Dark Matter; Modification of $f(R)$ or Wimps Miracle

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ABSTRACT

The identity of dark matter is one of the key outstanding problems in both particle and astrophysics. In this thesis, I review some candidates of dark matter, especially WIMPs (weakly interacting massive particles) which is one of the best candidate so it is called that WIMPs miracle. In addition of this, there are also some theories of modification of gravity, by changing the law of gravity, it could be possible that the dark matter observations are explained. Until the dark matter particle is detected, there is some room for uncertainty. So we should consider every part of the problem and solve it. Dark matter problem is covering a large area so every possibility is important. So $f(R)$ gravity is also reviewed in this thesis and some theories are considered as a possible solution of dark matter problem. Finally we highlight that, even in the case of WIMPs or another particles solution, $f(R)$ gravity is also can be used for this problem. However, last words will be said by experiments.

Keywords: Dark Matter, Weakly Interacting Particles(WIMPs), $f(R)$ Gravity, Cosmology.

ÖZ

Karanlık Maddenin gizemi hem astrofizik hem de parçacık fiziği açısından çok önemli bir problemdir. Bu tezde, karanlık madde adaylarını, özellikle WIMPs mucizesi olarak bilinen ve en güçlü aday olan WIMPs(zayıf etkileşen kütleli parçacık)i inceledim. Karanlık Maddenin gizemini çözmeye çalışan bundan başka teoriler de vardır, örneğin gravitasyonun modifiye edilmesi gibi. Karanlık madde parçacıkları deneylerde bulunana kadar, bu gizem sürecektir. Biz bu gizemi her açıdan inceleyip çözüm üretmemiz gerekir çünkü çok geniş bir alanı kapsayan bu alan için her bir ihtimal bile çok önemlidir. Karanlık maddeyi açıklamaya aday olan $f(R)$ gravitasyon teorisini de bu tezde inceledik ve çözüm üretmeye çalıştık. Sonuç olarak, gerek WIMPs olsun gerekse başka parçacıklar olsun, $f(R)$ gravistasyon teorisi de güzel bir alternatiftir ve kullanılabilir. Tabii hangisinin doğru olduğunu deneyler onaylaycak.

Anahtar Kelimeler: Karanlık Madde, Zayıf Etkileşen Kütleli Parçacıklar(WIMPs), $f(R)$ Gravistasyon Teorisi, Kozmoloji.

To My Family

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INTRODUCTION

1.1 Observational Evidences for Dark Matter

Dark Matter is a very strange form of matter. It does not interact with known objects because of the lack of electromagnetic interaction. However, it has a mass and we know that it exists.The main reason for this view is from gravitational interactions on the visible matter which holds galaxies and galaxies of clusters together. The idea of the existence of a new form of matter escaping electromagnetic detections, traces back at least to 1915. The name "dark matter" was firstly used by the Jacobus Kapteyn in 1922 in his studies of the motions of stars in our galaxy [2]. He found that there is no need of Dark Matter within and around the Solar System. In 1923, Jan Oort arrived to an opposite conclusion that there should be twice as much dark matter as visible matter in the Solar vicinity. This is the first claim of evidence for dark matter. However, since later observations have disproved this early claim, in 1933 the discovery of dark matter is usually credited to Fritz Zwicky who made the first correct claim about the existence of dark matter.He concluded that the velocities of the galaxies in the Coma cluster [3] are much larger than the expected one. Zwicky found that orbital velocities are almost a factor of ten larger than expected from the total mass of the cluster [16]. Zwicky concluded that, in order to hold galaxies together in the cluster, the cluster should contain huge amounts of some Dark (invisible) matter. The galaxy rotation curve v, where r marks the distance from the galaxy's center.

$$\frac{v^2}{r} = G\frac{M(r)}{r^2}\,, \tag{1.1}$$

where M(r) is the mass inside radius r and $G_N = 6.7\times10^8 cm^3 g^{-1} sec^{-2}$. After rearranged the above equation reads

$$v = \sqrt{\frac{GM(r)}{r}} \tag{1.2}$$

If the mass is only inside the radius, the velocity should decrease when there is large value of radius as

$$v \propto 1/\sqrt{r} \tag{1.3}$$

, where $G = 1$.

However, for values of r bigger than the size of the galactic bulge, the velocity of stars remains approximately constant[15]. It means that there is a some unknown energy there such that

$$M(r) \propto r \tag{1.4}$$

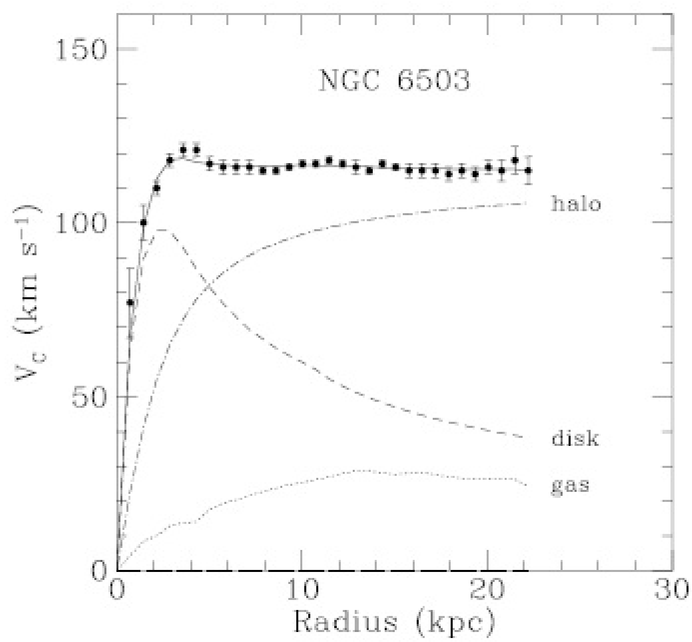

Figure 1.1: Distribution of dark matter in NGC 6503

Distribution of dark matter in NGC 6503. Rotation curve of NGC 6503. The curves labeled gas, disk, and halo represent the contributions from the mass in gas, stars, and a dark halo. This plot of the rotation curve is from Bertone et al. (2005b) using data from Begeman et al. (1991) [15]

For an orbit around a compact central mass, such as planets in the Solar system, we get Kepler's third law which is given at Eqn.(1.3).

As an example if the star is placed in the center of a galaxy, that is the mass density of a galaxy decreases as a power law i.e

$$\rho \propto r^{-n} \tag{1.5}$$

The mass inside radius r is

$$M(r) \propto \int r^2 r^{-n} dr \propto r^{3-n} \tag{1.6}$$

Thus the rotation velocity is

$$v(r) = r^{1-n/2} \tag{1.7}$$

Near the center of the galaxy, observed rotation curves of a galaxy v(r) increase with r from Eqn.(1.3), however then suddenly becomes $v(r) \propto$ constant. From the Eqn.(1.5), the density profile is

$$\rho \propto r^{-2} \tag{1.8}$$

On the other side, the density of stars falls faster when goes to the edges of the galaxy. The total mass from stars, gas and dust clouds, is very very small so that they are not able to provide the rotational speed at large distances. This means that there is another invisible, massive matter. This matter would be only dominant in the outer parts. The dark component forms a dark halo which holds galaxies and galaxies of clusters together.

At galactic scales, there is also additional evidence for dark matter which is based on the mass modeling of the detailed rotation curves [4]. An evidence of Dark Matter from cluster of galaxies was first noticed, as mentioned already, by [3] in 1933. On the cluster scales, the Ω_M is today's mass density including baryonic mass and dark matter [5, 6]

$$\Omega_M \approx 0.2 - 0.3 \tag{1.9}$$

As known that matter relic density component is bounded by the observation of the large scale structure.As reported by Sloan Digital Sky Survey (SDSS) $\Omega_M h^2 = 0.135$ for $\Omega_b h^2 = 0.025$ [7]. The mass density of baryons and the mass density of matter in the universe by using WMAP data are found to be [13, 8]

$$\Omega_b h^2 = 0.0223 \tag{1.10}$$

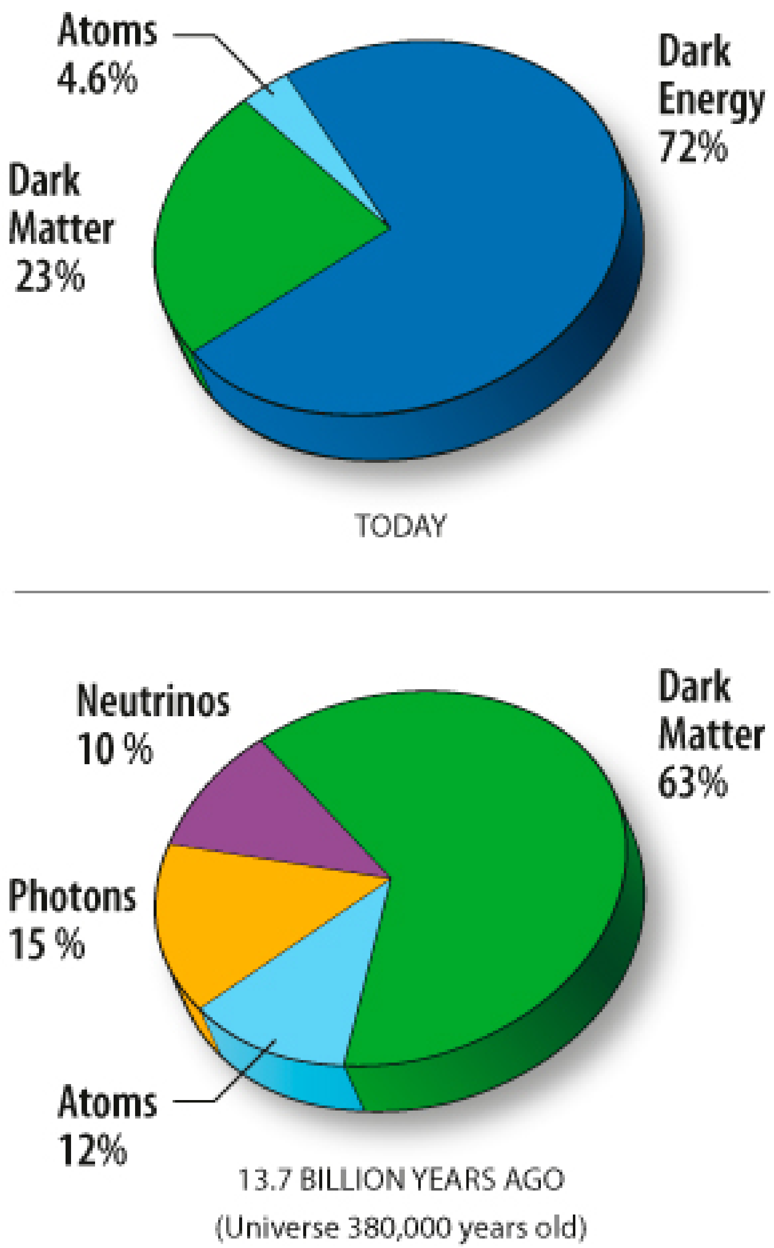

Figure 1.2: WMAP data

WMAP data. Credit: NASA / WMAP Science Team [9]

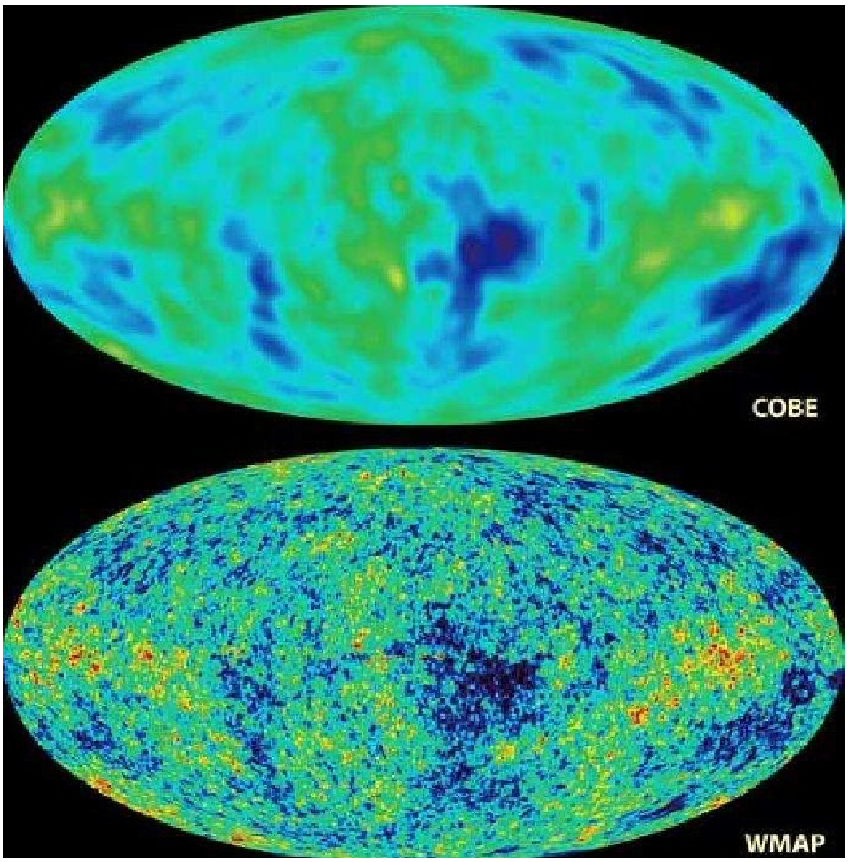

Figure 1.3: CMB Temperature fluctuations

CMB Temperature fluctuations. Image from NASA / WMAP Science Team [9].

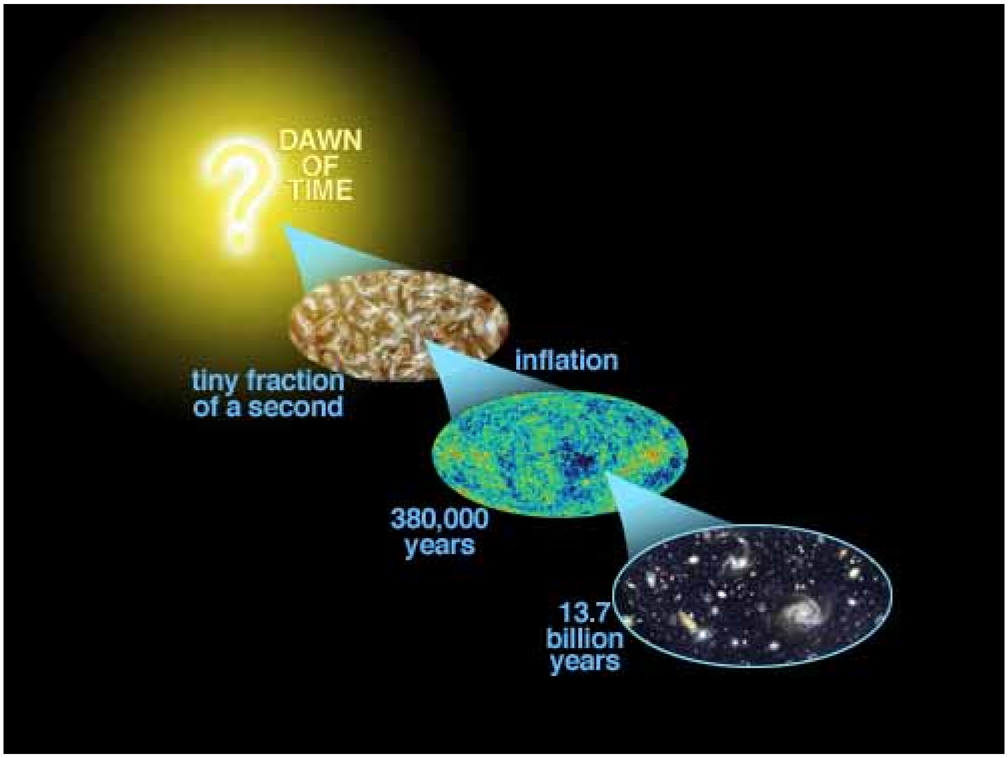

Figure 1.4: WMAP observes the first light of the universe

WMAP observes the first light of the universe. Credit: NASA / WMAP Science Team [9]

$$\Omega_M h^2 = 0.127 \tag{1.11}$$

From WMAP‘s data and large scale structe observations‘s data for relic abundance are consistent. In physical cosmology, Big Bang nucleosynthesis provides us also a constraint[10]

The difference between $\Omega_M h^2$ and $\Omega_b h^2$ is the clue of the existence of the non-baryonic dark matter.

There are many experiments which support the existence of Dark Matter such as CMB anisotropy, galaxy surveys, etc. so we think that there is Dark Matter. Unfortunately, the nature of dark matter is now an open problem. So this is a very hot topic in physics.

Furthermore, there are some additional constraints on the amount of dark matter in the Universe. We know very well from the concordance cosmology measurements(it's basically a combination of looking at the microwave background fluctuations, the baryon acoustic oscillations, and the supernova brightness versus redshift relation)what are the relative fractions of dark energy and mass in the Universe. We also have pretty good constraints from primordial abundances of the elements on what the relative fractions are for baryonic and non-baryonic dark matter because if there were too high a density of protons in the early universe, then there would have been more fusion in the early universe[17].

1.2 Baryonic Dark Matter(BDM)

MACHO(massive compact halo object); planet-like objects, plasma and other compact objects are the candidates for BDM. Other examples of this class of dark matter candidates include primordial black holes created during the big bang, neutron stars, white dwarf stars and various exotic stable configurations of quantum fields, such as non-topological solitons. Gravitational Microlensing is one of the ways to detect these objects. However, this method only gives us the information about the mass, distance and velocity of them and nothing more.

Heavier, relatively faint objects such as old white dwarfs and mini black holes,are the most important candidates for this BDM. If there are primordial mini black holes produced in the big bang before BBN(Big Bang Nucleosynthesis), they would not contribute to Ω_b.

$0.07 M_\odot$ is required by a star to start to shine as a star by igniting thermonuclear fusion. Brown dwarfs are not completely dark because they radiate faint thermal radiation. However, the typical mass of these MACHOs should be $\approx 0.5 M_\odot$, which is much larger than the brown dwarf mass. In addition, white dwarf has a mass which is suitable for dark matter but there is not enough number of observed white dwarfs. Hence, BDM is not as good candidates for dark matter as nonbaryonic dark matter.

1.3 Nonbaryonic Dark Matter

Hot Dark Matter (HDM) and Cold Dark Matter (CDM) is considered as a nonbaryonic dark matter. Other more special options have been considered, such as for example warm dark matter (WDM).

Dark matter particles that are called HDM, are relativistic at the time of matter-radiation equality time. This requirement implies that their mass has to be lower than about 100 eV. Their thermal velocities, when structure formation begins are large because of the small mass. In contrast, CDM has large mass so its velocities are negligible. Candidates between hot and cold are called warm dark matter.

Effect of the pull of gravity causes to form galaxies and galaxy clusters from early homogeneously distributed. This is called structure formation. Today the best way to differentiate between HDM, WDM and CDM is through the observed large-scale

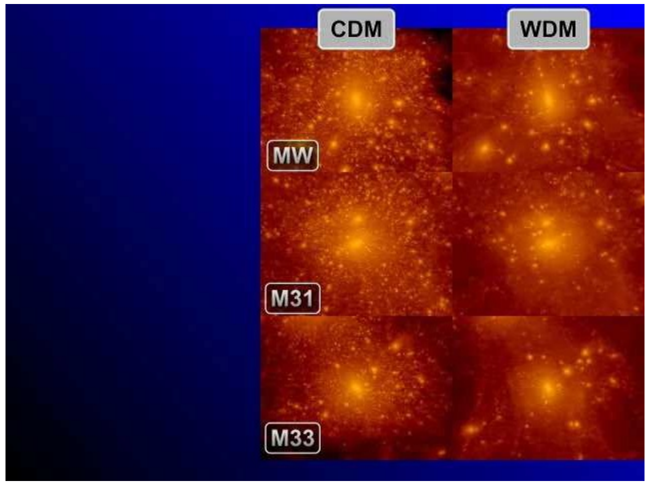

Figure 1.5: Comparison of the expected halo of the MilkyWay and the galaxies M31 and M33 in CDM and WDM models

Comparison of the expected halo of the MilkyWay and the galaxies M31 and M33 in CDM and WDM models. From http://www.clues-project.org/images/darkmatter.html.

structure, i.e. CMB which shows the seeds of structure. We show in Fig. (1.3) the results of a simulation of the halo of dark matter around the Milky Way and two other galaxies. For CDM, there is more substructure and satellites around the galaxy, while their formation is suppressed for WDM. According to observations, there is an order of magnitude less satellites observed around the Milky Way than predicted by CDM models. However, the observations are not complete, and the discrepancy may also be due to other causes than WDM.

The most common candidates for non-baryonic dark matter are Weakly Interacting Massive Particles, or WIMPs [1]. Cold dark matter has a most important candidate which is WIMPs. Because they have neutral charge and they are long lived particles. Only weak force and gravitational force effect them. Large amount of WIMPs are produced and then the interaction rate of WIMPs was decreasing so decoupling increasing between them in the thermal bath of the early universe when cools down, like neutrinos, but are much heavier, so that they are a form of CDM. After the decoupling, their relic density remains constant as consistent with experiments. This is called WIMPs miracle. Furthermore the interactions of some dark matter candidates are stronger or weaker. For example, gravitinos have only gravitational interactions, while TIMPs (Technicolour Interacting Massive Particles) have much stronger interactions.

Neutralino (bino+wino+2higgsino) lightest supersymmetric particle is the best candidate for WIMP in Minimal Supersymmetric Standard Model (MSSM)[11, 12].

The other candidate for WIMP is the lightest Kaluza Klein (LKP) which is coming from Universal extra-dimensions (UED) model .

1.3.1 Hot Dark Matter (HDM)

For the HDM neutrinos are candidates. Energy density today is dominated by their rest masses so to make a significant contribution to the density parameter, neutrinos should have a rest mass about 1 eV.

The present upper limit to HDM in the form of massive neutrinos leads to the conservative mass limit from WMAP data.

$$\sum m_\nu \leq 0.62eV \tag{1.12}$$

Therefore the maximum contribution of neutrinos is $\Omega_\nu h^2 \leq 0.02$, at most the same order of magnitude as baryonic matter. If neutrinos were the dominant form of matter, there would be a lower limit on their mass from constraints on the phase space density, called the Tremaine-Gunn limit. Essentially, in order to achieve a certain rotation velocity for galaxies, you need a certain amount of mass inside a given volume, and the Pauli exclusion principle constrains the number of particles you can pack inside a given volume. Even though we know that neutrinos are a subdominant component of dark matter, the Tremaine-Gunn limit applies to any fermionic dark matter candidate, even if its distribution is not thermal. (There is no similar lower limit on the mass of a bosonic dark matter particle.)

1.3.2 Cold Dark Matter (CDM)

Most of the matter in the universe has to be CDM because of data on large scale structure combined with structure formation theory. WIMPs (Weakly Interacting Massive Particles) are the main candidates of CDM.

Dark matter candidates are with masses in the 100 GeV to TeV range. Particles with GeV to TeV masses and weak-scale interaction cross sections are known as Weakly Interacting Massive Particles (WIMPs). There are also another candidates for DM. However, WIMP dark matter is compelling for two main reasons:

1. In the early universe, when WIMPs were in thermal equilibrium annihilated with each other with a weak scale cross section. As the universe expanded and has frozen out, WIMPs cooled to obtain a relic density today which is equal to the measured dark matter density.

As mentioned before that this is known as the WIMP Miracle.

2. The other reason is that WIMP dark matter are present in many of the favored extensions of the Standard Model.For example, the Minimal Supersymmetric Standard Model (MSSM) has a favourite WIMP candidate.

Chapter 2

EXTENDED GRAVITY: $f(R)$ GRAVITY

2.1 Introduction to $f(R)$ Gravity

By changing the law of gravity, it could be possible that the observations are explained. Until the dark matter particle is detected, there is some room for uncertainty. The problem for modified gravity is that there are so many different observations for dark matter, in different physical systems: motions of stars in galaxies, motions of galaxies in clusters, gravitational lensing, large-scale structure, CMB anisotropies and so on. Gravity has to be adjusted in a different manner for these different observations, and the resulting models are somewhat contrived. Expressed another way, the dark matter scenario is very predictive: the simple hypothesis of a massive particle with weak couplings to itself and to the Standard Model particles explains a number of disparate observations and has made successful predictions.

In 1919 and 1922 respectively by Weyl and Eddington [19, 20] gravitational action firstly studied. However, until 1960s, there was not any indications of complicating the gravitational action. Because of cosmological problems such as the dark energy problem, dark matter problem, this type of modification is needed to fix the problems. At $f(R)$ theory, the main idea is to take action as a function instead of a constant curvature such as:

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[f(R)] \tag{2.1}$$

By using the $f(R)$ function , it provides usage of many different properties, which are very important for explaining unknown matters such as dark matter and dark energy. For instance, considering f as a series expansion i.e [22, 23, 24, 25, 26, 27]

$$f(R) = ... + \frac{\alpha_2}{R^2} + \frac{\alpha_1}{R} - 2\Lambda + R + \frac{R^2}{\beta_2} + \frac{R^3}{\beta_3} \tag{2.2}$$

where α_i and β_j coefficients have the appropriate dimensions. So it can be seen that a number of interesting terms appear. [21]

Without matter action is given above, now after adding matter on the action, our $f(R)$ action is

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}[f(R) + S_M(g_{\mu\nu}, \psi)]. \tag{2.3}$$

Varying the action with respect to $g_{\mu\nu}$ provides

$$f_R R_{\mu\nu} - \frac{1}{2} f g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + g_{\mu\nu} \Box f_R = \kappa T_{\mu\nu} \tag{2.4}$$

where $f_R = \frac{df}{dR}$, $\Box f_R = \frac{1}{\sqrt{-g}} \partial_\mu \left(\sqrt{-g} \partial^\mu \left(\frac{df}{dR} \right) \right)$ and $\nabla^\nu \nabla_\mu f_R = g^{\alpha\nu} \left[(f_R)_{,\mu,\alpha} - \Gamma^m_{\mu\alpha} (f_R)_{,m} \right]$

So

$$\Box f_R = \Box \frac{df}{dR} = \frac{1}{\sqrt{-g}} \partial_1 \left(\sqrt{-g} g^{11} \partial_1 \left(\frac{df}{dR} \right) \right) = \frac{1}{\sqrt{-g}} \partial_1 \left(\sqrt{-g} g^{11} \left(\frac{df}{dR} \right)' \right) \tag{2.5}$$

$$\nabla^0 \nabla_0 f_R = \nabla^0 \nabla_0 \frac{df}{dR} = -\frac{1}{2} g^{00} \Gamma^1_{00} \left(\frac{df}{dR} \right)' = -\frac{1}{2} g^{00} g^{11} g_{00,1} \left(\frac{df}{dR} \right)' \tag{2.6}$$

$$\nabla^1 \nabla_1 f_R = \nabla^1 \nabla_1 \frac{df}{dR} = g^{11} \nabla_1 \nabla_1 \frac{df}{dR} = g^{11} \left(\frac{df}{dR} \right)'' - g^{11} \Gamma^1_{11} \left(\frac{df}{dR} \right)' \tag{2.7}$$

$$\nabla^2 \nabla_2 f_R = -g^{22} \Gamma^1_{22} \left(\frac{df}{dR} \right)' = \frac{1}{2} g^{22} g^{11} g_{22,1} \left(\frac{df}{dR} \right)' \tag{2.8}$$

in which a prime means d/dr and its variation with respect to $g_{\mu\nu}$ yields, after some manipulations and modulo surface terms, the field equation

$$f_R R_{\mu\nu} - \frac{f(R)}{2} g_{\mu\nu} = \nabla_\mu \nabla_\nu f_R - g_{\mu\nu} \Box f_R + \kappa T_{\mu\nu} \tag{2.9}$$

where ∇_μ is the covariant derivative associated with Levi-Civita connection of the metric, and $\Box = \nabla^\mu \nabla_\mu$.

These equations above are fourth order partial differential equations in the metric. The alternative name "fourth order gravity" used for this theories because in the first two terms on the right hand side fourth order derivatives appear.

Furthermore, the contraction of the above equation yield the trace equation

$$f_R R - 2f + 3\Box f_R = \kappa T \tag{2.10}$$

where $T = T^\mu_\mu = 0$

the trace equation can be used to simplify the field equations and then keep it as a constraint equation.

Therefore

$$f_R R^\mu_\nu - \frac{1}{4} \delta^\mu_\nu \left(f_R R - \Box f_R - \kappa T \right) - \nabla^\nu \nabla_\nu f_R = \kappa T^\mu_\nu \tag{2.11}$$

The trace of this equation is

$$f_R R - 2f(R) + 3\Box f_R = 8\pi G T \tag{2.12}$$

where $T = g^{\mu\nu} T_{\mu\nu}$

Here we can see that when $T = 0$, not anymore says that $R = 0$ or constant.

Another important thing is that when $R = constant$ and $T_{\mu\nu} = 0$, it reduces to

$$f_R R - 2f(R) = 0 \tag{2.13}$$

Maximally symmetric solutions is affected by f function, and they will be Minkowski, de Sitter or anti-de Sitter. Hence, We can now write the field equations in the form of Einstein equations as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\kappa T_{\mu\nu}}{f_R} + g_{\mu\nu} \frac{[f(R) - R f_R]}{2 f_R} + \frac{[\nabla_\mu \nabla^\nu f_R - g_{\mu\nu} \Box f_R]}{f_R} \tag{2.14}$$

After some algebra the above equation, takes the form

$$F(R)R_{\mu\nu} - \nabla_\mu \nabla^\nu F(R) - kT_{\mu\nu} = -g_{\mu\nu}(\frac{1}{2}f(R) - \nabla_\alpha \nabla^\alpha F(R)) \tag{2.15}$$

where $F(R) = df/dR = f_R$.

For solving above equation we need a metric and its components[21] . Here we are using spherically symmetric metric with radial components B(r) and X(r) :

$$ds^2 = -B(r)dt^2 + \frac{X(r)}{B(r)}dr^2 + r^2 d\Omega^2 \tag{2.16}$$

Ricci scalar of this metric can be written as

$$R(r) = \frac{2}{r^2} + \frac{X'}{X^2}(\frac{B'}{2} + \frac{2B}{r}) - \frac{1}{X}(B'' + \frac{4B'}{r} + \frac{2B}{r^2}) \tag{2.17}$$

For empty space if we solve the Einstein Equation with the form of $f(R)$ gravity, two independent field equations are found :

$$\frac{X'}{X} = \frac{2rF''}{2F + rF'} \tag{2.18}$$

$$B'' + (\frac{F'}{F} - \frac{X'}{2X})B' - \frac{2}{r}(\frac{F'}{F} - \frac{X'}{2X})B - \frac{2}{r^2}X = 0 \tag{2.19}$$

where the prime means d/dr. After that are choose our ansatz with constants F_0, X_0 and B_0 as

$$F(r) = F_0 r^n, X(r) = X_0 r^\alpha, B(r) = B_0 r^\alpha \tag{2.20}$$

Constans are bounded by two constraints as:

$$\alpha = \frac{2n(n-1)}{n+2}, X_0 = [1 + \frac{n(2-\alpha)}{2} - \frac{\alpha^2}{4}]B_0. \tag{2.21}$$

After that

$$R(r) = \frac{2R_0(n,\alpha)}{r^2} \tag{2.22}$$

is obtained.

where

$$R_0(n,\alpha) = 1 - \frac{1+\frac{\alpha}{2}+\frac{\alpha^2}{8}}{1+n(1-\frac{\alpha}{2})-\frac{\alpha^2}{4}} \tag{2.23}$$

This will be the well known solution of Schwarzschild when α or n goes to zero. After using those constraints and definitions, an asymptotic form of $f(R)$ where Λ_0= constant of integration is found as

$$f(R) = \frac{f_0(2R_0)^{(\frac{n}{2})}}{1-(\frac{n}{2})} R^{(1-\frac{n}{2})} + \Lambda_0 \tag{2.24}$$

In the paper of A Model of $f(R)$ Gravity as an Alternative for Dark Matter in Spiral Galaxies they took $F_0 = 1$ when n is going to zero and action goes to $F_0 R + \Lambda_0$.

For the application of this in the rotation curve, geodesic equation is used in the weak field approximation. The test particle is rotating around the central mass and obtains as

$$\ddot{r} + \Gamma^r_{tt} = 0 \tag{2.25}$$

where the dot means d/dt. After using the affine connection which we found by using metric :

$$\Gamma_{tt}{}^{r} = \frac{1}{2}\frac{B\,B'}{X}$$
$$\Gamma_{tr}{}^{t} = \frac{1}{2}\frac{B'}{B}$$
$$\Gamma_{rr}{}^{r} = \frac{1}{2}\frac{X'\,B - X\,B'}{B\,X}$$
$$\Gamma_{r\theta}{}^{\theta} = \frac{1}{r}$$
$$\Gamma_{r\phi}{}^{\phi} = \frac{1}{r}$$
$$\Gamma_{\theta\theta}{}^{r} = \frac{B\,r}{X}$$
$$\Gamma_{\theta\phi}{}^{\phi} = \frac{\cos(\theta)}{\sin(\theta)}$$
$$\Gamma_{\phi\phi}{}^{r} = -\frac{B\,r\,(\cos(\theta)-1)\,(\cos(\theta)+1)}{X}$$
$$\Gamma_{\phi\phi}{}^{\theta} = -\sin(\theta)\cos(\theta)$$

(2.26)

velocity for a test particle is obtained as

$$v^2 = \frac{rc^2 B' B}{X} = \frac{c_0^2 r^{\alpha}}{2} \tag{2.27}$$

After considering some assumption for the large distances from center of galaxy, such as using $r^{\alpha} = 1 + \alpha \ln(r)$ where α is smaller than 1, one finds that

$$v_{\infty}^2 = \frac{c_0^2}{2} \tag{2.28}$$

For taking $B_0 = 2\sqrt{\mu M_{galaxy}}$ where μ is the proportional coefficient with mass inverse dimension and M is mass of galaxy, hence asymptotic velocity is found

$$v_{\infty}^2 = c^2 \alpha \sqrt{\mu M_{galaxy}} \tag{2.29}$$

where α and μ depend on experimental datas. For instance, if this values taken as $\alpha^2 \mu = (v_{\infty}/c)^4 / M_{galaxy} = 10^{-54} kg^{-1}$, the velocity is found exactly $200 kms^{-1}$ and this value is consistent with Tully-Fisher‘s relation. [21]

Furthermore, another good testing is made by using the Modified Newtonian dynamics (MOND).

$$\alpha^2\mu = 0.98 \times 10^{-54} kg^{-1} \tag{2.30}$$

for the velocity of typical galaxy.

There are many difficulties of this theory. One example which gained a lot of publicity a few years ago is the Bullet cluster. A solution of the Dark matter conundrum in terms of a new elemantary particle is quite successful to explain the Bullet cluster, while explanations in terms of modified gravity models encounter serious difficulties. Another problem in this way of solution is that ansatz is choosen by hand and not naturally comes out.

Chapter 3

DARK MATTER PRODUCTION

3.1 Thermal Production

WIMPs and many other dark matter candidates are in thermal equilibrium in the early universe and decouple once their interactions become too weak to keep them in equilibrium. Those particles are called thermal relics, as their density today is determined by the thermal equilibrium of the early universe. When the annihilation processes of WIMPs into SM particles and vice versa happend at equal rates, there is a equilibrium abundance. When the Universe cooled down and the rate of expansion of the universe H exceeds the annihilation rate, the WIMPs effectively decouple from the remaining SM particles. So the equilibrium abundance drops exponentially until the freeze-out point.The abundance of cosmological relics remained almost constant until today [18].

If the candidate is stable or has a long lifetime, the number of particles is conserved after it decouples, so the number density falls like a^{-3}. Specifically, we use the Lee-Weinberg Equation [29]. It describes annihilation and creation of χ particles [28].

$$\frac{dn_\chi}{dt} + 3Hn_\chi = - < \sigma v > (n_\chi^2 - n_{\chi,eqn}^2) \tag{3.1}$$

with $n_{\chi,eqn}^2$,the number density of the relic particles in equilibrium. Furthermore Hubble constant make expanding universe effect. The thermal average of the annihilation cross section σ multiplied with the relative velocity v of the two annihilating χ particles. The first (second) term on the right-hand side (RHS) of the equation describes the decrease (increase) of the number density due to annihilation into (production from) lighter particles. Kinetic equilibrium is satisfied by the Lee-Weinberg equation for χ

Now we use Hubble constant effect which provide universe expansion by considering the evolution of the number of particles in a portion of comoving volume given by

$$N = n_\chi R^3 \tag{3.2}$$

We can then introduce the convenient quantity

$$\bar{N} = \frac{n_\chi R^3}{N(0)} \tag{3.3}$$

such that $N\bar{(}0) = 1$.

In addition, since the interaction term usually depend explicitly upon temperature rather than time, we introduce the $x = \frac{m}{T}$, the scaled inverse temperature.

During the radiation dominated period of the universe, thermal production of WIMPs takes also place . For this time, one can find the expansion rate

$$H = \sqrt{\frac{8\Pi^3 g g_\chi T^2}{90}} \tag{3.4}$$

and then

$$\Longrightarrow H(x=1) = m^2 \sqrt{\frac{8\pi^3 g g_\chi}{90}} \tag{3.5}$$

where $H(x) = \frac{H(x=1)}{x^2}$ and where $_\chi$ refer the number of intrinsic degrees of freedom such a like that spin or color for χ particle.

Hence we can write the last format of our equation a

$$\frac{d\bar{N}}{dt} = -\frac{<\sigma v> N(0)}{R^3 H x}(\bar{N}^2 - \bar{N}_{eqn}^2) \tag{3.6}$$

where

$$\frac{N(0)}{R^3} = \frac{3\zeta[3] m^3}{4\Pi^2 x^3} \tag{3.7}$$

After all little tricks , our Lee-Weinberg equation can be recast as

$$\frac{d\bar{N}}{dt} = -\frac{\Psi(\chi)}{x^2}(\bar{N}^2 - \bar{N}_{eqn}^2) \tag{3.8}$$

where we defined

$$\Psi(\chi) = \frac{3\zeta[3]}{4\Pi^2}\sqrt{\frac{45}{4\Pi^3}}\frac{g_\chi}{\sqrt{g_\chi}}mM_{Pl} < \sigma v >= 0.055\frac{g_\chi}{\sqrt{g_\chi}}mM_{Pl}\sigma_0 \tag{3.9}$$

where ζ is EulerRiemann zeta function To integrate the Lee-Weinberg equation, we need to have an expression for the equilibrium number density in comoving volume.Once the particle is non-relativistic, the difference in statics is not important.

The general equation of equilibrium number density in comoving volume is

$$N(\bar{x})_{eqn} = \frac{2}{3\zeta[3]}\int_0^\infty \frac{y^2}{1+\exp^{\sqrt{x^2+y^2}}}dy \tag{3.10}$$

For the nonrelativistic case at low temperatures $T \gg m_\chi$ one obtains,

$$N(\bar{x})_{eqn,NR} = \frac{2}{3\zeta[3]}\frac{\Pi}{2}x^{3/2}\exp^{-x} \tag{3.11}$$

At great value of temperatures, dark matter quickly is destroyed by its own antiparticle. Shortly after that, until $\Gamma_\chi = N(\bar{x}) < \sigma v > \ll H$, T has dropped too much which is under the value of $m_\chi(T << m_\chi)$, also the number density of dark matter drops exponentially. The temperature at which the particle decouples (The time when the number of particles reaches this constant value) is called freeze-out temperature T_F. Hence the number density per comoving volume becomes almost constant because dark matter particles are no longer able to annihilate efficiently.

An approximate solution for the relic abundance is given by

$$\bar{N}(t) \equiv \bar{N}(\infty) \cong \frac{x_f}{\Psi} \tag{3.12}$$

. where $x_f = \frac{m}{T_f}$ and T_f is the so called ‘freeze-out‘ temperature and approximately, for the typical case $x_f \succ\succ 1$,one has that x_f is a solution of

$$\sqrt{x_f} e^{x_f} = \Psi \tag{3.13}$$

We can then finally calculate the contribution of χ to the energy density parameter finding a well know result

$$\Omega_\chi h^2 \cong 4 \times 10^{-10} \frac{GeV^{-2}}{\sigma_0} \tag{3.14}$$

It is intriguing that so called “WIMPS”(e.g. the lightest supersymmetric particle) dark matter particles seem to reproduce naturally the right abundance since they have a weak cross section

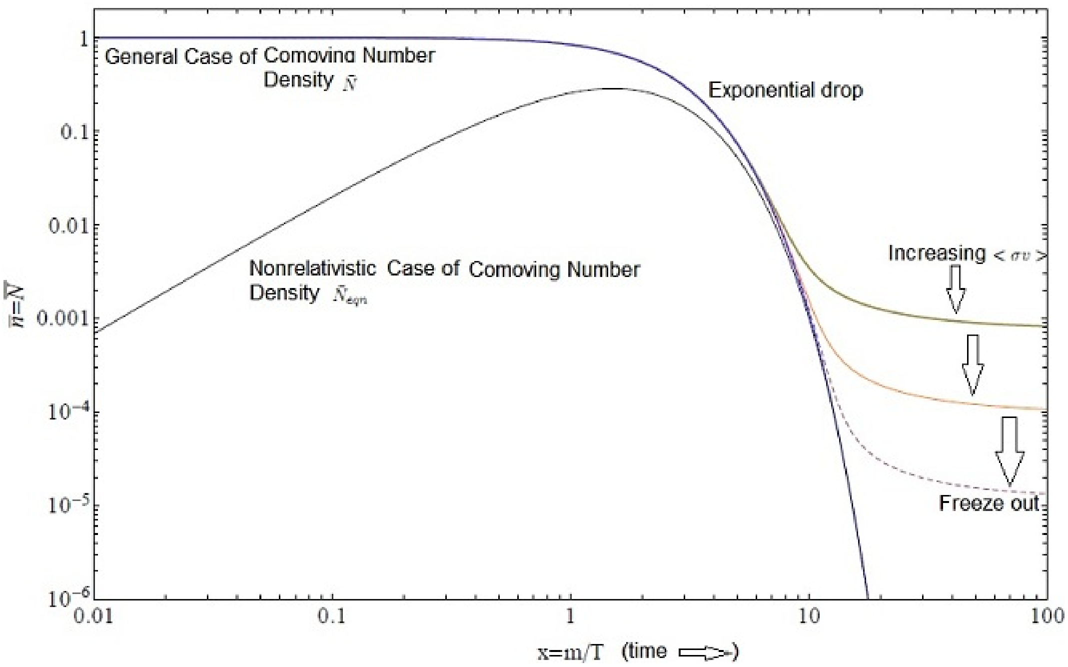

Figure 3.1: Evolution of $\bar{N}$ and $\bar{N}_{eqn}$ as function of x

Evolution of $\bar{N}$ and $\bar{N}_{eqn}$ as function of x for m =100 GeV, $g_\chi = 2$, g =90 (constant),

$\sigma_0 = 10^{-14}, 10^{-15} and 10^{-16}$, $M_{Pl} = 1.9 \times 10^{19} GeV^2$

$$\sigma_0 \cong \frac{\alpha^2}{m^2} \cong 10^{-9}(\frac{100GeV}{m^2}) \tag{3.15}$$

and masses m_x m_{ew} $100GeV$. This observations is called the "WIMP miracle" and typically considered as an encouraging point supporting WIMPS as Dark Matter candidates.

The LW eqn. can be solved by approximately analytic calculation or numerical plotting by using Mathematica.We have plotted those equation by using both of the ways. As you can see Fig. 2.1.

LW Eqn., the termal production, is the most traditional mechanism but that many other mechanisms have been considered in the literature such as non thermal production of very massive particles(so called WIMPZILLAS) at preheating or even right-handed neutrino oscillations.

Chapter 4

CONCLUSION

The observational evidence for dark matter is overwhelming and now we are entering one of the perhaps most exciting periods in physics history where a wide array of experiments may have the potential to unveil its nature.In this thesis I review some important concepts about dark matter. One of the best motivated candidates for dark matter so far is the WIMPS which I mentioned before. In this thesis the focus was on dwarf spheroidal galaxies, promising targets that are assumed to be strongly dark matter dominated systems due to their high mass-tolight ratios up to few hundred solar units. Dwarf galaxies of the Milky Way have the advantage of relative proximity and low astrophysical background as they do not lie within the galactic plane. Great importance was given to studying the properties of these systems and how their dark matter profiles can be derived from kinematical studies. However a precise estimation of the density distribution is a challenging task and due to too little data available in particular the shape of the dark matter halo inner regions remains poorly known. Cusped profiles suggested from simulations like the Navarro-Frenk-White profile are not ruled out in general although observations seem to favor profiles with a flat inner core.

The two investigated dwarf spheroidal galaxies Sagittarius and Canis Major are particularly attractive targets as they are the closest dwarf galaxies discovered so far. The uncertainty concerning their inner region is reflected in the plots of sensitivity limits for both investigated dwarf galaxies, where the distances between the two lines of a cored and a cusped profile are about one order of magnitude.

Better opportunities for a significant dark matter detection could be obtained for instance by the next generation ground-based CTA (Cherenkov Telescope Array) experiment with bigger effective areas due to many Cherenkov telescopes in the order of 50 and also IceCube experiment . Here, stereoscopy will be possible for small energies. Moreover the space based ray telescope FERMI will yield interesting results in the future.Apart from Sagittarius and Canis Major, there could be promising faint dwarf galaxies that are not discovered yet. However, there should be as many undiscovered dwarf galaxies in the other direction, too, which, when discovered, will provide interesting targets for the search of dark matter.

In chapter 2 we gave a simple example of $f(R)$ extended gravity in which the effect of DM is not due to exotic sources but due to the wrong choice of Lagrangian. Einstein‘s model, which extends that of Newton is characterized by the specific choice $f(R) = R$. In the $f(R)$ model this has been modified to different polynomial powers of R. As an example the choice $f(R) \approx R^{1-\frac{n}{2}}$ where $n \ll 1$, describes asymptotic behaviours of flat rotation curves and experimental results of Tully-Fisher (i.e. $v^4 \approx$ Mass). For n=0 the model reduces to the Einstein‘s general relativity model which fails to explain flat rotation curves unless supplemented by exotic sources.

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