d-dimensional non-asymptotically flat thin-shell wormholes in Einstein-Yang-Mills-Dilaton gravity

S. Habib Mazharimousavi,[*] M. Halilsoy,[†] and Z. Amirabi[‡]

Department of Physics, Eastern Mediterranean University, G. Magusa, north Cyprus,
Mersin 10, Turkey. Tel: 0090 392 6301067, Fax: 0090 3692 365 1604.

Thin-shell wormholes in Einstein-Yang-Mills-dilaton (EYMD) gravity are considered. We show that a non-asymptotically flat (NAF) black hole solution of the d-dimensional EYMD theory provides stable thin-shell wormholes which are supported entirely by exotic matter. The presence of dilaton makes the spacetime naturally NAF, and with our conclusion it remains still open to construct wormholes supported by normal matter between two such spacetimes.

I. INTRODUCTION

The original aim of a spacetime wormhole was to connect two distinct, asymptotically flat (AF) spacetimes, or two distant regions in the same AF spacetime [1, 2]. In making such a short cut travel possible it is crucial that the traveller doesn't encounter event horizons of black holes. A thin-shell may support such a wormhole provided it has the proper source to resist against the gravitational collapse. An exotic matter, which fails to satisfy the energy conditions has been used extensively to provide maintenance of such wormholes. The non-physical source of energy is confined on rather thin spherical shells, simply to invoke justification from quantum theory. More recently, however, it has been shown that without such resort, thin-shell wormholes can be constructed entirely from normal matter obeying the energy conditions [3]. Further, such wormholes established on realistic matter may be stable against radial, linear perturbations. Clearly this implies that existence of wormholes may be an undeniable reality in our universe.

In 4−dimensional (4d) Einstein-Maxwell (EM) theory it was not possible to employ normal matter in the construction / maintenance of thin-shell wormholes. In 5d, with the Gauss-Bonnet (GB) extension of EM theory [4], it was further proved that stable, thin-shell wormholes supported by normal matter is possible provided the GB parameter takes negative values, i.e. $\alpha < 0$ [3]. Is this true also for different sources such as Yang-Mills (YM) fields when considered in Einstein-Gauss-Bonnet (EGB) gravity? The answer, to the best of our knowledge, is not in the affirmative.

In this Letter we employ dilaton field beside YM field to investigate the reality of such thin-shell wormholes. In doing this it should be remembered that the strong coupling of dilaton turns the spacetime into a non-asymptotically flat (NAF) one. This is a digression from the original idea of a wormhole since we have to revise the advantages of an AF spacetime. We remind, however, that we have already enough familiarity with the cases of NAF spacetimes, the best known one being the de (anti)-Sitter. For this reason we extend the concept of a wormhole in an AF spacetime to a NAF one through the prominent source of a dilaton. Such a wormhole will provide traversability from one NAF to another NAF spacetime. For this purpose we employ d-dimensional solutions of Einstein-Yang-Mills-dilaton (EYMD) theory found recently [5, 6], and investigate thin-shell construction in such geometries. It turns out that in the dilatonic thin-shells the required source must be exotic. Our findings show, against our expectation that neither the YM charge, nor the dilatonic parameter have significant effect on the negative energy density of the thin-shell. A deeper reasoning should be sought in the Kaluza-Klein reduction procedure in which dilaton is created from higher dimensional EM theory. The latter needs also an exotic matter to support and maintain a thin-shell wormhole. To have anything but exotic, with reference to our past experience, the dilaton must vanish and additional geometrical structures, such as Gauss-Bonnet (GB) terms must be added.

The Letter is organized as follows. In Sec. II we give a brief review of the d-dimensional EYMD gravity. Dynamic thin-shell wormhole construction and stability analysis is presented in Sec. III. The paper ends with our conclusion, which appears in Sec. IV.

[*]Electronic address: habib.mazhari@emu.edu.tr
[†]Electronic address: mustafa.halilsoy@emu.edu.tr
[‡]Electronic address: zahra.amirabi@emu.edu.tr

II. A BRIEF REVIEW OF HIGHER DIMENSIONAL EYMD GRAVITY

We consider the $d-$dimensional action in the EYMD theory as $(G = 1)$

$$
\begin{aligned}
S &= -\frac{1}{16\pi} \int_{\mathcal{M}} d^d x \sqrt{-g} \left(R - \frac{4}{d-2} \left(\boldsymbol{\nabla}\Phi \right)^2 + \mathcal{L}\left(\Phi\right) \right), \\
\mathcal{L}\left(\Phi\right) &= -e^{-4\alpha\Phi/(d-2)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}),
\end{aligned} \tag{1}
$$

where

$$
\mathbf{Tr}(.) = \sum_{a=1}^{6} (.) , \tag{2}
$$

Φ is the dilaton scalar potential, the parameter α denotes the coupling between dilaton and Yang-Mills (YM) field and as usual R is the Ricci scalar. The YM field $2-$forms $\mathbf{F}^{(a)} = F_{\mu\nu}^{(a)} dx^\mu \wedge dx^\nu$ are given by [5, 6]

$$
\mathbf{F}^{(a)} = \mathbf{dA}^{(a)} + \frac{1}{2\sigma} C_{(b)(c)}^{(a)} \mathbf{A}^{(b)} \wedge \mathbf{A}^{(c)} \tag{3}
$$

in which $C_{(b)(c)}^{(a)}$ are the structure constants, σ is a coupling constant and the YM potential $1-$forms are given by

$$
\mathbf{A}^{(a)} = \frac{Q}{r^2} \left(x_i dx_j - x_j dx_i \right), \quad Q = \text{charge}, \quad r^2 = \sum_{i=1}^{d-1} x_i^2, \quad 2 \le j+1 \le i \le d-1, \quad \text{and} \quad 1 \le a \le (d-1)(d-2)/2, \tag{4}
$$

which is the higher dimensional Wu-Yang ansatz [5, 6]. The field equations, after varying the action, are given by

$$
\mathbf{d} \left(e^{-4\alpha\Phi/(d-2)\star} \mathbf{F}^{(a)} \right) + \frac{1}{\sigma} C_{(b)(c)}^{(a)} e^{-4\alpha\Phi/(d-2)} \mathbf{A}^{(b)} \wedge^\star \mathbf{F}^{(c)} = 0, \tag{5}
$$

$$
R_{\mu\nu} = \frac{4}{d-2} \partial_\mu \Phi \partial_\nu \Phi + 2e^{-4\alpha\Phi/(d-2)} \left[\mathbf{Tr} \left(F_{\mu\lambda}^{(a)} F_\nu^{(a)\ \lambda} \right) - \frac{1}{2(d-2)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}) g_{\mu\nu} \right], \tag{6}
$$

$$
\nabla^2 \Phi = -\frac{1}{2} \alpha e^{-4\alpha\Phi/(d-2)} \mathbf{Tr}(F_{\lambda\sigma}^{(a)} F^{(a)\lambda\sigma}), \tag{7}
$$

in which $R_{\mu\nu}$ is the Ricci tensor and the hodge star * means duality. As it was shown in Ref. [5, 6], these equations admit black hole solution in the form of

$$
ds^2 = -f\left(r\right) dt^2 + \frac{dr^2}{f\left(r\right)} + h\left(r\right)^2 d\Omega_{d-2}^2, \tag{8}
$$

where $d\Omega_{d-2}^2$ is the line element on S^{d-2} and the solution can be summarized as follows

$$
\Phi = -\frac{(d-2)}{2} \frac{\alpha \ln r}{\alpha^2 + 1}, \quad h\left(r\right) = A r^{\frac{\alpha^2}{\alpha^2+1}}, \quad f\left(r\right) = \Xi \left(1 - \left(\frac{r_h}{r} \right)^{\frac{(d-3)\alpha^2+1}{\alpha^2+1}} \right) r^{\frac{2}{\alpha^2+1}}. \tag{9}
$$

We abbreviate here

$$
\Xi = \frac{(d-3)}{((d-3)\alpha^2 + 1) Q^2}, \quad A^2 = Q^2 \left(\alpha^2 + 1 \right), \quad r_h = \left(\frac{4\left(\alpha^2 + 1\right) M}{(d-2) \Xi \alpha^2 A^{d-2}} \right) \tag{10}
$$

and r_h stands for the radius of event horizon. Here M implies the quasilocal mass (see [6] and the references therein).

III.   DYNAMIC THIN-SHELL WORMHOLES IN EYMD GRAVITY

Consider two copies of the EYMD spacetime

$$M^{\pm} = \{r^{\pm} \geq a, \quad a > r_h\} \tag{11}$$

and paste them at the boundary hypersurface $\Sigma^{\pm} = \{r^{\pm} = a, \ a > r_h\}$. These surfaces are identified on $r = a$ with a surface energy-momentum of a thin-shell such that geodesic completeness holds. Following the Darmois-Israel formalism [7] in terms of the original coordinates $x^{\gamma} = (t, r, \theta_1, \theta_2, ...)$ (i.e. on M) the induced metric $\xi^i = (\tau, \theta_1, \theta_2, ...)$ on Σ is given by (Latin indices run over the induced coordinates i.e., $\{1, 2, ..., d-1\}$ and Greek indices run over the original manifold's coordinates i.e., $\{1, 2, ..., d\}$)

$$g_{ij} = \frac{\partial x^{\alpha}}{\partial \xi^i} \frac{\partial x^{\beta}}{\partial \xi^j} g_{\alpha\beta}. \tag{12}$$

We shall study now the dynamic thin-shell wormhole by letting the throat radius to be a function of proper time τ, i.e. $a = a(\tau)$. (A $4-$dimensional formalism similar to these section can be found in Ref.[8]). One can easily show that the induced metric is in the form

$$g_{ij} = \mathrm{diag}\left(-1, h\left(a\left(\tau\right)\right)^2, h\left(a\left(\tau\right)\right)^2 \sin^2\theta_1, h\left(a\left(\tau\right)\right)^2 \sin^2\theta_1 \sin^2\theta_2, ...\right). \tag{13}$$

The parametric equation of the hypersurface Σ is given by

$$F\left(r, a\left(\tau\right)\right) = r - a\left(\tau\right) = 0, \tag{14}$$

and the normal unit vectors to $M^{\pm}$ defined by

$$n_{\gamma} = \left(\pm \left|g^{\alpha\beta}\frac{\partial F}{\partial x^{\alpha}}\frac{\partial F}{\partial x^{\beta}}\right|^{-1/2}\frac{\partial F}{\partial x^{\gamma}}\right)_{r=a}, \tag{15}$$

are found as follows

$$n_t = \pm\left(\left|g^{tt}\left(\frac{\partial a\left(\tau\right)}{\partial t}\right)^2 + g^{rr}\right|^{-1/2}\frac{\partial F}{\partial t}\right)_{r=a}. \tag{16}$$

Upon using

$$\left(\frac{\partial t}{\partial \tau}\right)^2 = \frac{1}{f\left(a\right)}\left(1 + \frac{1}{f\left(a\right)}\dot{a}^2\right), \tag{17}$$

it implies

$$n_t = \pm\left(-\dot{a}\right). \tag{18}$$

Similarly one finds that

$$n_r = \pm\left(\left|g^{tt}\frac{\partial F}{\partial t}\frac{\partial F}{\partial t} + g^{rr}\frac{\partial F}{\partial r}\frac{\partial F}{\partial r}\right|^{-1/2}\frac{\partial F}{\partial r}\right)_{r=a} = \pm\left(\frac{\sqrt{f\left(a\right)+\dot{a}^2}}{f\left(a\right)}\right), \text{ and } n_{\theta_i} = 0, \text{ for all } \theta_i. \tag{19}$$

After the unit $d-$normal, one finds the extrinsic curvature tensor components from the definition

$$K_{ij}^{\pm} = -n_{\gamma}^{\pm}\left(\frac{\partial^2 x^{\gamma}}{\partial \xi^i \partial \xi^j} + \Gamma_{\alpha\beta}^{\gamma}\frac{\partial x^{\alpha}}{\partial \xi^i}\frac{\partial x^{\beta}}{\partial \xi^j}\right)_{r=a}. \tag{20}$$

It follows that

$$K_{\tau\tau}^{\pm} = -n_t^{\pm}\left(\frac{\partial^2 t}{\partial \tau^2} + \Gamma_{\alpha\beta}^t\frac{\partial x^{\alpha}}{\partial \tau}\frac{\partial x^{\beta}}{\partial \tau}\right)_{r=a} - n_r^{\pm}\left(\frac{\partial^2 r}{\partial \tau^2} + \Gamma_{\alpha\beta}^r\frac{\partial x^{\alpha}}{\partial \tau}\frac{\partial x^{\beta}}{\partial \tau}\right)_{r=a} =$$

$$-n_t^{\pm}\left(\frac{\partial^2 t}{\partial \tau^2} + 2\Gamma_{tr}^t\frac{\partial t}{\partial \tau}\frac{\partial r}{\partial \tau}\right)_{r=a} - n_r^{\pm}\left(\frac{\partial^2 r}{\partial \tau^2} + \Gamma_{tt}^r\frac{\partial t}{\partial \tau}\frac{\partial t}{\partial \tau} + \Gamma_{rr}^r\frac{\partial r}{\partial \tau}\frac{\partial r}{\partial \tau}\right)_{r=a} = \pm\left(-\frac{f' + 2\ddot{a}}{2\sqrt{f+\dot{a}^2}}\right), \tag{21}$$

Also

$$K^{\pm}_{\theta_i \theta_i} = -n^{\pm}_{\gamma} \left(\frac{\partial^2 x^{\gamma}}{\partial \theta_i^2} + \Gamma^{\gamma}_{\alpha\beta} \frac{\partial x^{\alpha}}{\partial \theta_i} \frac{\partial x^{\beta}}{\partial \theta_i} \right)_{r=a} = \pm \sqrt{f(a) + \dot{a}^2} hh'. \tag{22}$$

In sum, we have

$$\langle K_{ij} \rangle = \mathrm{diag} \left(-\frac{f' + 2\ddot{a}}{\sqrt{f + \dot{a}^2}}, 2\sqrt{f(a) + \dot{a}^2} hh', 2\sqrt{f(a) + \dot{a}^2} hh' \sin^2 \theta_1, 2\sqrt{f(a) + \dot{a}^2} hh' \sin^2 \theta_1 \sin^2 \theta_2, ... \right), \tag{23}$$

which implies

$$\left\langle K_i^j \right\rangle = \mathrm{diag} \left(\frac{f' + 2\ddot{a}}{\sqrt{f + \dot{a}^2}}, 2\sqrt{f(a) + \dot{a}^2} \frac{h'}{h}, 2\sqrt{f(a) + \dot{a}^2} \frac{h'}{h}, 2\sqrt{f(a) + \dot{a}^2} \frac{h'}{h}, ... \right) \tag{24}$$

and therefore

$$K = \mathrm{Trace} \left\langle K_i^j \right\rangle = \left\langle K_i^i \right\rangle = \frac{f' + 2\ddot{a}}{\sqrt{f + \dot{a}^2}} + 2(d-2) \sqrt{f(a) + \dot{a}^2} \frac{h'}{h}. \tag{25}$$

The surface energy-momentum components of the thin-shell are [9, 10]

$$S_i^j = -\frac{1}{8\pi} \left(\left\langle K_i^j \right\rangle - K\delta_i^j \right) \tag{26}$$

which yield

$$\sigma = -S_{\tau}^{\tau} = -\frac{(d-2)}{4\pi} \left(\sqrt{f(a) + \dot{a}^2} \frac{h'}{h} \right), \tag{27}$$

$$S_{\theta_i}^{\theta_i} = p_{\theta_i} = \frac{1}{8\pi} \left(\frac{f' + 2\ddot{a}}{\sqrt{f(a) + \dot{a}^2}} + 2(d-3) \sqrt{f(a) + \dot{a}^2} \frac{h'}{h} \right). \tag{28}$$

By substitution one can show that the energy conservation takes the form

$$\frac{d}{d\tau} (\sigma \mathcal{A}) + p \frac{d}{d\tau} (\mathcal{A}) = -\frac{(d-2)}{4\pi} \frac{h''}{h} \dot{a} \mathcal{A} \sqrt{f(a) + \dot{a}^2} \neq 0, \tag{29}$$

in which $\mathcal{A} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)} h(a)^{d-2}$ is the area of the thin-shell. In other words, due to the exchange with the bulk spacetime, the energy on the shell is not conserved.

Let us note that we adopt the junction conditions of general relativity which can be justified by the fact that the dilaton field Φ and its normal derivative are both continuous across the shell. Similar approach has been followed by different authors [11] which can be justified easily by integrating the dilaton equation (7) across the throat radius $a_0 \pm \epsilon$, in the limit as $\epsilon \to 0$. Since the singularity and horizons all reside deliberately at distances $r < a_0$, the contribution from the dilaton field and its derivative to the thin-shell source vanishes. We note that this is different in the case of Brans-Dicke (BD) scalar field, which has structural difference compared with the dilaton field [12]. To say the least among others, the exponential coupling of dilaton with the gauge field makes it short ranged whereas BD field is long ranged. To calculate the amount of exotic matter needed to construct the traversable wormhole we use the integral

$$\Omega = \int \sqrt{-g} \left(\rho + p_r \right) dx^{d-1}. \tag{30}$$

For a thin-shell wormhole $p_r = 0$ and $\rho = \sigma \delta(r - a)$, where $\delta(r - a)$ is the Dirac delta function. In static configuration, a simple calculation gives

$$\Omega = \int_0^{2\pi} \int_0^{\pi} ... \int_0^{\pi} \int_0^{\infty} \sqrt{-g} \sigma \delta(r - a) \, dr d\theta_1 d\theta_2 ... d\theta_{d-2} = \frac{2\pi^{\frac{d-1}{2}}}{\Gamma\left(\frac{d-1}{2}\right)} h(a)^{d-2} \sigma(a). \tag{31}$$

We aim now to apply a small radial perturbation around the radius of equilibrium a_0 and to investigate the behavior of the throat under this perturbation. For this perturbation we consider the radial pressure of the thin-shell to be a linear function of the energy density, i.e., [13]

$$p = p_0 + \beta^2 \left(\sigma - \sigma_0\right).$$ (32)

Here p_0 and σ_0 are the radial pressure and energy density of the thin-shell in the static configuration of radius a_0 which are given by

$$\sigma_0 = -\frac{(d-2)}{4\pi}\left(\sqrt{f\left(a_0\right)}\frac{h'\left(a_0\right)}{h\left(a_0\right)}\right),$$ (33)

$$p_0 = \frac{1}{8\pi}\left(\frac{f'\left(a_0\right)}{\sqrt{f\left(a_0\right)}} + 2\left(d-3\right)\sqrt{f\left(a_0\right)}\frac{h'\left(a_0\right)}{h\left(a_0\right)}\right),$$ (34)

and β^2 is a constant related to the speed of sound. By substituting (32) into (31), one finds a first order differential equation for $\sigma\left(a\right)$ which is given by

$$\sigma'\left(a\right) + \left(d-2\right)\left(\sigma + p\right)\frac{h'\left(a\right)}{h\left(a\right)} = \frac{h''\left(a\right)}{h'\left(a\right)}\sigma\left(a\right),$$ (35)

or equivalently

$$\sigma'\left(a\right) + \sigma\left(a\right)\left[\left(d-2\right)\left(1+\beta^2\right)\frac{h'\left(a\right)}{h\left(a\right)} - \frac{h''\left(a\right)}{h'\left(a\right)}\right] = \left(d-2\right)\left(\sigma_0\beta^2 - p_0\right)\frac{h'\left(a\right)}{h\left(a\right)}.$$ (36)

This equation, for the case of EYMD wormhole introduced before can be expressed as

$$r\sigma'\left(a\right) + \xi_1\sigma\left(a\right) = \xi_2$$ (37)

in which

$$\xi_1 = \frac{1 + \left(d-2\right)\alpha^2\left(1+\beta^2\right)}{\alpha^2 + 1},$$ (38)

$$\xi_2 = \frac{\left(d-2\right)\alpha^2\left(\sigma_0\beta^2 - p_0\right)}{\alpha^2 + 1}.$$ (39)

This equation admits a solution in the form of

$$\sigma\left(a\right) = \frac{\xi_2}{\xi_1} + \left(\sigma_0 - \frac{\xi_2}{\xi_1}\right)\left(\frac{a_0}{a}\right)^{\xi_1},$$ (40)

which obviously at $a = a_0$ is nothing but σ_0. In terms of the metric function $f\left(a\right)$, in a dynamic case $\sigma\left(a\right)$ was given by (35) which after equating with the latter expression we obtain

$$\dot{a}^2 + V\left(a\right) = 0.$$ (41)

Here the potential function $V\left(a\right)$ is given by

$$V\left(a\right) = f\left(a\right) - \frac{16\pi^2 a^2}{\left(d-2\right)^2}\left(\frac{\alpha^2 + 1}{\alpha^2}\right)^2\left[\frac{\xi_2}{\xi_1} + \left(\sigma_0 - \frac{\xi_2}{\xi_1}\right)\left(\frac{a_0}{a}\right)^{\xi_1}\right]^2.$$ (42)

We notice that $V\left(a\right)$, and more tediously $V'\left(a\right)$, both vanish at $a = a_0$. The stability requirement for equilibrium reduces therefore to the determination of the regions in which $V''(a_0) > 0$. For this reason we shall proceed through numerical analysis to see whether stability regions/ islands develop and under what conditions. Let us note that our figures refer to $d = 5$, for $d > 5$ we observed that no significant changes take place. In Fig.s 1-2 we show the stability regions in terms of β and wormhole throat a_0. (Note that we use scalings $\Omega = Q^2\tilde{\Omega}$, $f = \frac{\tilde{f}}{Q^2}$ and $p_0 = \frac{1}{|Q|}\tilde{p}_0$ in terms of the YM charge Q and we plot $(\tilde{\Omega}, \tilde{f}, \tilde{p}_0)$). We also show $\tilde{\Omega}$ in terms of a_0 for different values of α. As stated before, our wormhole is supported entirely by negative energy, $\tilde{\Omega} < 0$ on the shell. For changing dilatonic parameter α the change in $\tilde{\Omega}$ is visible in the plots shown. For fixed mass, decreasing α improves $\tilde{\Omega}$ toward zero line but yet in the $(-)$ domain. The pressure on the shell increases with increasing α. It is noticed from the dark stability regions that no stable region forms for $|\beta| < 1$, i.e. below the speed of light. Consideration of dimensions $d > 5$, doesn't change the behaviors of $d = 5$ much, this can be seen by further plots which we shall ignore in this study.

IV. CONCLUSION

We have introduced $d-$dimensional thin-shell wormholes in EYMD gravity supported by exotic matter. Although in quantum theory such matter finds acceptance on reasonable grounds in classical physics we should be more cautious. By choosing finely-tuned parameters of mass and dilaton parameter α we provide the stability of such wormholes against linear, radial perturbations. For any finite YM charge which amounts to scale the energy and pressure no significant difference is observed in the plots. It should be added that extreme YM charge, such as $Q \to 0(\infty)$, results in extreme energy (but still $(-)$) and pressure conditions. When the YM and dilaton fields vanish we recover the previously known results. Inclusion of dilaton converts the spacetime from asymptotically flat (AF) to non-asymptotically flat (NAF) case and only exotic matter can support such a wormhole. We recall that wormholes in NAF universe (without dilaton) is not a new idea [14]. Accordingly, the mass is no more that of ADM but rather the quasi-local mass is implied. Vanishing of dilaton brings us to EYM theory, which may be supplemented with modified gravity to pave the way toward wormholes with normal matter. We have shown elsewhere that inclusion of Gauss-Bonnet (GB) term accomplishes this task and provides stable, thin-shell wormholes with normal (i.e. non-exotic) matter [15].

---

[1] M. S. Morris and K. S. Thorne, Am. J. Phys. **56**, 395 (1988).
[2] M. Visser, Lorantzian Wormholes (AIP Press, Newyork, 1996).
[3] M. G. Richarte and C. Simeone Phys. Rev. D **76**, 087502 (2007); Erratum-ibid **77**, 089903 (2008)E; S. H. Mazharimousavi, M. Halilsoy and Z. Amirabi, Phys. Rev. D **81**, 104002 (2010).
[4] B. Zwiebach, Phys. Lett. B **156**, 315(1985); D. G. Boulware and S. Deser, Phys. Rev. Lett., **55**, 2656(1985); R. G. Cai and K. S. Soh, Phys. Rev. D **59**, 044013 (1999); R. G. Cai, ibid. **65**, 084014(2002); R. Aros, R. Troncoso and J. Zanelli, Phys. Rev. D **63**, 084015(2001); Y. M. Cho and I. P. Neupane, Phys. Rev. D **66**, 024044(2002); M. H. Dehghani, Phys. Rev. D **67,** 064017(2003).
[5] S. H. Mazharimousavi and M. Halilsoy, Phys. Rev. D **76**, 087501 (2007).
[6] S. H. Mazharimousavi, M. Halilsoy and Z. Amirabi, Gen. Relativ. Gravit. **42**, 261 (2010).
[7] G. Darmois, Mémorial des Sciences Mathématiques, Fascicule XXV (Gauthier-Villars, Paris, 1927), Chap. V; W. Israel, Nuovo Cimento B **44**, 1(1966); B **48**, 463(E)(1967); P. Musgrave and K. Lake, Class. Quant. Grav. **13**, 1885 (1996).
[8] E. F. Eiroa, Phys. Pev. D **78**, 024018 (2008).
[9] N. Sen, Ann. Phys. (Leipzig) 73 (1924) 365; K. Lanczos, Ann. Phys. (Leipzig) 74 (1924) 518; W. Israel, Nuovo Cimento B 44 (1966) 1; W. Israel, Nuovo Cimento B 48 (1967) 463, Erratum.
[10] S. C. Davis, Phys. Rev. D, **67**, 024030 (2003); F. S. N. Lobo, Gen. Rel. Grav. **37**, 2023 (2005); K. K. Nandi, Y. Z. Zhang and K. B. Vijaya Kumar, Phys. Rev. D **70**, 127503 (2004).
[11] E. F. Eiroa and C. Simeone, Phys. Rev. D **71**, 127501 (2005);
A. A. Usmani, Z. Hasan, F. Rahaman, Sk. A. Rakib, Saibal Ray and Peter K. F. Kuhfittig, Gen. Rel. Grav. (2010) in press, Doi: 10.1007/s10714-010-1044-y.
[12] L. A. Anchordoqui, S. E. Perez Bergliaffa, and D. F. Torres, Phys. Rev. D **55**, 5226 (1997);
F. Eiroa, M. G. Richarte, and C. Simeone, Phys. Lett. A **373** 1 (2008), Erratum-ibid. **373**, 2399 (2009);
F. Dahia and C. Romero, Phys. Rev. D **60**, 104019 (1999).
[13] P. R. Brady, J. Louko and E. Poisson, Phys. Rev. D **44**, 1891(1991); E. Poisson and M. Visser, Phys. Rev. D **52**, 7318(1995).
[14] O. B. Zaslavskii, Phys. Rev. D 72, 061303(R) (2005).
[15] S. Habib Mazharimousavi, M. Halilsoy and Z. Amirabi, arXiv:1007.4627.

Figure Captions:

Figure 1: The plot of $\tilde{f}(a_0) = Q^2 f(a_0)$, $\tilde{\Omega}(a_0) = \Omega(a_0)/Q^2$ and pressure $\tilde{p}_0 = p_0 |Q|$, for $d = 5$. The shaded inscribed part shows the stable regions (i.e. $V''(a_0) > 0$) in the (β, a_0) diagram. Fig.1a) for $M = 0.1, \alpha = 2.0$, Fig.1b) for $M = 0.10, \alpha = 1.0$, and Fig. 1c) for $M = 0.05, \alpha = 1.0$. It is seen that the Ω behavior doesn't differ much in this range of parameters.

Figure 2: Similar plots for $\tilde{f}(a_0)$, $\tilde{\Omega}(a_0)$ and pressure $\tilde{p}_0$, (again for $d = 5$), for 2a) $M = 0.10$, $\alpha = 0.4$ and 2b) $M = 0.10$, $\alpha = 0.2$. Decreasing α values improves $\tilde{\Omega}(a_0)$ slightly which still lies in the $(-)$ domain. Smaller α implies also less pressure ($\tilde{p}_0$) on the shell. Stable regions ($V''(a_0) > 0$) are shown, versus (β, a_0) in dark.

This figure "FIGURE1a.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1005.2953v3

This figure "FIGURE1b.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1005.2953v3

This figure "FIGURE1c.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1005.2953v3

This figure "FIGURE2a.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1005.2953v3

This figure "FIGURE2b.jpg" is available in "jpg" format from:

http://arxiv.org/ps/1005.2953v3