferring tension on a horizontal direction which better under a vertical arch. Depending on the kind of the transfer arch, both the above mentioned surfaces bring the same result of transferring a vertical arch along a horizontal direct line which is perpendicular to the vertical arch. The cylindrical form can be circular, elliptical, or hyperbolic. Transferring a right paraboloid by rotating it upwards or downwards and on another perpendicular paraboloid which rotates downwards creates a surface called hyperbolic elliptical. This surface is appropriate for covering the rectangular surface. The hyperbolic elliptical is the first form which had been used for making thin shells (1907). When an incomplete circular cylindrical shell has been constructed as a roof structure, the main loading on the structure



which consists of self weight (Dead Load), snow loading, and wind loading should be considered.

The general theories of cylindrical shells originated in the period 1930-1940. The initial of the several approximate bending theories was put forward during this period by Finsterwalder (1932, 1933), and was applied to the analysis of several concrete shell roofs built in Germany (1920, 1930), though, this theory is applicable only to long shells. Roughly the same time the first theory present was a rigorous and concise one which was applicable to cylindrical shells by Flügge (1934). The theory presented by Dischinger (1935) had the added advantage of being more suitable for design. A simpler theory than those of Flügge and Dischinger was developed by Donnell (1933, 1934), but with a more general applicability than that of Finsterwalder.

Donnell's theory was tested by several researchers (e.g. Kármán and Tsien 1941, Jenkins 1947, Hoff 1955), with the result that the accuracy of this theory for the analysis of cylindrical shell roofs was proved. Donnell's theory was developed in the subsequent years due to the independent work of several investigators such as Morley (1959, 1960) and Wang et al. (1974), making this theory one of the best solutions for short shells. It should be noted that the impact of imperfections on cylinders under uniform external pressure is relatively moderate (Budiansky 1967; Budiansky and Amazigo 1968) and is normally accounted for by a constant factor for all kinds of applications (geometry, boundary conditions, etc.). Theory and its limitations are found in the work of Goldenveizer (1961) and Zingoni (1997).

### **2.4.3.3 General Theories of Dome Shells**

The dome shape has been around in architecture and design for hundreds of years. Nearly every one of the famous architectural structures in the world have dome and vaulted shapes. The shape of the dome is based on spherical geometry and was taken from the shell form of some elements of nature. The dome shell behaviors follow the ideas of using curvature, a thin surface to establish strength and stability. The dome provides a structure with highly redundant strength and flexibility features. When a dome shell has been constructed as a roof structure, the main loading on the structure which consists of self weight (Dead Load), snow loading, and wind loading should be considered.

### **2.4.4 General Theories of Stiffened Shells**

Stiffened shells, and in particular stiffened duopitch, cylindrical, dome shells, are usually very efficient structures that have various applications in civil engineering. Considerable effort has therefore been devoted to buckling analysis and experimental studies. Latest summaries of these studies were given by Singer and Baruch (1966), Burns and Almroth (1966), Milligan et al. (1966) and Singer (1972). In recent decades, the wide use of closely stiffened cylindrical shells has led to even more extensive theoretical and experimental investigations (e.g. Croll 1985, Kendrick 1985, Green and Nelson 1982, Walker et al. 1982, Dowling and Harding 1982). Useful information can also be found in the work of Ellinas et al. (1984) or Galambos (1998). Stiffened cylindrical shells subjected to external pressure or axial compression can fail in one of the three modes: local shell instability (elastic or inelastic), global instability (elastic or inelastic) and axi-symmetric or non-symmetric plastic collapse. Moreover, Buckling of reticulated shell structure was studied by Steven E (1970) and Hutechinson W (1970).

For instance, a stiffened shell structure can fail either by local buckling of the curved panels between the stiffeners or the stiffeners themselves, or by general instability of their reinforcing members. This aspect has been the focus of most buckling and post-buckling studies of stiffened shells (e.g. Baruch et al. 1966, Burns and Almroth 1966, Milligan et al. 1966, Singer 1972, Gerard 1961, Crawford and Burns 1963, Singer 1969). The smeared stiffener theory is characterized by the employment of a simplified mathematical model to represent the stiffeners. The stiffeners are “smeared”, or “distributed”, over the entire shell. With some appropriate assumptions (Baruch and Singer 1963, Singer et al. 1966, 1967), this theory has been found to be a satisfactory approach for closely stiffened shells that fail by general instability, since the effect of the discreteness of stiffeners is usually negligible (Singer 2004). The model can be employed in a simplified linear theory, as used in most studies (e.g. Thielemann and Eslinger 1965, Baruch et al. 1966, Singer et al. 1966; Bushnell 1989), or in more sophisticated numerical solutions, in which nonlinear effects are taken into consideration.

## **2.5 Techniques for Shell Buckling Experiments**

### **2.5.1 Introduction**

The problem of stability of single layer domes was first considered by Kloppel and Schardt (1962). The elastic buckling of the reticulated shell has been studied by many investigators. Two main approaches have been used in developing computational method to evaluate the stability. The first was the continuous shell analogy, in which the behavior of reticulated shell is approximated by the behavior of a continuous shell having equivalent properties with discrete shell. Secondly, the discrete mathematical models have been developed recently mainly due to the fact that they require extensive numerical calculations which were only possible after the

development of the powerful electronic computers and very efficient programs. In this way the names of See (1983), Kani (1993), Mc Connel (1978), Rothert (1984), Ramm (1980), Ricks (1984), Borri (1985), Wempner (1971), Crisfield (1983), and Papadrakakis (1993) must be mentioned.

Despite the increasing ease of complex numerical modeling as a result of advances in computer technology and numerical methods, experimental research still acts a significant role in understanding shell behavior. This is particularly so if the structure includes features of considerable uncertainty such as poorly defined boundary conditions or complex joints. Shell buckling experiments, despite a long history, is still a serious challenge to researchers. Test programs with the main objective being to aid in the design of reinforced concrete shells was given by Billington and Harris (1981).

### **2.5.2 Techniques for Metal Shells**

The successful implementation of a particular experimental study may very well hinge on the techniques adopted in the model fabrication. Many sources of errors inherent in any given fabrication process can significantly influence the structural behavior of shell buckling tests, which may be very sensitive to small details, especially, the geometric imperfections in the shell. Therefore, high quality shell models are desirable to reduce the large scatter generally exhibited by test results. In addition, shell models must be made neither too large to avoid inconvenience in laboratory tests nor too small to avoid difficulty in representing the prototype structures. When choosing the material, extreme care shall be exercised so that the buckling and collapse behavior of real shells can be appropriately represented. Many techniques have been developed for the fabrication of good quality shell models, among which are electroforming of nickel or copper, rolling and seaming of Mylar

sheets, thermal vacuum forming of plastics (such as PVC, polyethylene, Lexan, and other thermal forming plastics), spin casting of plastics (such as polymer and epoxy resins), casting of resins, and cold working (such as spinning, explosive forming, and hydroforming) of metal, machining and special forging. Further detailed information regarding fabrication methods of interest can be found in Babcock (1974) and Singer et al. (2002). It should be noted that the discussions here are focused on cylindrical shell models. However, the applicability of techniques described in this respect to other types of shells is pointed out.

Apart from efforts aimed at the fabrication of small-scale shell models, realistic models with the same material and fabrication technique as their full-scale counterparts have been built by some investigators, for example, Sturm (1941), Wilson and Newmark (1933) and Harris et al. (1957). This is of particular importance in the development of design guidelines in civil and ocean engineering. To obtain models with geometric imperfections and weld-induced residual stresses which are similar to those in real structures, efforts were made by many researchers, such as Dowling and Harding (1982), Dowling et al. (1982), Green and Nelson (1982), Miller (1982), Walker et al. (1982), Scott et al. (1987), Knoedel et al. (1995), Berry (1997), Chryssanthopoulos et al. (1997), Schmidt and Swadlo (1996, 1997), Schimdt and Winterstetter (1999), Berry et al. (2000), Chryssanthopoulos and Poggi (2001), Zhao (2001), Song (2002) and Lin (2004). High quality welding work is essential in controlling the quality of the metallic shell models. The TIG (tungsten inert gas) welding process, has long been used in fabricating model metal shells (e.g. Green and Nelson 1982; Walker et al. 1982; Scott et al. 1987), as it gives the best welds for thin steel sheets. More recently, the development of pulsed TIG welding, which has the advantages of lower total heat input and better heat dissipation, further

enhanced the process to produce smooth, clean and well controlled welds, especially for very thin sheet metals (Knoedel et al. 1995; Berry 1997; Teng et al. 2001). The equipment required for TIG welding includes a generator, gas tanks, a welding torch, electrodes and safety masks, all of which are easily acquired, even in a very small laboratory. Moreover, automatic TIG welding was also employed by some researchers (Berry 1997; Teng et al. 2001; Lin 2004). As a further measure to achieve high quality welding, the former should be machined in or covered with copper sheets, to provide a non-sticky backing as well as a heat sink to reduce welding distortions and welding residual stresses (e.g. Berry 1997; Teng et al. 2001; Lin 2004). Geometric imperfections, boundary and loading conditions have a considerable influence on the buckling load of a shell. Nowadays, geometric imperfection measurements have become a necessary step of shell buckling experiments, and a variety of imperfection measurement techniques has been developed. Reviews of these techniques have been provided by Singer and Abramovich (1995) and Singer et al. (2002). Boundary and loading conditions are also crucial and should be properly designed in shell buckling experiments. An appropriate load transfer is usually much more difficult to achieve than a reliable boundary scheme. More detailed information can be found in the publications of Babcock (1974) and Singer et al. (2002).

## **2.6 Numerical Techniques for Shell Buckling Analyses**

### **2.6.1 Introduction**

The difficulties in conducting dependable buckling experiments and in deriving analytical solutions for most shell buckling problems have led engineering researchers to look for an ease alternative to forecast shell buckling behavior. For this reason, it is not surprising that the shell buckling study area was one of the first

groups of engineering researchers to hold the application of modern computer methods. Since the 1960s, great advances in computer technology have been achieved. The processing speed has increased dramatically and memory and storage capacities have expanded rapidly. Accordingly, quite a lot of complicated general purpose finite element packages are now commercially available and widely used, which has provided the ease and assurance in the numerical studies of shell buckling problems.

One of these programs is the ANSYS general purpose finite element package (ANSYS 2007). All different types of analyses, as summarized in Table 2.1 (ENV 1993-1-6 1999), can be written as a FORTRAN program in ANSYS. The new Eurocode 3 (ENV1993-1-6 1999) also recommends the direct use of powerful computer analysis methods in stability design and assessment as a feasible alternative and provides recommendations on the conduct of such analyses and interpretation of results for use. The recommendations also provide guidance to numerical analyses of shells by researchers in advanced numerical simulations for the development of structural understanding and simple design methods.

### **2.6.2 Linear and Nonlinear Division Analysis**

The buckling analysis can be used as the simplest numerical buckling analysis to find the critical load of a shell structure. Two types of Eigen value buckling analyses are available, linear analysis and nonlinear analysis. The linear analysis is a buckling analysis where the effect of pre buckling deformation is ignored. By difference, in a nonlinear analysis this effect is considered. On occasion, negative Eigen value indicates that the structure would buckle if the load were applied in the opposite direction. Classic examples include a plate under shear loading, where the plate can buckle at the same value for positive and negative applied loads. A linear structure

analysis is used for majority of structures. But there are some structures with softening response for which the application of linear analysis is unsafe. The problem for the designer is the decision to choose between linear and non linear analysis, and so the decision to opt for a nonlinear analysis is very important one. A nonlinear analysis is performed on the nonlinearly deformed state, generally also for the perfect geometry.

### **2.6.3 Nonlinear Analysis**

The buckling and post buckling responses of shell under combined loads are highly complicated, where the load displacement response shows a negative stiffness in the post buckling range and the structure must release strain energy to remain in equilibrium. The load magnitude is used as an additional unknown computed as a load proportionality factor multiplying the reference loads. In nonlinear analysis of shells, it is important to consider the effect of geometric imperfections in an appropriate manner. The 'equivalent geometric imperfection' concept was adopted in Eurocode 3 (ENV 1993-1-6 1999) to consider the effects of all imperfections on shell stability. In such analyses, the choice of imperfection pattern is important. If the most unfavorable pattern cannot be readily identified beyond reasonable doubt, the analysis should be performed repeatedly for different imperfection patterns. In particular, the code recommends the use of the Eigen mode affine pattern (e.g. in the form of the linear or non-linear bifurcation mode) unless a different unfavorable pattern can be justified. It should be noted that, the amplitude of the equivalent geometric imperfection should be chosen with consideration of fabrication quality and the effects of non geometric imperfections.



## **2.7 Folded Plate**

### **2.7.1 Introduction**

The ease in forming plane surfaces plate is the distinguishing characteristic. It is the simplest of all the shell structures. Folded plate construction is a surface structure constructed from individual plane surface, or plates, joined together to form a composite surface. Therefore, they are more adaptable to smaller areas than curved surfaces. A folded plate is often used for horizontal slab and has much less steel and concrete for the same spans. Some advantage may be gained by increasing the thickness of the slab just at the valleys. Folded plate will perform as a hunched beam and as an I-section plate girder. The structure above may have a simple span, or multiple spans of varying length, or the folded plate may cantilever from the supports without a stiffener at the end.

### **2.7.2 History of Folded Plates**

In the nineteen sixties, the study and construction of reinforced concrete reached what was very likely their peak. Within this category of structures, so called folded plates, special consideration merits for their thickness and flat surfaces. As professor Cassinello found out for the first time, on one hand folded plates differ from other thin shells in that they do not have the properties of curvature. On the other hand, their membrane behavior has close resemblance to corrugated plates and cylindrical shells. The underlying idea is quite simple: longer span can be accommodated with relatively small increase in weight by enlarging the level arm of the structure. The top and bottom chords of each stated slab house the main reinforcements while the shear stresses are absorbed across the sloping sides (1974). F. Candela distinguishes prismatic structures and folded slabs from other thin shells in that they are “subjected to a combination of membrane and bending forces” (1970) and American engineer

Milo Ketchum, specializing in the design and construction of such structure, wrote with respect to their advantages that: “the analysis was straightforward, used methods with which I was familiar, and the structural elements were those we used for other structures. It is possible to analyze folded plates with more precision than barrel shells (1990). Ketchum also contributed to christening this type of structures: I always disliked the name “hipped plate”, and when I was chairman of an ASCE committee, I was at least partially responsible for changing the name to “folded plate” (1990)”.

Nonetheless, except for the simplest folded which are admittedly the most common cases and forms with parallel horizontal folds, regardless of Ketchum’s remarks, to the contrary a general theory applicable to the structural analysis of such plates is anything but straightforward. Wilby C.B. summarized the key milestones in the history of the analytical study of such structures in the following terms:

The principle was first used in Germany by Ehlers in 1924, not for roofs but for large coal bunkers and he published a paper on the structural analysis in 1930. Then in 1932, Gruber published an analysis in German. In the next few years many Europeans – Craemer, Ohlig, Girkman and Vlasov (1939) amongst them – made contributions to this subject. The Europeans’ theories were generally complex and arduous for designer use. Since 1945 simplified methods have been developed in the USA by Winter & Pei (1947), Gaafar (1953), Simpson (1958), by Whitney (1959) adapting the method by Girkman, by Traum (1959), by Parme (1960) and by Goble (1964).

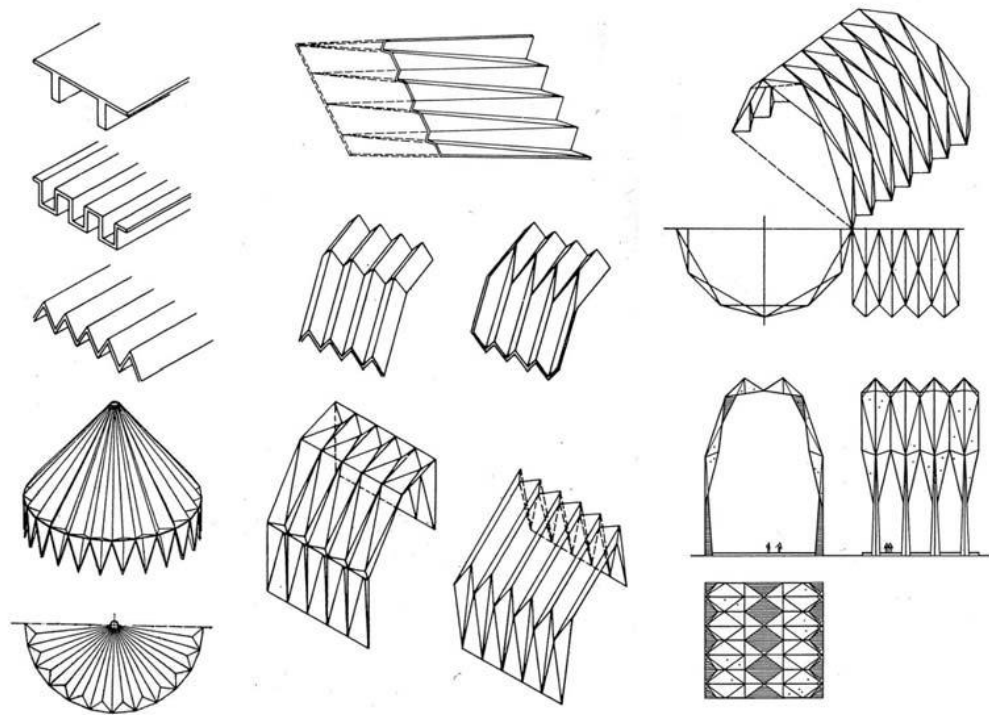


Figure 2.1: Example of folded structures (Angerer 1974, Engel 2001).

The enormous variety of imaginable forms for such structures has been depicted in a number of texts (Fig2.1). The ones actually erected, however, generally adopt the simplest forms, although certain very complex combinations have on occasion been used (St Josef Church at Neub Weckhoven, Germany, 1966-67, Bauwelt 1967, p. 912; Gadet Chapel, USAF Academy, 1963). The layouts devised to build domed forms also merit mention, although since such designs generally include curved elements, they cannot be regarded to be pure folded plates. Polyhedral forms can naturally also be considered to be folded structures consisting in polygonal facets totally or partially comprising such a surface. Yet another beneficial property is: “their advantage, in comparison to shells, is that formwork for flat surfaces entail fewer difficulties” (Angerer 1972, p. 51). It should nonetheless be borne in mind in this respect that such: “Simplified formwork intensifies the risk of buckling,

inasmuch as non-curvature makes such shells more sensitive to this effect". For this reason, they are "more limited in terms of amplitude and loading" (Cassinello 1960, p. 542) than curved forms.

### **2.7.3 Folded Plate Construction**

Folded plates are surface constructed from individual plane surfaces, or plates, joined together to form a composite surface. Such construction type has been extensively used in the construction of long span roof systems because of its economy and interesting architectural appearance. The proposed system can be used for a variety of structural purposes. Such a combination will result in a strong and efficient structure with many advantages over traditional forms of construction.

### **2.7.4 Folded Plate Characters**

The structural action of a folded plate is different from slab action. The roof surfaces span as a slab in the direction transverse to the fold lines, with the fold lines serving as lines of support for the transverse slab strips. The reaction of each slab strip is applied to any given fold line. The in-plane component for a plate at one fold line is added to that adjacent fold line of the same plate and to the in-plane component of the surface load to obtain the total in-plane load applied to the plate.

#### **2.7.4.1 Folded plate Duopitched Form**

The principle workings in a folded plate structure are illustrated in duopitch form. They consist of: (1) the inclined plates, (2) stiffeners to carry the loads to the supports and to hold the plates in line. The span of the structure is the greater distance between columns and the bay width is the distance between similar structural units. The structure above is a two-segment folded plate. If several units were placed side by side, the edge plates should be omitted except for the first and last plate. The folded plate structure above may have a simple span (Figure 2.2), or

multiple spans of varying length, or the folded plate may cantilever from the supports without a stiffener at the end.

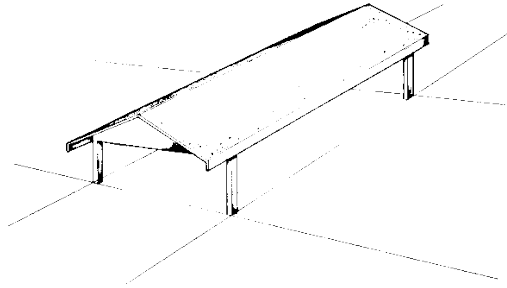


Figure 2.2: Doupitch roof in concrete folded plate.

#### **2.7.4.2 Folded plate in Arch model**

Arch folded plate structure is suitable for quite long spans and forms for the concrete can be used many times because each unit can be made self supporting (Figure 2.3). All of the different section shapes of folded plates are possible with this type of structure. As in the folded plate shapes, an edge plate is required for the outside member. Placing of concrete on the sheer slope at the springing of the arches may be a problem unless blown on concrete is used or the lower portion of the shell may be precast on the ground and lifted into place.

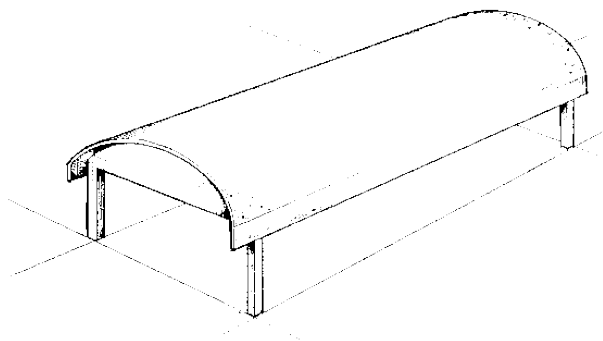


Figure 2.3: Arch folded plate concrete.

### 2.7.4.3 Folded plate dome

Domes may be constructed with many planes so they resemble the facets of a diamond (Figure 2.4). The structural problem in designing these shells is to provide enough angle between the planes so that an actual rib is formed which will be stiff enough to support the plane surface. Usually it is best to start with a spherical translation surface or other mathematical surface and have all the intersections lie on this surface. If not, there may be discontinuities in the layout of intersections which make or destroy the visual effect and make the structure more difficult to design.

This dome can be of much greater span because the span of the individual slab elements is less. A dome hexagonal in plan can be made continuous with all the adjacent units if it is necessary to cover a large area.

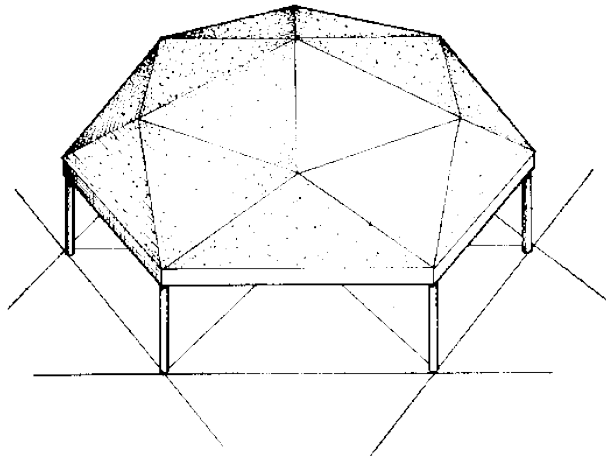


Figure 2.4: Dome folded plate concrete.

## Chapter 3

### DESIGN AND METHODS

#### 3.1 Methodology of Design

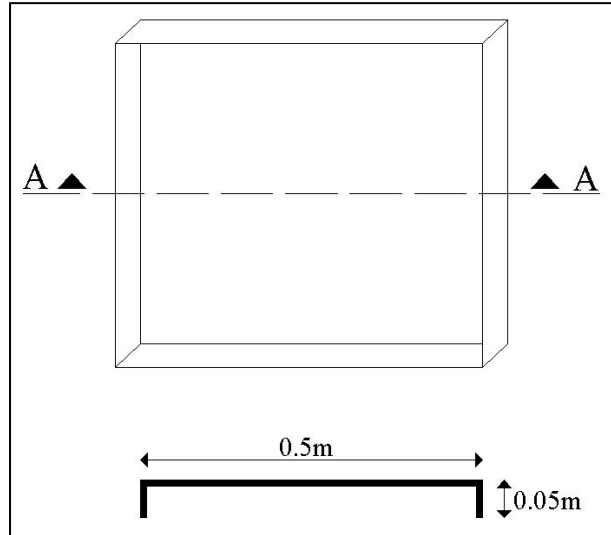
##### 3.1.1 Model Design

For assessment of shell structure, the first step is to design different models with different sizes to be assessed to next step. The models used in this study are basic forms for each kind of space structure roofs (Billington, D.P. 1965; Bulson, P.S 1976; Calladine, C.R. 1983; Chin, J. 1959; Farshad, M. 1992; Fung, Y.C., Sechler, E.E. 1974; Jenkis, R.S 1947; Ketchum, M. 1990; Makowski, Z.S. 1984; Mindham, C.N. 2006; Nooshin, H. 1991; Suzuki, t. 1989; Wong, H.T., Teng, J.G. 2004). Among the available literature, this study similar to that of Ketchum, M. (1990) and Wong, H.T., Teng, J.G. (2001). Consequently, the basic character of folded plate for design models same as Ketchum, M. (1990) and Wong, H.T., Teng, J.G. (2001), compared the results of analysis carried out on these models by using nonlinear static for check buckling behavior in each models.

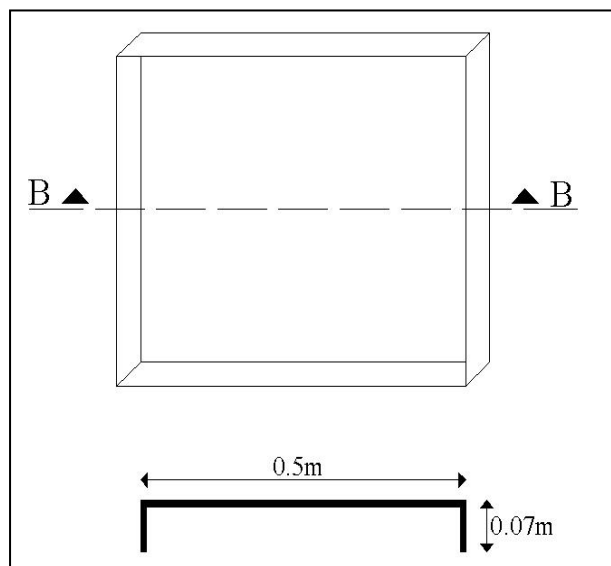
The shell models are made up of three different metal boxes and the geometrical information for these boxes are as follows:

- Metal box with 5cm I section edges, each box has 0.5m length and breadth and I section edges with 0.05m height and 0.03m thickness.
- Metal box with 7cm I section edges, each box has 0.5m length and breadth and I section edge width with 0.07m height and 0.03m thicknesses.

- Metal box with L section edges, each box has 0.5m length and breadth and L section edge width with 0.05m height and 0.02 edge and 0.03m thicknesses.

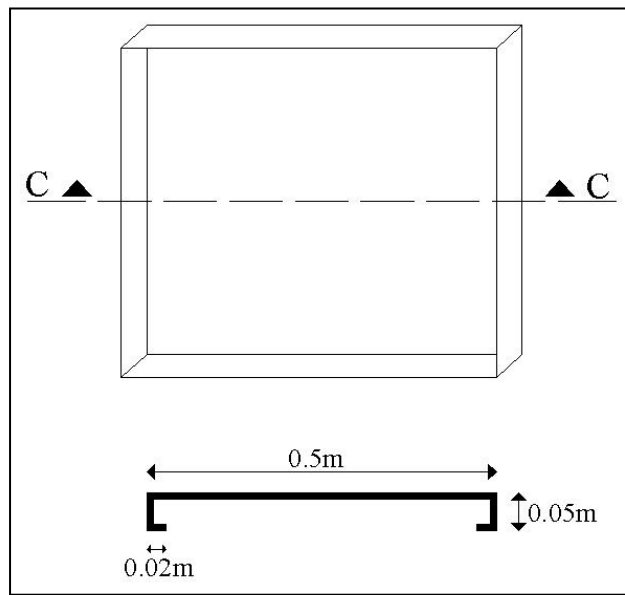


Cross-section A-A



Cross-section B-B





Cross-section C-C

Figure 3.1: The boxes that were used for the models.

- The models used in this study are: duopitch roof (Figure 3.2), cylindrical roof (Figure 3.3) and Spherical dome roof (Figure 3.4)

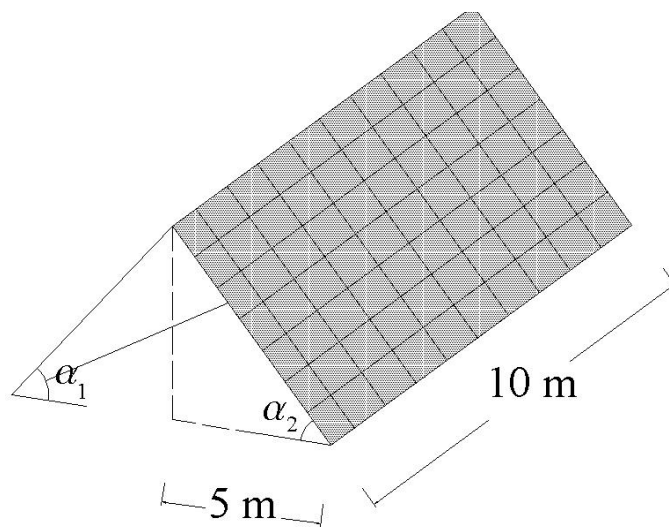


Figure 3.2: Duopitch roof.

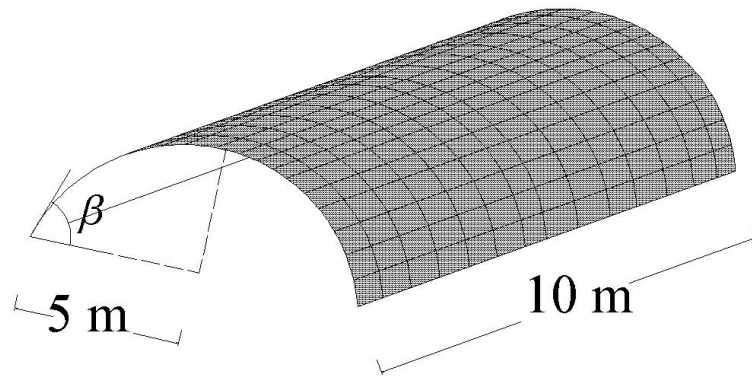


Figure 3.3: Cylindrical roof.

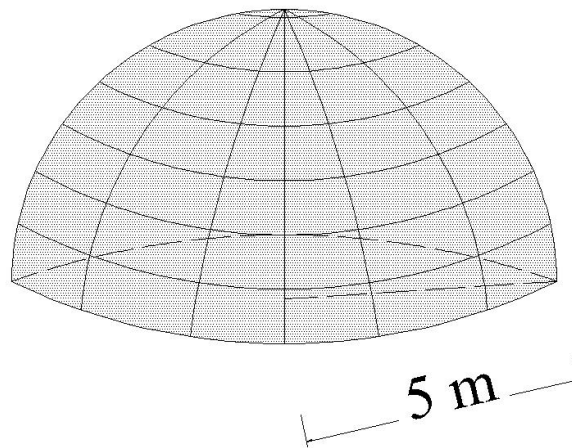


Figure 3.4: Spherical dome roof.

- For duopitch roof and cylindrical roof each model has 5m span and 10m length, and 10m diameter for spherical dome and started analysis as per the models of Ketchum, M. (1990).

### 3.2 Load used

The loads used in this study are dead load, snow load and wind load according to the Eurocode (ENV 1991-2-3).

1.35D

1.35D+1.5L

D±1.5W

1.35D±1.5W

1.35D+1.5L±0.9W

1.35D±1.5W+1.05L

### **3.2.1 Dead Load**

Dead load used for the forms is the self weight of each model, which is obtained from Ansys program.

### **3.2.2 Snow Load on Roof**

The snow load used in this study is obtained from Eurocode3 (Env 1991-2-3), and was considered as live load for the shell roofs.

The snow load on a roof shall be determined from:

$$S = \mu_i c_e c_t s_k \quad (3.1)$$

Where

$\mu_i$  is the snow load shape coefficient (Table3.1)

$s_k$  is the characteristic value of the snow load on the ground [ $\text{kN}/\text{m}^2$ ]

$c_e$  is the exposure coefficient, which usually has the value 1.0

$c_t$  is the thermal coefficient, which usually has the value 1.0

Table 3.1: Snow load shape coefficient – duopitch roofs (Eurocode3 (ENV1991-2-3)).

Angle of pitch of roof	$0^\circ \leq \alpha \leq 15^\circ$	$15^\circ < \alpha \leq 30^\circ$	$30^\circ < \alpha < 60^\circ$	$\alpha \geq 60^\circ$
Shape coefficient $\mu_1$	0.8	0.8	$0.8(0.6 - \alpha)/30$	0.0
Shape coefficient $\mu_2$	0.8	$0.8 + 0.6(\alpha - 15)/30$	$1.1(60 - \alpha)/30$	0.0

### 3.2.2.1 Snow Load on Duopitch Roofs

The value of the snow load shape coefficients for the duopitch roofs is detailed in this section and are summarized in Figure 3.5

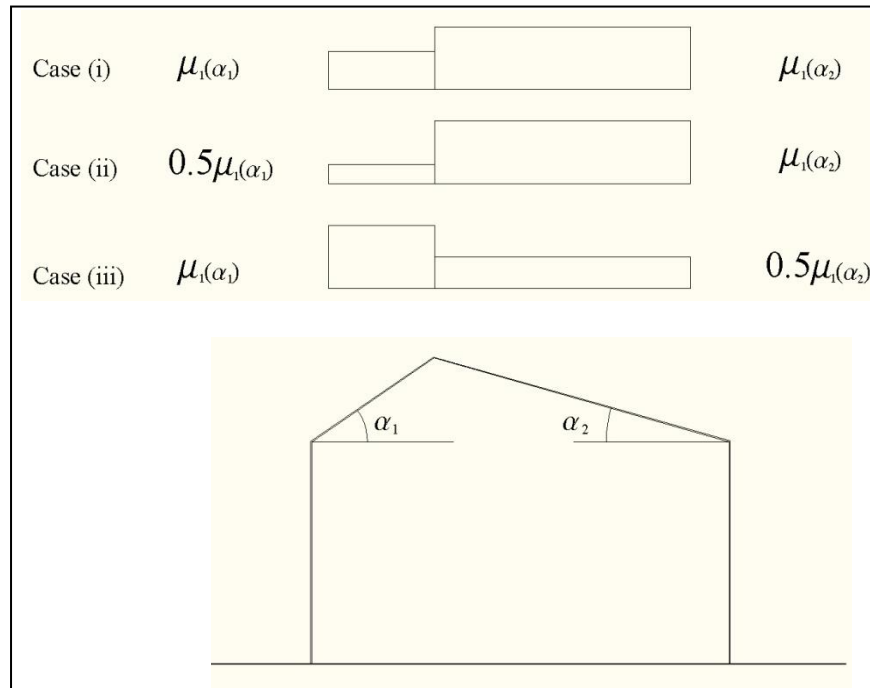


Figure 3.5: Snow load shape coefficients – duopitch roofs (Eurocode3 (ENV1991-1-3)).

The snow load shape coefficient for duopitch roofs are given in Table 3.1. It is assumed that the snow is not prevented from sliding off the roof.

### 3.2.2.2 Snow Load on Cylindrical Roofs

For cylindrical roofs either the uniform or metric snow loads, whichever produces the most adverse effect, should be consider (Figure 3.6).. The values of the snow load shape coefficients summarized in Figure 3.7. It is assumed that snow is not prevented from sliding on the roof.

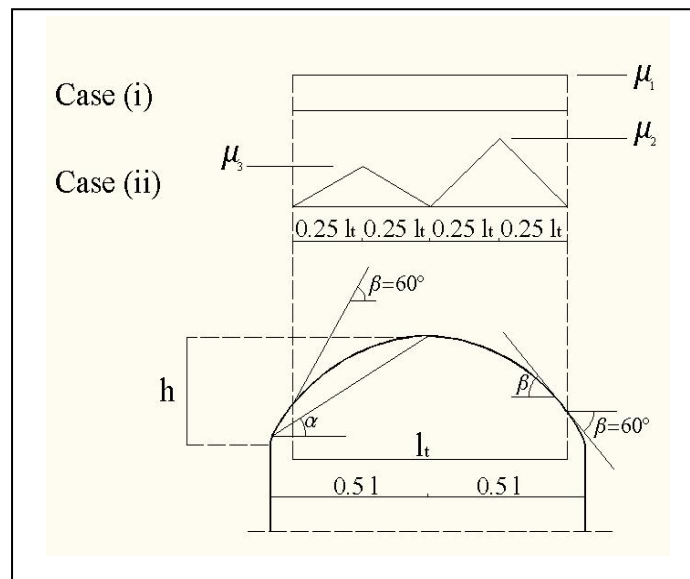


Figure 3.6: Snow load shape coefficients – cylindrical roof (Eurocode3 (ENV1991-2-3)).

$$* l_t \leq 1$$

The snow load shape coefficients are determined as follows:

$$\text{For } \beta \leq 60^\circ, \mu_1 = 0.8 \quad (1)$$

$$\mu_2 = 0.2 + 10 h/l_t \quad \text{With the restriction } \mu_2 \leq [0.2]$$

$$\mu_3 = 0.5 \mu_2$$

$$\text{For } \beta > 60^\circ, \mu_1 = \mu_2 = \mu_3 = 0 \quad (2)$$

At any point on the cross section  $\beta$  is the angle between the horizontal and the tangent to the curve at that point.

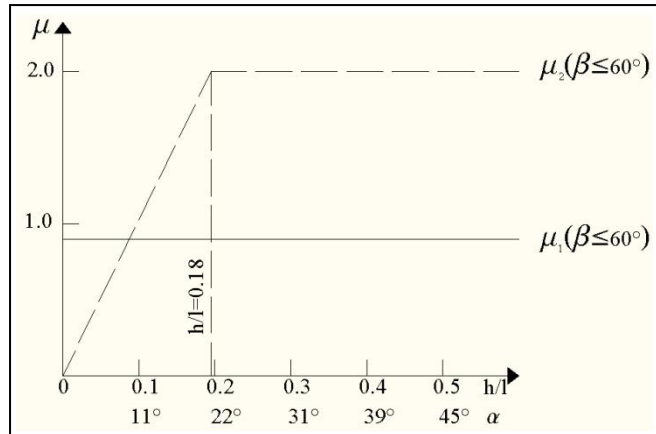


Figure 3.7: Snow load shape coefficient for cylindrical roofs of differing rise to span ratios (Eurocode3 (ENV1991-2-3)).

### 3.2.2.3 Snow Load on Dome Shape Roofs

For dome roofs either the uniform or asymmetric snow loads shown in Figure 3.8 should be considered.

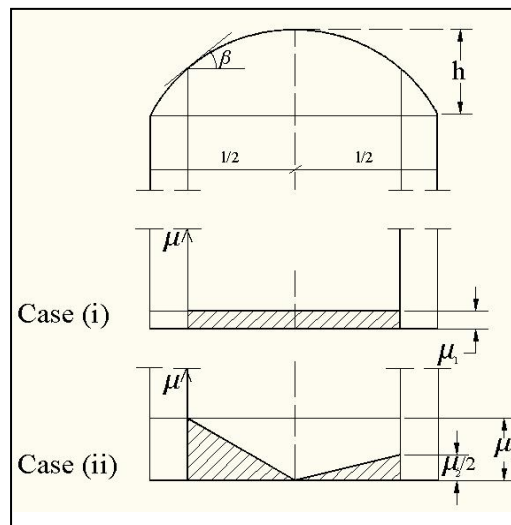


Figure 3.8: Snow load shape coefficient-dome roofs (Eurocode3 (ENV1991-2-3)).

$$\mu_1 = 0.8$$

$$\mu_2 = 0.3 + 10 h/l$$

Restriction:

$$\mu_2 \leq 2.3$$

$\mu = 0$  if  $\beta > 60^\circ$

### 3.2.3 Wind Pressure on Surfaces

The wind pressure is valued for surfaces which are sufficiently rigid to neglect their resonant vibrations caused by the wind. However, if a natural frequency of vibration of the surface is low, these vibrations may become significant, and they shall be taken into account. For wind pressure should consider to: 1) External pressure 2) internal pressure.

#### 3.2.3.1 External Pressure

The wind pressure,  $w_e$ , acting on the external surfaces of the structure can be obtained from:

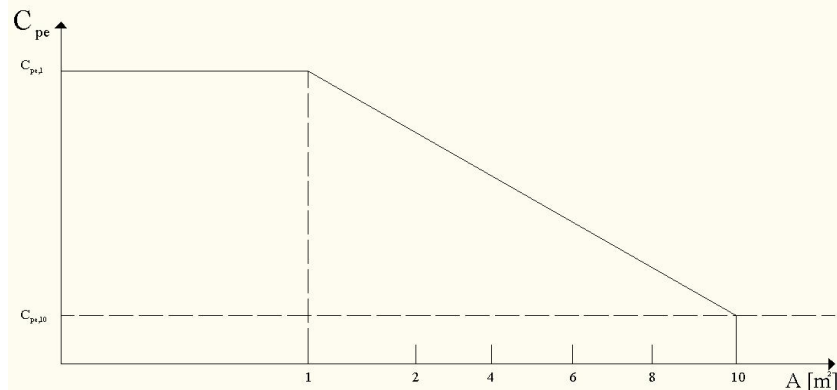
$$w_e = q_{ref} c_e(z_e) c_{pe} \quad (3.2)$$

Where

$c_{pe}$  external pressure coefficient derived from Table 3.2

Table 3.2: External pressure coefficient  $c_{pe}$  for building which depends on the size of the loaded area  $A$ .

$c_{pe} = c_{pe,1}$	$A \leq 1 \text{ m}^2$
$c_{pe} = c_{pe,1} + (c_{pe,10} - c_{pe,1}) \log_{10} A$	$1 \text{ m}^2 < A < 10 \text{ m}^2$
$c_{pe} = c_{pe,10}$	$A > 10 \text{ m}^2$



The loaded area is the area of the structure which produces the wind action in the section to be calculated. Considering the type of the  $c_{pe}$  in Table 3.2 can find value of this. Each shape of the roofs has particular method for finding value of the  $c_{pe}$ .

$q_{ref}$  reference mean wind velocity pressure and shall be determined from:

$$q_{ref} = \frac{\rho}{2} u_{ref}^2 \quad (3.3)$$

Where

$u_{ref}$  is reference wind velocity and shall be determined from:

$$u_{ref} = c_{DIR} \cdot c_{TEM} \cdot c_{ALT} \cdot u_{ref.0}$$

Where

$u_{ref.0}$  basic value of the reference wind velocity at 10 m above sea level

$c_{DIR}$  direction factor to be taken as 1.0

$c_{TEM}$  temporary factor taken as 1.0

$c_{ALT}$  altitude factor to be taken as 1.0

With consideration wind velocity in north part of Tehran can obtain:

$$u_{ref.0} = 100 \text{ km/h}$$

As result

$$u_{ref} = 100 \text{ km/h}$$

$$\rho = 1 \text{ kg}$$

$$\text{Consequently } q_{ref} = 50 \text{ kg. km/h}$$

This is constant for all of the roofs which consider in the study.

$c_e(z)$  exposure coefficient accounting to the terrain and height above ground

Z reference height given the terrains and regions information (Table 3.3).



Table 3.3: Terrain category (Eurocode3 (ENV1991-2-3)).

I	Rough open sea lakeshore with at least 5 km fetch upwind smooth flat country without obstacles
II	Farmland with boundary hedges. Occasional small farm structure. Houses or tress
III	Suburban or industrial areas and permanent forests
IV	Urban area in which at least 15% of the surfaces is covered with buildings and their average height exceeds 15 m

Z reference height defined in Figure 3.9 as appropriate to the relevant pressure coefficient ( $z = z_e$  for external).

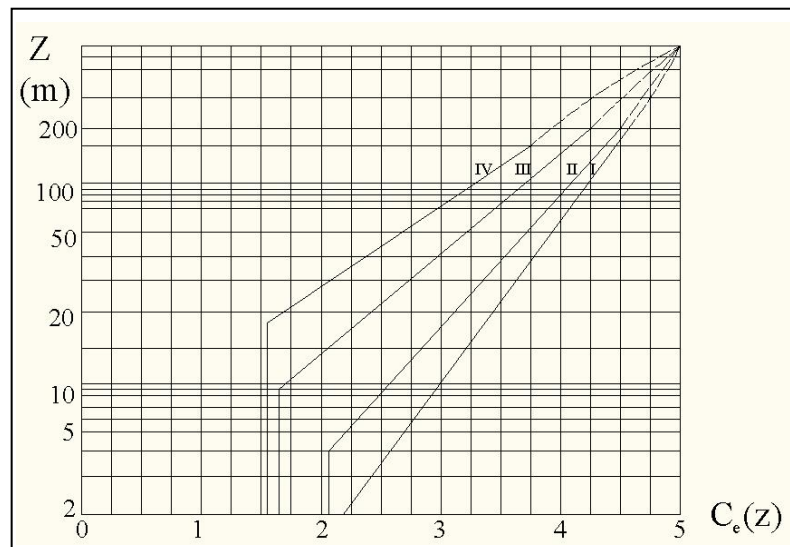


Figure 3.9: External pressure coefficient  $C_{pe}$  for building depending on the size of the loaded area A.

### 3.2.3.1.1 External Pressure for Duopitch Roofs

External pressure for duopitch roof is used in this study and 2 directions  $0^\circ$  and  $90^\circ$  was used. Every direction has a particular behavior on their surfaces. In duopitch roof, surface is divided into certain surfaces as shown in Figure 3.10, Figure 3.11 and every part has determinate loading.

For finding  $c_{pe}$  in both directions, each surface should be specified from Table 3.4 and Table 3.5.

Table 3.4: External pressure coefficient for duopitch roofs: wind direction  $0^\circ$ .

Pitch angle $\alpha$	F		G		H		I		J	
	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$
$-45^\circ$	-0.6		-0.6		-0.8		-0.7		-1.0	-1.5
$-30^\circ$	-1.1	-2.0	-0.8	-1.5	-0.8		-0.6		-0.8	-1.4
$-15^\circ$	-2.5	-2.8	-1.3	-2.0	-0.9	-1.2	-0.5		-0.7	-1.2
$-5^\circ$	-2.3	-2.5	-1.2	-2.0	-0.8	-1.2	-0.3		-0.3	
$5^\circ$	-1.7	-2.5	-1.2	-2.0	0.6	-1.2	-0.3		-0.3	
$15^\circ$	-0.9	-2.0	-0.8	-1.5	-0.3		-0.4		-1.0	-1.5
	+0.2		+0.2		+0.2					
$30^\circ$	-0.5	-1.5	-0.5	-1.5	+0.2		-0.4		-0.5	
	+0.7		+0.7		+0.4					
$45^\circ$	+0.7		+0.7		+0.6		-0.2		-0.3	
$60^\circ$	+0.7		+0.7		+0.7		-0.2		-0.3	
$75^\circ$	+0.8		+0.8		+0.8		-0.2		-0.3	

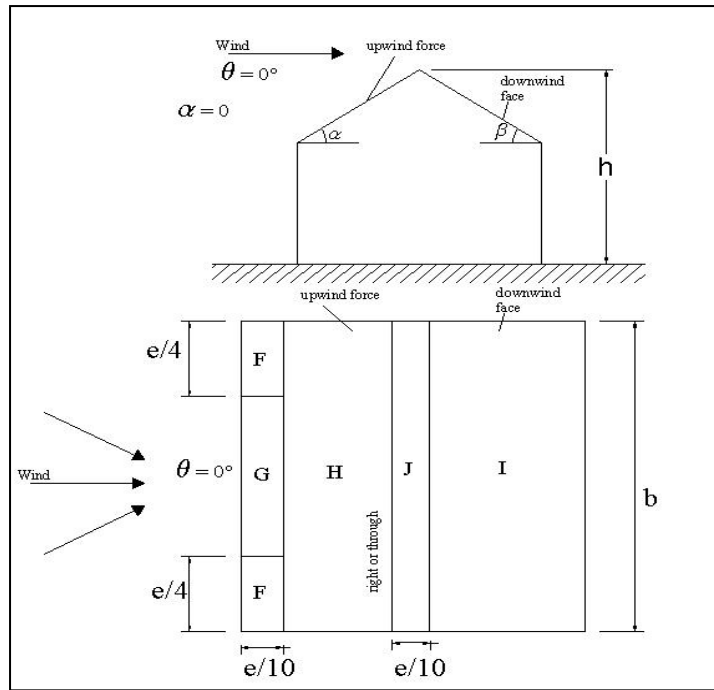


Figure 3.10: Key for duopitch roofs: Wind direction  $\theta = 0^\circ$ .

Table 3.5: External pressure coefficient for duopitch roofs: Wind direction  $90^\circ$ .

Pitch angle $\alpha$	F		G		H		I	
	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$	$C_{pe,10}$	$C_{pe,1}$
$-45^\circ$	-1.4	-2.0	-1.2	-2.0	-1	-1.3	-0.9	-1.2
$-30^\circ$	-1.5	-2.1	-1.2	-2.0	-1	-1.3	-0.9	-1.2
$-15^\circ$	-1.9	-2.5	-1.2	-2.0	-0.8	-1.2	-0.8	-1.2
$-5^\circ$	-1.8	-2.5	-1.2	-2.0	-0.7	-1.2	-0.6	-1.2
$5^\circ$	-1.6	-2.2	-1.3	-2.0	-0.7	-1.2	-0.5	
$15^\circ$	-1.3	-2	-1.3	-2.0	-0.6	-1.2	-0.5	
$30^\circ$	-1.1	-1.5	-1.4	-2.0	-0.8	-1.2	-0.5	
$45^\circ$	-1.1	-1.5	-1.4	-2.0	-0.9	-1.2	-0.5	
$60^\circ$	-1.1	-1.5	-1.2	-2.0	-0.8	-1.0	-0.5	
$75^\circ$	-1.1	-1.5	-1.2	-2.0	-0.8	-1.0	-0.5	

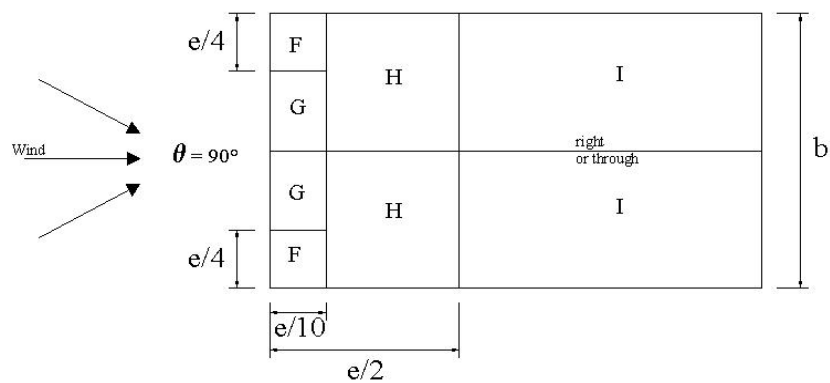


Figure 3.11: Key for duopitch roofs: Wind direction  $\theta = 90^\circ$ .

### 3.2.3.1.2 External Pressure for Cylindrical Roofs

In this study the key for loaded areas and reference height is given in Figure 3.12 with the pressure coefficient.

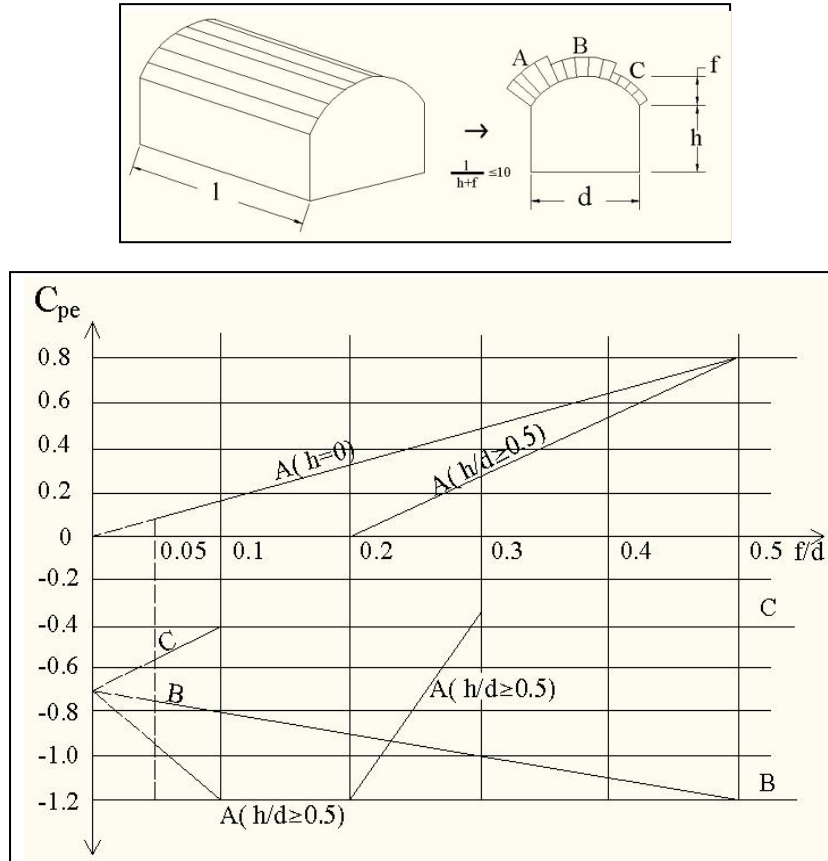


Figure 3.12: External pressure coefficient  $C_{pe,10}$  for vaulted roofs with rectangular base and  $l/(h + f) \leq 10$ .

- If  $0 \leq h/d \leq 0.5$ ,  $C_{pe,10}$  is obtained by linear interpolation.
- If  $0.2 \leq h/d \leq 0.3$  and  $h/d \leq 0.5$

### 3.2.3.1.3 External Pressure for Dome Roofs

In this study the key for loaded areas and reference height is given in Figure 3.13 with the pressure coefficient.

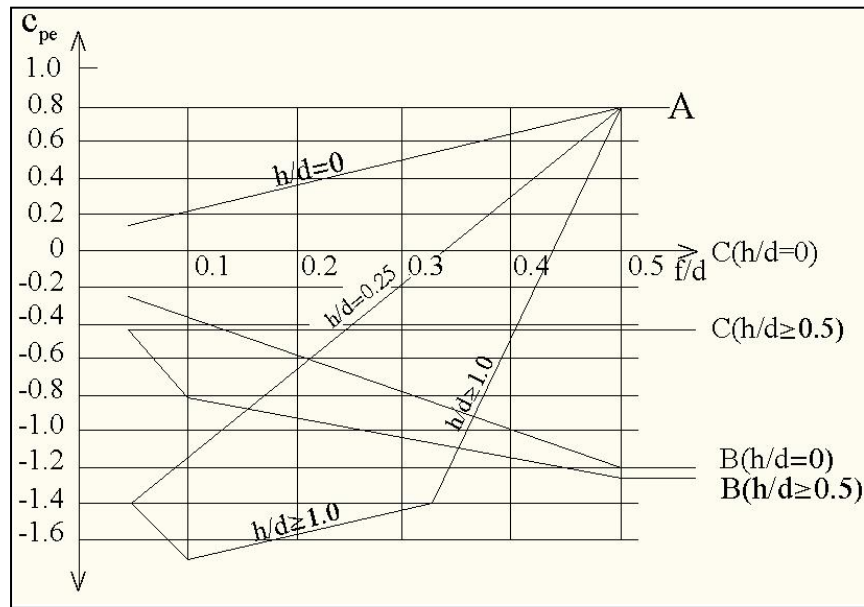
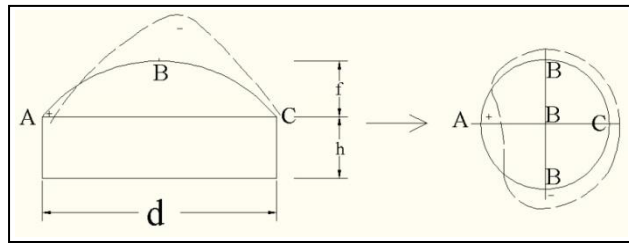


Figure 3.13: External pressure coefficient  $c_{pe,10}$  for dome with circular base.

- 1- Reference height:  $z_e = h + f/2$
- 2-  $c_{pe,10}$  is constant along arcs of circle, intersections of the sphere and of planes perpendicular to the wind, it can be determined as a first approximation by linear interpolation between the values in A, B and C along the arcs of circles parallel to the wind. In the values of  $c_{pe,10}$  in A if  $0 < h/d < 1$  and B or C if  $0 < h/d < 0.5$  can be obtained by linear interpolation in the above Figure4.4.

### 3.2.3.2 Internal Pressure

The wind pressure acting on the internal surfaces of a structure,  $w_i$ , shall be obtained from:

$$w_i = q_{ref} c_e(z_e) c_{pi} \quad (3.4)$$

$c_{pi}$  internal pressure coefficient derived from Figure 4.1 and is function of the opening.

The internal pressure coefficient  $c_{pi}$  for building without internal partition is given in Figure 3.14 and is a function of the opening ratio,  $\mu$ , which is defined as:

$$\mu = \frac{\sum \text{opening at the leeward and wind parallel side}}{\sum \text{opening at the wind ward ,leeward and wind parallel sides}}$$

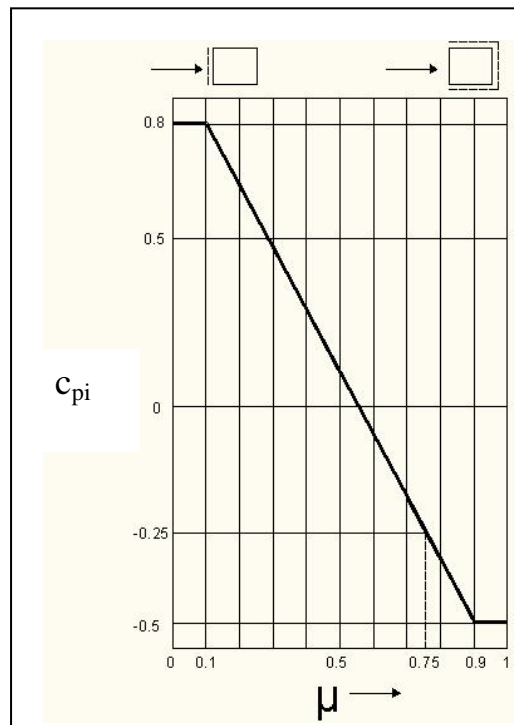


Figure 3.14: International pressure coefficient  $c_{pi}$  for building with openings.

In this study there was no opening in the roof, the structure is just considered to be a roof. For this reason the internal pressure was not calculated.

### 3.3 Criteria Design

ENV (2005) was used as steel frame design code. ENV 1991-2-3 Eurocode1 was used as loading code.

### 3.4 Design Software

Engineers routinely use the Finite Element Method (FEM) to solve everyday problems of stress, deformation, heat transfer, fluid flow, electromagnetic, etc. using Commercial as well as special purpose computer codes.

The ANSYS software is one of the most mature, widely distributed and popular commercial FEM (Finite Element Method) programs available. In continuous use and refinement since 1970, its long history of development has resulted in a code with a vast range of capabilities.

ANSYS version 11 has been used for the design of models as it is a strong computer program for design and analysis of space structure.

### 3.5 Design Material

For steel material property, this study has referred to ENV 1993 which covers the design of structures fabricated from steel material, and it one of the most widely used steel construction manuals worldwide.

The steel properties are given below:

- Modulus of elasticity:  $E = 21 \text{ kg/m}^2$
- Poisson's ratio:  $\nu = 0.3$
- Coefficient of linear thermal expansion:  $\alpha = 12 \times 10^{-6} \text{ per } ^\circ\text{C}$
- Unit mass:  $\rho = 7850 \text{ kg/m}^3$
- Nominal values of yield strength:  $f_y = 27.5 \text{ kg/m}^2$

- Ultimate tensile strength:  $f_u = 43 \text{ kg/m}^2$

### **3.6 Connection**

The panels connections were assumed to be pinned according to Teng, J.G (2001).

### **3.7 Design Result**

After modeling the frames, they were subjected to loads and designed according to ENV (1991-2-3) and the results can be found in chapter 5 in three different models for duopitch, cylindrical and spherical dome roofs.

### **3.8 Buckling Design**

After the design and analysis of the models the buckling check was done for each model explained in chapter 4 where failure pattern for each form was calculated.



## Chapter 4

### BUCKLING ANALYSIS

#### 4.1 Consideration of Nonlinear Behavior in space structure

A structure can tolerate stresses caused by its self weight but this is not enough to say that the structure will be safe for applied loads. For this reason one generally should consider additional loads to check the safety of the structure. Additional loads generally have more influence on the ductility of material, and therefore should be considered in nonlinear behavior. Consideration of nonlinear structural behavior for space structures is necessary due to the presence of geometric and material nonlinearities, use of thin materials and changes in the boundary conditions and structural integrity.

As it is mentioned above, deformation and ductility are main keys of nonlinear parameters in shell structure. Nonlinear parameters (displacement and factor of safety material) can all be extracted from the bilinear kinematic hardening of the frame with mathematical equations, as shown in Figure 4.1.

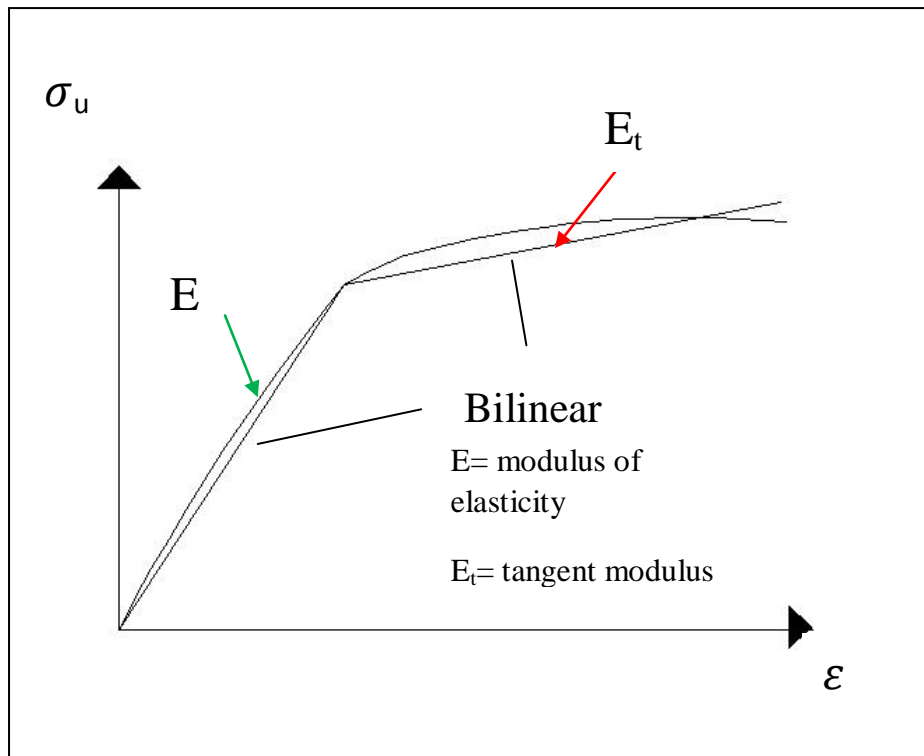


Figure 4.1: Bilinear kinematic hardening for stress and strain of material.

## 4.2 Method of Analysis

Among the four methods of linear static, linear dynamic, nonlinear static and nonlinear dynamic nonlinear static analysis was used for this study. The reasons for this choice can be explained as follows; nonlinear calculation is more precise than linear calculation and it can specify the feature and form of failure pattern of the material (shows changes from the elastic to the plastic condition).

Considering the behavior of shell structure, linear procedures can only be used with acceptable accuracy when the structure behaves completely elastically.

Hence, the linear procedure has limited use, whilst the former requires more computer analysis time, consideration of the location of the site (fault) and the local wind characteristic, the latter requires less computer analysis time and do not need

consideration of the site location. It ought to be noted that the aim of this study is to compare the behaviors between the straight and curved models of the shells, regardless of the connection detail, but with snow and wind loading for a special region northern part of Thran in Iran. The research by Liang, Q.Q. and Uy, B. (1998), Makowski, Z.S. (1984), Ketchum, M. (1990) have shown that nonlinear buckling analysis is highly capable to handle analysis of shells structures when there are lateral loads applied to the structure. On a research similar to this study, Miller, C.D. (1982) used nonlinear buckling tests for ductility assessment in cylindrical form.

As a result, the nonlinear static procedure has been chosen to be used for this research specifically to compare the failure pattern among different models.

### **4.3 Software for Computer Analysis**

There are a number of computer programs that are capable of doing buckling analysis on shell structure. ANSYS v11 (2008) which is used in this study, is one of the program with capability of analyzing nonlinear buckling in the shell structures.

### **4.4 Buckling and the Method of Analysis in ANSYS**

Techniques for pre-buckling and collapse load analysis include:

- Linear Eigenvalue Buckling
- Nonlinear Buckling Analysis

#### **4.3.1 Linear Eigenvalue Buckling**

Eigenvalue buckling analysis predicts the theoretical buckling strength (the bifurcation point) of an ideal linear elastic structure.

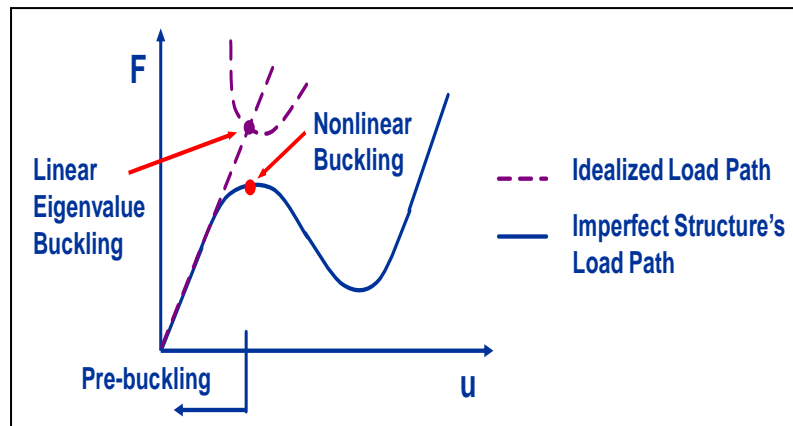


Figure 4.2: Linear buckling for the structure (Willam, K. J., and Warnke, E. D. (1974)).

The eigenvalue formulation determines the bifurcation points of a structure. However, imperfections and nonlinear behavior may prevent most of the real world structures from achieving their theoretical elastic buckling strength. Eigenvalue buckling generally yields unconservative results and should be used with caution.

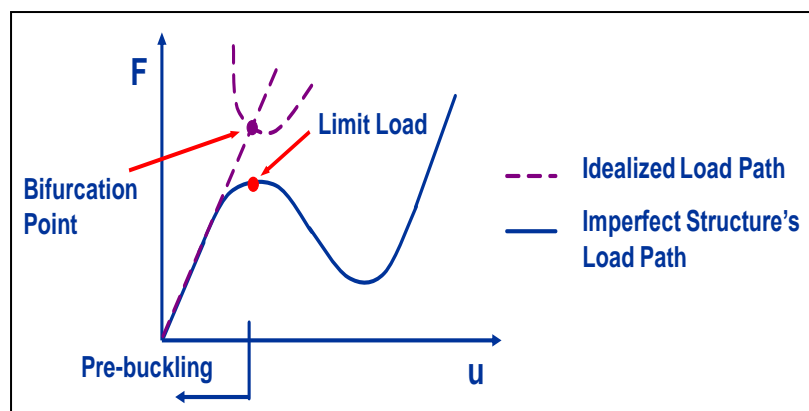


Figure 4.3: Limit load for the structure (Willam, K. J., and Warnke, E. D. (1974)).

Although eigenvalue buckling typically yields unconservative results there are two advantages of performing a linear buckling analysis:

- Relatively inexpensive (fast) analysis.

- The buckled mode shapes can be used as an initial geometric imperfection for a nonlinear buckling analysis in order to provide more realistic results.

A linear buckling analysis is based on a classic eigenvalue problem. To develop the eigenvalue problem, first of all one should solve the load-displacement relationship for a linear elastic pre-buckling load state  $\{P_0\}$ ; i.e. given the  $\{P_0\}$  solve for:

$$\{P_0\} = [K_e]\{u_0\} \quad (4.1)$$

To obtain

$\{u_0\}$  = the displacements resulting from the applied load  $\{P_0\}$

$\{s\}$  = the stresses resulting from  $\{u_0\}$

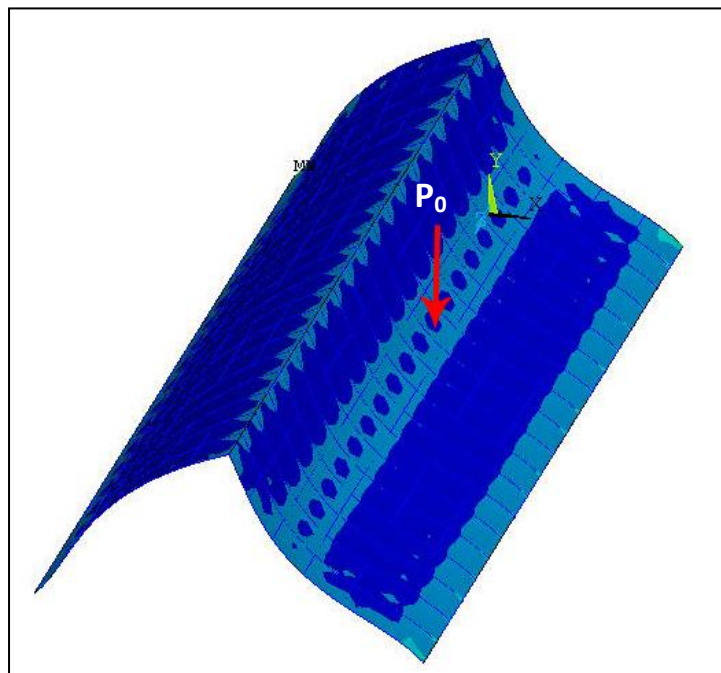


Figure 4.4: An example for Eigen buckling.

Assuming that the pre-buckling displacements are small, the incremental equilibrium equations at an arbitrary state ( $\{P\}$ ,  $\{u\}$ ,  $\{s\}$ ) are given by

$$\{\Delta p\} = [[K_e] + [K_\sigma(\sigma)]]\{\Delta u\} \quad (4.2)$$

where

$[K_e]$  = elastic stiffness matrix

$[K_\sigma(\sigma)]$  = initial stress matrix evaluated at the stress  $\{\sigma\}$

Assuming pre-buckling behavior is a linear function of the applied load  $\{P_0\}$ ,

$$\{p\} = \lambda \{p_0\}$$

$$\{u\} = \lambda \{u_0\}$$

$$\{\sigma\} = \lambda \{\sigma_0\}$$

Then it can be show that

$$\{k_\sigma(\sigma)\} = \lambda \{K_\sigma(\sigma)\} \quad (4.3)$$

Thus, the incremental equilibrium equations expressed for the entire pre-buckling range become

$$\{\Delta p\} = [[K_e] + \lambda [K_\sigma(\sigma_0)]]\{\Delta u\} \quad (4.4)$$

At the onset of instability (the buckling load  $\{P_{cr}\}$ ), the structure can exhibit a change in deformation  $\{\Delta u\}$  in the case of  $\{\Delta p\} \cong 0$ .

By substituting the above expression into the previously given incremental equilibrium equations for the pre-buckling range

$$[[K_e]+\lambda[K_\sigma(\sigma_0)]]\{\Delta u\}=\{0\} \quad (4.5)$$

The above relation represents a classic eigenvalue problem.

In order to satisfy the previous relationship, must have

$$\det[[K_e]+\lambda[K_\sigma(\sigma_0)]]\{\Delta u\}=0 \quad (4.6)$$

An eigenvalue buckling analysis includes the following four main steps:

- Build the Model
- Obtain the Static Solution with Pre-stress
- Obtain the Eigenvalue Buckling Solution
- Review the Results

#### **4.3.2 Non-linear Buckling Analysis**

Shown below is a generalized nonlinear load deflection curve. This Figure 4.5 illustrates the idealized load path, an imperfect structure's load path and the actual dynamic response of the structure.

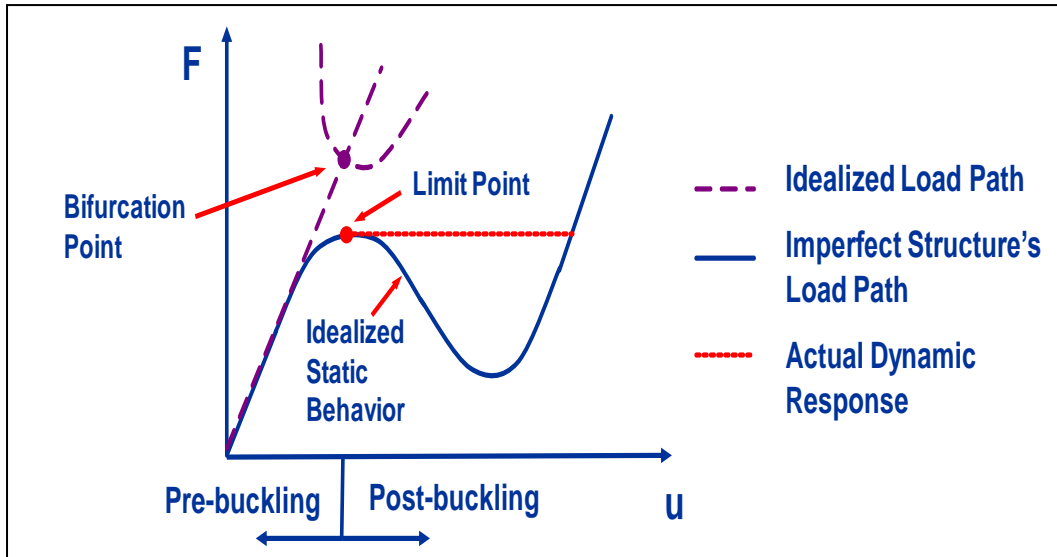


Figure 4.5: Nonlinear load deflection curve (Willam, K. J., and Warnke, E. D. (1974)).

There are various analysis techniques available for calculating the nonlinear static force deflection response of a structure. These techniques include:

1. Load Control
2. Displacement Control
3. Arc-Length Method

#### 4.3.2.1 Load Control

Consider the snap through analysis of the shallow arch shown below Figure 4.6. When the solution to a problem is performed with incrementally applied forces ( $F$ ) the solution is performed using load control.



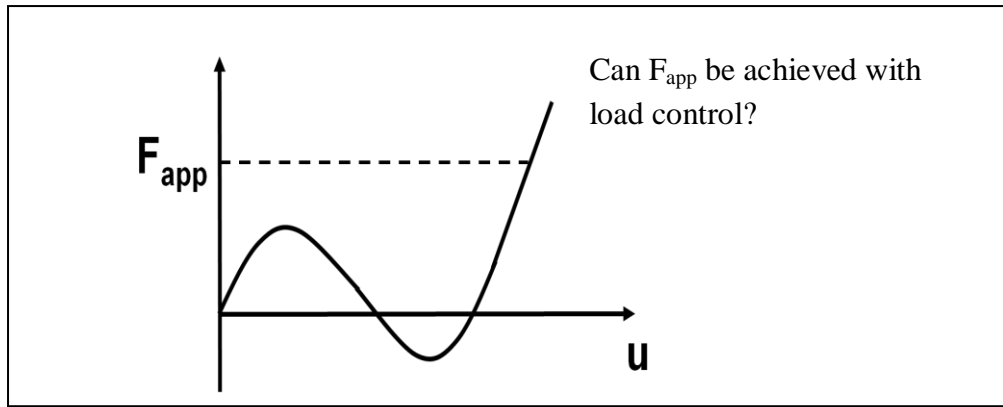


Figure 4.6: Nonlinear load control curve for  $F_{app}$  (Willam, K. J., and Warnke, E. D. (1974)).

The difficulty of using load control with the Newton-Raphson is that the solution cannot progress once passed the point of instability. At the point of instability ( $F_{cr}$ ) the tangent stiffness matrix  $K^T$  is singular. Using load control, the Newton-Raphson method will not converge. However, this type of analysis can be useful to characterize the pre-buckling behavior of a structure.

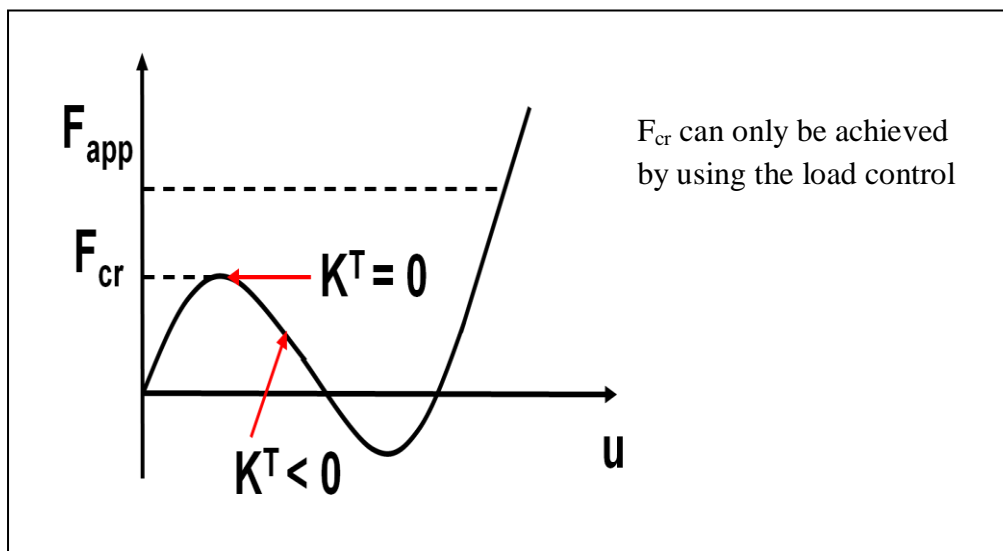


Figure 4.7: Nonlinear load control curve for  $F_{cr}$  (Willam, K. J., and Warnke, E. D. (1974)).

#### 4.3.2.2 Displacement Control

When the arch is loaded with an incrementally applied displacement, as opposed to a force, the solution is performed using displacement control. The advantage of displacement control is that it produces a stable solution beyond  $F_{cr}$ . (The imposed displacement provides an additional constraint at the point of instability.)

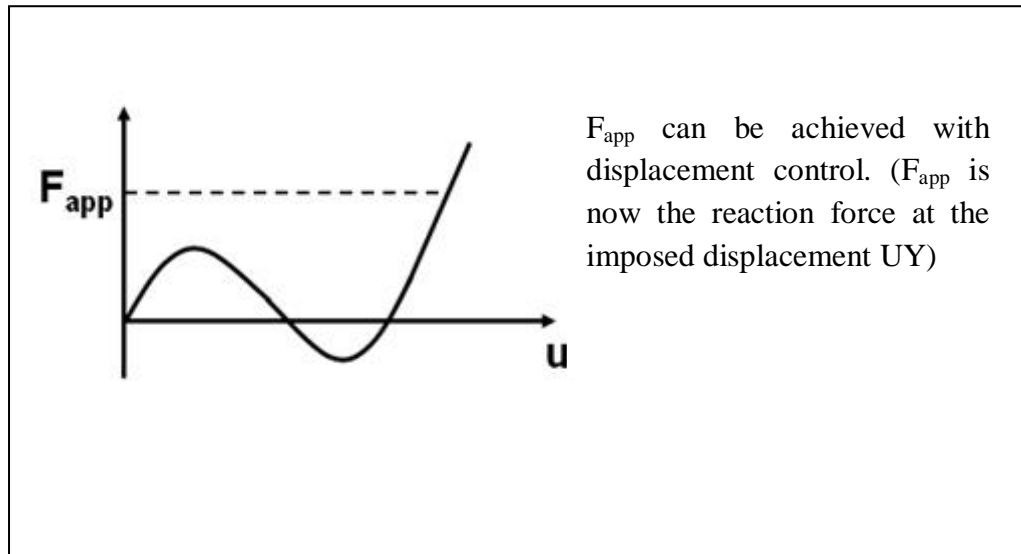


Figure 4.8: Nonlinear displacement control curve for  $F_{app}$  (Willam, K. J., and Warnke, E. D. (1974)).

The disadvantage of displacement control is that it only works when you know what displacements to impose. If the arch is loaded with a pressure load as opposed to a concentrated force the displacement control is not possible.

#### 4.3.2.3 Arc-Length Method

The arc-length is a solution method used to obtain numerically stable solutions for problems with instabilities ( $K^T \rightarrow 0$ ), or negative tangent stiffness ( $K^T < 0$ ).

The arc-length method can be used for *static* problems with proportional loading.

Although the arc-length method can solve problems with a complicated force deflection response, it is best suited to solve problems where the response is smooth without sudden bifurcation points.

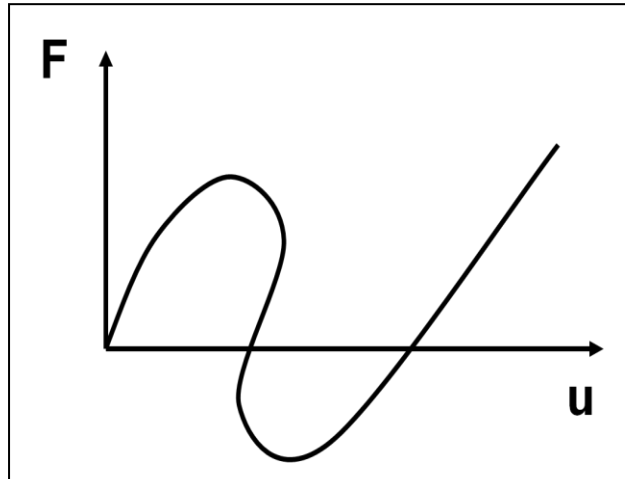


Figure 4.9: Nonlinear Arc-Length curve (Willam, K. J., and Warnke, E. D. (1974)).

The arc-length method solves for load and displacement simultaneously in a manner similar to the Newton-Raphson method. However, an additional unknown is introduced in the solution, the load factor  $\lambda$  ( $-1 < \lambda < 1$ ). The force equilibrium equations can then be rewritten as,

$$[\mathbf{K}^T]\{\Delta\mathbf{u}\} = \lambda \{\mathbf{F}^a\} - \{\mathbf{F}^{nr}\} \quad (4.7)$$

In order to accommodate the additional unknown a constraint equation must be introduced the arc-length  $\ell$ . The arc-length relates to the load factor  $\lambda$  and the displacement increments  $\{\Delta\mathbf{u}\}$  in the arc-length iterations.

Note that the Arc-Length Method reduces to the full Newton-Raphson method if this constraint  $\ell$  is removed.

Another way to view the difference between Arc-Length and (full) Newton-Raphson method is that Newton-Raphson methods use a *fixed* applied load vector  $\{F^a\}$  per substep. Arc-Length methods, on the other hand, use a *variable* load vector  $\lambda\{F^a\}$  per substep.

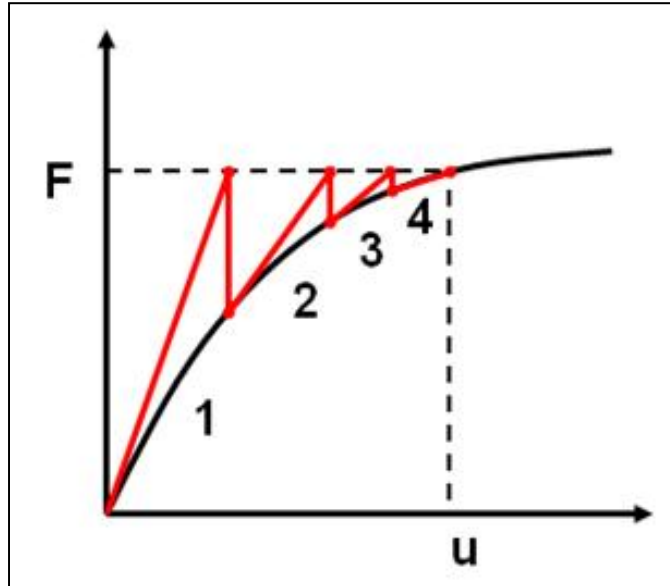


Figure 4.10: Newton-Raphson method (Willam, K. J., and Warnke, E. D. (1974)).

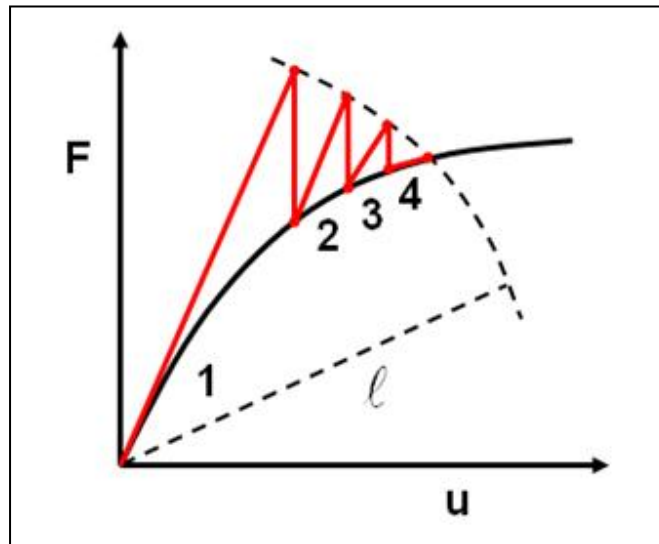


Figure 4.11: Arc-Length method (Willam, K. J., and Warnke, E. D. (1974)).

The arc-length relates to the incremental load factor  $\Delta\lambda$  to the incremental displacement  $\Delta u$  via a spherical arc. Figure 4.10 Figure 4.11 are the incremental load factor  $\Delta\lambda$  and the incremental displacement  $\Delta u$  for the arc-length method with the full Newton-Raphson.

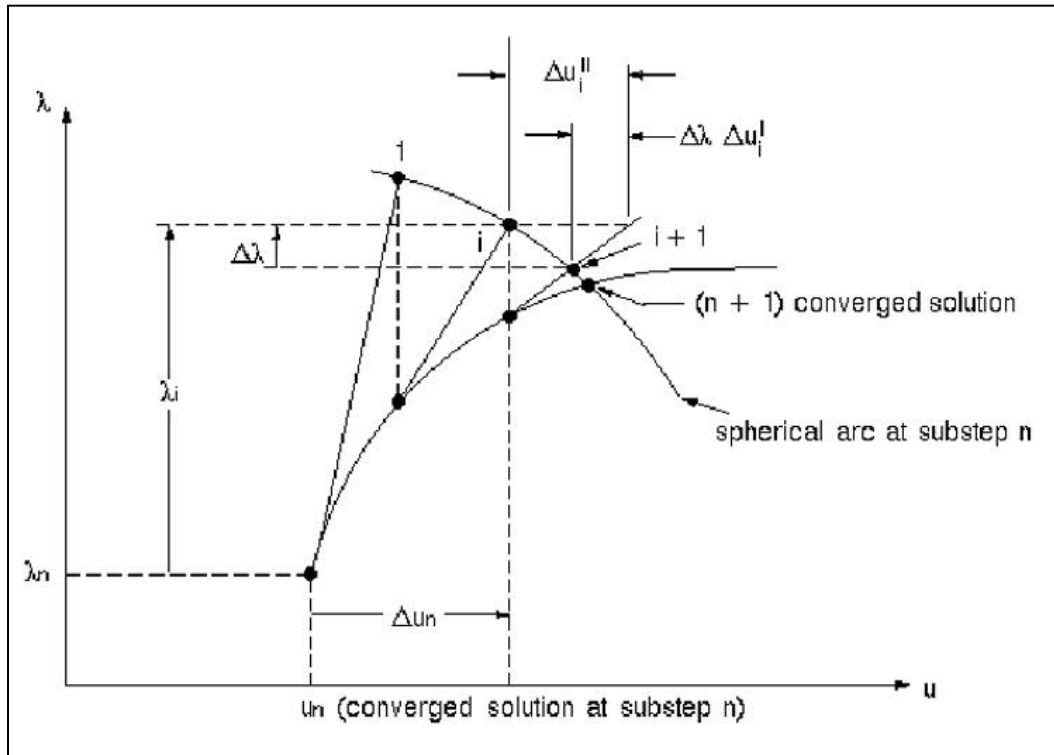


Figure 4.12: Arc-Length radius =  $\sqrt{\Delta u_n^2 + \lambda^2}$  (Willam, K. J., and Warnke, E. D. (1974)).

By forcing the arc-length iterations to converge along a spherical arc which intersects with the equilibrium path, solutions can be obtained for structures undergoing zero or negative stiffness behaviors.

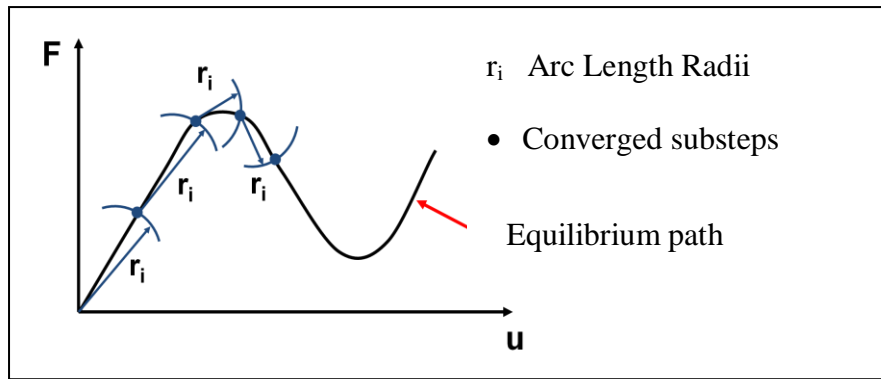


Figure 4.13: Nonlinear curve for structures undergoing zero or negative stiffness behaviors.

#### 4.3.2.4 Summary of the Three Nonlinear Buckling Techniques

Load control, displacement control and arc-length method are summarized below.

These are three techniques used in the solution of nonlinear static buckling problems.

Table 4.1: Compare the three techniques of nonlinear analysis (Willam, K. J., and Warnke, E. D. (1974)).

Loading	Solution Method	Pre-Buckling	Post-Buckling	Restriction
Load Control	Newton-Raphson Method	yes	No	Response must have a one-to-one relationship
Displacement Control	Newton-Raphson Method	yes	yes	Response must have a one-to-one relationship with respect to the displacement. Sometimes, imposed displacement are not
Either	Arc-Length Method	yes	yes	The Arch-Length constraint must be satisfied. May not handle response.

A nonlinear buckling analysis employs a nonlinear static analysis with gradually increasing loads to seek the load level at which a structure becomes unstable.

Using a nonlinear buckling analysis you can include features such as initial imperfections, plastic behavior, large-deformation response and other nonlinear behavior.

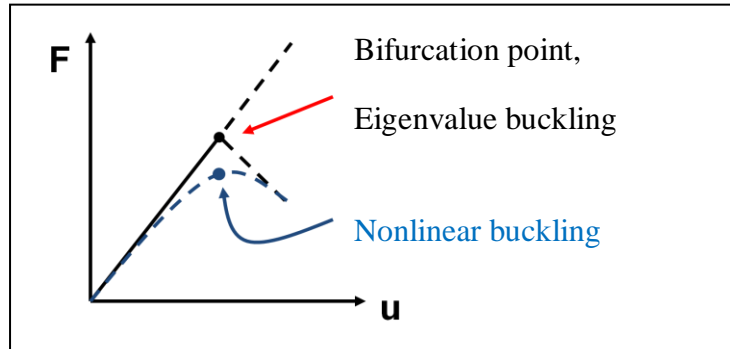


Figure 4.14: Comparison of three kinds of buckling in one curve (Willam, K. J., and Warnke, E. D. (1974)).

In a nonlinear buckling analysis, the goal is to find the first limit point (the largest value of the load before the solution becomes unstable). The arc-length method can be used to follow the post-buckling behavior.

Nonlinear buckling is more accurate than eigenvalue buckling, and is therefore recommended for the design or evaluation of structures.

## Chapter 5

### ANALYSIS OF MODELS

#### 5.1 Introduction

To consider and compare the three models of structure with one another, they all should have the same conditions, i.e. the plates in all models have the same material and stable base form as it is already explained in Chapter 3. The stability needs to comply with Eurocode1 (ENV 1993-1-1).

#### 5.2 Description of Material

Fe 430 is the material obtained from Eurocode 1. The below given Bilinear Kinematic Hardening in Figure 5.1 indicates the extent of the stress and strain of the selected material taking into account of the self coefficient of 1.1.

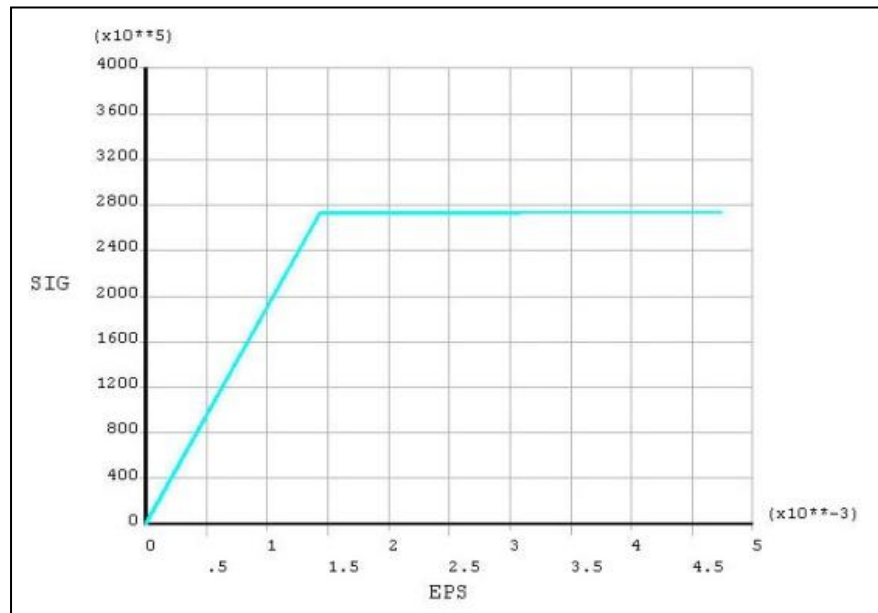


Figure 5.1: Stress and strain for Fe 430.



### 5.3 Description of Membrane

Regarding what has been considered about the properties of the membranes in Chapter 3, the selected ones have the dimensions of 50 cm x 50 cm x 0.3 cm with the edges of I sections and L sections. I & L sections are ideal forms as per the previous experiences. However, in fact the sections of membranes need to be separately analyzed with respect to the types, sizes, stresses, and failure patterns.

The membrane shown in Figure 5.2 has one end fixed and the other end is free. In order to determine the point of failure and the load carrying capacity of the two unit loads 3000N (maximum load could tolerate in elastic time) were applied at the corners of the free end (according to the tests carried out by Teng, J.G. and Rotter, J.M. (2004) test).

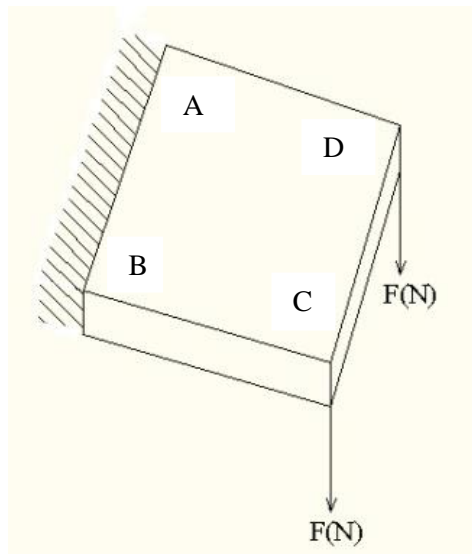


Figure 5.2: Schematic of view of the test carried out by Teng, J.G. and Rotter, J.M. (2004) test.

Test similar to the one carried out by teng, J.G and Rotter, J.M (2004) were performed for the elements considered in this study. For this experiment, first of all a

linear analysis (Eigen Value) was carried out in order to obtain the deformation of the first mode. To have assurances about the stiffness of the structure, the membrane was subjected to deformations applied with the coefficient of 0.01. Then the applied loads are multiplied by an appropriate coefficient (obtained from linear solution) and the non linear solution was performed. The critical load and tensions applied were determined in three forms of membrane with regards to their geometric properties, which have been stated in Chapter3. The tested elements had different stresses.

As can be seen the Figures 5.3, 5.4 and 5.5, the buckling happened mostly at the edges of the membranes.

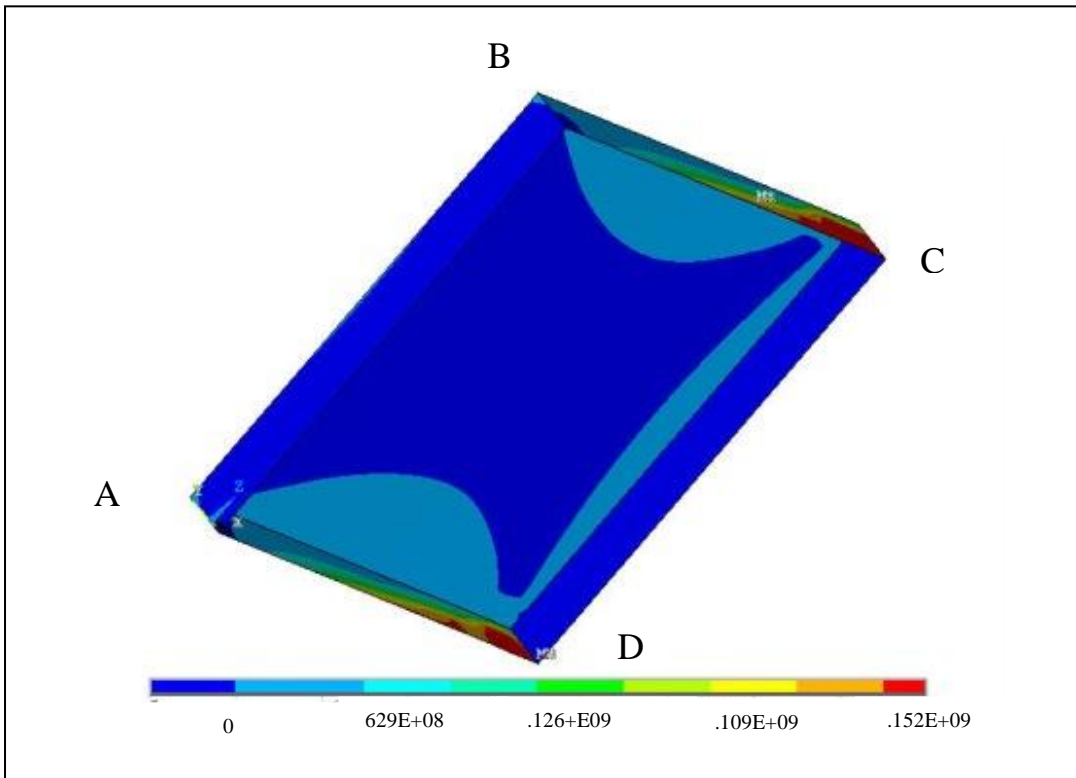


Figure 5.3: Buckling behavior of element with 5cm edges (N/mm<sup>2</sup>)

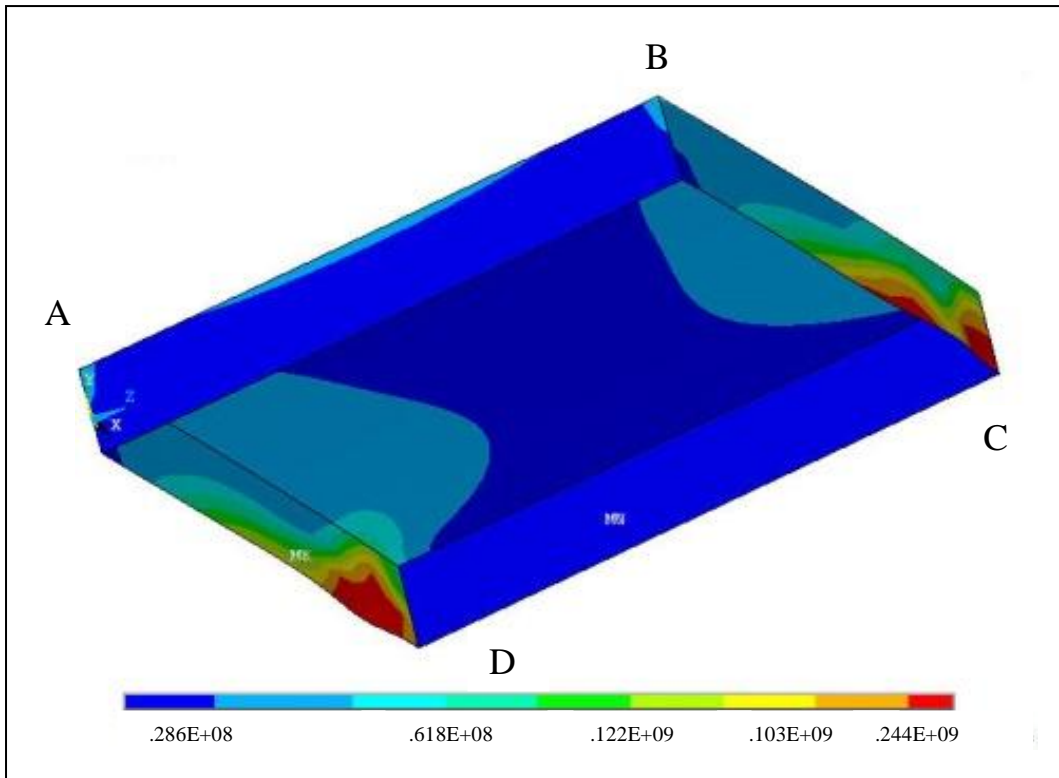


Figure 5.4: Buckling behavior of element with 7cm edges (N/mm<sup>2</sup>)

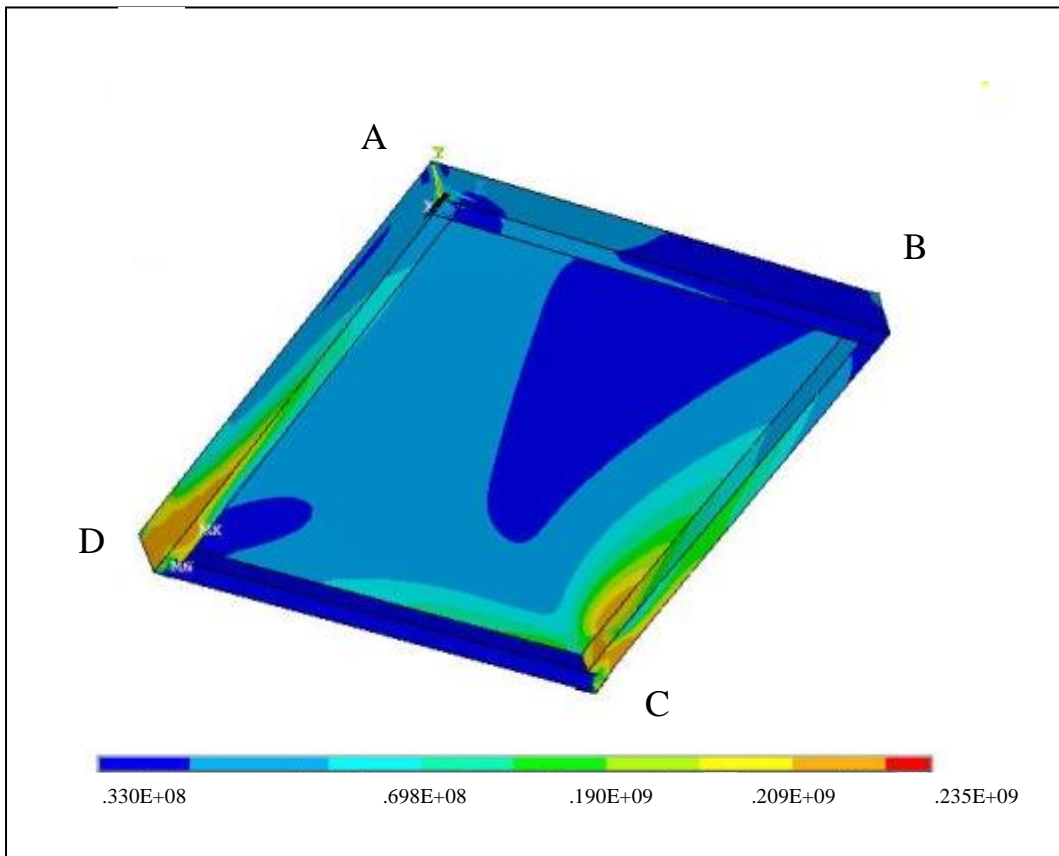


Figure 5.5: Buckling behavior of element with L section edges (N/mm<sup>2</sup>)

The displacement of the three elements is compared in Figure 5.7.

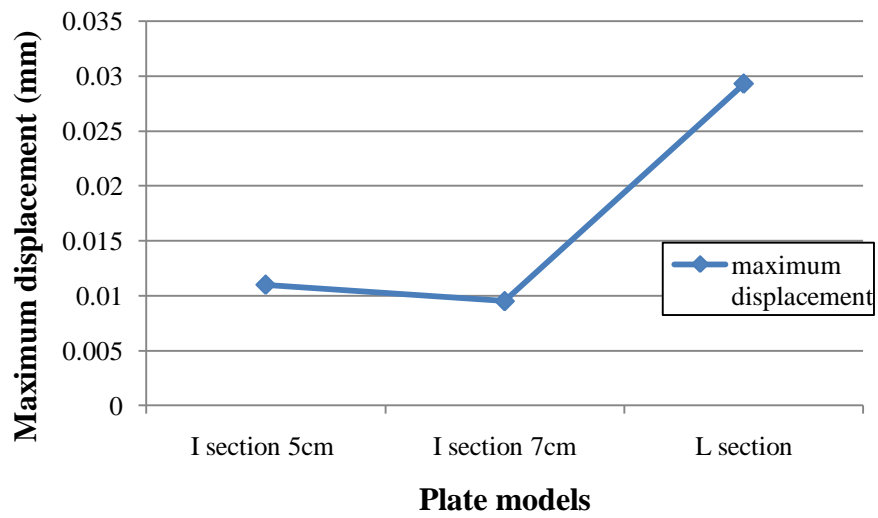


Figure 5.6: Maximum displacement in three model elements.

The maximum force for the three elements is showed in Figure 5.7.

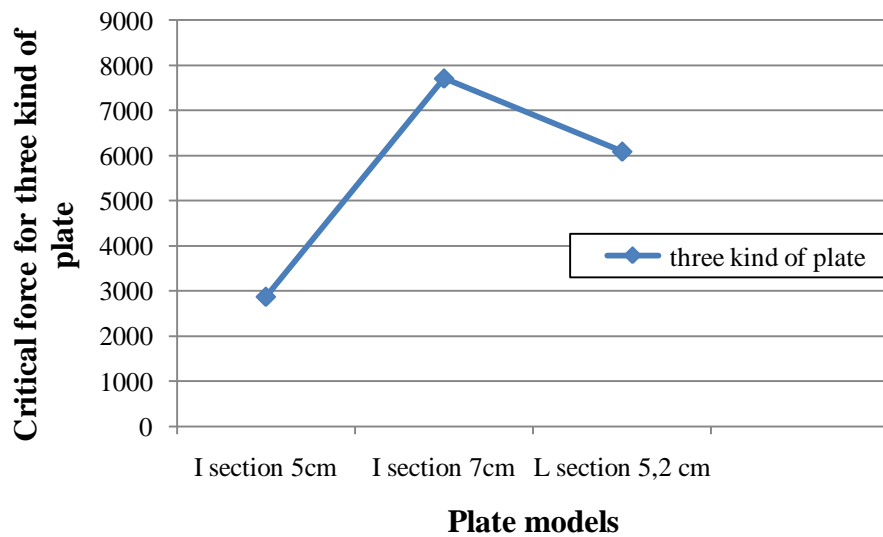


Figure 5.7: Eigen buckling critical force ( $F_{cr}$ ) for three kinds of elements.

Comparing the results of the experiments among the elements, the following can be noted the resistance of L are similar to those of I section with 5 cm and 7 cm. The

starting point is the design of three models with membranes having 5 cm and 7 cm I sections.

## 5.4 Result of Stress Analysis

### 5.4.1 Maximum Span

Each of the three models of the shell can be designed by using different maximum spans, which depends on the form of the shell Figure 5.8 shows the maximum span for each model.

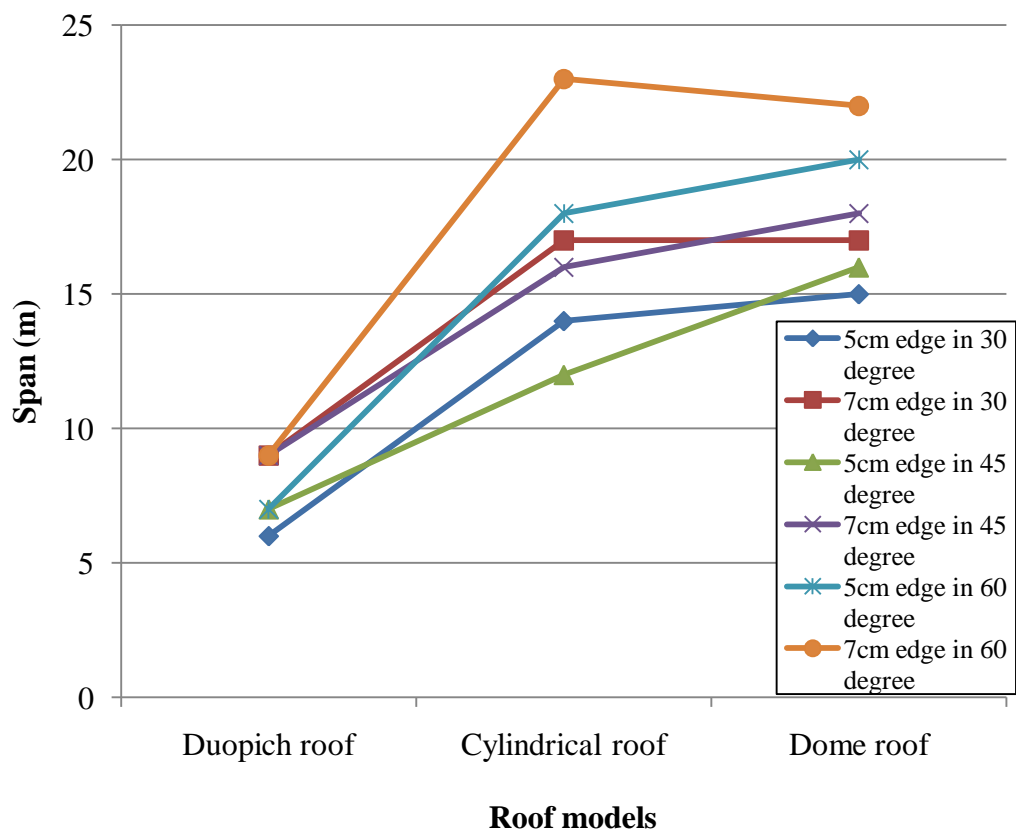


Figure 5.8: Maximum span for each models with 5cm and 7cm edge.

### 5.4.2 Comparison of the Displacements in three Models

The displacement of each model is determined after applying loads as per Eurocode (wind load, snow load, and dead load). In Figure 5.9, the maximum displacement in three models of structure is shown for three roof angles of 30°, 45° and 60° .

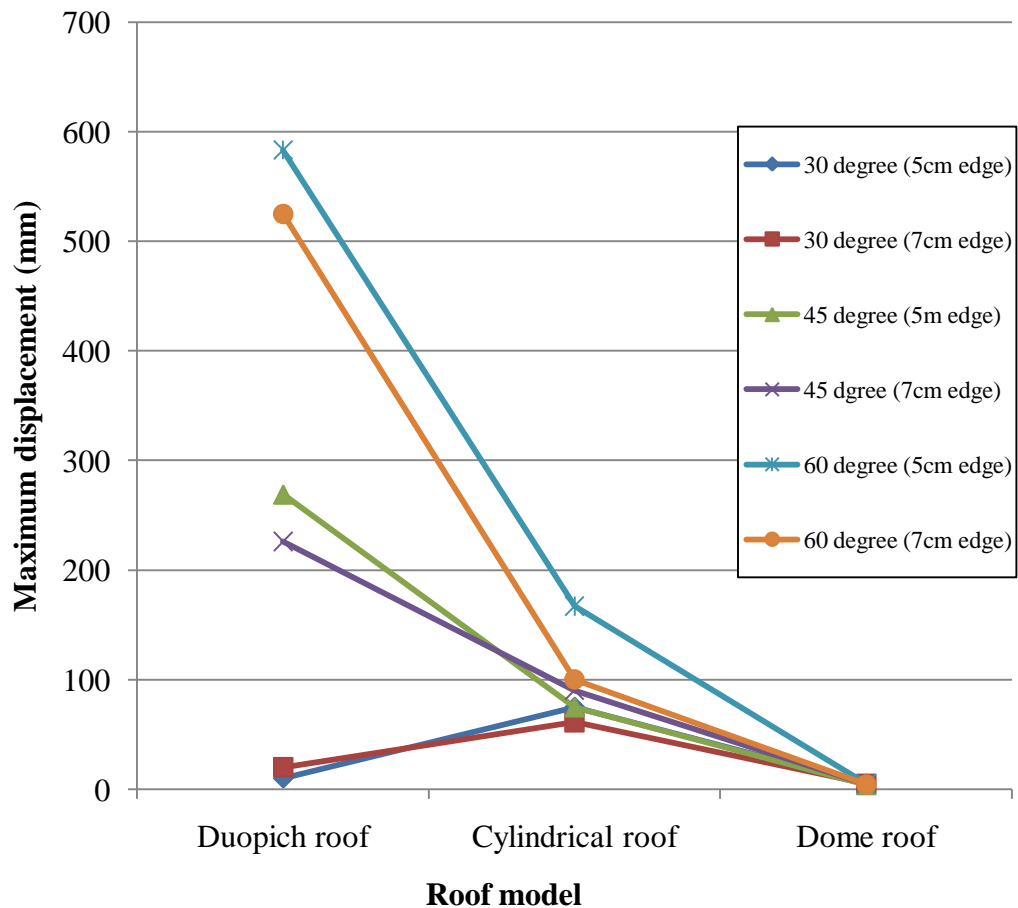


Figure 5.9: Maximum displacement for three types of shell roofs having different angles and different edge models.

### 5.4.3 Comparison of Stress in Three Models of Structure

To study the stress in structures in this thesis, Eurocode 1 has been used in the conditions before the structure has reached the plastic form in order to have a precise analysis of structure in elastic conditions. The value of the stress in elastic conditions

approximately specifies both the limit of resistance and the scope of design in each model of structure.

In all shell structures, angle and span are the most important factors influencing the value of maximum stress. Therefore, the value of maximum stress is calculated in three angles of 30°, 45° and 60° for three models of structure. The below Figure 5.11, 5.12, 5.13, show and compare the different values of maximum stress in three models of structure.

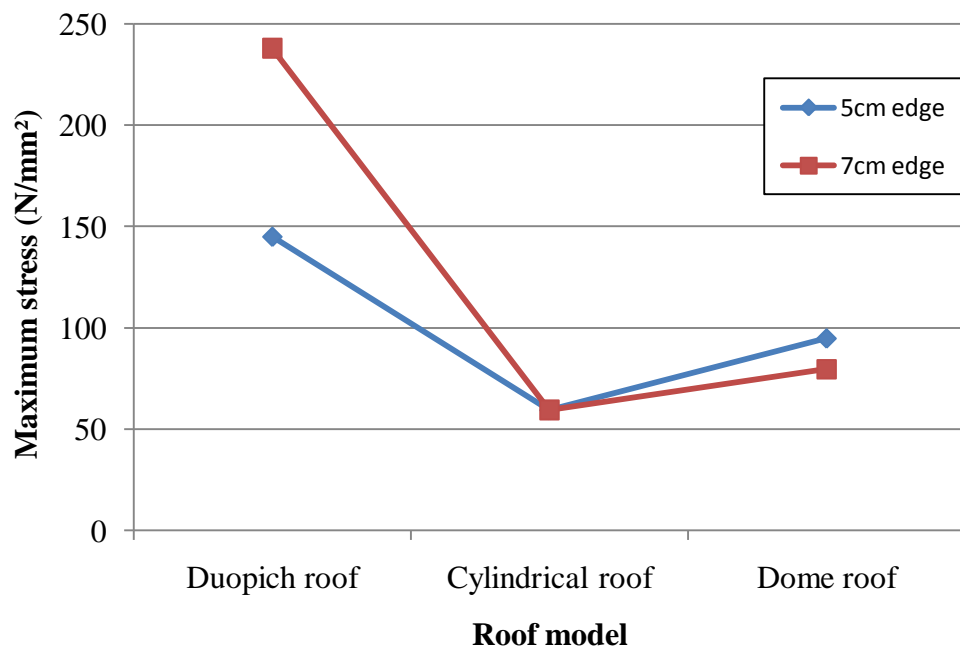


Figure 5.10: Maximum stress for different shell roof models with 30° angles.

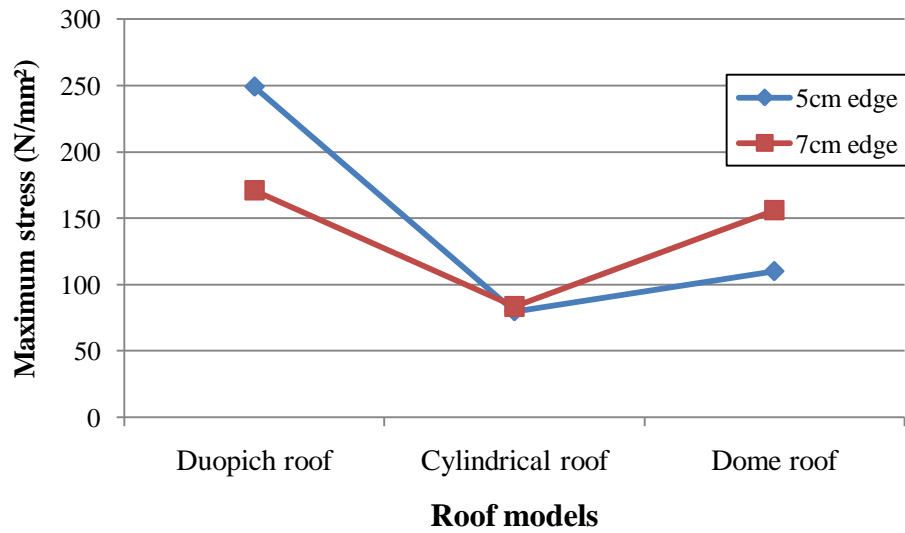


Figure 5.12: Maximum stresses for different shell roof models with 45° angles.

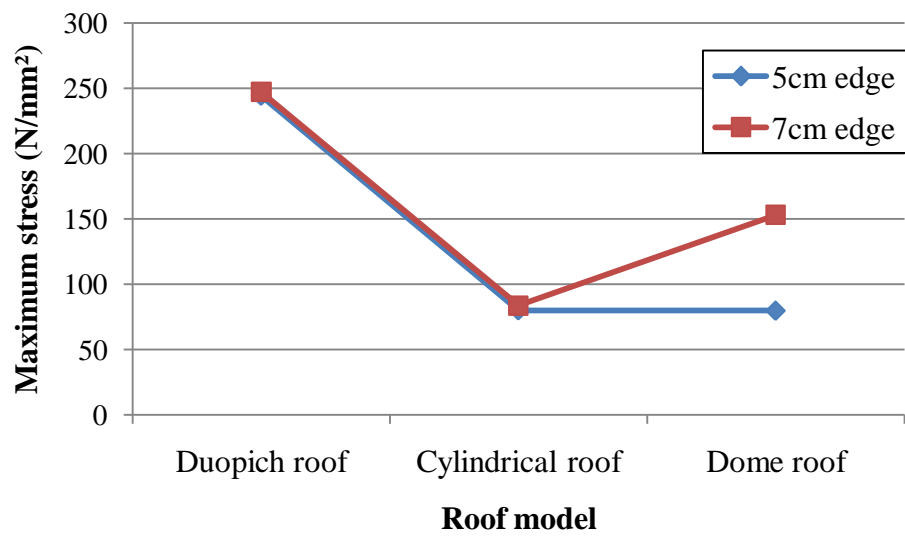


Figure 5.13: Maximum stresses for different shell roof models with 60° angles.



## 5.5 Results of Buckling Analysis

### 5.5.1 Linear Method of Determining the Value of Critical Load

Each structure could tolerate the stresses mention to the previous section. However, structure may not be sufficiently stable when subject to lateral loads. As a result, the structures need to be strong and stable to tolerate the probable lateral loads.

For this study, in order to obtain the critical load, 1000 N point load is applied to the structure according to Chapter 4 so that the tolerance of the structure can be checked in linear solution (Eigen Value). As a result of this solution, the span of the three models was reduced when compared to those values in maximum stress and the scope of the design became more limited. After specifying the structure at stiffness with linear solution, the critical force can be determined. Figure 5.14, 5.15 and 5.16, indicate the critical force for each of the three models with three angles different roof of 30°, 45°, and 60° .

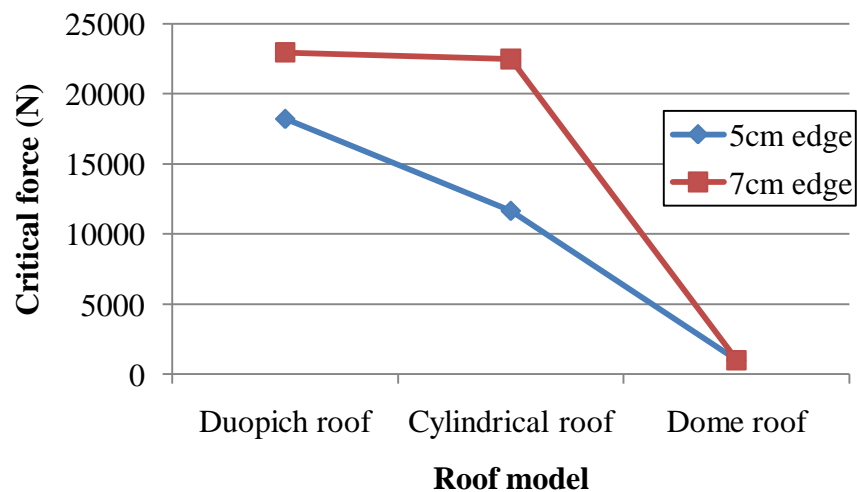


Figure 5.14: Eigen buckling critical force ( $F_{cr}$ ) in 30° angles.

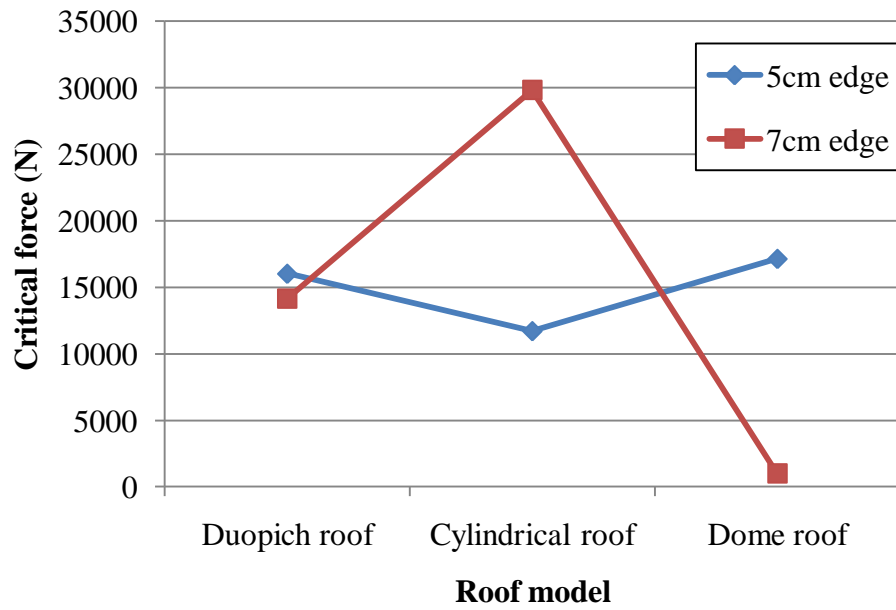


Figure 5.15: Eigen buckling critical point ( $F_{cr}$ ) for 45° roof angles.

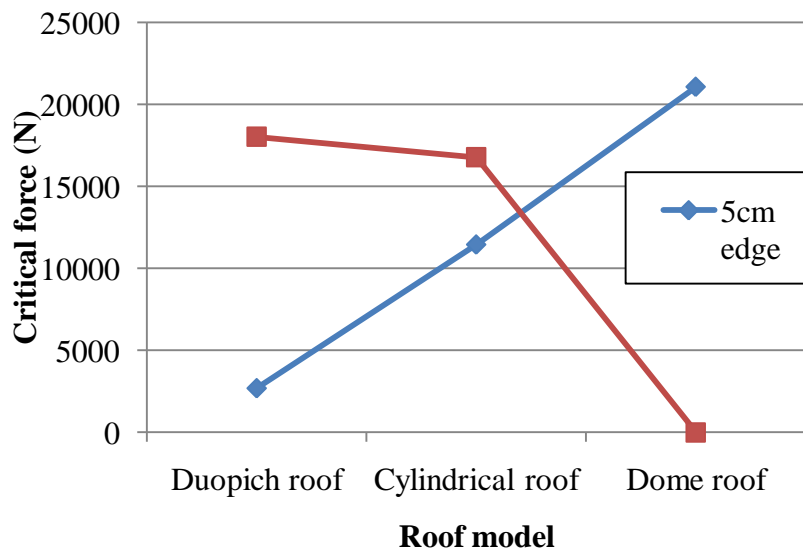


Figure 5.16: Eigen buckling critical point ( $F_{cr}$ ) for 60° roof angles.

### 5.5.2 Determination of the Load Multiplier in Structures

After doing the linear solution, it can be concluded that each structure has the load tolerance with specific resistance coefficient. These coefficients are shown in Table 5.1 for the three models in three angles of 30°, 45° and 60°.

Table 5.1: Load multiplier for different angles in Eigen buckling.

Linear load multiplier (Eigen buckling)									
Model	Duopitch roof			Cylindrical roof			Dome roof		
	30°	45°	60°	30°	45°	60°	30°	45°	60°
5cm edge	4.5953	3.4473	3.366	1.36	1.75	1.54	1.05	1.439	1.53
7cm edge	3.266	3.2883	4.23	1.956	1.72	1.595	1.05	1.02	*

### 5.5.3 Global Non-linear Buckling in Structures

As it was mentioned in Chapter 4, the load multiplier is greater than the actual condition and the obtained coefficients display the extent to which the structure can tolerate. To determine the real value of the structural stiffness, the non-linear buckling needs to be considered in order to find out the point in which the structure faces failure.

In non-linear solution for global buckling, the value of imposed load is increased from 1 to 5 times (according to the coefficients already obtained in Table 5.1). The maximum loads that the models could Table 5.2 show the real multipliers for the three models using three different angles of 30°, 45° and 60°. One of the results of obtained from these analysis is that most of the structures can tolerate up to 5 times of the increased load. (The conditions in which the structures were studied were the limit structural tolerance).

Table 5.2: Global nonlinear buckling multiplier for all models with particular edges and angles.

Nonlinear load multiplier (global buckling)									
Model	Duopitch roof			Cylindrical roof			Dome roof		
	30°	45°	60°	30°	45°	60°	30°	45°	60°
5cm edge	-	-	-	1.04	0.97	0.915	-	0.88	1
7cm edge	0.808	1.152	0.90206	1.365	0.99558	0.5268	-	0.98	*

(-) this shape shows that structure can tolerate 5 times load.

(\*) this shape shows that the structure cannot tolerate strain the load and it is not economical.

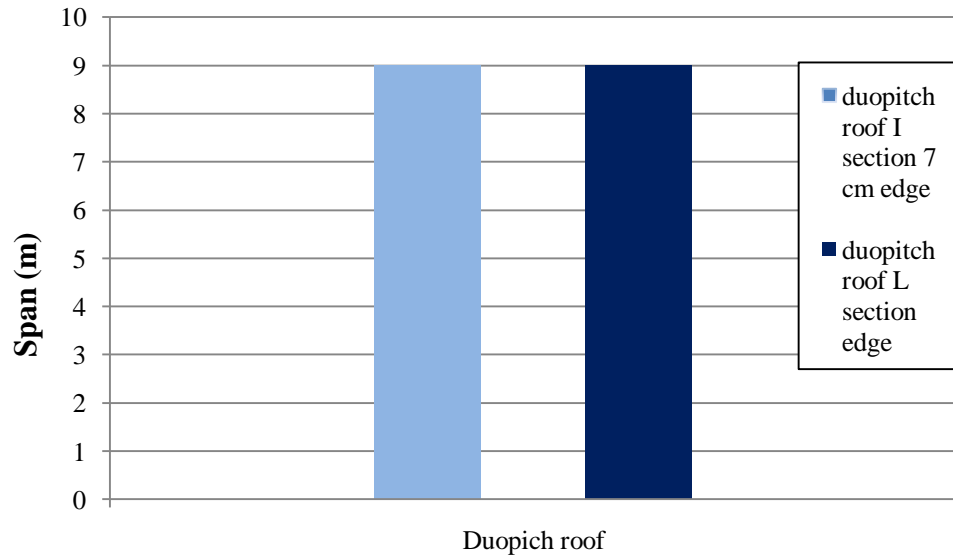
The dome roof with asterix (\*) is using heavy and yield at 23 m span. It is more efficient and economical to use 5cm edge 30°, 45° or 60° or 7 cm edge 30°, 45° dome roof.

#### 5.5.4 Comparing the Result of Using L and I Section Edges

The three most critical models of Duopitch 30°, Cylindrical 60° and Dome Shape 45° made of elements with I sections edges, and subjected to non-linear buckling analysis to find failure point in the structures. The elements of these models were changed to L shape section in accordance with the specifications mentioned in chapter 3, section 3.1 and then the results of analysis were compared with those results of I shape sections.

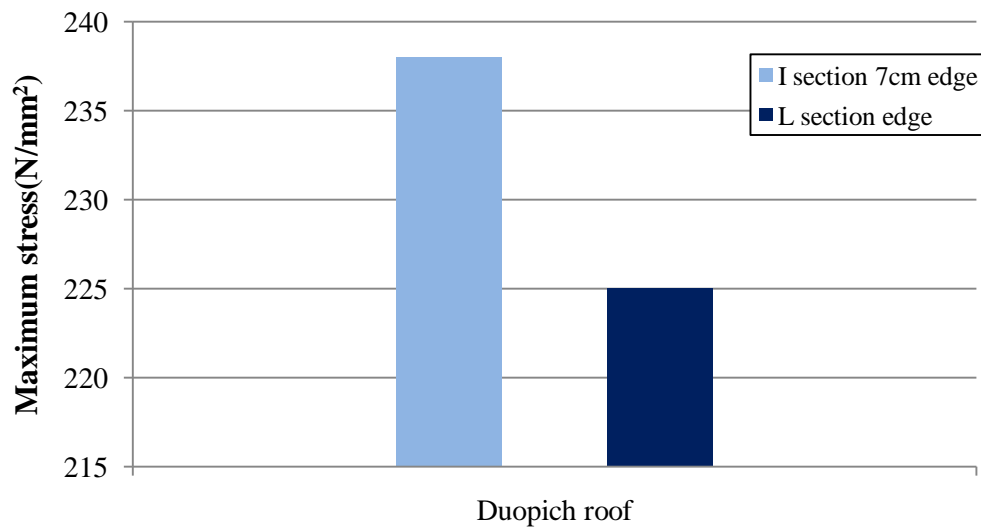
#### 5.5.5 Comparing the Results of I Section and L Section in Duopitch Roof

Considering the load multiplier of the critical model of duopitch roof in Table 5.2 the 30° angles with 7cm edge had lower stiffness than the other angles and edge sizes. Figures 5.17, 5.18, 5.19, 5.20 show the result of the comparisons among the I section plate and L section plate in duopitch roofs.



**Duopich roof with I section and L section plate**

Figure 5.17: Global nonlinear buckling multiplier for all models with particular edges and angles.



**Duopich roof with I section and L section**

Figure 5.18: Comparison of the maximum stresses between I section and L section plate with 30° angle.

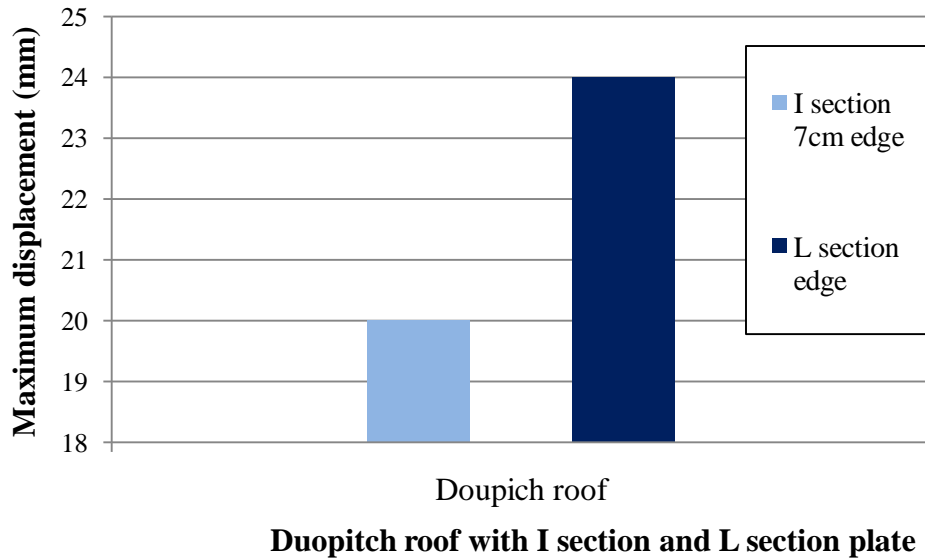


Figure 5.19: Comparison of the displacement between I section and L section edge in duopitch roof with 30° angle.

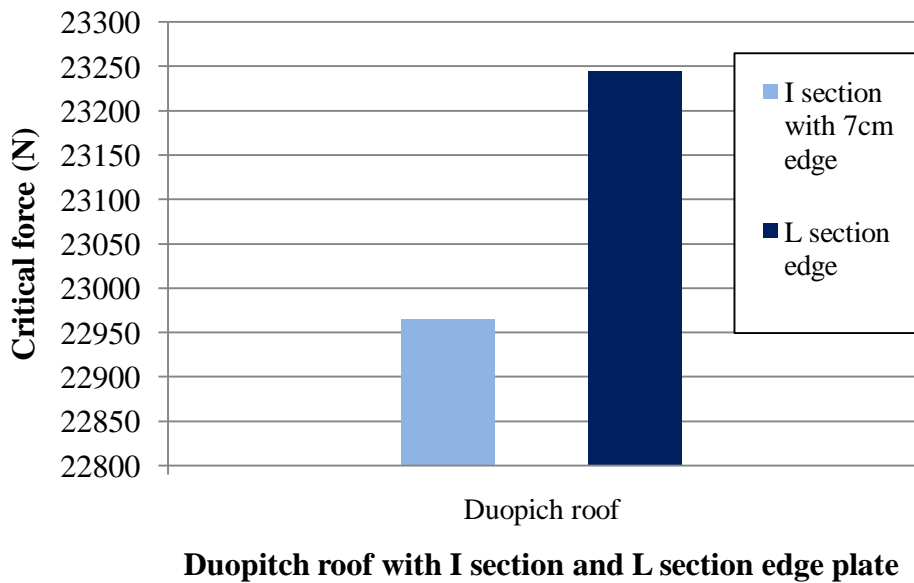
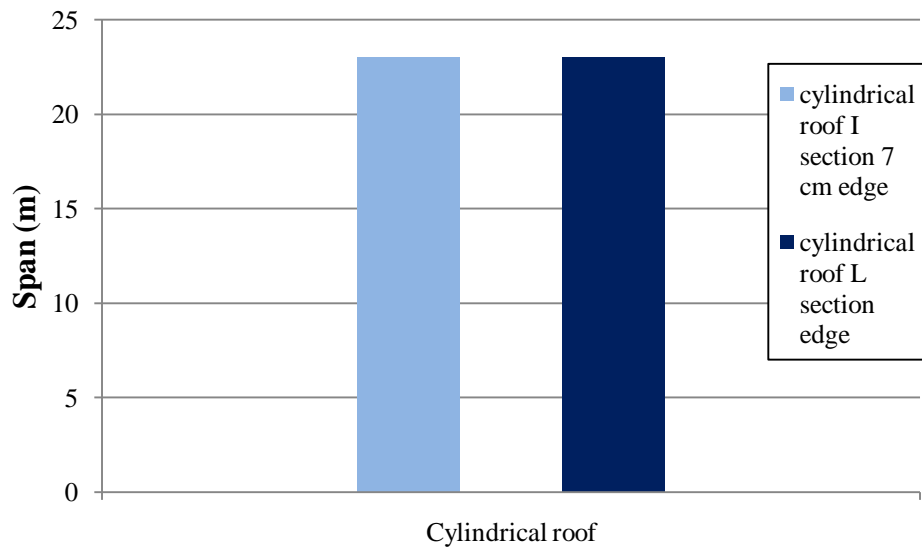


Figure 5.20: Comparison of the critical force ( $F_{cr}$ ) between I section and L section edge in duopitch roof with 30° angle.

### 5.5.6 Comparing the Result of I Section and L Section in Cylindrical Roof

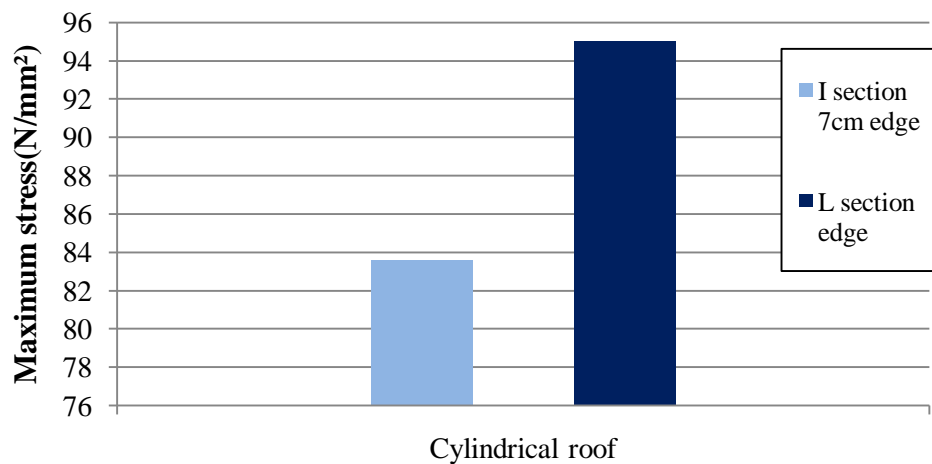
Considering the load multiplier of the critical model of cylindrical roof in Table 5.2 with 60° angles with 7cm edge has lower stiffness than the other angles and edge

size. Figures 5.21, 5.22, 5.23, 5.24 show the result of the comparisons among the I section plate and L section plate in cylindrical roofs.



**Cylindrical roof with I section and L section plates**

Figure 5.21: Compare the spans between I section and L section plate with 60° angle.



**Duopitch roof with I section and L section plates**

Figure 5.22: Compare the maximum stresses between I section and L section plate with 60° angle.

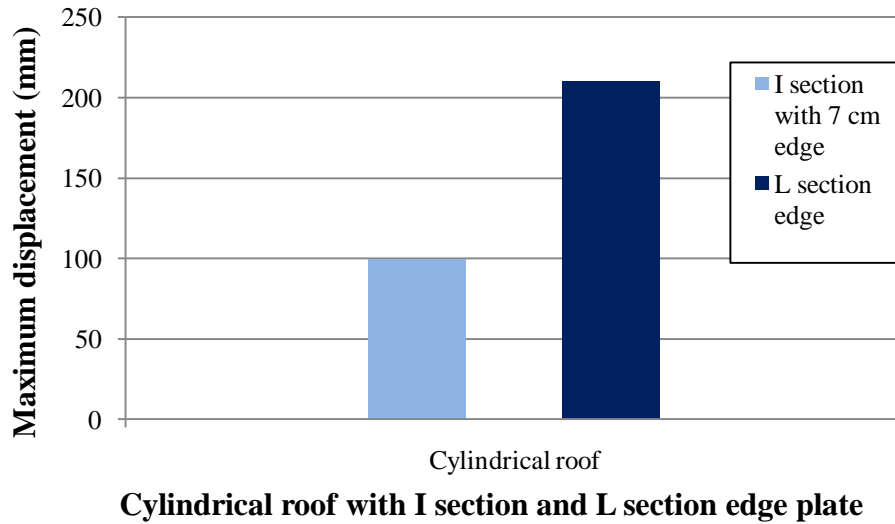


Figure 5.23: Comparison of the displacement between I section and L section edge in cylindrical roof with 60° angles.

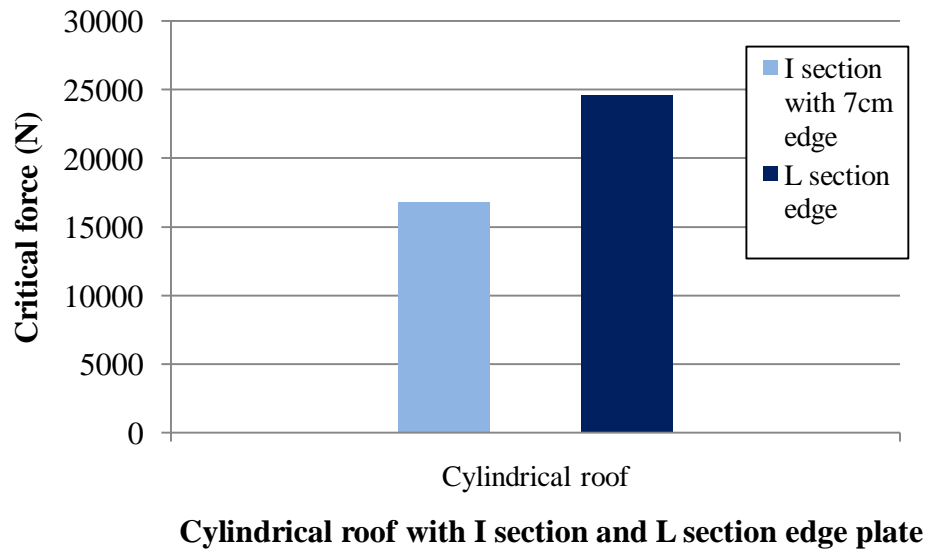


Figure 5.24: Comparison of the critical force ( $F_{cr}$ ) between I section and L section edge in cylindrical roof with 60° angles.



### 5.5.7 Comparison Static Values of I Section and L Section in Dome Roofs

Considering the load multiplier of the critical model of dome roof in Table 5.2 the 45° angle with 7cm edge has lower stiffness than the other angles and edge sizes. Figures 5.25, 5.26, 5.27, 5.28 show the result of the comparisons among the I section plate and L section plate in dome roofs.

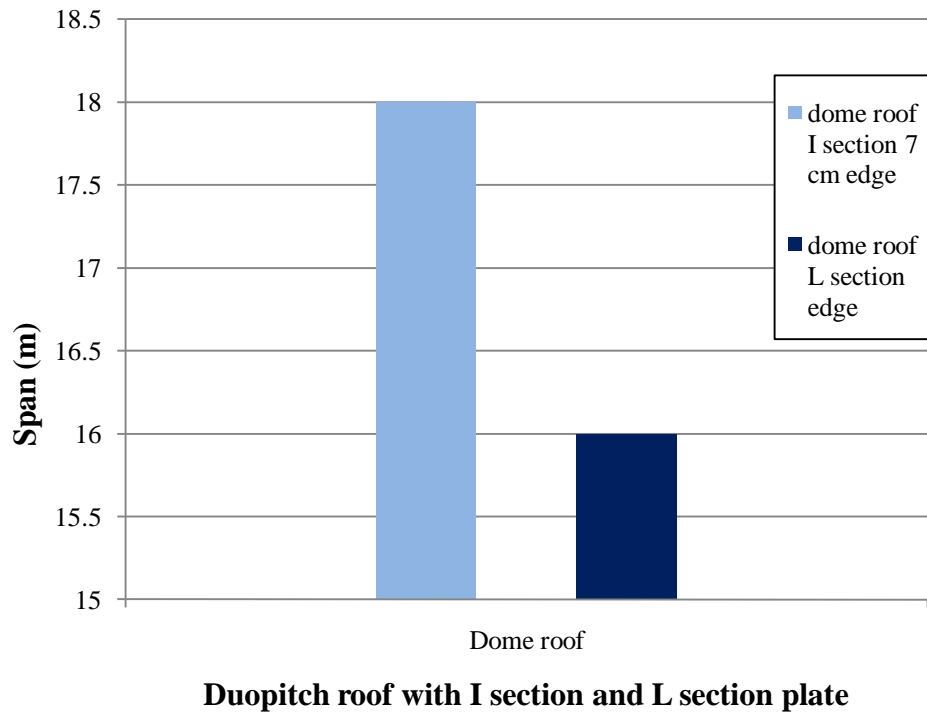


Figure 5.25: Comparison of the spans between I section and L section plate with 45° angles.

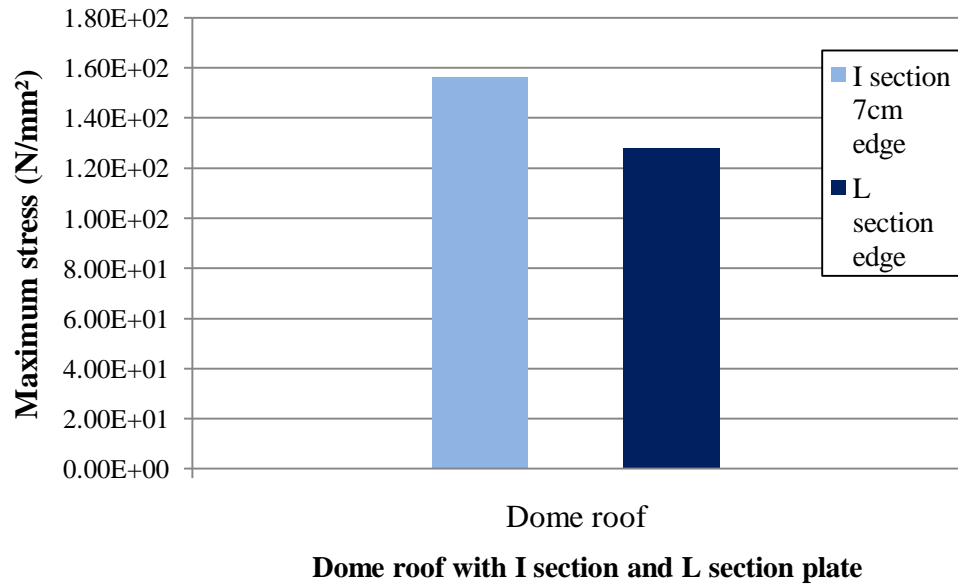


Figure 2.26: Comparison of the maximum stresses between I section and L section plate with 45° angles.

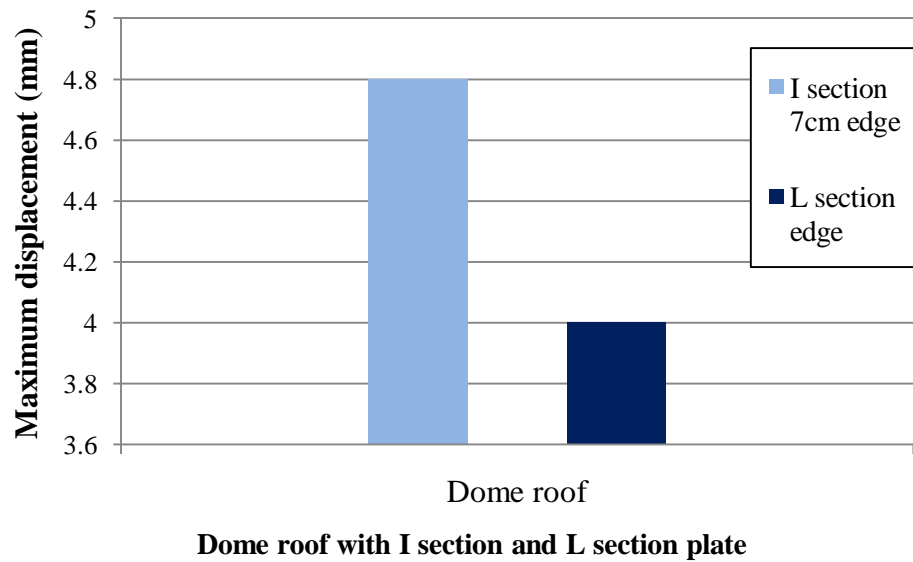


Figure 5.27: Comparison of the displacements between I section and L section edge in dome roof with 45° angles.

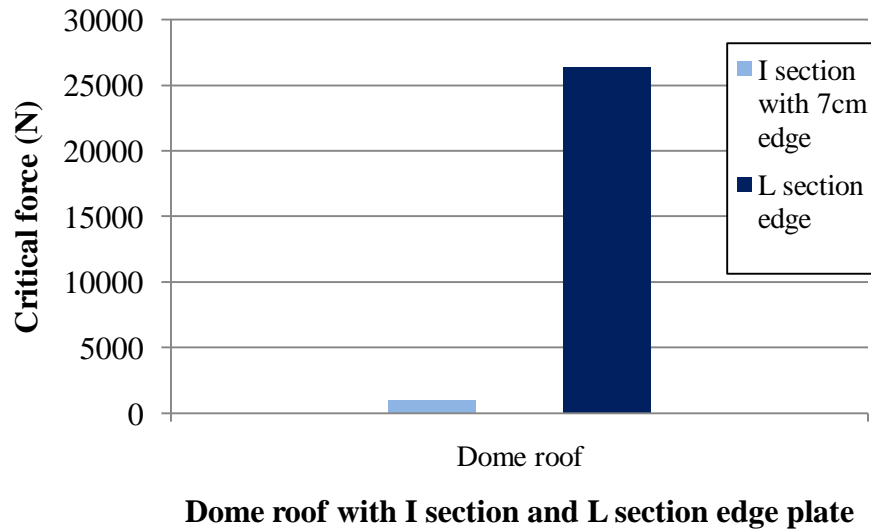


Figure 5.28: Comparison of the critical force ( $F_{cr}$ ) between I section and L section edge in cylindrical roof with  $45^\circ$  angles.

### 5.5.8 Determination of the Load Multiplier in I Section and L Section Plate

In this study linear buckling (Eigen buckling) is used for checking the stiffness of the structures. In this way the stability of the structure and how much load it could tolerate can be when subject to multiple loads (point load).

Table 5.3: Load multiplier for models with I section and L section edge plates in Eigen buckling.

Model	Linear load multiplier (Eigen buckling) for shell roofs		
	Duopitch $30^\circ$	Cylindrical $60^\circ$	Dome $45^\circ$
7cm edge	3.266	1.02	1.595
L section	3.32	1.5	1.405

### 5.5.9 Global Non-linear Buckling in I Section and L Section Plate

After checking the stiffness of the structure in linear buckling, the results are compared in global buckling. In global buckling 5 times the normal load was applied to check the stiffness of the structure in real time, Table 5.4 shows the results.

Table 5.4: Global nonlinear buckling multiplier for I section and L section edge plate models and angles.

Model	Non-linear load multiplier (global buckling) for shell roofs		
	Duopitch 30°	Cylindrical 60°	Dome 45°
7cm edge	0.808	0.98	0.5268
L section	0.31	0.26	1.4

## CHAPTER 6

# RESULTS, DISCUSSIONS AND CONCLUSIONS AND RECOMMENDATIONS

### 6.1 Result and Discussions

Three kinds of plates with standard measurements each of which have special specifications are used for modeling shell roofs in this thesis. It can be found in the comparison of the plates that the plates with the edges of 5 cm is less resistant than the plates with 7 cm, but lighter and smaller structures can be designed using the plates with 5cm edge. Moreover, being light, the structures can better resist the buckling.

The plate with 7 cm edge tolerates heavier loads than the ones with 5 cm edge, but the amount of buckling is more on the edges for this plate. In order to minimize the buckling on the edges, the plates with 7 cm edges were replaced with the plates with L section edges. This change in plate's geometrical characteristics and therefore the changes in the characteristics of the shell form cause the structure to better resist the forces but does not guarantee the structure to be able to have longer span. When the plate with 7cm edges is compared to the one with L section edges, it can be found that the amount of displacement on the surface of the plates with L section edges is several times as much as that of the plates with I section edges. As a result, all these aspects should be taken into consideration while designing structures.

When comparing the models, it is found that the 60° angle can help increasing the span of the structures since there is no snow load on them. However, it is not sufficient to have a strong structure because structures with shell roofs having smaller angles are heavier and need to tolerate more of their self weight. Therefore, the optimal angle is different for different forms of shell roof structures.

Comparing the stresses in the shell roof structures with two forms of plates with I section edges of 5 cm and 7 cm, it is found that the dome roofs have less stresses because of having curving in two directions and when the angle is 30°, minimum stresses can be obtained from inside the structure.

Every space structure has to tolerate loads of up to three times the expected loads which may be suddenly imposed on the structure. That is why the amount of appropriate stress in structure is not sufficient to say that the design is successful. After checking the structure with plates having I section edges and L section edges, it becomes clear that the structural stability has been changed, e.g. the dome roof can be designed with a span of up to 25m while the appropriate stress is imposed. When the additional loads occur in the dome structure, it causes the span to be reduced to 20m in order to attain a dome structure with higher load resistance.

## **6.2 Conclusion**

Shell roof structures have the ability to cover large openings without using column and beam. The three types of shell roofs, cylindrical, dome and duopitch can be used to compare the behavior of straight model and curve model of shells structure. Each of these shapes made with three types of metal plates. For these structures checking self stress is not enough. In addition they should be checked for buckling.

In this research, it can be concluded that to evaluate ideal conditions, the optimal angle should be found by considering different angles and three kinds of plates for each type of shell roof structure.

For curved shell roof structures, changing the forms of plates caused the structure to be more resistant against the sudden point load. However, the structure was not able to resist when subject to additional loads.

An important point which can be noted as a result of this research is that the curving of the roof very much helps to increase the resistance of the structure against loads. That is why the weak point of duopitch roof compared to the other roofs is the fact that it does not have curving in its form which causes displacement when a normal load is imposed on its surface. As a result, the duopitch roof has smaller span compared to the roofs of the other models, but curving in for the cylindrical roof causes the structure to be more resistant against the surface displacement. Generally speaking, it can be said that as the angle increases then the displacement occurs in the surface of the structures.

### **6.3 Recommendations**

In this study by reviewing the straight and curved shell roof structures with various forms, shell roof structures with higher resistance and better efficiency can be designed in future. These types of structures and their connections can be developed in future to be more stable with longer spans and more economical. Structure can be designed as such that it can be constructed in a location but at a later date it can be dismantled and used in another location.

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