Teaching-Learning Based Algorithm for Numerical Dynamic Multi-Objective Optimization Problems

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ABSTRACT

Teaching–Learning-Based Optimization (TLBO) algorithm has become an alternative optimization method in a great number of applications in different fields of engineering and science since it has been introduced in 2011. Teaching-learning-based optimization (TLBO) is a population-based metaheuristic examination algorithm stimulated by the teaching and learning procedure in a classroom environment. TLBO with its comparatively reasonable performances outperforms some of the well-known metaheuristics concerning constrained benchmark tasks, controlled mechanical schemes, and nonstop non-linear numerical optimization problems. In the TLBO algorithm's variants, all the learners have an equal chance of receiving information from the teacher and from each other as learners by interacting with each other in the class. The Experimental results of the TLBO method are tested on the set of CEC2018 dynamic multi-objective optimization benchmark problems and the computed results show that TLBO offers promising outcomes with diverse dynamic features and changing environments compared with other algorithms. It works well with producing a good class of population when alterations happen for pursuing the influential Pareto-optimal set efficiently for refining population conjunction and multiplicity. TLBO extracted improved or equal quality solutions compared to other evolutionary algorithms. It is a promising alternative for the solution of difficult dynamic multi-objective optimization problems.

Keywords: Dynamic Multi-Objective Optimization Problems, Optimization Algorithms, Teaching Learning Based Optimization, Pareto-Front, Pareto-Set

ÖZ

Öğretme-Öğrenme Tabanlı Optimizasyon (TLBO) algoritması, 2011 yılında tanıtıldığından beri mühendislik ve bilimin önemli sayıdaki farklı alanlarında altenatif bir optimizasyon yöntemi olmuştur. Öğretme-öğrenme tabanlı optimizasyon (TLBO), sınıf ortamında öğretme ve öğrenme süreci üzerine kurgulanan popülasyon tabanlı bir metasezgisel arama algoritmasıdır. Karşılaştırmalı makul başarımlarıyla TLBO, kısıtlamalı kalite testi problemleri, kontrollü mekanik şemalar ve kesintisiz doğrusal olmayan sayısal optimizasyon problemlerinin çözülmesinde iyi bilinen bazı metasezgisel yöntemlerden daha üstün başarım göstermektedir. TLBO algoritmasının değişik uygulamalarında, tüm öğrenenler sınıfta birbirleriyle etkileşime girerek öğretmenden ve birbirlerinden eşit bilgi alma şansına sahiptirler. TLBO yönteminin deneysel sonuçları, CEC2018 konferansında yayınlanan dinamik çok amaçlı kalite testi optimizasyon problemleri kümesi üzerinde sınanmıştır ve hesaplanan sonuçlar TLBO'nun diğer algoritmalara kıyasla çeşitli dinamik özellikler ve değişen ortamlarla umut verici sonuçlar sunduğunu göstermektedir. Problem koşulları değiştiğinde, algoritma iyi bir nüfus yapısı ve çeşitliliği sağlayarak Pareto-optimal çözümler kümesini verimli bir şekilde takip eder. TLBO'nun başarımı diğer evrimsel algoritmalarla karşılaştırıldığında iyileştirilmiş veya aynı kalitede çözümler elde ettiği görülür. Algoritma zor dinamik çok amaçlı optimizasyon problemlerinin çözümü için iyi bir alternatiftir.

Anahtar Kelimeler: Dinamik Çok Amaçlı Optimizasyon Problemleri, Optimizasyon Algoritmaları, Öğretme-Öğrenme Tabanlı Optimizasyon, Pareto-Ön, Pareto-Küme

DEDICATION

This work is dedicated to my mother ‘HUDA’ the great guidance I have in my life who I consider that Allahu bsutu alrrizq for me in the picture of Huda indeed. ‘HUDA’ is the merciful leader who has created the unconventional love road that is full of compassion and discipline. Thank you for your faith and your highest state of wisdom. Thank you for fighting for us and never give up. Thank you Huda.

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LIST OF ABBREVIATIONS

CEC	IEEE Congress on Evolutionary Computation
CLONALG	Clonal Selection Algorithm
CRO	Chemical Reaction Optimization
DEMO	Dynamic Evolutionary Multi-Objective Optimization
DMOAs	Dynamic Multi-Objective Optimization Algorithms
DMOOPS	Dynamic Multi-Objective Optimization Problems
DMOPs	Dynamic Multi-Objective Problems
EAs	Evolutionary Algorithms
EMO	Evolutionary Multi-objective Optimization
FBP	Fitting based Prediction Algorithm
GA	Genetic Algorithm
HS	Harmony Search
IGD	Inverted Generational Distance
LCA	League Championship Algorithm
MIGD	Mean Inverted Generational Distance
MOE	Mixture of Experts
MOEA/D-FD	Multi-Objective Evolutionary Algorithm based on Decomposition for First Order Difference Model
MOEAs	Multi-Objective Evolutionary Algorithms
MOOPs	Multi-Objective Optimization Problems
MOP	Multi-objective Optimization Problem
MOTLBO	Multi-Objective Teaching Learning Based Optimization
MS	Metaheuristic search

NDS	Non-Dominated Sets
PF	Pareto-Front
POF	Pareto-Optimal Front
POS	Pareto-Optimal Set
PPS	Population Prediction Strategy
PS	Pareto-Set
PSO	Particle Swarm Optimization
RM-MEDA	Regularity Model based on Multi-objective Estimation of Distribution Algorithm
SA	Simulated Annealing
SCA	Sine Cosine Algorithm
TLBO	Teaching Learning Based Optimization
TrDMOEA	Transfer Learning Based DMOEA
TS	Tabu Search
VNS	Variable Neighborhood Search
WWO	Water Waves Optimization

Chapter 1

INTRODUCTION

1.1 Background of Study

Evolutionary multi-objective optimization (EMO) significantly expanded in recent years in the domain of scientific research [1]. As time goes on, a variety of efficient evolutionary algorithms (EAs) had been suggested to identify the best algorithms in order to conquer the obstacles in finding the ideal compromise of two or more objectives that depict a Multi-objective Optimization Problem (MOP). However, the majority of these techniques are effective for multi-objective problems MOPs with static structures or characteristics, which do not change significantly over time. However, due to their time-dependent characteristics, dynamic multi-objective problems (DMOPs) are extra challenging and exciting than constant multi-objective problems [2].

Dynamic multi-objective problems constrain three main challenges or experiments. First of all, environmental changes and the multiplicity of dynamics constrains different stages of complications to algorithms as well as there is no particular or singular change mechanism feedback that can entirely treat dynamics. Second, it is a difficult process to keep inclusion and variety, the key factors that population-based algorithms depend on for dynamics. Last but not least, algorithms are made more rigid in order to investigate the new search area due to diversity loss whenever there is no reaction time for environmental changes [3].

In order to carefully follow Pareto fronts or sets with temporal variation, algorithms for DMOPs must establish a solid stability within variety and convergence so that any surroundings modifications may be quickly addressed. All of these point to an urgent need for innovative approaches to DMOPs [4]. As trial and rating real parameter multi-objective EAs that operate within static parameters, benchmark frameworks with various levels of complexity have also recently been established [5].

However, despite the requests from a variety of application areas, relatively few analyses studies have been focused on either examining different challenges related with the DMOPs or applying evolutionary computing methods to address the dynamic multi-objective problems. In 2004, Farina et al. [6] the researcher took a big stride in this direction when they suggested a test-suite of five real-parameter dynamic MOPs. The authors of [7] encouraged EA scholars to investigate DMOPs in great detail, but this work has not yet attracted the same level of attention as that given to the disciplines of dynamic single-objective optimization problems and static MOPs [8].

Due to the search-variables' temporal variability, the landscapes of the objective functions, and/or the applicable restrictions, a MOP may exhibit dynamic behavior [9]. The Pareto-Front (PF) and Pareto-Set (PS) interactions, which are the primary complexity of MOPs, can be subjected to temporal variation to add dynamicity to the issue. Objective functions, constraint functions, and problem parameters all alter over time in a dynamic optimization problem. These issues frequently come up while solving genuine issues, particularly when tackling optimized control issues or issues that demand for on-line optimization. Two computational approaches are often employed.

The analysis of a solution against a variety of real-world instances of the dynamic problem creates an off-line optimization problem that may be used to establish the optimum control rules or regulations [10]. This method works well with problems that require too much computer power to be solved online by any optimization technique. The alternative strategy is a direct on-line optimization process. In such a scenario, the problem is considered to be stationary for a certain amount of time, and an optimization method is permitted to discover optimum or nearly optimal solutions for up to two problems during the stationary period [11].

A new optimization is then conducted out for the new time frame, and a current development is created based on the present problem scenario. The aim and objective are to develop effective optimization algorithms that can track the optimal solution(s) within a limited number of iterations in order to reduce the required time for fixing the problem and the approximation error, even though this process is approximate due to the static consideration of the problem during the time for optimization.

1.2 Aim of the Study

In this research, we consider utilizing the direct on-line optimization method formerly developed to address dynamic optimization problems with multiple objective functions. In the last ten years or so, the multi-objective optimization domain has seen a fundamental change including how problems are addressed when utilizing evolutionary computing techniques as opposed to conventional techniques. Since there are many Pareto-optimal solutions to these issues, evolutionary multi-objective optimization (EMO) techniques try to locate a large number of solutions that are as near to the real Pareto-optimal front (POF) as they may be in a single simulation run. These methodologies not only provide an accurate sense of the true POF's magnitude (ideal and nadir solutions), but they also show the front's shape [12] and reveal if there is a "knee" solution [13]. They also permit customers to investigate the obtained solutions to identify any intriguing traits of the best possible solutions [14]. Despite the EMO algorithm's value, there hasn't been much enthusiasm for expanding the concepts to address dynamic multi-objective optimization issues.

That subject is addressed in this work, in addition to a set of test cases for both continuous and discrete dynamic multi-objective optimization problems, as well as a general technique for solving such problems. The EMO scholars have examined several static (not altering during optimization) test cases that illustrate all sorts of challenges that an EMO algorithm may encounter into when convergent approaching the POF [15]. Several other substantial real-world applications actually require time-dependent (on-line) multi-objective optimization, where either the objective function and constraint or the affiliated problem parameters or both vary with time. As mentioned previously, these problems require a static optimization procedure, in which the task is to find a set of design variables to optimize the objective functions, which are static (iteration of the optimization process).

There aren't many EMO algorithms capable of solving these issues, and there aren't enough test problems to fully evaluate a dynamic evolutionary multi-objective optimization (DEMO) approach. Multi-Objective Optimization Problems (MOOPs) that are dynamically changing either the objective functions or the constraints are known as DMOOPS. This article focuses on bounded constraint DMOOPs, which are unconstrained DMOOPs with dynamically changing objectives and static boundary restrictions. The fact that this paper does not concentrate on MOOPs with noise should also be emphasized. An algorithm should be trialed on DMOOPs that test its ability to deal with unique challenges, such as tracking a Pareto-Optimal Front (POF) that shifts from convex to concave over time or locating a diverse set of solutions where the density of solutions shifts over time, in order to ascertain whether it can solve DMOOPs efficiently. Benchmark functions are those that fit this description [16].

The benchmark functions used for a comparative research have an impact on the outcomes and efficiency of the analysis. Therefore, consideration should be given while determining the benchmark functions. The unavailability of standardized benchmark functions, however, is one of the major issues in the field of DMOO [17]. Choosing which benchmark functions to utilize is therefore not an easy task to accomplish. Furthermore, there isn't presently a thorough description of DMOOPs in the literature. In order to enable a consistent comparison of Dynamic Multi-Objective Optimization Algorithms (DMOAs), this article provides a comprehensive examination of DMOOPs that were offered in the literature and suggests an optimal set of DMOO benchmark functions. These two primary goals were achieved by identifying the following sub-goals: determining if the DMOOPs already published in the literature are effectively evaluating the performance of DMOO algorithms identifying weaknesses of present DMOOPs and resolving the discovered weaknesses of current DMOOPs by proposing a strategy to produce DMOOPs with an isolated POF, an approach to develop DMOOPs with a deceptive POF, and introducing new DMOOPs with complex Pareto-Optimal Sets (POSs) [18, 19].

In the real world, there are many multi-objective optimization problems (MOPs) whose nature is dynamic, i.e., their objective functions, constraints, and/or parameters may change over time. Due to their dynamism, dynamic MOPs (DMOPs) provide serious challenges to evolutionary algorithms (EAs), as any change in the environment may have an effect on the objective vector, constraints, and/or parameters. The Pareto-optimal set (POS), which is a collection of mathematical solutions to MOPs, as well as the Pareto-optimal front (POF), which is a representation of POS in the objective space, may therefore evolve over time. In order to follow the shifting POF and/or POS and produce a series of estimates over time, optimization must be performed [20].

DMOPs can be characterized in several ways depending on the nature dynamisms involved [21]. Since many real-world applications, such as thermal scheduling [22] and circular antenna design [23], have at least two objectives that are in conflict with one another, or they are MOPs, there has been an increase in research interest in the area of evolutionary multi-objective optimization over the past few years. The objective of solving MOPs is to identify a group of tradeoff solutions rather than a single optimal solution because of their multi-objectivity.

An MOP can be categorized as a DMOP if it has time-dependent components. Planning, scheduling, and control are examples of several DMOPs that arise in nature in real life [24]. Performance metrics, test problems, dynamism classification and algorithm design are a few significant areas where contributions have been made in this field [25, 26]. The most crucial of them is algorithm design since it is the means by which DMOPs solve their issues [27].

1.3 Organization of the Thesis

The second Chapter represent a brief overview to optimization and TLBO algorithm with a literature review on Dynamic Evolutionary algorithm. Chapter 3 is the methodology part where it used in this thesis with a detail explanation. The experimental results and evaluations is going to be demonstrated at Chapter 4 that covers the outcomes and effects of the TLBO algorithm with discussion of the accomplished results. The work will concluded in chapter 5, short brief of the whole study and contribution for the future work based on the experimental results.

Chapter 2

LITERATURE REVIEW

2.1 Metaheuristic Search (MS) Algorithms

An appraisal of Population-based Meta-Heuristic strategies to enhance the computational efficiency of heuristic procedures. Laporte and Osman [28] made it clear throughout their research that over the years meta-heuristic is a mixture of different ideas that have been redefined for discovering a novel space search of new learning strategies to be used to assembly information with the purpose of obtaining optimal solutions and applications in which approximate algorithms, regularly termed as heuristic algorithms. They defined combinatorial optimization problems can be framed as consuming finite or countable infinite of unusual number of solutions and it is a mathematical study formally demarcated as the of finding an optimal gathering of separated objects typically finite in numbers [29]. They witnessed that there is a chance of finding the accurate solutions of combinatorial optimization that state to a remarkable progress in mathematical programming in computer technology meanwhile in the sense that still there is a real challenge that realistically it is not easy to solve large problem pattern in reasonable computation times, because of that challenge.

The scholars of heuristics have made an intensive research area of research directed to the development of many approximate algorithms where the concept of intelligently the decent initial solutions have been found by generating a materialistic random adaptive search spaces[30]. The learning strategies such as non-monotonic search strategies that goes from testing to receiving neighbors came from a presented complex neighborhoods mechanism such as ejection chains from the simulated annealing to get a synthesis moves with correct data organizations that built on the adaptive search spaces which gather data throughout the algorithms execution with the purpose of finding possibly best set of solutions rather than a single solution by all possible means [31, 32].

According to the passage of Voss et al. [33], for the manipulation of hard optimization problems, the ideal way to produce an efficient high quality set of solutions is a meta-heuristic that it might be high or low level actions, or a humble local exploration, or a manufacture process only. Especially with the new advances in the scheme and application to approximate solutions of meta-heuristics, they defined for each iteration an approximate means a broad or inadequate single solution or a collection of solutions [34].

However, researchers highlights that in metaheuristic computing there is a big room of research still seem not cover many issues and the field of metaheuristics has not reached precocity compared to mathematics, chemistry or physics fields [35]. As we know, metaheuristic algorithms can classified in two conducts, either population based or non-population based algorithms (Yang,2010) to know the difference between them in searching schemes for example, the population based algorithms use a set of strings to simulate the selected phenomenon behavior in the search area like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), which makes use of a large number of agents or particles, is just one of many such algorithms. Others include

Teaching-Learning-Based Optimization (TLBO), Sine Cosine Algorithm (SCA), Chemical Reaction Optimization (CRO), Harmony Search (HS), Clonal Selection Algorithm (CLONALG), League Championship Algorithm (LCA), and Sine Cosine Algorithm. Also described are Tabu Search (TS) and Variable Neighborhood Search (VNS), two non-population based metaheuristics [36].

In view of to the pervious element about the population based algorithms, Rao et al. (2011) presented the teaching-learning-based optimization (TLBO) algorithm which requires a population size and amount of generations for its working with such normal and common controlling parameters [37]. Because of TLBO successful implementations and high intensity, and analysis, after its introduction in 2011, TLBO algorithm has mentioned in so many metaheuristic research paper at international journals of Elsevier, Springer-Verlag, Taylor & Francis, and IEEE Transactions, in addition to international conferences. In the disciplines of economics, physics, chemistry, biotechnology, civil engineering, mechanical design, thermal engineering, manufacturing engineering, computer engineering, electronics engineering, and structural engineering. It has widely been recognized by scholars in advanced optimization [38].

2.2 Concept of Algorithm

The goal of teaching-learning-based optimization (TLBO) is to develop comprehensive solutions for continuous optimization problems with a minimum of computing work and a maximum of consistency. Researchers have been working on building an algorithm to be free from algorithm-specific parameters. TLBO is a result of the effort of researcher, since TLBO is a population-based algorithm, only common control parameters, such the number of generations and population size, are needed; it does not require any algorithm-dependent parameters [39].

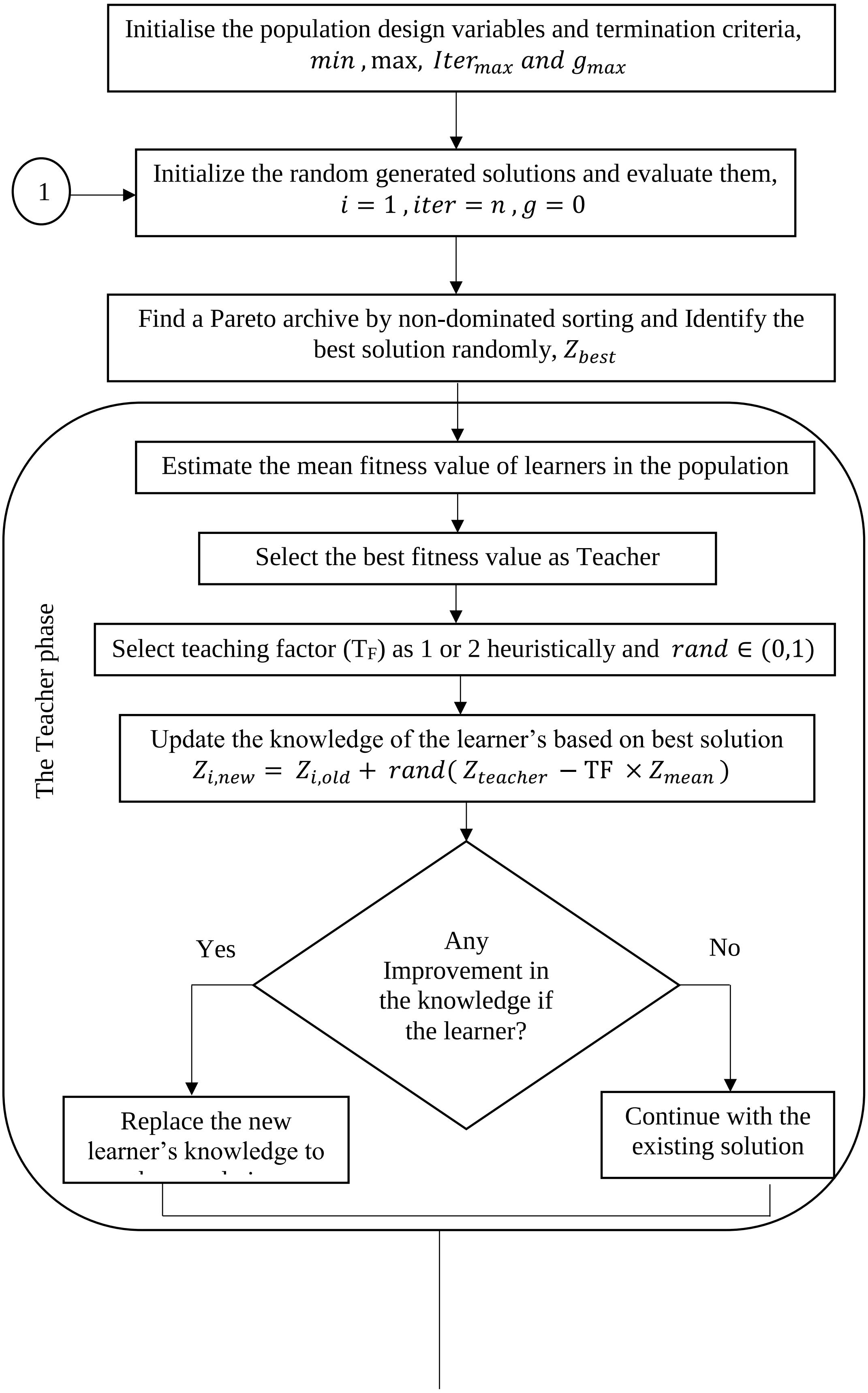
The TLBO algorithm is built on the outcome influence of a teacher on the how learners are productive in a class. The outcome can be described as the term of results or grades. Generally, the teacher is theorized as the highly learned person in the class who shares his or her knowledge with the learners and it shows how much quality of the teacher and knowledge that she/he has the more effective results on learners. As declared, the result is considered as a term of marks or grades so it is understandable that decent teachers educate learners such that they can have better results [40]. And above, to reach a good results, learners also can learn from communication and interaction between individuals, which aids in their outcomes.

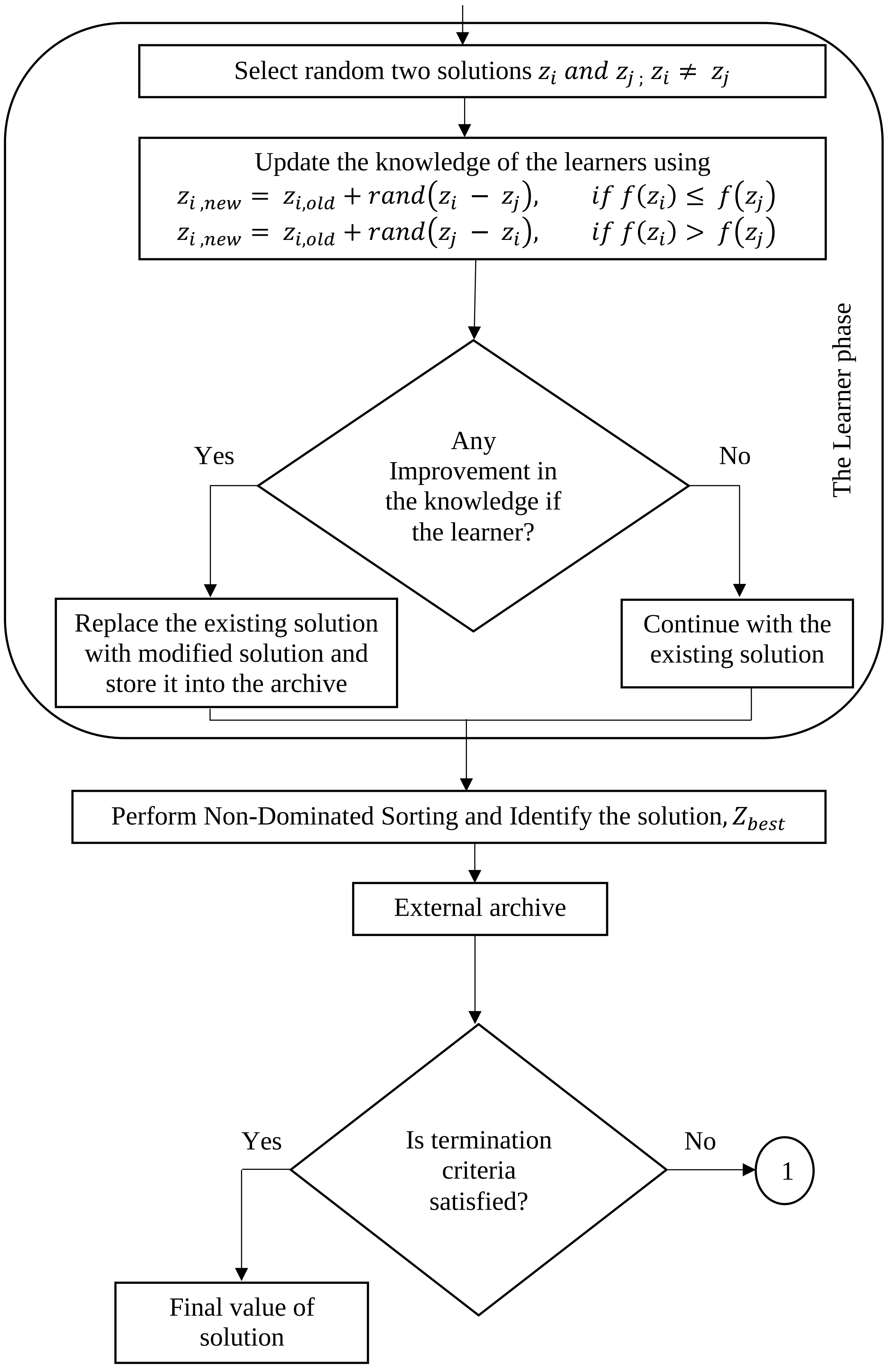
Algorithm1. Basic Teaching-learning-based optimization Algorithm

Begin
    Initialize NP (numbers of learners) and D (dimension);
    Initialize learners and evaluate them;
    while stopping condition is not met
        Choose the best learner as $X_{teacher}$;
        Calculate the mean X_{mean} of all learners;
        for each learners X_i
        // Teacher phase //
        $T_F = round[1 + rand(0,1)]$;
        Update the learner $X_{i,new} = X_{i,old} + rand.(X_{teacher} - T_F.x_{mean})$;
        Evaluate the new learner $X_{i,new}$;
        Accept $X_{i,new}$ if it is better than the old one $X_{i,old}$
        // learner phase //
        Randomly select another learner X_j which is different from X_i;
        Update the learner $X_{i,new} = X_{i,old} + rand.(x_i - x_j), if\ f(x_i) < f(x_j)$,
        $X_{i,new} = X_{i,old} + rand.(x_j - x_i), if\ f(x_i) < f(x_j)$, ;
        Evaluate the new learner $X_{i,new}$;
        Accept $X_{i,new}$ if it is better than the old one $X_{i,old}$;
      end for
    end while
End

2.2.1 Characteristics of Algorithm

The function of TLBO is a population-based method and n is considered as a population that is classified into dualistic phases, the ‘Teacher phase’ and ‘Learner phase’ that considered as diverse scheme of variables, m, of the optimization problem and offered to the learners as well. Essentially starting to the ‘Teacher phase’, the selection of the teacher quality depends on his/her capabilities. Teacher tries to improve the mean performance of the group of learners in their concerned topic. The best resolution in the whole population is considered as the teacher. Any given optimization problem involves parameters that are the designed variables in the objective function and the best solution of the given optimization problem is the best unbiased utility value. TLBO is alienated into dual fragments: the teacher part and the learner part [41].


Initialise the population design variables and termination criteria, min , max, $Iter_{max}$ and g_{max}
1
Initialize the random generated solutions and evaluate them, $i = 1$, $iter = n$, $g = 0$
Find a Pareto archive by non-dominated sorting and Identify the best solution randomly, Z_{best}
The Teacher phase
Estimate the mean fitness value of learners in the population
Select the best fitness value as Teacher
Select teaching factor (T_F) as 1 or 2 heuristically and $rand \in (0,1)$
Update the knowledge of the learner's based on best solution $Z_{i,new} = Z_{i,old} + rand(Z_{teacher} - \mathrm{TF} \times Z_{mean})$
Any Improvement in the knowledge if the learner?
Yes
No
Replace the new learner's knowledge to
Continue with the existing solution



Flowchart of Basic Teaching-learning-based optimization Algorithm [42]

2.3 The Teacher Phase

The first stages of the TLBO algorithm is the teacher phase, where the teacher teach and train learners. Throughout this stage depending on the teacher skills and knowledges, the instructor tries to increase the session's average score in the subject they are teaching. The greatest way to describe the excellent educator is to say that they represent the most knowledgeable member of the class or of the entire learning environment. During the learning process in a classroom or in the environment of learning, a teacher is the person who likely to have more knowledge more than the students. The teaching learning based optimization method is built on this characteristic, which is an inhabitant's scheme that begins as a recognized collection of resolutions recognized as inhabitants of optimization problems [41, 42].

The teacher who is the most knowledgeable and who can increase the students' grasp of a certain topic in class is deemed to have the greatest impartial utility attribute of inhabitants. However, each leaner has her/his own skills and capabilities to learn in the classroom, not all students are the same so in not likely and conceivable therefore for instructor to upsurge each student's level equally in the same way for each one because students are different have different knowledge levels. So the best way is to control or test students will be by the mean knowledge value. Afterward training and education process to the students, the teacher expected to raise the learners' average knowledge level to a higher average understanding value. If the If the resolution has any utility, it is for example symbolized as $X_{j.k,i}$which X_j means the jth learners enterprise adjustable objective function; $j = 1,2, \dots\dots, m$; and k symbolized the kth inhabitants participants who is a student, $k = 1, 2, \dots., n$; plus i symbolized the ith

restatement, $i = 1, 2, \ldots., G_{max}$, where iterations G_{max} is the quantity of all-out generation of iterations [41, 42].

A tutor who is the best learner from the beginning of the teacher phase has been recognized as the greatest resolution out of inhabitants to some optimization issue, and it is decided from the value of the objective function. Depending on the kind of optimization issue, the specified minimum value of the objective function is always the optimum answer, for instance if the problem is one of minimization. At any iteration allow x_k, i remain the greatest resolution i intended for whatever assessment of $f(x_k, i)$ is lowest amongst the inhabitants. That superlative resolution would become represented by $f(x_{best}, i)$.

Then compute that mean outcome $m_{j,i}$ of the students in a specific topic j . And how well the teacher is able to raise the average class performance in the subject that they are teaching depends entirely on their abilities and skills in the classroom. The increase of the current mean outcome of each topic that has been taught through a tutor intended for individually topic stands:

$$Difference\_Mean_{j,k,i} = r_{j,i}(X_{j,kbest,i} - T_F M_{j,i})$$

where $X_{j,kbest,i}$ is the outcome of the ultimate student who is the tutor in matter j, and depending on capability and skills of a teacher, "T_F" is the instruction aspect that selects the assessment of mean average to be altered.

The random number in the [0, 1] range is $r_{j,i}$. T_F's value can either be 1 or 2. T_F's value is chosen at random with the same possibility as:

$$T_F = round[1 + rand(0,1)\{2 - 1\}]$$

T_F is not a constraint in teaching learning based optimization technique, nor is that one assessment provided as an input to the algorithm; instead, the program uses the aforementioned equation to generate T_F's value at random.

This procedure executes well for T_F attributes of one and two, but it performs much better if the value of T_F varies as per equation above randomly, either value 1 or 2. Throughout that research of teaching learning based optimization technique focused on tutor stage, it is noticed that diverse attributes of T_F are tested by 25 numbers of runs. The mean solution is observed that the algorithm enhanced if the attribute of T_F diverges by way of comparison above arbitrarily, moreover attribute one or two. Rao et al. recommended that educating aspect of T_F attribute to be whichever one or else two subjected to a rounding-up standards specified using the equation above and it is only for simplifying purposes of the TLBO algorithm. Established for the $Difference\_Mean_{j,k,i}$, that current resolution is restructured on a tutor stage;

$$X_{j,k,i} = X_{j,k,i} + Difference\_Mean_{j,k,i}$$

Wherever $X_{j,k,i}$ remains restructured attribute of $X_{j,k,i}$. Admit $X_{j,k,i}$ whatever this one contributes w well utility attribute above $X_{j,k,i}$. Finally, of the tutor stage, entirely recognized function attributes are kept plus recorded to convert the response to the student stage.

2.4 The Learner Phase

It is the second stage of the teaching learning based optimization technique. The second stage called the interaction phase, exchanging information or sharing knowledge phase. In the class room or any learning environment process, learners upsurge their knowledge by communication among themselves. The learning environment process is not systematic or organized process because each learner cooperates and interrelates haphazardly through different students in the classroom for improving his or her knowledge. The student absorbs innovative ideas only whenever another students consume extra ideas and new information to share more than she/he ensures. Bearing in mind an inhabitant's mass of n.

Every student is connected through further students haphazardly, with any iteration i, randomly choice dualistic students X_P and X_q like at $X'_{P,i} \neq X'_{Q,i}$ where $X'_{P,i}$ and $X'_{Q,i}$ are the restructured values at the end of the teacher phase":

$$X''_{j,p,i} = X'_{j,p,i} + r_{j,i}\left(X'_{j,p,i} + X'_{j,Q,i}\right), if\ f\left(X'_{p,i}\right) < f\left(X'_{Q,i}\right),$$

$$X''_{j,p,i} = X'_{j,p,i} + r_{j,i}\left(X'_{j,Q,i} + X'_{j,P,i}\right), if\ f\left(X'_{Q,i}\right) < f\left(X'_{P,i}\right).$$

Whenever that offers such greater utility assessment, accept $X'_{j,p,i}$. At any iteration i , loops are used to carry out the learner phase.

Algorithm 2: Learning enthusiasm based learner phase

```
for k =1:n;
    Allow the current learner to be X'_{j,p,i} ;
    Randomly choice a different learner who is X''_{Q,i}, such that X'_{P,i} ≠ X'_{Q,i} ;
  if f(X'_{p,i}) < f(X'_{Q,i});
   for j=1:m;
                          X''_{j,p,i} = X'_{j,p,i} + r_{j,i}(X'_{j,p,i} − X'_{j,Q,i}).
   end for
else
  for j=1:m               X''_{j,p,i} = X'_{j,p,i} + r_{j,i}(X'_{j,Q,i} − X'_{j,P,i}),
    end for
    end if
end for
```

The effectively acknowledged operate attributes are retained like a contribution at the conclusion of the student stage during a tutor stage of that subsequent repetition [41].

2.5 Multi-Objective Teaching Learning Based Optimization (MOTLBO)

The multi-objective application programs are implemented and studied in greater detail since they accurately reflect the conditions that exist in the real world. Multi-objective tasks contain more than one objective, each of which is of the best range, as opposed to single-objective problems, which have only one objective and one optimal solution. Because of this, multi-objective problems have a set of ideal solutions known as the Pareto-Front, from which the designer must choose the best one based on the importance of the objective [42].

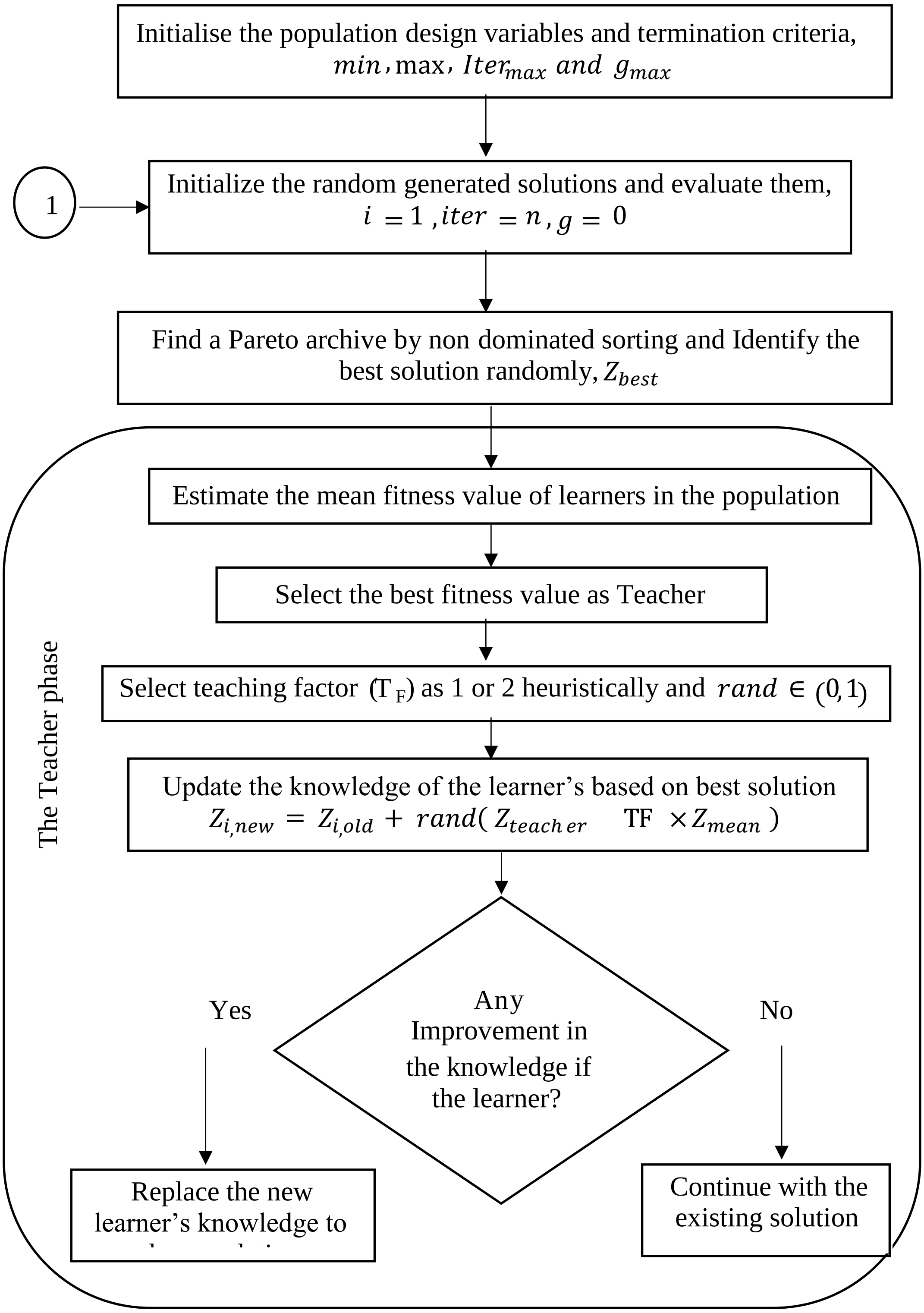
Metaheuristics have been widely used to address a variety of technical, scientific, and industrial issues, and have emerged as a potential best alternative to handle these multi-objective design problems. Because they are gradient free, can handle almost any kind of variable, and don't need any special information about problems, metaheuristics are efficient compared to conventional approaches. They are inspired by natural events. However, they are criticized because of flaws in their design that render them ineffective for issues involving global optimization, such as local optimal traps, premature convergence, the need to tune controlling parameters, least-quality solutions, etc. [43].

Global diversification and local intensification equalization are important aspects of an effective optimization algorithm that still need to be taken into consideration. According to a review of the research, increasing diversification decreases search intensity, and vice versa. However, a good balance between these two factors determines how effective an optimization algorithm is, which encourages researchers to keep looking for algorithms that work better [44]. The authors therefore presented a multi-objective teaching learning based optimization strategy for structuring optimization design issues that is based on the dominant approach in an effort to find a better and more global optimization method.

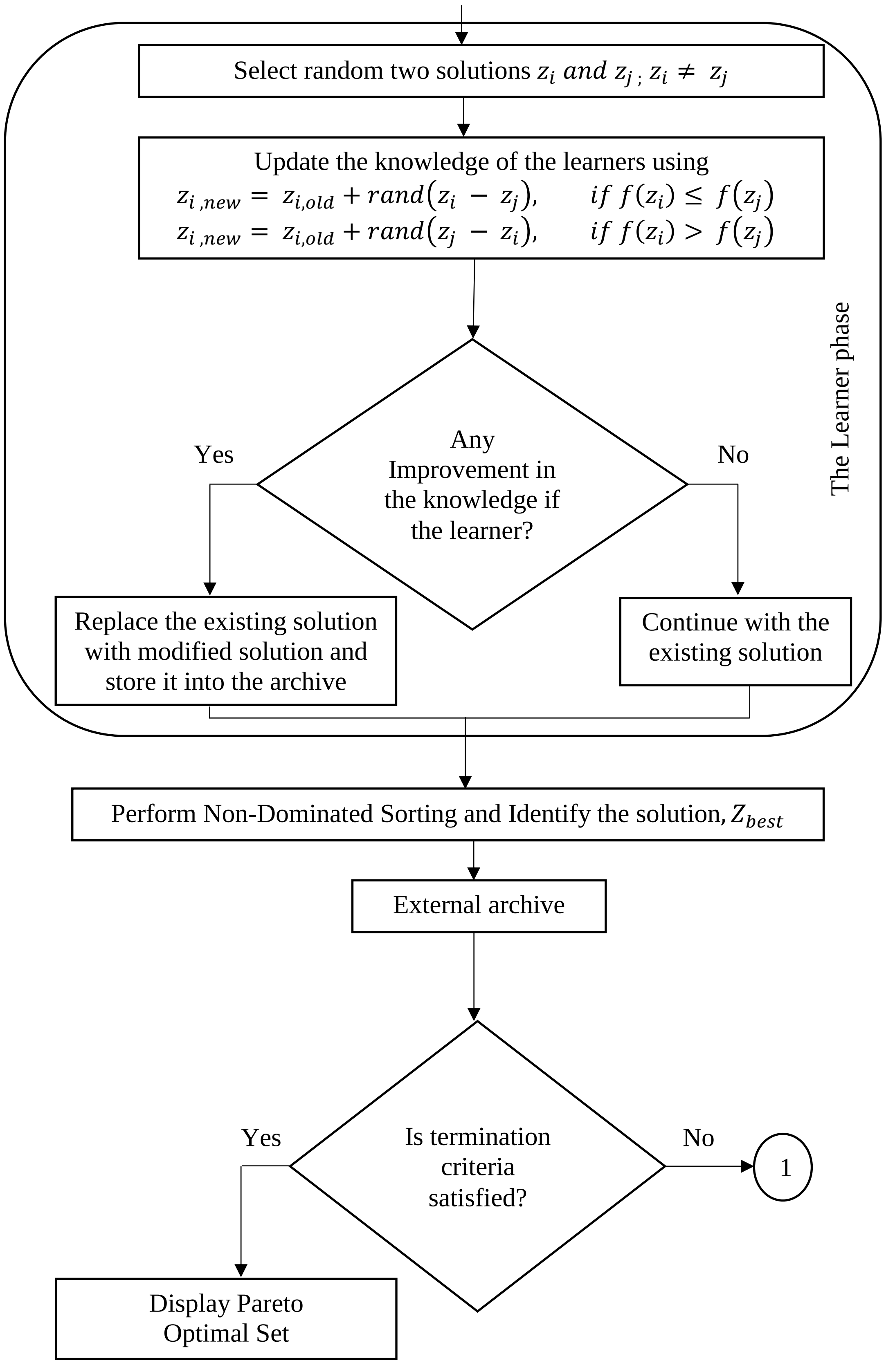
Subsequently for a specified enterprise resolutions x_1 and x_2 while prompts to f_1 and f_2 function vector, x_2 will be named to be dominated by x_1 if two requirements are met, the minimization goal will be achieved. For instance, the initial requirement is that all factors in f_1 are equal to or fewer than their equivalent factors in f_2 . One or more f_1 possible factors must expressly be fewer than their corresponding f_2 component in the second precondition.

Non-dominated sets (NDS) are a proposed solution that is not controlled by other members of the set after the defining of dominance. Multi-objective teaching learning based optimization is structured by the concept of classroom teaching environment which the teachers who are acknowledged as experts and have a major impact on the outcomes or grades of the students. With such technique, the knowledge that the teacher imparts to the class, as well as the interaction amongst students regarding their shared knowledge, has a significant impact on the learner's output. In multi-objective teaching learning based optimization, the population is equal to the group of learners. A specific function is given to them as design factors, fitness is equal to the grades achieved by the students. Then, a desired solution is identified and recognized as the teacher.

The multi-objective teaching learning based optimization structure produces a real-time resolution for each objective, controls the non-dominated sets then save them in an exterior record to produce Pareto front. Every time a new set of improved alternatives is generated, the exterior record is upgraded, the old resolution is discarded, and therefore only non-dominated sets remain in the archiving at the end. This helps produce a wide variety of Pareto optimal solutions. This method outperforms search strategy while instantaneously treating all non-dominated sets evenly. The goal is to maintain the diversity of the Pareto solution sets while producing a Pareto solution set that is close to the actual Pareto solutions.



Initialise the population design variables and termination criteria,
min, max, Itermax and gmax
1
Initialize the random generated solutions and evaluate them,
i = 1, iter = n, g = 0
Find a Pareto archive by non dominated sorting and Identify the best solution randomly, Zbest
Estimate the mean fitness value of learners in the population
Select the best fitness value as Teacher
Select teaching factor (TF) as 1 or 2 heuristically and rand ∈ (0,1)
Update the knowledge of the learner's based on best solution
Zi,new = Zi,old + rand(Zteacher TF × Zmean)
The Teacher phase
Yes
Any Improvement in the knowledge if the learner?
No
Replace the new learner's knowledge to
Continue with the existing solution



Flow chart of Multi-Objective Teaching Learning Based Optimization (MOTLBO) Algorithm [42].

Chapter 3

METHODOLOGY

3.1 Overview

In this study, testing functions are significant to confirm and equate the act of optimization algorithms. The test of consistency, effectiveness and justification of optimization algorithms is commonly supported out by using a selected group of common standard benchmarks. There are many benchmark functions reported in the search area which they have a major importance to algorithm analysis because they help for improved comprehension of the advantages and disadvantages of evolutionary algorithms. Benchmark functions have varied properties and features which open a way to correctly test optimization algorithms in such an equitable and balanced method so, benchmark functions are very ultimate for testing because they have no standard set or list [44].

With determination, we have studied and compiled a total of 14 benchmark functions that are incorporating various properties. These 14 benchmark functions symbolize various formulation settings, like period reliant on Pareto-Set hyperbolic geometry and Pareto-Front forms, irregular Pareto-Front shapes, large-scale of variables, dis-connectivity, and dynamically changed environment and so on. We want to advance study into evolutionary dynamic multi-objective optimization by transmission learning which can reprocess previous experiences to solve related source problems, Dynamic multi-objective problems (DMOPs) are widely used in evolutionary optimization. We want build optimization and see how good it performs and compares multi-objective TLBO to others [45].

With any environmental changes with the DMOPs types of problem, directly the environmental changes can be sensed with an experimental review of a random population member so, there is a necessary desire to handle and trace the balance between time restriction and time changing to carefully track Pareto fronts on DMOPs. The first part of the formulation should result a true Pareto-front and then concepts the benchmark equivalent to that, staring by getting an initial population for better performance. Resolving the DMOPs should be solved by a few computational costs and get a good final approximation population member to reach to the best solution.

With benchmark problems, we know the real PF that are only partial points depending on rather we have continuous or discrete input variables it can only be points or at the end would be a line, but we can see it's the best solutions resulting out of the optimization problem we have [45].

3.2 MTLBO Operation

In this segment, the multi-objective stimulating benchmarks is going to be studied and compared with other inhabitants founded well experienced approaches to rate the productivity of multi-objective teaching learning based optimization. In order to resolve the 14 benchmark problems at hand, the multi-objective TLBO simulates the processes of instruction and learning in a classroom environment. Both tutor and the student stage are into advancement. Getting a population from the classroom environment, staring from teacher phase which by nominating learners from the population to get to identify which one is the best graded student who would have been nominated to be the teacher. The teacher has two main responsibilities that are training and grading. Firstly, the teacher is in charge for teaching the learners. Secondly, the teacher is responsible for improving the mean grade in the classroom [42, 45].

Secondly, going through the learner phase which it is about randomization chooses and interactions. Each learner chooses an acquaintance at random with whom to interact. While the interaction part, the two learners learn from each other and share their knowledge together. During this phase, our objective is to increase the mean grade in the classroom [46]. These processes are reiterated according to the stopping criteria in the benchmark problems parameters is met.

3.3 Configure Parameters

The parameters have taken into account in the experiment from their original papers [45]. There are two key constraints of teaching learning based optimization method that has been included which they are classification parameters and control parameters. The inhabitants dimension , n was fixed to one hundred for both multi-objective cases which they are from problem DF1 to problem DF9 and three-objective cases which they are from problem DF10 to problem DF14.

Number of variables and strength of change n_t both are equivalent towards ten. The number of changes is equivalent towards 30. Independent runs are equal to 20 on independent times on each test occurrence. Depending on the changing of environments, if environments change very quickly the number of frequency of change T_t is going to be 10, if it is the opposite, it is going be 30. Stopping conditions and the number of executions is $100(30T_t + 50)$ where T_t is the number of frequency of changes, with TLBO algorithm with all possible changes during the process should be covered especially during the learner phase, it ends with an assigned number of generations before the first environmental change occurs [47]. Additional constraints. All the constraints in the associated algorithms castoff the matching sets as in their main papers [45].

Algorithm 3 :Pseudo Code of MTLBO Algorithm

for i=1 to D
$a \leftarrow selesct\ a\ random\ interger\ in\ (1\ ,2\ , \ldots .\ , NP)$
Initialize learners and evaluate them;
$M(i) = X''(i)$;
End
for k=1 to NP
if $r_1 < 0.5$ // **Teaching**
for i=1 to D
if $r_2 <$ SP
$T_F(i) = round(1 + r_2)$;
$X_{new}\ (i) = X_k(i) + r_3.\left(x_{best}(i) - T_F(i).M(i)\right)$;,
End
Endfor
else // **learning from each other between learners**
for i=1 to D
$r \leftarrow selesct\ a\ random\ interger\ in\ (1\ ,2\ , \ldots .\ , NP)$
if
$M(i) = X''(i)$;
End
for k=1 to NP
if $r_1 < 0.5$ // **Teaching**
for i=1 to D
if $X_r\ is\ better\ than\ X_k$
$X_{new}\ (i) = X_k(i) + r_4.\left(x_r(i) - x_k(i)\right)$;,
Else
$X_{new}\ (i) = X_k(i) + r_5.\left(x_k(i) - x_r(i)\right)$;,
End
Endif
end
End
if $X_{new}\ is\ better\ than\ X_k$
$X_k = X_{new}$
End

That inhabitant's dimension (Pareto-Set) plus the max quantity of group of iterations GN_{max} stand among the controller constraints by teaching learning based optimization method.

Set up student X as the inhabitants starting point = $\{X_{11}, X_{12}, \ldots \ldots X_{i,k}, \ldots . X_{n,m}\}$, which $X_{i,k}$ represents the assigned sum amount of confrontation of i on k. The sum amount of confrontation were produced by using;

$X_{i,k} = X_{\min i,k} + rand(0,1)(X_{\max i,k} - X_{\min i,k})$ Which $X_{\min i,k}$ plus $X_{\max i,k}$ Accordingly, a greater and inferior limits serve as symbols for $X_{i,k}$, and the random integer in the range [0, 1] is represented by rand (0, 1). It should be noted that all students who identify as $X_{i,k}$ must adhere to the requirements [48].

To accomplish the main objective of the process in an experiment, each learner's grade is first valued. This is done by converting the decision variables of the learner's $X_{i,k}$ value into the objective function, allowing us to obtain the objective value, which is solely based on the learner's grade. Next, a maximum evaluation student is chosen to be the tutor plus their grade is customary with X_{best}.

The first teacher phase which it is all about improvement and grading, m is going to be the mean grade of the classroom. During the teacher phase, the teacher tries to improve the average evaluation on students to his/her level X_{best}. Certainly, there is a difference in knowledge and experience among tutor and students that may remain stated with $Difference\_Mean_{.} = r(X_{best} - T_F M)$ where T_F is the education aspect, its value is selected as one otherwise two, plus it was unsystematically definite by an equivalent possibility $T_F = \text{round } [1 + \text{rand } (0,1)\ (2-1)]$. T_F chooses an average evaluation to stay altered, plus r is an unsystematic quantity into zero and one variety. From that above difference attributes, $X_{new,i,k} = X_{i,k} + Difference\_Mean_{.}$ where the current resolution X is modernized into a tutor stage. After updating that solution, it is now necessary to evaluate it to see if X_{new} offers a well utility attribute than X plus complies with a control parameters. If yes, use X_{new} to update the solution; if not, stick with X [48].

The second step, known as the learner phase, is where students choose other students at random from whom they would share their expertise through interaction. For the purpose of enhancing their learning experiences, students are certified to exchange information at random with other students throughout this phase. If the other students in the class have a greater level of comprehension than the learner has, then the learner will undoubtedly pick up new information and content this manner.

On the other hand, if the other learners do not have additional expertise, the student will not acquire any new knowledge or information. To execute this phase, we are going to let X_a be the other learner who has been selected randomly. The learning of X (recipient learner) from X_a (selected learner who has more knowledge) is mathematically conveyed as in

$$X_{new,i,k} = \{ X_{i,k} + \mathrm{r}\,(X_{i,k} - X_{a,i,k})\,, if\ f\left(X_{i,k}\right) < f\left(X_{a,i,k}\right),$$

$$X_{new,i,k} = \{ X_{i,k} + \mathrm{r}\,(X_{a,i,k} - X_{i,k})\,, if\ f\left(X_{i,k}\right) \geq f\left(X_{a,i,k}\right),$$

Learners who has higher evaluation among the original student in addition to the recently created student is going to remain acknowledged concurrently with that instructor phase [48].

The teaching learning based optimization algorithm completes plus productions of the existing resolution that has been regarded as great resolution from the group of iterations, when a maximum number by generations is achieved, if the stopping conditions are satisfied. If not, assess each student's grade before moving on to the next instructor phase [48].

3.4 Compared Algorithms

In determining problematic comprehensive optimization problems, there are so many surveys in the literature only works for the comparison between optimization algorithms to search and identify which algorithms is going reach the best solution. From the original paper intended for resolving dynamic multi-objective problems , a methodology for testing problems which the set of the 14 benchmark functions has been tested with serval algorithms with their variety of different dynamic characteristics with keeping the same existing approaches and parameter settings as the proposed parameters from the original paper [45]. Here are some brief descriptions of these algorithms to know their techniques and to get to know what the difference between these algorithms to compare them with multi-objective TLBO algorithm after been done with the experimental results.

3.4.1 Population Prediction Strategy (PPS)

In both cooperative DMOOs, singular DOOs and cooperative, prediction-based techniques have proven useful for predicting some inaccessible sites [49]. By accounting for the presence of continuously occurring dynamic multi-objective optimization problems, researchers expand this indicator to compute a whole population [50]. The key clue of Population Prediction Strategy (PPS) is to split of the PS that is separated within dual measures which are core population objection as well as a multiplex [51].

In order to calculate the next center and estimate the next center's manifold population, a structure of population center points is retained. So, Population Prediction Strategy possibly will set a full population by uniting the anticipated point and projected manifold once a modification is sensed [52]. A period sequence of earlier population centers will be used to identify another inhabitant's midpoint, the auto regression (AR) model has been used [53].

Correspondingly, old diversified population are also recycled to calculate new diversified population. At that point, an innovative inhabitance resolve to be accumulated grounded on the expected inhabitance center besides diversified population [54].

3.4.2 Transfer Learning Based DMOEA (TrDMOEA)

The inhabitants based evolutionary algorithms and transfer learning are both utilized within transmission knowledge founded interactive several priorities generative technique, which is the framework for attempting to solve interactive several priorities generative optimization issues. An essential factor for understanding here is that algorithmic framework effect at diverse periods have altered deliveries for producing an operative examination population. TrDMOEA feats the transfer learning method by means of an instrument to produce an actual preliminary population group through reprocessing previous understanding to rapidity the evolutionary procedure. Therefore, some inhabitants founded multi-objective procedures may advantage by its addition deprived of requiring a substantial adjustments, and there are no significant changes required [55].

3.4.3 Mixture of Experts (MOE)

There is always a request to improve the population prediction framework because it is challenging when it comes to the distribution of suitable loads to respectively of the mechanisms at diverse times [56]. The loads assigned to the prediction mechanisms will change as a result of the dynamic development and the mechanism with a better estimate accuracy must be assumed advanced load more than other mechanisms. So, the mixture of experts (MOE) framework considered to a modest operative collaborative construction that allows proficient converting among mechanisms in the collaborative based on their virtual act for producing strong POS and improving the complete prediction excellence in cooperation with dynamic multi objective optimization problems [57].

A MOE found to be a widely held ensemble structural design in machine learning because of its quickness, strength, and accurateness [58] in resolving multifaceted reversion, organization problems, language demonstrating and system identification difficulties. Initially, the MOE framework is intended for static data depend on statistical individuality of the training sets [59]. The MOE framework correspondingly has been effectively utilized to several time sequence modeling presentations, in addition to actual time controller and graphic applications [60].

3.4.4 Multi Objective Evolutionary Algorithm based on Decomposition for First Order Difference Model (MOEA/D-FD)

The initial command derivative method that relies on Multi Objective Evolutionary Algorithm based on Decomposition algorithm (MOEA/D-FD) is a common resolution structure for algorithm. This algorithm corrupts a MOOP within a numeral by a particular reasonable enhancement substitute problems otherwise modest MOOPs in addition at that time procedures a heuristic examination to enhance these optimization substitute problems concurrently and accommodatingly [61].

In particular, MOEA/D-FD algorithm operates old data to calculate the position of the innovative POS subsequently an alteration is sensed. The innovative population is collected of dual types of resolutions which they are the dated and the anticipated results. The association of population intermediate point outlines as an expected direction. In the direction of creating the innovative population expanded so it should be consistently circulated characters nominated from the earlier population are utilized in the prediction procedure [62].

3.4.5 Fitting based Prediction Algorithm (FBP)

For the purpose of continuous research of attempting to interactive several priorities generative problems through a revised estimation predicated optimization system, researchers have proposed an algorithm in 2021 called Fitting based prediction algorithm as an innovative and incorporating a novel machinery that holds a consistency pattern built on a multi-"regularity model-based multi-objective estimation of distribution algorithm (RM-MEDA) " is integrated into suchlike maximizing several objectives in dynamical situations[63].

An approach that includes two stages for forecasting non-dominated solutions, modeling and the curve fitting scheme, the prediction constructed response mechanism targets to produce a great value population when changes occur. It primarily consists of three different subpopulations for effectively following the affecting Pareto optimal set. For each section has an essential part for generate well feature population, refining whichever assortment or conjunction, when a modification take place in the background surroundings.

The primary subpopulation is shaped by a modest linear forecast symbol with dual diverse phase dimensions. A few novel selection entities created by the fitting-based prediction technique are included in the following subpopulation. By implementing a current selection strategy, the next subgroup is created, providing virtually functional examination entities for improving population congregation and multiplicity. The outcome of the experiment that made with an innovative fitting based prediction (FBP) algorithm have reasonable success paralleled with utilizing a few cutting-edge techniques and presented an anticipated procedure has a respectable following capability and replies rapid to environmental alterations [45].

Chapter 4

EXPERIMENTAL RESULT AND EVALUATIONS

4.1 Introduction

A thorough explanation of the algorithms employed in this thesis is presented in this chapter. Implementation appraisal of the TLBO algorithm and the demonstration of the equivalent achievement established separately from the usual meta-heuristics is to be assumed within the point of reference of tested united of CEC2018 contest on DMOPs [64]. It contains information on benchmark functions, performance measures, comparing methodologies, parameter settings, and monetary outcomes. The suggested value for each parameter remains the same based on the literature.

4.2 CEC2018 Benchmark Dynamic Multi-objective Optimization Test Problems

CEC2018 benchmark DMOPs has a challenging difficulties that accords with numerous goals and phases subjected possessions [64]. Throughout the testing phase with all the different algorithms not only multi-objective TLBO, there are three main challenges which first is the environmental alterations where a selection of dynamics position changed stages of problems to algorithms plus there is not any solitary alteration response mechanism that is able to process all dynamics.

Additional, multiplicity which it is the main sensitive incentive of population based algorithms that it is difficult to be fully preserved. Lastly, Time limit on dynamic multi-objective problems necessitates algorithms to spread a respectable stability between diversity and convergence where with any environmental changes are able to be on time promptly controlled to be strictly pursue time fluctuating Pareto fronts or sets that sometimes is rather tight for algorithms. CEC2018 Benchmark problems are important for test and better understanding to TLBO and other algorithm analysis.

CEC2018 benchmark problems are total of fourteen functions are layering varied possessions including period reliant on Pareto-Front and Pareto-Set hyperbolic geometry, ambiguous Pareto-Front forms, dis-connectivity and hyperextension. To test the multi-objective TLBO algorithm on the benchmark problems, we have been implemented in MATLAB code. The CEC2018 proposed test set called DF that have 9 nine with two objectives from DF1 till DF9 and 5 three objective problems from DF10 till DF14. The key dynamic features of CEC2018 Benchmark problems that separately problem contains are presented in Table 4.1 [64].

Table 4.1: The key dynamic features of CEC2018 Benchmark problems

Problems	No. objectives	Dynamic Functions	Notes
DF1	2	Concavity and convexity are mixed, optimum location.	Dynamic Pareto-Front and Pareto-Set
DF2	2	Variable associated to position changed, optimum location	Static convex Pareto-Front, dynamic Pareto-Set, severe diversity loss.
DF3	2	Concavity and convexity are mixed, Varying connection, and optimum location.	"Dynamic Pareto-Front and Pareto-Set".
DF4	2	"Varying connection, Pareto-Front portions, bounds of Pareto-Set".	"Dynamic Pareto-Front and Pareto-Set".
DF5	2	Quantity of knee portions, optimum location.	"Dynamic Pareto-Front and Pareto-Set".
DF6	2	Concavity and convexity are mixed, multimodality, optimum location.	"Dynamic Pareto-Front and Pareto-Set".
DF7	2	"Pareto-Front portions, optimum location".	Convex Pareto-Front, static Pareto-Set centroid,

			dynamic Pareto-Front and PS.
DF8	2	Concavity and convexity are mixed, distribution of solutions, location of optima.	Static Pareto-Set centroid, dynamic Pareto-Front and PS, varying connection.
DF9	2	Quantity of disconnected Pareto-Front segments, optimum location.	“Dynamic Pareto-Set and Pareto-Front, varying connection“.
DF10	3	Concavity and convexity are mixed, optimum location.	“Dynamic Pareto-Set and Pareto-Front, varying connection“.
DF11	3	Size of Pareto-Front portions, Pareto-Front portions, optimum location.	“Dynamic Pareto-Set and Pareto-Front, concave Pareto-Front, varying connection“.
DF12	3	Quantity of Pareto-Front holes, optimum location.	“Dynamic Pareto-Set and Pareto-Front, concave Pareto-Front, varying connection“.
DF13	3	“Quantity of disconnected Pareto-Front segments, optimum location“.	“Dynamic Pareto-Set and Pareto-Front, the Pareto-Front can be a continuous convex or concave segment, or several disconnected segments“.
DF14	3	Degenerate Pareto-Front, Quantity of knee portions, optimum location.	Dynamic Pareto-Set and Pareto-Front, varying connection.

4.2.1 Problems Definition

The subsequent representations are extensively utilized in each problem definition such as M is a quantity of objects that alterations starting by two to three depends on each problem number objectives of each problem, n is the sum of deciding factors, x_i “is the i-th“deciding factors, f_i “is the i-th“aim factors, T is the a production reverse, T_t is frequency of change, n_t is the degree of the alteration and t represents time instant [45].

4.2.1.1 DF1

Due of its small dynamic on the Pareto-Set and its Pareto-Front geometric deviations from contour to convex hull and conversely, the DF1 issue is utilized to measure the capability of concavity distinctions to be pursued.

4.2.1.2 DF2

DF2 has a straightforward dynamic into PS, and the second problem's PF residues indeclinable through the time period. Yet, the adjustment of the station connected decision variable x_r is dynamic challenging because it able to make severe multiplicity damage to population. Towards a solution to DF2, must consider to maintain and boost techniques to reach a well diversity preservation.

4.2.1.3 DF3

The characteristics of DF3 is a combination of convexity or concavity, inconstant connections, and position of optimal solution. This problem is utilized to test the measure the tracing capability of concavity or convexity distinctions and time fluctuating inconstant relations. The features of DF3 which are concavity or convexity contrasts, and the variables gets associated when the time is completed.

4.2.1.4 DF4

The dynamic problem DF4 has changing aspects on equally the PF plus PS. Throughout the time, the distance and location of the Pareto-Set deviations. The extension and bending of the Pareto-Front division has a time fluctuating.

4.2.1.5 DF5

This DF5 problem's remarks are dynamic Pareto-Front and Pareto-Set. For Pareto-Set is relatively common, but the Pareto-Front has geometries changing through the course of time phase. Because one the main challenging through my testing that sometimes the Pareto-Front can be linear and through other times it comprises numerous domestic concave or convex section that because it is reliant on time.

4.2.1.6 DF6

Remarkably for DF6 problem, The DF6 problem's design has produced a clear knowledge of how well algorithms function when their properties are satisfied dynamically. Naturally, Pareto-Front geometry changes with time, and Pareto-Front have knee facts and extended conclusions.

4.2.1.7 DF7

Irregularly, the Pareto-Front series of DF7 is measured and have alterations over time. Although, Pareto-Set is dynamic but PS center points stays not effected or changed. Throughout the research, it has been found that this type of assets is sometimes problematic for center points that based on prediction techniques.

4.2.1.8 DF8

Through the study of DF8 problem, it has a fixed Pareto-Set center points. It is not easy to approximate the Pareto-Set because it contrasts over time. The complete PF geometrical adjustments among concavity-convexity, plus comprises hyperextension states, so it gets a circulation of solutions.

4.2.1.9 DF9

The DF9 is the last problem with two objectives problems. DF9 has subordinated between variables. There are time-variable amounts of broken Pareto-Front sections in this issue.

4.2.1.10 DF10

The DF10 problem has three number of objectives. The most obvious and evident tasks of this DF10 problem postures to for example with multi-objective TLBO algorithm is to find a way to preserve consistency of resolutions on the desperately formed

Pareto-Front at approximately period stages. However, it has a fixed and not changed Pareto-Set center points in malice of the difference of the Pareto-Set situation. Its Pareto-Front geometry alterations from convexity to concavity, and vice versa.

4.2.1.11 DF11

With the DF11 problem features are the extent of Pareto-Front state the time changing decrease or increase of the Pareto-Front section. Moreover, the Pareto-Front transfers concluded over time absent from and near to the derivation.

4.2.1.12 DF12

The DF12 problem has a time changing amount of Pareto-Front gaps that may cause a difficultly for degeneration and decomposition reaction with working mass vectors procedures because mass vectors are missed if they occur to go through the gaps.

4.2.1.13 DF13

The DF13 problem produces an equally nonstop and disengaged Pareto-Front geometries. The amount of separated Pareto-Front sections contrasts with time. The DF13 problem is cooperative for an improved recognition of the influence of dis-connectivity on procedures.

4.2.1.14 DF14

The last problem is DF14 that its dynamics is the altering measurement and dimension of the Pareto-Front. The Pareto-Front may be deteriorated into a 1-D various. Once the Pareto-Front is not debased, the size measurement of the 2-D Pareto-Front various changes through time and then the sum number of knee states deviations consequently.

4.3 Experimental Setup

The apparatus for the experiment in the problem testing were done with the Windows 10 with 12.0 GB RAM and Intel (R) Core(TM) i5-4210U CPU @ 1.70GHz 2.40GHz. TLBO was coded in MATLAB R2020a.

4.4 Experimental Results

Through experimental evaluation of the CEC2018 benchmark functions, this section evaluates the development of the TLBO technique. Teaching-Learning-Based Optimization utilized dual dynamic operatives which a tutor and learning stage to reach to the best solutions [65]. The TLBO versions give all students an equal opportunity to receive knowledge by a tutor during tutor stage or by different students during the learner phase while interacting with one another in the surroundings. The goal is to obtain multi-objective TLBO algorithm outputs that fundamentally differ from those produced by other algorithms, such as PPS, TrDMOEA, MOE, MOEA/D-FD and FBP. All parameter settings retain the equivalent with the proposal according to the works [45]. The quantity calculated by the iteration and inhabitants dimension n are equals towards one hundred.

In benchmarking functions that are set to different values, the magnitude of change n_t and the periodicity of transformation T_t both considered to be critical factors. Compared methods are PPS, TrDMOEA, MOE, MOEA/D-FD, FBP and TLBO is measured by the performance indicators mean inverted generational distance (MIGD) where average inverted generational distance (IGD) value of each generation calculated by using all obtained values.

At the beginning, as it shows in the table 4.2 "the mean and standard deviation values of mean computed for the inverted generational distance" only by TLBO algorithm while T_t= ten or thirty and n_t=ten.

Table 4.2: Mean and standard deviation values of MIGD obtained by TLBO.

Problems	T_t	MIGD (mean(std.))
DF1	10	5.547559e-04(1.152445335e-06)
	30	5.550688e-04(1.130365634e-06)
DF2	10	4.977080e-04(1.187683974e-19)
	30	4.977080e-04(1.187683974e-19)
DF3	10	5.533056e-04(3.345533385e-06)
	30	5.522105e-04 (2.342609196e-06)
DF4	10	1.158211e-01(1.134807594e-03)
	30	1.160380e-01(6.285690996e-04)
DF5	10	5.450574e-04(0.000000e+00)
	30	5.617553e-04(1.187684e-19)
DF6	10	5.345085e-04(1.725729403e-04)
	30	5.872768e-04(1.745424612e-04)
DF7	10	8.691893e-04(1.564040065e-04)
	30	9.113349e-04(1.400583352e-04)
DF8	10	5.921441e-04(3.573923657e-05)
	30	5.398238e-04(5.678381361e-05)
DF9	10	1.114255e-03(1.706886289e-04)
	30	1.132780e-03(1.775577148e-04)
DF10	10	3.108685e-04(5.173776964e-05)
	30	3.538017e-04(3.269924124e-06)
DF11	10	8.548749e-03(2.970231882e-03)
	30	7.459446e-03(2.415660505e-03)
DF12	10	4.331565e-04(4.730526588e-05)

	30	4.327370e-04(4.704364441e-05)
DF13	10	6.686750e-03(8.302527373e-03)
	30	6.518140e-03(5.016824512e-03)
DF14	10	1.936050e-04(7.184213433e-05)
	30	2.163864e-04(5.544607406e-05)

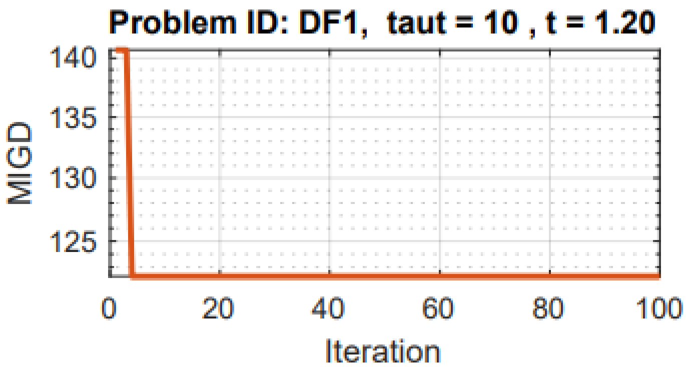


Figure 4.1: The Plot of PF with T_t=10 for DF1

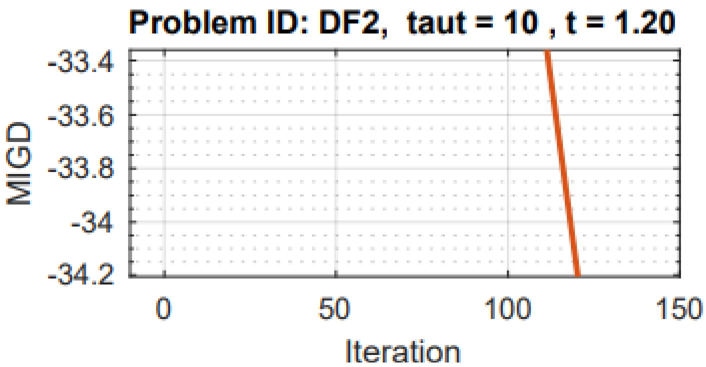


Figure 4.2: The Plot of PF with T_t=10 for DF2

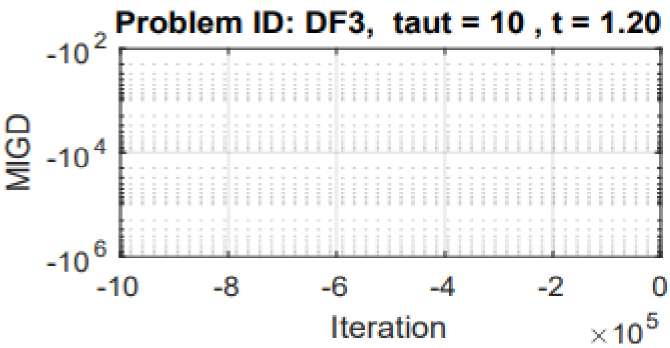


Figure 4.3: The Plot of PF with T_t=10 for DF3

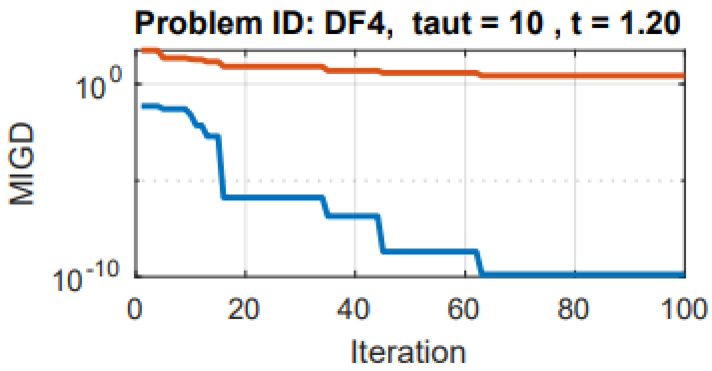


Figure 4.4: The Plot of PF with T_t=10 for DF4

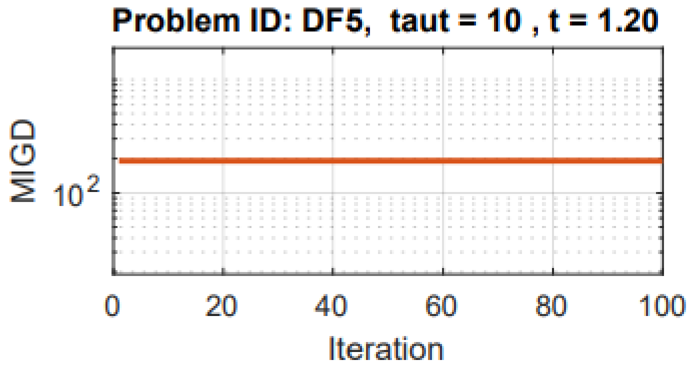


Figure 4.5: The Plot of PF with T_t=10 for DF5

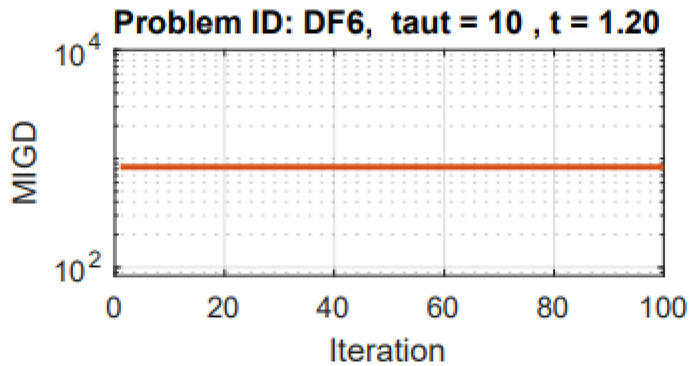


Figure 4.6: The Plot of PF with T_t=10 for DF6

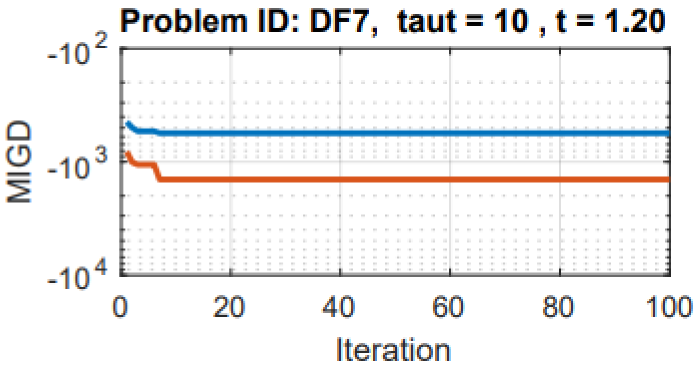


Figure 4.7: The Plot of PF with T_t=10 for DF7

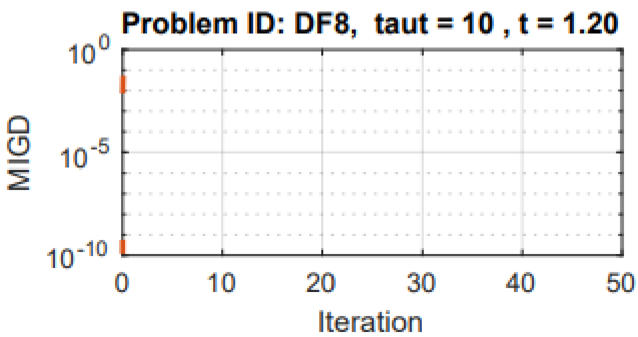


Figure 4.8: The Plot of PF with T_t=10 for DF8

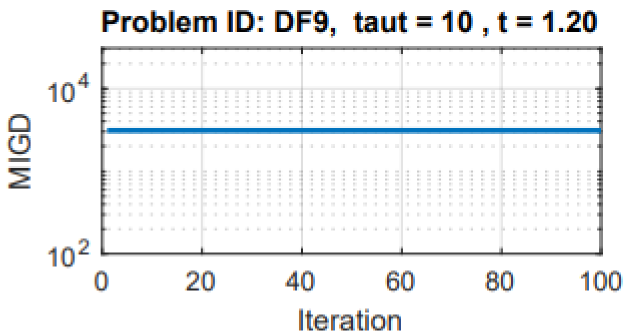


Figure 4.9: The Plot of PF with T_t=10 for DF9

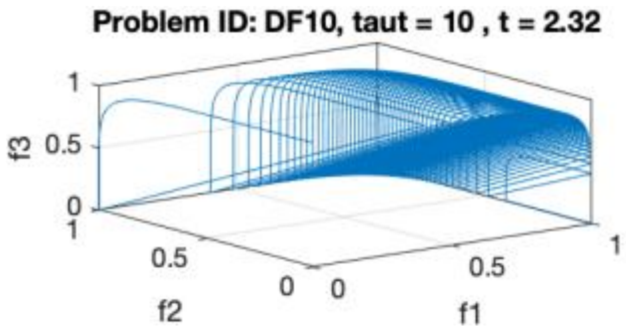


Figure 4.10: The Plot of PF with T_t=10 for DF10

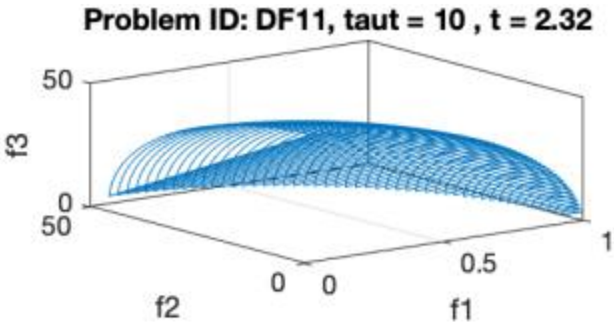


Figure 4.11: The Plot of PF with T_t=10 for DF11

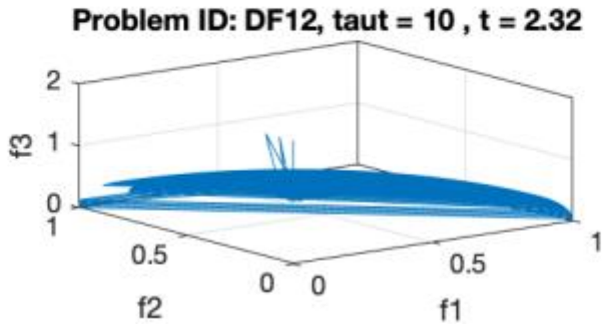


Figure 4.12: The Plot of PF with T_t=10 for DF12

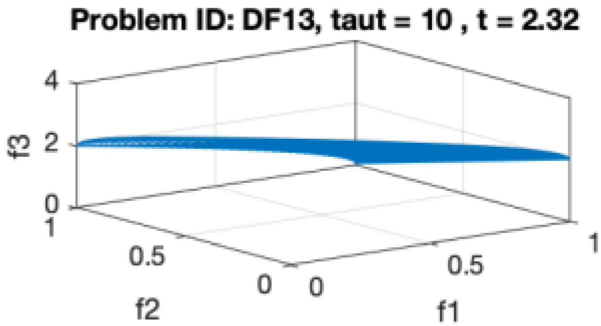


Figure 4.13: The Plot of PF with T_t=10 for DF13

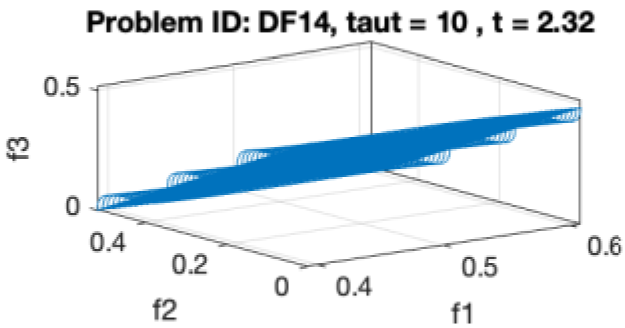


Figure 4.14: The Plot of PF with T_t=10 for DF14

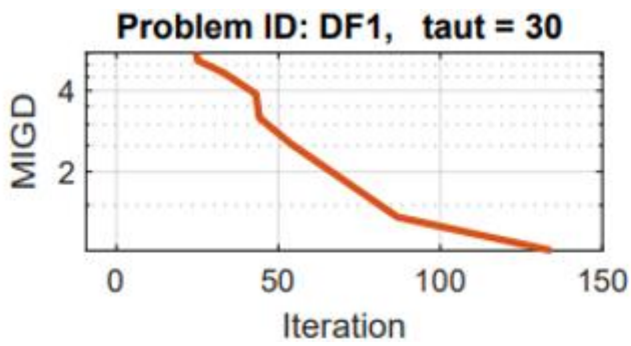


Figure 4.15: The Plot of PF with T_t=30 for DF1

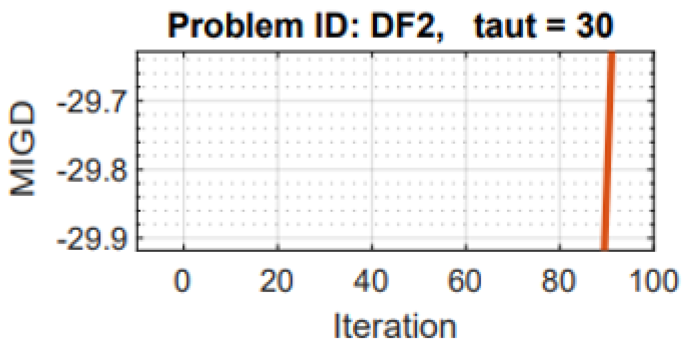


Figure 4.16: The Plot of PF with T_t=30 for DF2

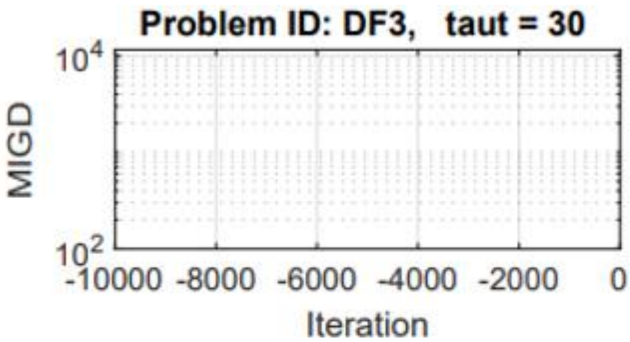


Figure 4.17: The Plot of PF with T_t=30 for DF3

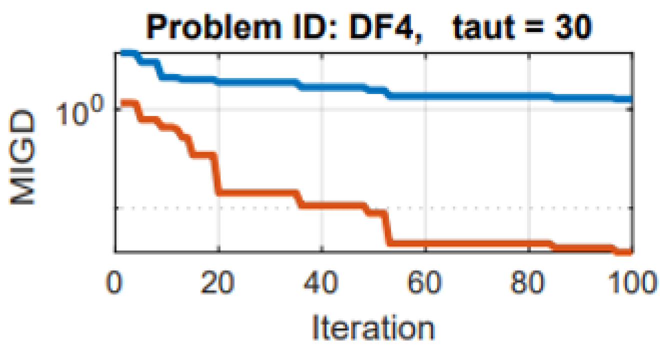


Figure 4.18: The Plot of PF with T_t=30 for DF4

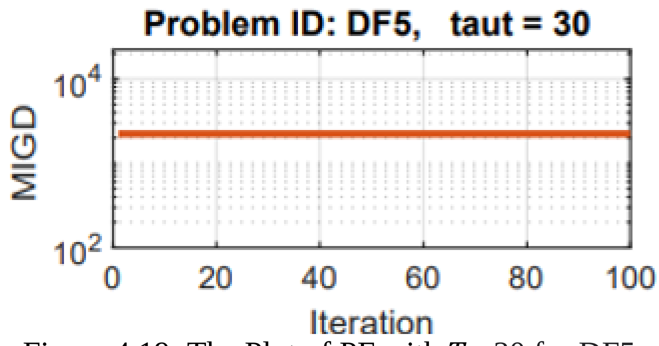


Figure 4.19: The Plot of PF with T_t=30 for DF5

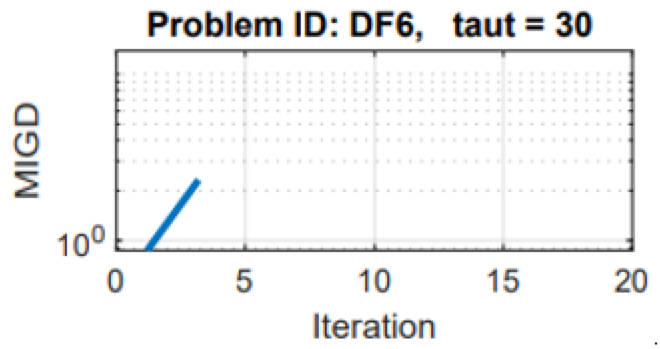


Figure 4.20: The Plot of PF with T_t=30 for DF6

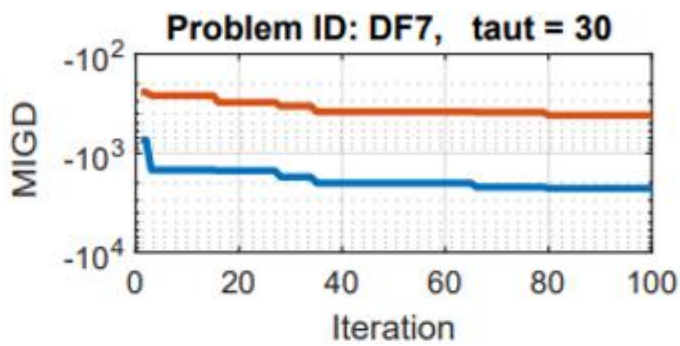


Figure 4.21: The Plot of PF with T_t=30 for DF7

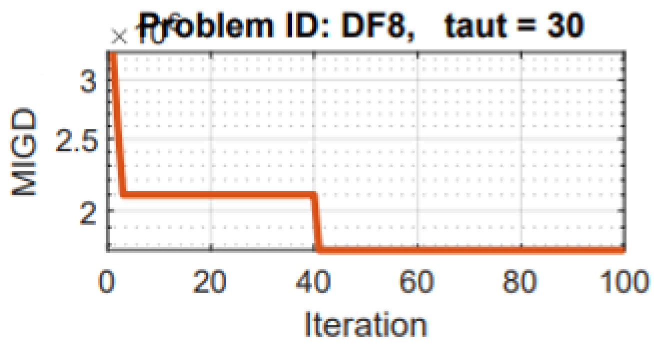


Figure 4.22: The Plot of PF with T_t=30 for DF8

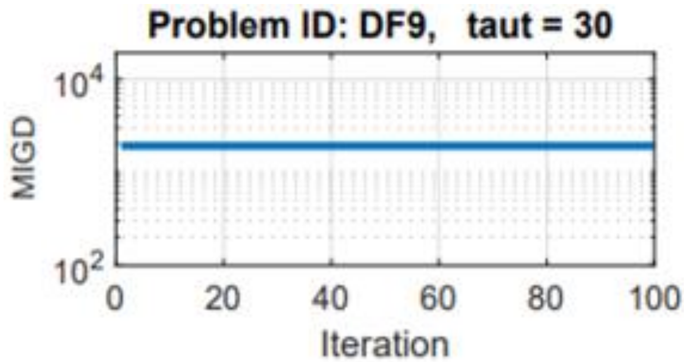


Figure 4.23: The Plot of PF with T_t=30 for DF9

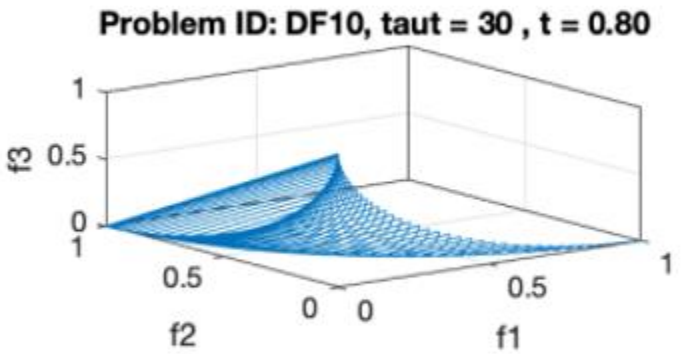


Figure 4.24: The Plot of PF with T_t=30 for DF10

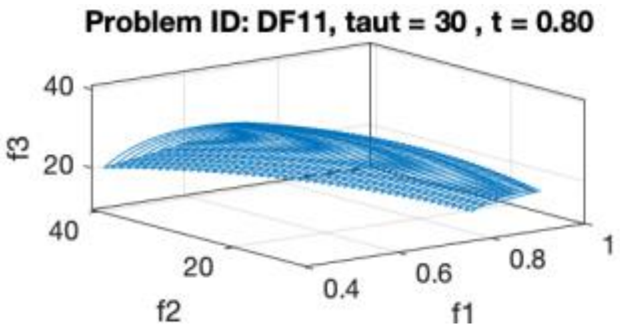


Figure 4.25: The Plot of PF with T_t=30 for DF11

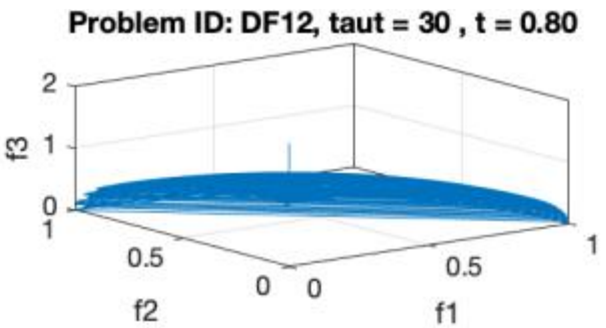


Figure 4.26: The Plot of PF with T_t=30 for DF12

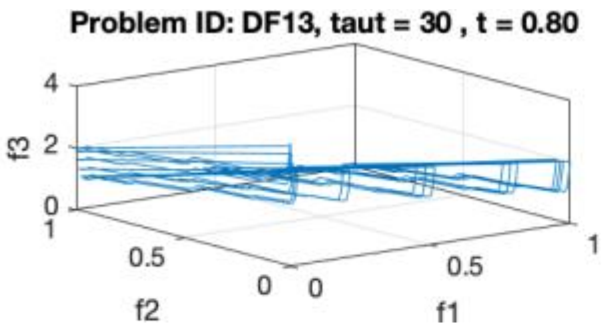


Figure 4.27: The Plot of PF with T_t=30 for DF13

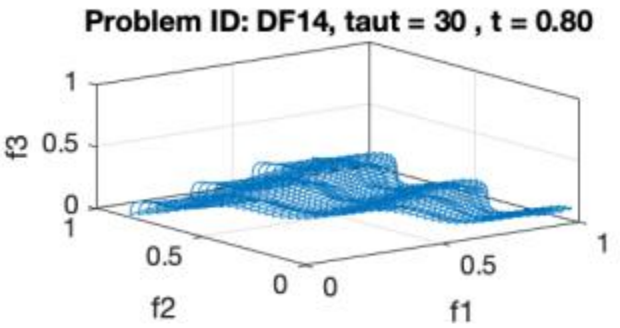


Figure 4.28: The Plot of PF with T_t=30 for DF14

As the evaluation of multi-objective TLBO as its own without comparing TLBO with other algorithms, TLBO algorithm has sculpted a potential function for itself for such an advanced optimization problem. The TLBO algorithm is very recent, yet it has a promising future for solving dynamic optimization issues. The algorithm detailed parameter are less concept which consider as one of the algorithm's appealing characteristics. It is relatively easy to use and administer, and it may produce optimal resolutions within reasonably a reduced amount of operate estimations.

Table 4.3: Mean and standard deviation values of MIGD obtained by six algorithms for $(n_t\ ,\ T_t)$ = (5,20) .

	Algorithms					
Pro. No.	TrDMOEA	PPS	MOE	MOEA/D-FD	FBP	TLBO
DF1	1.777e-2(2.139e-3)	3.668e-1(7.186e-2)	6.941e-3(3.613e-4)	1.179e-2(1.764e-4)	1.065e-2(7.992e-4)	5.543746e-04 (9.791749339e-07)
DF2	6.565e-3(6.454e-4)	2.440e-1(5.131e-2)	1.323e-2(4.886e-4)	1.073e-2(3.404e-4)	4.170e-2(3.022e-3)	4.977080e-04 (1.187683974e-19)
DF3	5.734e-2(1.981e-2)	1.797e-1(1.494e-1)	1.364e-1(1.182e-4)	4.606e-2(4.499e-3)	1.882e-2(6.951e-3)	5.503783e-04 (3.190313768e-06)

DF4	5.872e-1(1.742e-3)	1.370e-1(1.003e-2)	0.145e+0(2.667e-3)	0.986e-1(2.085e-3)	10.635e-1(2.613e-2)	1.147962e-01 (1.260227187e-03)
DF5	2.808e-2(3.792e-4)	3.723e-1(1.041e-1)	1.533e+0(1.063e-3)	2.027e-2(2.061e-4)	1.541e-2(9.571e-4)	5.452075e-04 (2.534679334e-07)
DF6	9.798e-1(2.154e-1)	7.797e+0(8.883e-3)	3.381e+2(2.248e+1)	5.414e+3(4.384e-4)	5.863e-3(2.991e-1)	5.941529e-04 (1.775116595e-04)
DF7	3.829e-2(1.287e-3)	5.436e-2(1.662e-2)	6.859e+2(2.18564e-3)	9.758e-1(2.863e-3)	2.177e-3(2.135e-2)	8.684413e-04 (1.596559291e-04)
DF8	8.208e-2(4.023e-4)	5.431e-1(1.111e-1)	2.657e-1(1.315e-2)	6.731e-1(2.418e-2)	4.728e-1(2.973e-3)	5.861553e-04 (5.905181419e-05)
DF9	9.792e-2(2.423e-3)	1.931e-1(1.144e-2)	2.318e+1(3.182e-4)	8.535e-2(3.959e-2)	10.468e-3(7.891e-2)	1.135222e-03 (1.787326635e-04)
DF10	2.804e-1(6.160e-3)	7.591e-1(2.447e-1)	2.632e-2(5.677e-3)	0.977e-2(4.411e-3)	2.167e-1(2.765e-2)	3.579481e-04 (5.897937628e-06)
DF11	2.846e-1(3.159e-2)	4.123e-1(1.151e-1)	8.630e-1(2.535e-3)	7.414e-1(4.128e-4)	7.474e-1(1.225e-3)	6.305027e-03 (2.954590475e-03)
DF12	3.266e-1(1.545e-2)	5.148e-1(4.258e-1)	3.813e-2(2.401e-2)	8.731e-2(3.021e-1)	1.683e-2(5.338e-4)	4.336009e-04 (4.930979508e-05)
DF13	1.659e-1(2.258e-3)	1.552e-1(1.963e-2)	1.633e+1(2.352e-3)	1.641e-2(2.475e-1)	2.719e-2(5.578e-2)	1.256808e-02 (7.562334478e-03)
DF14	7.204e-2(3.136e-4)	3.668e-1(7.186e-2)	2.148e+1(3.127e-4)	2.282e-0(2.850e-3)	6.612e-0(2.421e-2)	2.517544e-04 (7.239825673e-05)

Table 4.4: Mean and standard deviation values of MIGD obtained by six algorithms for (n_t , T_t) = (10,10) .

	Algorithms					
Pro. No.	TrDMOEA	PPS	MOE	MOEA/D-FD	FBP	TLBO
DF1	8.431e-2(9.136e-2)	3.729e-1(6.416e-2)	1.487e-2(1.322e-3)	9.522e-3(1.371e-4)	5.844e-3(2.682e-4)	5.547559e-04(1.152445335e-06)
DF2	8.149e-3(5.478e-4)	2.261e-1(4.723e-2)	3.718e-2(2.567e-3)	1.097e-2(2.093e-4)	3.937e-2(4.765e-3)	4.977080e-04(1.187683974e-19)
DF3	3.358e-2(1.542e-2)	1.460e-1(1.323e-1)	1.447e-1(9.352e-4)	3.386e-2(2.023e-3)	1.162e-2(5.596e-3)	5.533056e-04(3.345533385e-06)
DF4	5.644e-1(1.407e-1)	1.162e-1(1.222e-2)	2.217e+1(2.849e-2)	2.069e-0(2.686e-3)	8.138e-3(1.5140e-4)	1.158211e-01(1.134807594e-03)
DF5	2.583e-2(4.359e-3)	3.628e-1(9.444e-2)	1.228e+0(2.129e-3)	1.455e-2(2.501e-4)	6.832e-3(7.967e-4)	5.450574e-04(0.000000e+00)
DF6	1.209e+0(2.704e-1)	7.477e+0(7.384e-1)	4.581e+0(6.182e-1)	5.080e+0(5.327e-1)	6.140e-1(4.412e-2)	5.345085e-04(1.725729403e-04)
DF7	3.546e-2(9.378e-4)	6.051e-2(1.616e-2)	2.429e+1(4.465e-2)	9.106e-0(2.418e-1)	8.874e-2(3.261e-3)	8.691893e-04(1.564040065e-04)
DF8	7.954e-2(7.347e-3)	1.569e-2(1.487e-4)	2.782e-0(5.910e-3)	3.053e-1(2.051e-2)	6.263e-4(6.123e-5)	5.921441e-04(3.573923657e-05)
DF9	7.540e-2(1.771e-2)	4.713e-1(1.280e-1)	1.078e+0(3.807e-3)	8.732e-2(1.473e-2)	6.755e-2(5.406e-3)	1.114255e-03(1.706886289e-04)

DF10	2.775e-1(1.289e-2)	1.816e-1(1.075e-3)	2.631e-0(3.819e-2)	2.652e-0(3.667e-1)	2.124e-0(4.832e-4)	3.108685e-04(5.173776964e-05)
DF11	2.877e-1(1.657e-2)	6.551e-1(2.509e-3)	2.141e-2(4.126e-4)	5.373e-2(3.384e-4)	5.247e-0(4.549e-9)	8.548749e-03(2.970231882e-03)
DF12	2.448e-0(3.281e-2)	1.231e-0(9.512e-2)	2.261e-0(4.656e-0)	9.526e-0(2.738e-3)	2.924e-1(1.074e-2)"	4.331565e-04(4.730526588e-05)
DF13	1.542e-1(7.866e-3)	4.057e-1(2.940e-2)	2.107e+0(1.301e-2)	2.239e-1(5.951e-3)	1.500e-1(4.638e-3)	6.686750e-03(8.302527373e-03)
DF14	6.943e-2(3.387e-3)	1.620e-1(2.177e-2)	9.391e-2(4.883e-2)	2.221e-0(4.208e-3)	4.362e-0(2.728e-4)	1.936050e-04(7.184213433e-05)

Table 4.5: Mean and standard deviation values of MIGD obtained by six algorithms for (n_t , T_t) = (10,20) .

	Algorithms					
Pro. No.	TrDMOEA	PPS	MOE	MOEA/D-FD	FBP	TLBO
DF1	5.809e-2(1.091e-3)	2.479e-1(5.558e-2)	7.599e-3(1.148e-3)	6.546e-3(1.322e-4)	6.461e-2(3.306e-4)	5.6939e-04 (0.0000e+0)
DF2	8.197e-3(2.152e-5)	1.508e-1(4.644e-2)	1.060e-2(6.531e-4)	9.222e-3(1.670e-4)	3.182e-2(2.665e-3)	5.1594e-04 (0.0000e+0)
DF3	8.895e-2(1.175e-3)	1.662e-1(1.457e-1)	2.527e-4(2.578e-3)"	9.217e-2(2.145e-3)	6.927e-1(8.572e-2)	5.7222e-04 (1.1877e-19)
DF4	5.588e-1(1.801e-2)	4.808e-1(1.193e-1)	2.145e+1(8.477e-2)	3.351e-0(5.528e-3)	5.139e-0(2.782e-1)	1.1906e-01 (1.5202e-17)
DF5	3.118e-2(2.894e-4)	1.044e-1(2.204e-2)	1.221e+0(6.439e-4)	2.580e-2(1.771e-4)	2.219e-2(4.738e-4)	5.6176e-04 (1.1877e-19)
DF6	1.953e+0(2.570e-1)	4.032e+0(6.981e-0)	4.864e+1(7.947e-2)	3.235e+1(1.072e+0)	4.933e-0(4.439e-3)	5.966829e-04 (1.786581410e-04)
DF7	7.679e-2(9.822e-3)	8.795e-2(8.222e-3)	3.310e+0(1.900e-4)	1.241e-1(1.096e-2)	5.523e-2(3.420e-4)	7.832935e-04 (3.603890691e-05)
DF8	8.472e-2(2.789e-5)	1.390e-1(3.522e-3)	2.481e-0(7.381e-3)	2.374e-0(4.063e-2)	2.210e-0(2.143e-5)	6.143387e-(04 1.651854004e-06)
DF9	6.534e-2(1.396e-2)	2.970e-1(1.167e-1)	2.176e+1(4.813e-3)	7.396e-2(2.394e-3)	4.867e-1(4.271e-2)	1.135430e-03 (1.785319387e-04)
DF10	2.867e-1(1.412e-2)	2.491e-1(7.535e-3)	1.341e-0(2.384e-4)"	3.147e-0(1.227e-2)	1.276e-0(3.241e-2)	3.423863e-(04 5.211456590e-05)
DF11	2.507e-1(2.641e-2)	7.639e-1(1.988e-3)	8.318e-0(5.672e-3)	8.480e-0(3.787e-4)	6.782e-0(2.814e-2)	5.803690e-(03 2.999773089e-03)
DF12	3.369e-1(1.431e-2)	3.099e-1(1.016e-0)	2.127e-0(2.826e-2)	9.348e-1(2.987e-7)	4.219e-0(5.384e-6)	1.704444e-(03 1.324239560e-05)
DF13	1.715e-1(2.208e-3)	2.996e-1(1.908e-2)	3.108e+1(8.515e-2)	1.782e-0(2.351e-3)	2.814e-0(7.854e-2)	1.704478e-(02 8.152212585e-03)

DF14	7.995e-2(7.074e-3)	1.095e-1(9.542e-2)	7.231e-0(1.476e-2)	2.427e-0(4.208e-2)	5.674e-1(1.472e-4)	2.678613e-(04 7.164322933e-05)

The last type is the testing application is to apply a different values of T_t which are 50 and 100 to see how the TLBO algorithms would react. Afterwards, an illustration of the PS and PF with different values of generation counter such as t equal to 2.32, 1.16, 0.80, 0.60, 0.48, 0.40, 0.36, 0.32, 0.28, and 0.24.

Table 4.6: Mean and standard deviation values of MIGD obtained by TLBO with (n_t)=50 and (T_t) = (50 , 100) .

Problems	T_t	MIGD (mean(std.))
DF1	50	5.555170e-04(1.275803424e-06)
	100	5.550347e-04(1.079853918e-06)
DF2	50	4.977080e-04(1.187683974e-19)
	100	4.977080e-04(1.187683974e-19)
DF3	50	5.543418e-04(1.064060698e-06)
	100	5.557810e-04(1.875932157e-07)
DF4	50	1.167124e-01(5.018026136e-04)
	100	1.164366e-01(3.912790726e-04)
DF5	50	5.451564e-04(2.423722409e-07)
	100	5.451564e-04(2.423722409e-07)
DF6	50	5.625005e-04(1.634959366e-04)
	100	4.894502e-04(1.300569701e-04)
DF7	50	9.758247e-04(1.076847121e-04)
	100	1.060530e-03(6.514668883e-05)
DF8	50	5.729837e-04(4.178359762e-05)

	100	6.156862e-04(2.100617468e-06)
DF9	50 100	1.128461e-03(1.754995529e-04) 1.099799e-03(1.630417988e-04)
DF10	50 100	3.508884e-04(1.294910333e-06) 3.493700e-04(3.311522570e-07)
DF11	50 100	8.970584e-03(1.588173787e-03) 1.044719e-02(7.957767881e-04)
DF12	50 100	4.339508e-04(4.761091793e-05) 4.328435e-04(4.715653091e-05)
DF13	50 100	3.078063e-03(1.798106942e-03) 4.041419e-04(0.000000000e+00)
DF14	50 100	1.746447e-04(2.873042227e-05) 1.477516e-04(1.425645886e-05)

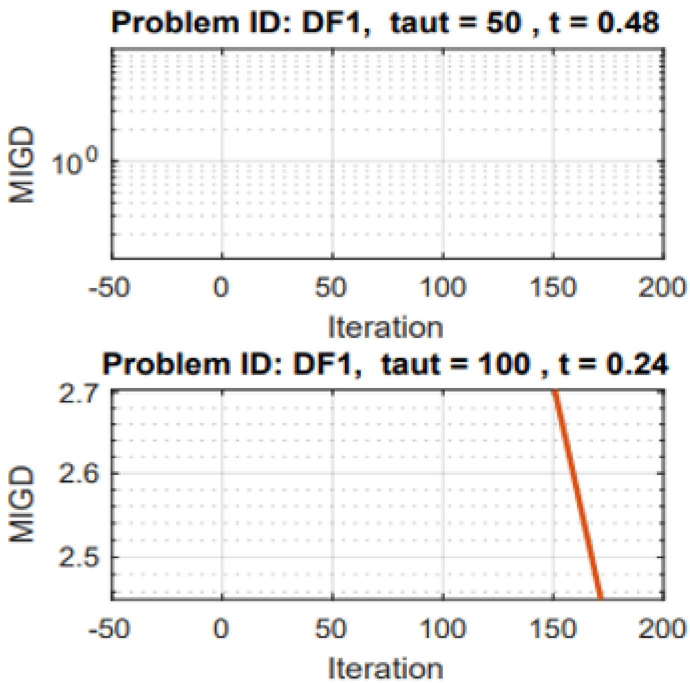


Figure 4.29: The Plot of PF with T_t= (50, 100) for DF1

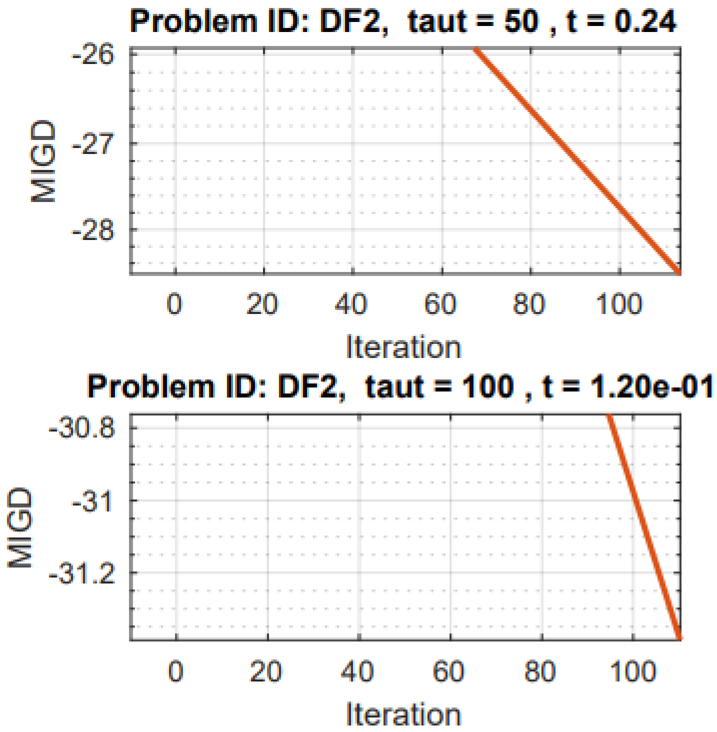


Figure 4.30: The Plot of PF with T_t= (50, 100) for DF2

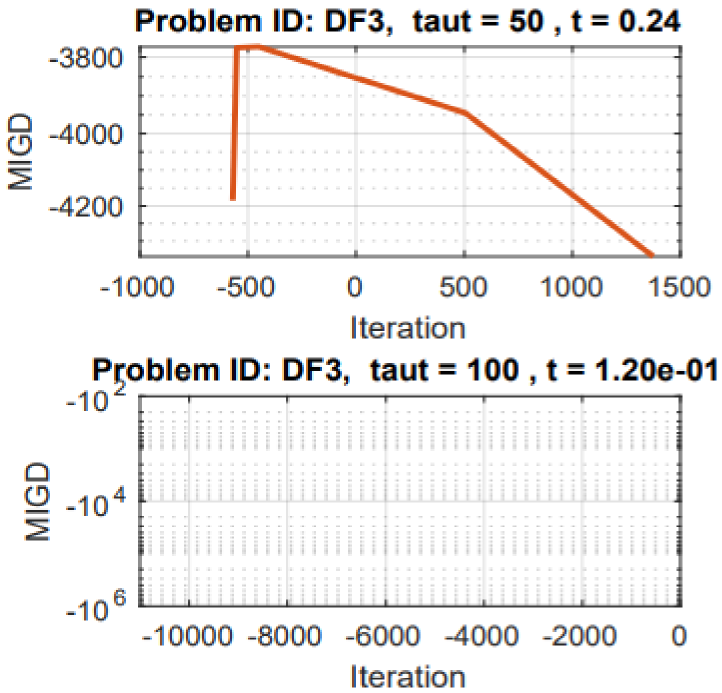


Figure 4.31: The Plot of PF with T_t= (50, 100) for DF3

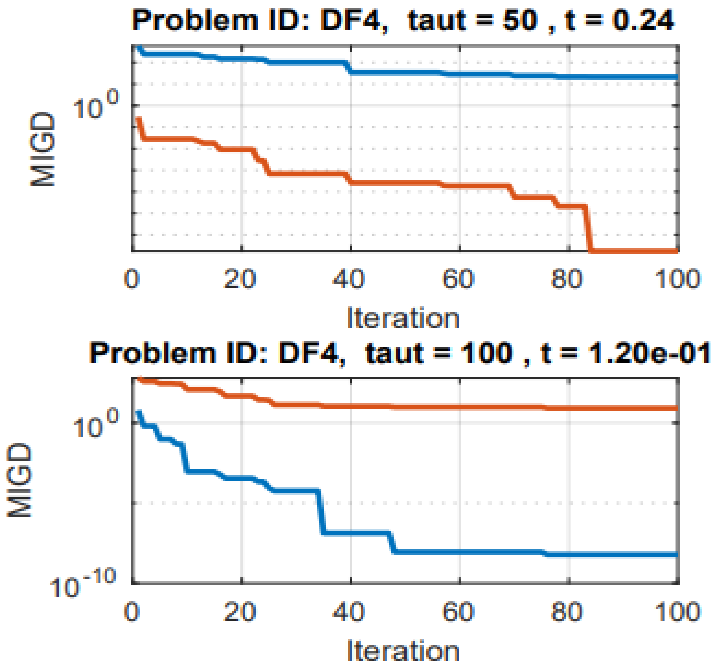


Figure 4.32: The Plot of PF with T_t= (50, 100) for DF4

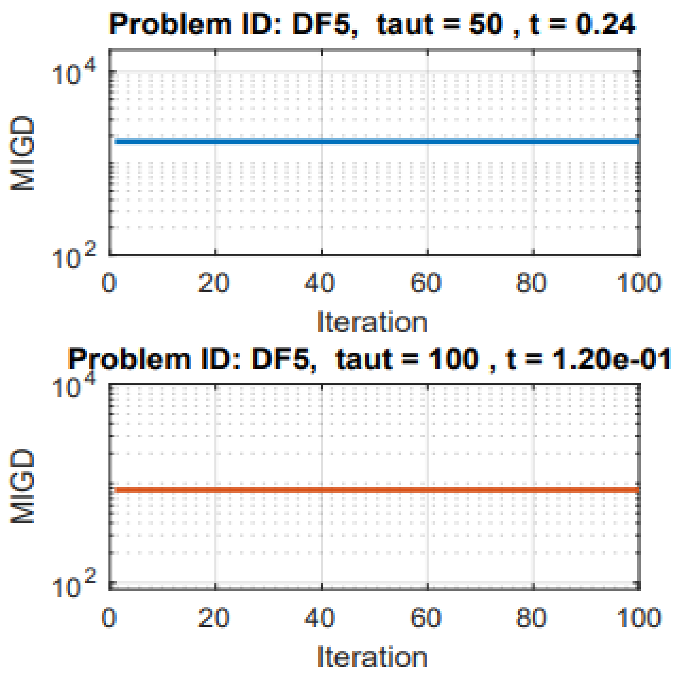


Figure 4.33: The Plot of PF with T_t= (50, 100) for DF5

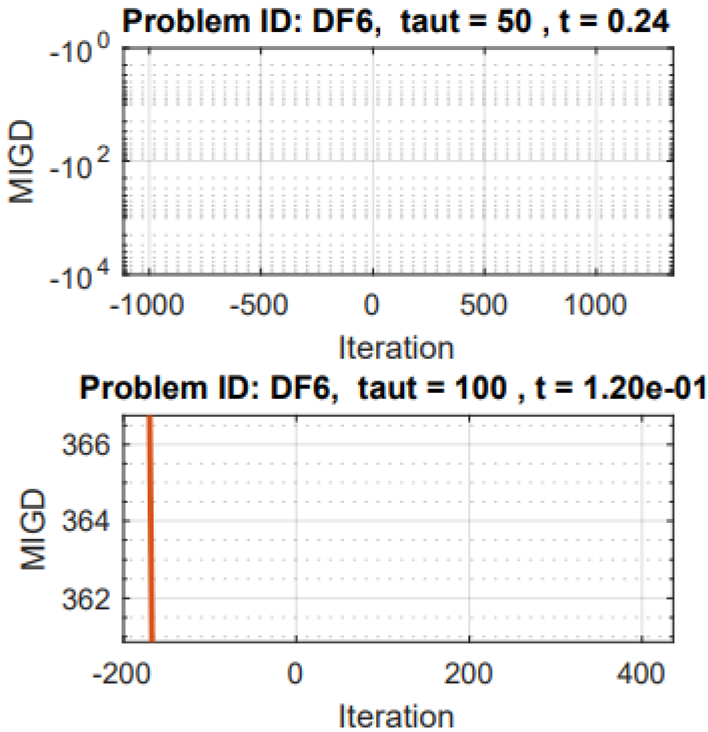


Figure 4.34: The Plot of PF with T_t= (50, 100) for DF6

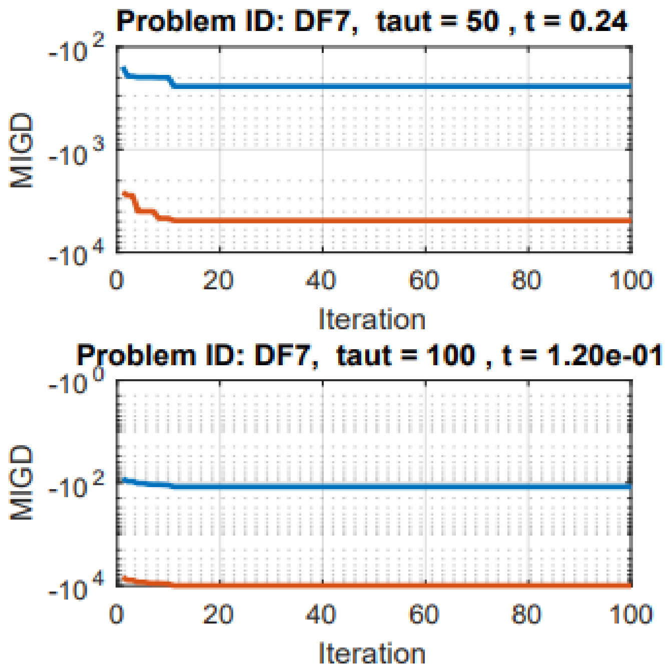


Figure 4.35: The Plot of PF with T_t= (50, 100) for DF7

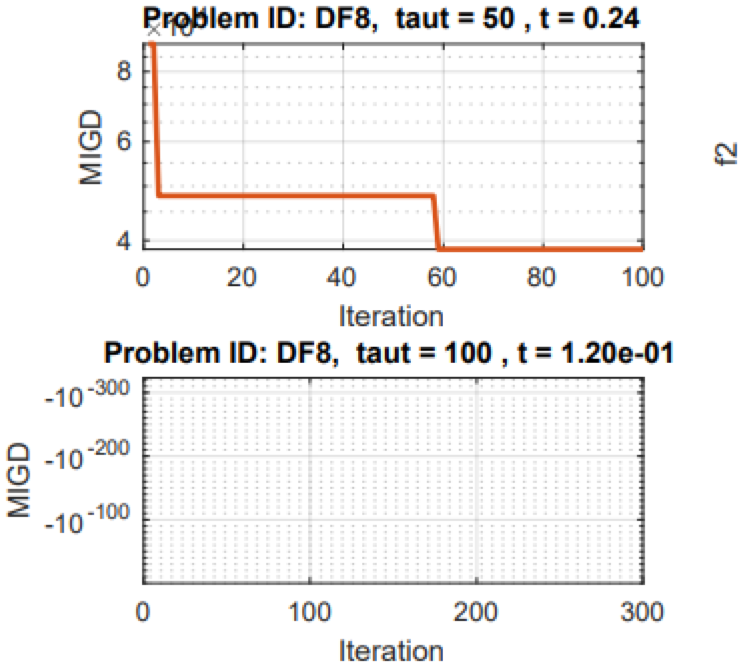


Figure 4.36: The Plot of PF with T_t= (50, 100) for DF8

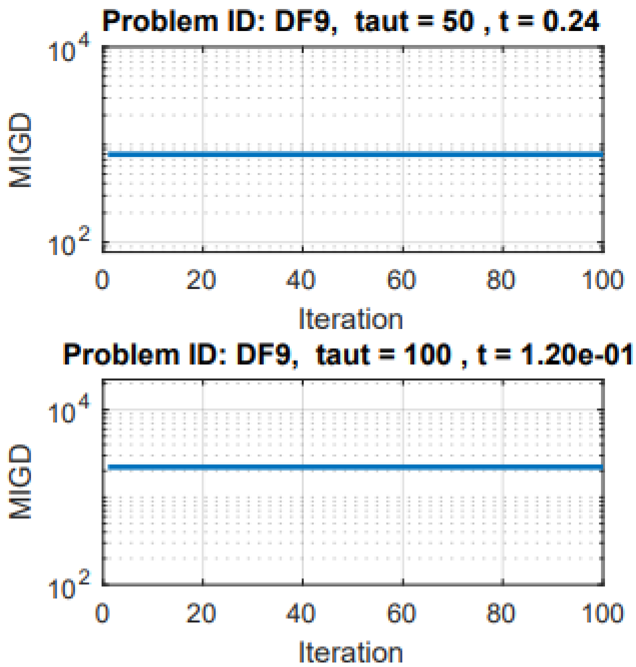


Figure 4.37: The Plot of PF with T_t= (50, 100) for DF9

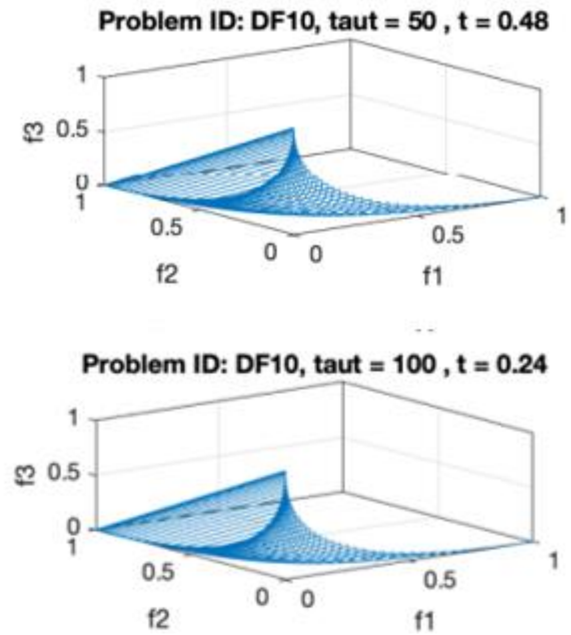


Figure 4.38: The Plot of PF with T_t= (50, 100) for DF10

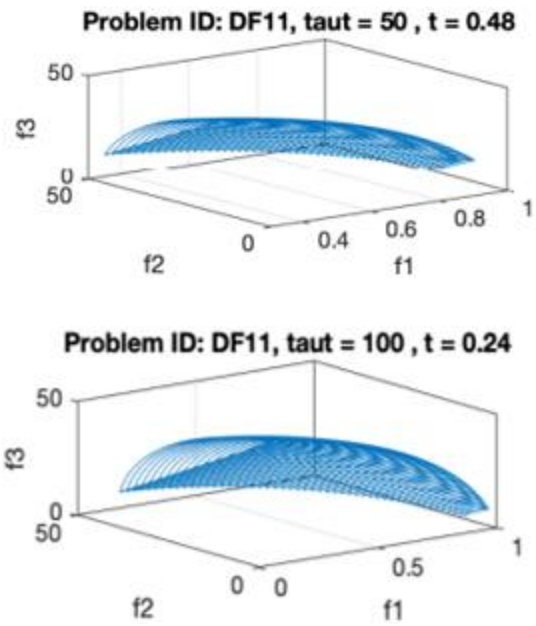


Figure 4.39: The Plot of PF with T_t= (50, 100) for DF11

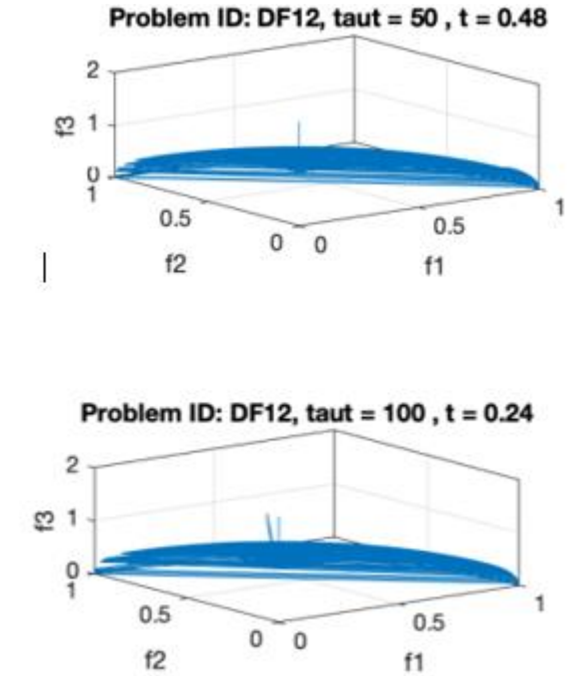


Figure 4.40: The Plot of PF with T_t= (50, 100) for DF12

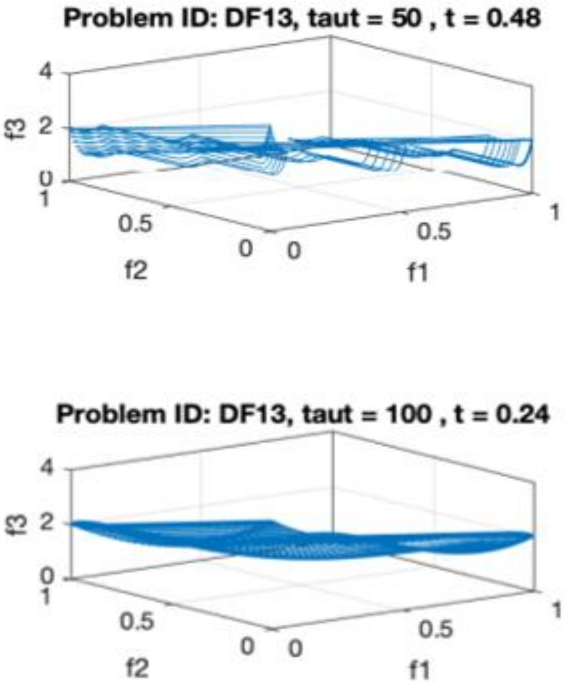


Figure 4.41: The Plot of PF with T_t= (50, 100) for DF13

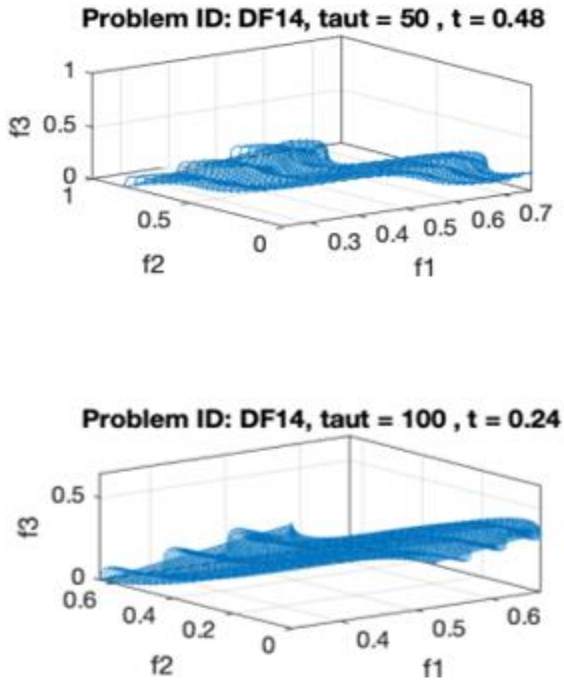


Figure 4.42: The Plot of PF with T_t= (50, 100) for DF14

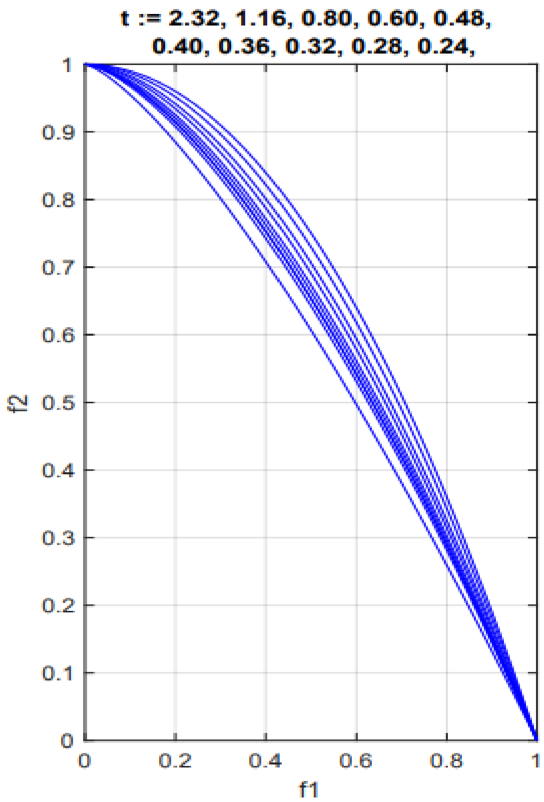


Figure 4.43: Illustration of PS and PF with different generation counter τ for DF1

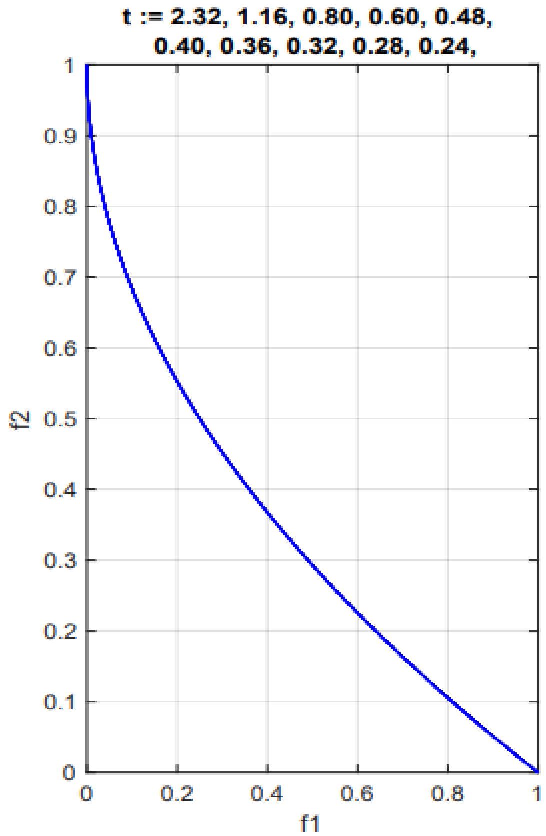


Figure 4.44: Illustration of PS and PF with different generation counter τ for DF2

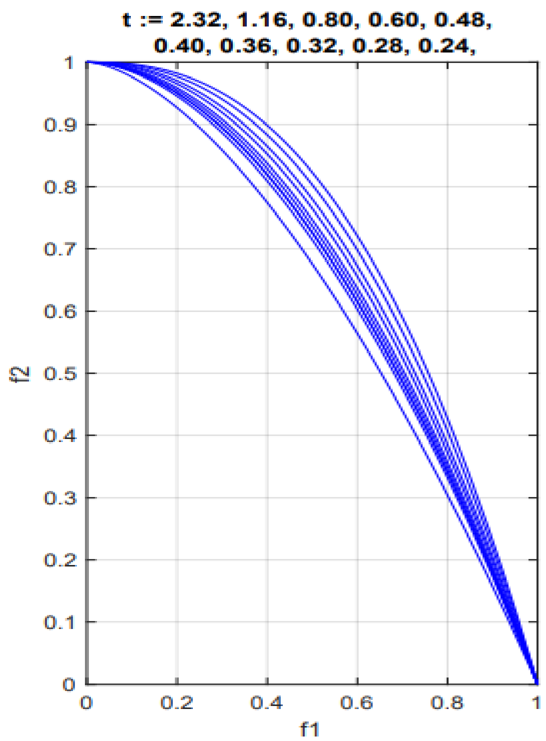


Figure 4.45: Illustration of PS and PF with different generation counter τ for DF3

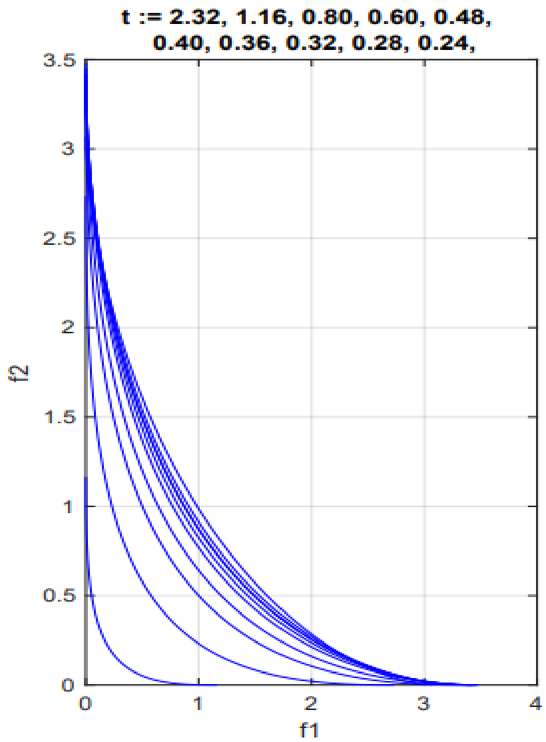


Figure 4.46: Illustration of PS and PF with different generation counter τ for DF4

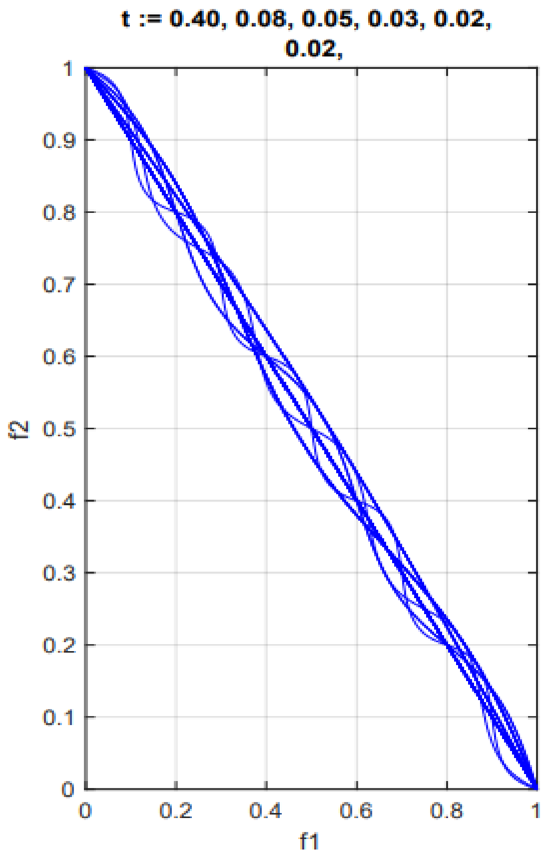


Figure 4.47: Illustration of PS and PF with different generation counter τ for DF5

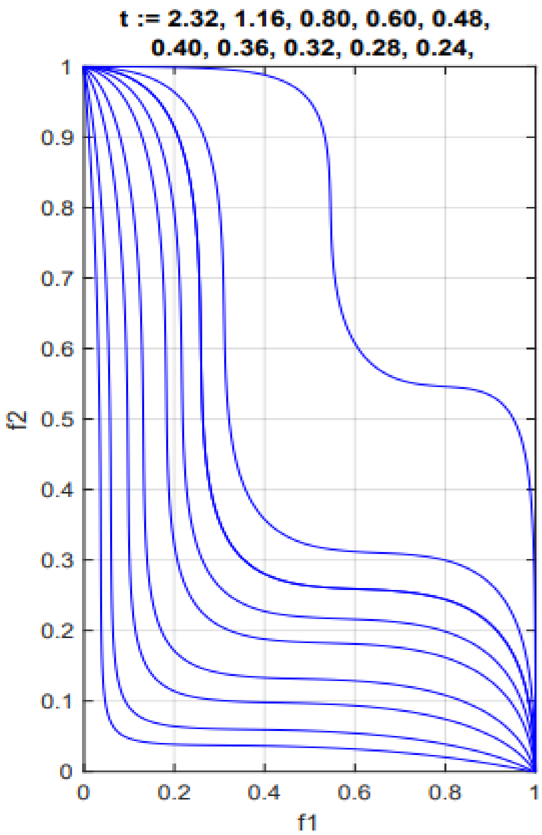


Figure 4.48: Illustration of PS and PF with different generation counter τ for DF6

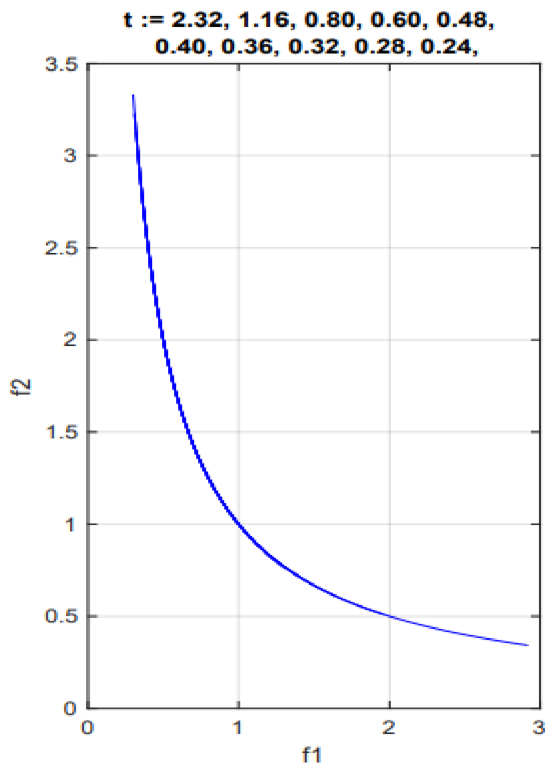


Figure 4.49: Illustration of PS and PF with different generation counter τ for DF7

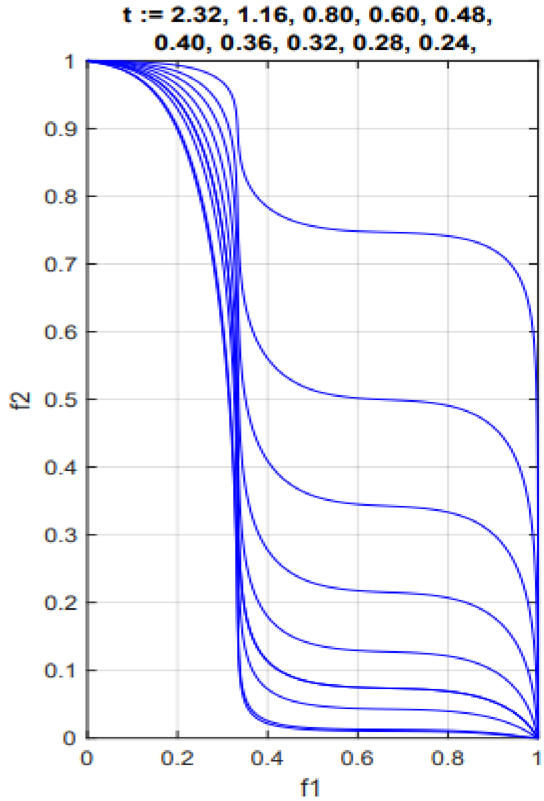


Figure 4.50: Illustration of PS and PF with different generation counter τ for DF8

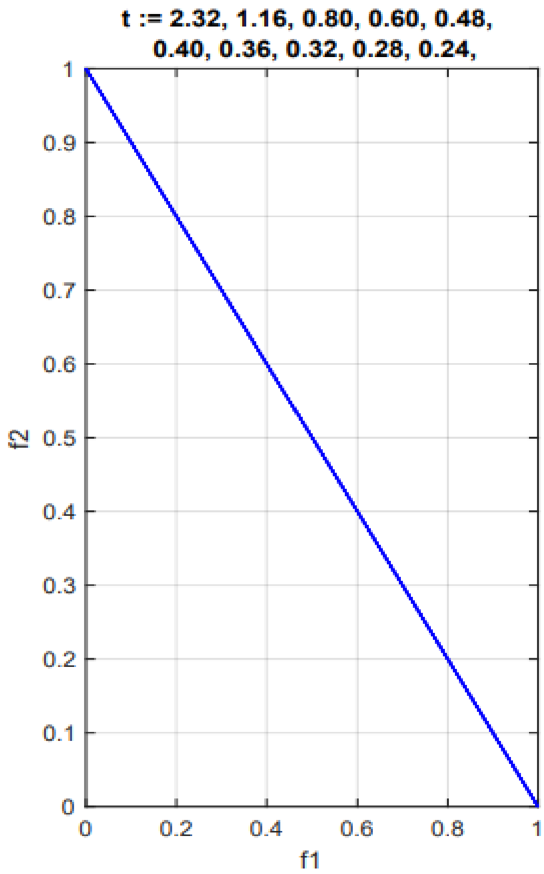


Figure 4.51: Illustration of PS and PF with different generation counter τ for DF9

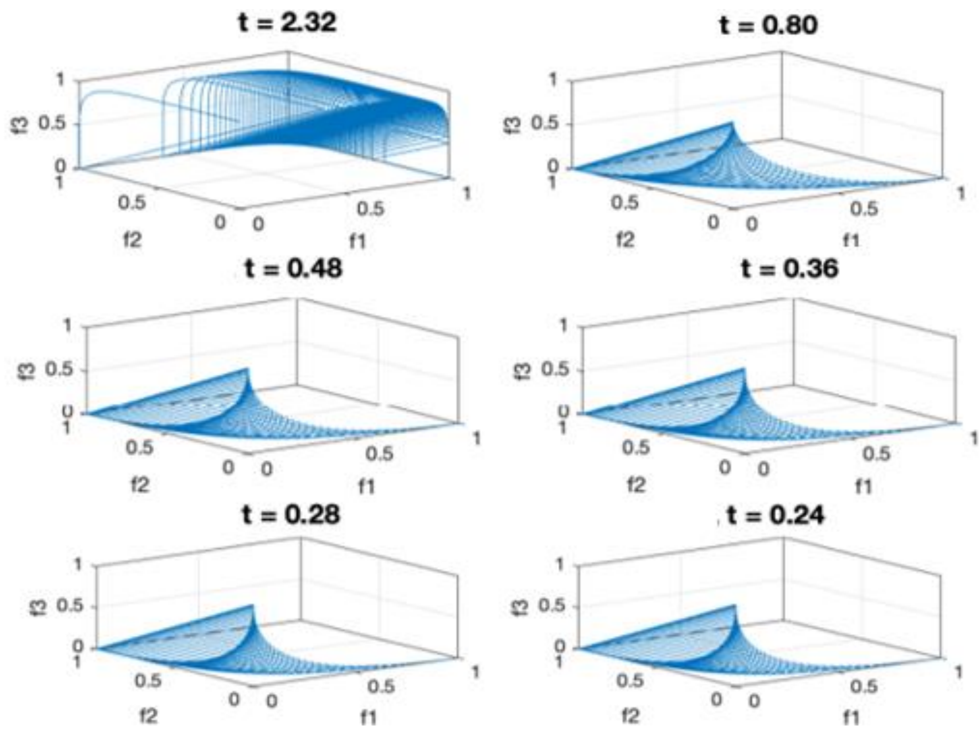


Figure 4.52: Illustration of PS and PF with different generation counter τ for DF10

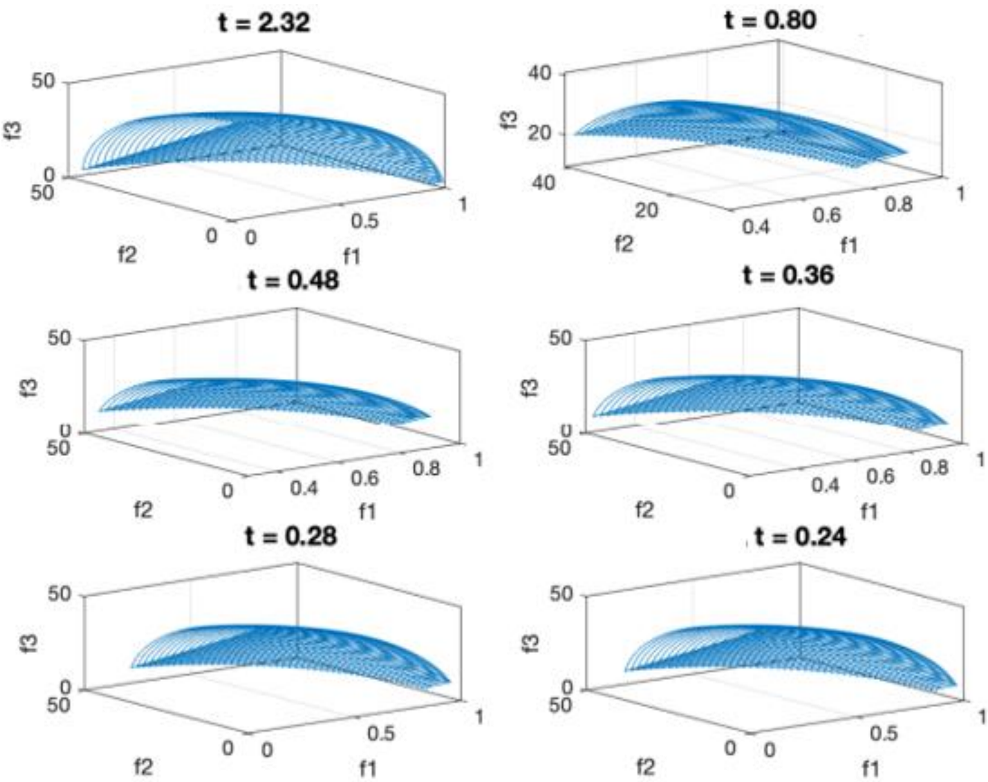


Figure 4.53: Illustration of PS and PF with different generation counter τ for DF11

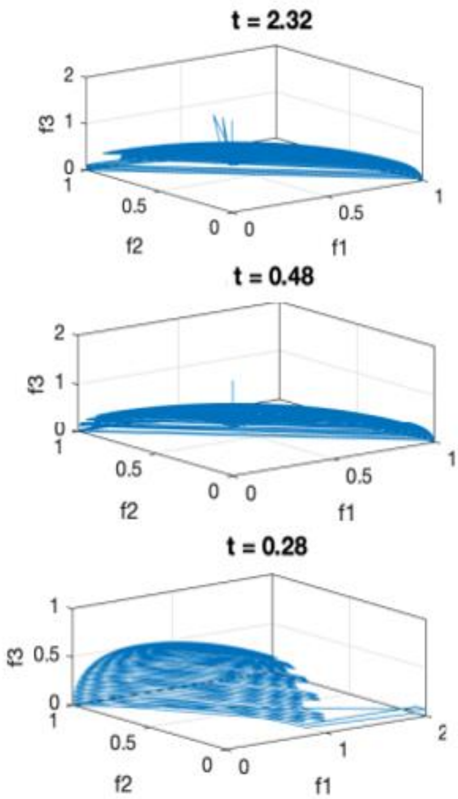


Figure 4.54: Illustration of PS and PF with different generation counter τ for DF12

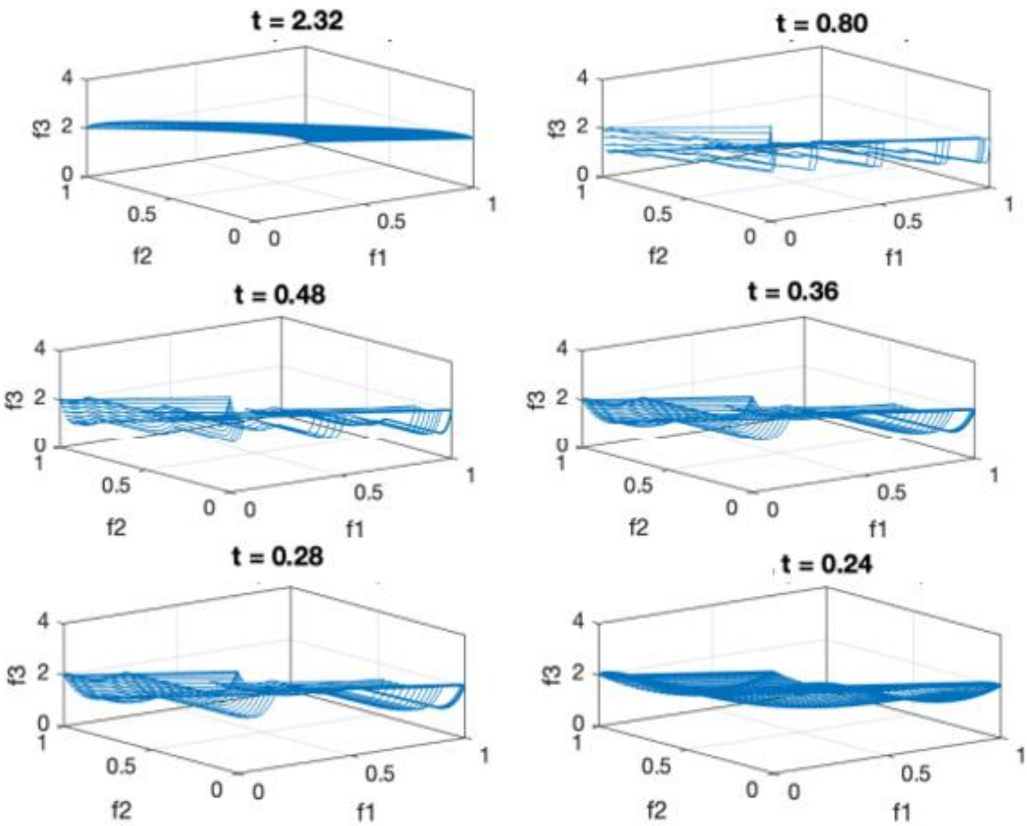


Figure 4.55: Illustration of PS and PF with different generation counter τ for DF13

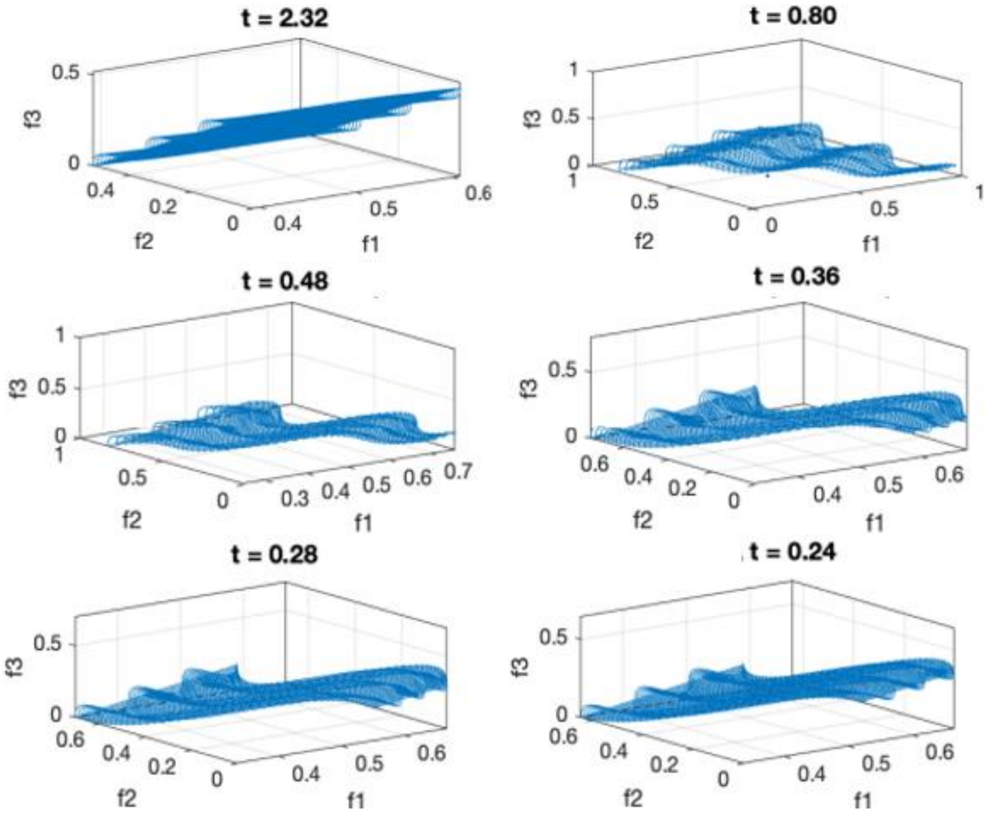


Figure 4.56: Illustration of PS and PF with different generation counter τ for DF14

Even though the results of TLBO algorithm In table 4.4 are quite similar to FBP algorithm, we can when the values of the $(n_t\ , T_t)$ = (10,10) are the same and the TLBO algorithm were assigned as the fixed parameters with very low population size as compared to that of 1000 in case of FBP algorithm and the number iterations $f100(30t_t + 50)$ fitness evaluations where 500 fitness evaluations are conducted before the initial environmental modification progress occurs.

So the multi-objective TLBO algorithm needs more iterations for consistency and has converged the optimum result in hundred iteration only. Whereas, TLBO algorithm has demonstrated its supremacy in standings of faster convergence proportion for MOOPs through coalescing entirely the goals from teacher phase to the learners phase with mutual procedure parameters bounds will be found that contents all the differing objectives. As TLBO is a parameter less algorithm. The results has united very fast because with TLBO positive outcomes were just united and achieved for optimization wherever the fitness distance association was high.

TLBO has outdone the five algorithms for seven benchmark functions: DF1, DF5, DF6, DF7, DF11, DF12 and DF13 despite the fact it could be diverse with a proper range of parameters is critical for the getting of the ideal response because its method mechanism on the repercussions of the inspiration of a teacher on learners. Similar to environment stimulated procedures, teaching-learning-based optimization algorithm's technique was built which utilize an inhabitants of resolutions to continue to the total result.

Teaching-learning-based optimization algorithm customs the much more effective remedy for the incarnation to alteration the current solution in the population, thus continuing to increase the convergence rate. The most important feature that multi-objective TLBO has no division to the population, it utilizes the average attribute of the inhabitants for apprise such resolution by presenting the perception of number of teachers.

Teaching learning based optimization algorithm displays a well act with less computational power for problems of a high dimensionality by evaluating the outcome of the sum of teachers on the fitness assessment of the experimentation and objective performance on actual big measurement problems like DF10, DF11, DF12, DF13 and DF14. TBLO algorithm is made for minimization of the entire operational period of all key transmits. For the evaluation results documented in Table 4.3, Table 4.4 and Table 4.5 implies that presume nothing exists as the optimal values for all the test functions like DF2 and DF3 are not affected the time parameter value with problem DF6 and DF8 are barley affected but DF5 and DF9 are steady while other dimensional cases like DF10, DF11, DF12, DF13 and DF14 are affected by this parameter too much.

Although it is not always the best, this testing experimentation funds that TLBO has superior performance in comparison to other options when with the proposed FBP. Separately from the above examination, to consider the presentation of the compared dynamic multi-objective algorithm PPS, TrDMOEA, MOE, MOEA/D-FD and FBP, They are all built with the same reaction technique used in TLBO that simulates the outcomes comprise mean values, standard deviation evaluate problems based on MIGD results which MIGD is more consistent to differentiate among algorithms to make statement of the overall effectiveness of the compared machine learning algorithms because of the influence of the changes of the frequency which make it difficult to find a descent value calculations to the POF where TLBO is less responsive and vulnerable to the frequency and severity of change. That’s why it displays more consistent capability and mends quicker from dynamic environments adjusts that shown in the plots for the CEC2018 dynamic problems.

Chapter 5

CONCLUSION

The objective of this thesis to test with the most recent development TLBO. It mergers the Teacher and Learner dual essential stages for the multi-objective resolve problems with diverse features, that consequence in total ideal resolutions. Many publications have addressed in their discussions that TLBO is an inventions for superior or comparable results that has greater speed over others evolutionary algorithms, which TLBO has less parameters because of that, it can be more simply functional by experts for explaining problematic manufacturing design complications. These entitlements were retested during the experiment on CEC2018 proposed dynamic benchmark problems. In this thesis, the test was made within the elementary Teaching-learning-based optimization algorithm and demonstrated such an efficiency thanks to the algorithm's design and its structure.

The application of Teaching-learning-based optimization technique has wanted to keep its exact constraint fewer notion in operational processes or else the operator would be confronted with the load of alteration TLBO precise constraints with the general controller constraints. There is no optimization algorithm available in the researcher domain has ever necessitated that it is the most effective optimization algorithm currently in use neither TLBO algorithm but as a fact TLBO algorithm has showing that it is straightforward to apply because it has no limited or boundaries which outlines the possibility of the optimization process and it delivers the optimal consequences in a reasonably less number of assignment appraisals. So, there is no requirement to have any uncertainties on these features, as the TLBO algorithm has recognized itself with a different point of view when it comes with testing with benchmark problems.

Throughout our testing procuress the algorithm results in outcome non-dominated sets near to the true Pareto resolutions relatively with improved the range amongst results set. Numerical trials prove that multi-objective teaching learning based optimization outdoes the other algorithms mentioning to examination correctness and junction degree. It gets the optimal resolution with minimum computer work that shows its strength in comparison to the other MS. It has verified its control as far as speed convergence frequency as newly industrialized algorithm is successfully applied to cases like the dimensionality problems similar to the last ones in CEC2018 problems like DF10 , DF11, DF12 , DF13 and DF14. For the TLBO algorithm path of the processing to get to the solution that is upgraded and compressed in the teacher phase along with the learner phase so if we can put this the multi-objective TLBO computed and calculated two times multiple by the population size multiple by the number of generations with receiving the total function evaluations performed for duplication removal.

Rao and Patel utilized that formulation method to sum the several system operates assessments in the meantime pointing experimentations with TLBO algorithm. As that component assessments obligatory for replication subtraction so Rao and Patel (2012) rationally come close to the equal for diverse population sizes while no one of the previous researchers have come close to the he idea of component assessments are necessary towards removing duplication. While associating the potential effectiveness of diverse algorithms on numerous benchmark utilities. The new forward thinking optimization Teaching-learning-based technique that has been created on the outcome of the impact of a tutor on the outcome of students in a classroom. The algorithm has proven itself in the engineering fields because it can be utilized in a high amount of constraints and multi-objective function.

In the future, we'd want to rate TLBO’s consideration consequently manipulation controls by involving created statistics with different settings, and associate them with other optimization algorithm to classify the true assessment and misuse influences of TLBO about the algorithm’s internal mechanisms and an improved justification of the obtainable results may be given to the field of scientific research.

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