Dynamics Analysis of Viscoelastic Sandwiched Structures Integrated with Aluminum Sheets by Using Finite Element Method

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ABSTRACT

Passive vibration attenuation of modern mechanical structures is one of the most essential technologies applied to the arsenal of modern mechanical structures. In this thesis, dynamics analysis is performed on viscoelastic (VE) beam and plate sandwich structures. The proposed structures are composed of a VE core and aluminum face sheets as the substrate layers on both sides of the structure. The small-strain VE material is modeled based on the complex constant moduli model. In the modal analysis, the model effective mass analysis is performed to investigate its dominant mode shape and its sweet spot at resonance. Then, in the harmonic analysis, the resonance frequency is obtained to evaluate the performance of the VE sandwich structures via the maximum deformation. A comparison between the results of an analysis of viscoelastic sandwiched structures with subsequent done using FE models. A numerical application is accomplished to develop the integral shear finite element and modal assembly of the stiffness element and mass element established according to strain energy and the Hamilton principle. The harmonic analysis is done using ABAQUS® software with both C3D20RH®, CPS8R where C3D20RH indicates a 3-D element with 20 nodes with reduced integration technique and hybrid formulation and CPS8R indicates 2-D plane strain with eight nodes with reduced integration technique, the results are compared against analytical solutions form the literature, a parametric study is done on the beam's core thickness to study the effect on the damping characteristics of the beam, the results showed that the modal developed in MATLAB® achieved the least error compared to the analytical solution from the literature and then comes the Abaqus® software solution, also the parametric study showed that the natural frequency of the beam does not change with the increase of the core thickness, however, the loss factor is directly proportional to the thickness of the viscoelastic layer of the beam.

Keywords: Viscoelastic, Sandwich Beam/Plate, Dynamics Analysis, Complex Constant Moduli, Finite Element Method, Shear Deformation Theory

ÖZ

Modern mekanik yapıların pasif titreşim azaltması, modern mekanik yapıların cephaneliğine uygulanan en temel teknolojilerden biridir. Bu tezde, viskoelastik (VE) kiriş ve plakalı sandviç yapılar üzerinde dinamik analiz yapılmıştır. Önerilen yapılar, yapının her iki tarafında alt katman katmanları olarak bir VE çekirdeği ve alüminyum yüzey levhalarından oluşur. Küçük gerilimli VE malzemesi, karmaşık sabit modül modeline dayalı olarak modellenmiştir. Modal analizde, baskın mod şeklini ve rezonanstaki tatlı noktasını araştırmak için model etkin kütle analizi yapılır. Daha sonra harmonik analizde, VE sandviç yapıların performansını maksimum deformasyon yoluyla değerlendirmek için rezonans frekansı elde edilir. Daha sonra FE modelleri kullanılarak yapılan viskoelastik sandviç yapıların bir analizinin sonuçları arasında bir karşılaştırma. Gerinim enerjisi ve Hamilton ilkesine göre oluşturulan rijitlik elemanı ve kütle elemanının integral kesme sonlu elemanını ve modal montajını geliştirmek için sayısal bir uygulama gerçekleştirilmiştir. Harmonik analiz, hem C3D20RH® hem de CPS8R ile Abaqus® yazılımı kullanılarak yapılır; burada C3D20RH, azaltılmış entegrasyon tekniği ve hibrit formülasyon ile 20 düğümlü bir 3-D elemanı belirtir ve CPS8R, azaltılmış entegrasyon tekniği ile sekiz düğümlü 2-D düzlem gerinimini gösterir, sonuçlar literatürdeki analitik çözümlerle karşılaştırıldı, kirişin sönümleme özellikleri üzerindeki etkisini incelemek için kirişin çekirdek kalınlığı üzerinde parametrik bir çalışma yapıldı, sonuçlar MATLAB®'da geliştirilen modun analitik ile karşılaştırıldığında en az hataya ulaştığını gösterdi. Literatürden bir çözüm ve ardından ABAQUS® yazılım çözümü geliyor, ayrıca parametrik çalışma, kirişin doğal frekansının çekirdek kalınlığının artmasıyla değişmediğini, ancak kayıp faktörünün viskoelastiğin kalınlığı ile doğru orantılı olduğunu gösterdi. ışın tabakası.

Anahtar Kelimeler: Viskoelastik, Sandviç Kiriş/Plaka, Dinamik Analiz, Karmaşık Sabit Modül, Sonlu Elemanlar Metodu, Kayma Deformasyon Teorisi

To my beloved parents and my wife

Without whom none of my success would be possible

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First and foremost I am deeply indebted to my supervisor, Asst. Prof. Babak Safaei for his invaluable advice, continuous support, and patience during my Master's study. His immense knowledge and plentiful experience have encouraged me in all the time of my academic research and daily life.

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$\{d^e\}$	Total Set of The Nodal Displacements
K^e	Total Element Stiffness Matrix
$u_i(x,t)$	Axial Displacement of The Centroid Ith Layer
$w_i\ (x,t)$	Transverse Displacement of The Centroid Ith Layer
DOF	Degree of Freedom
FE	Finite Element
FRF	Frequency Response Function
FSDT	First Order Shear Deformation Theory
t_1	Thickness of The Aluminum Constraining Layer
t_2	Thickness of The Core Layer
t_3	Thickness of The Aluminum Base Layer
VE	Viscoelastic
θ	Rotation Angle of The VE Sandwich Beam
φ	Shear Angle of The VE Sandwich Beam
γ	Shear Strain of The VE Sandwich Beam

Chapter 1

INTRODUCTION

1.1 Background

The keys to reaching an optimum design of modern mechanical structures with ideal performance are lightweight and noise control of the structures which directly leads to the wide applications of thin-walled structures. Thin-walled structures can be represented as thin beams and thin plates that are more widely used in many engineering applications such as aerospace, automobile, and civil engineering [1]. Since Thin-walled structures are "light" and "thin” it is more likely to create vibrations under external excitation, which will compromise the structure's reliability, accuracy, quality, and life of the structure, as well as cause vibration damage to the human body and noise pollution. As a result, the thin-walled structure's vibration must be controlled.

Applying damping to thin-walled structures to control their vibrations is a widely utilized strategy in engineering practice [2]. Among them, viscoelastic materials, which have good damping qualities, are most widely utilized in structural vibration and noise reduction [3]. Thus, it is considered a potent passive damping treatment technique to reduce vibration and vibration-induced noise. Moreover, it suppresses the instability of the structure and decreases the vibration amplitude to prevent structural damage from fatigue [4]. Furthermore, many studies showed that After the viscoelastic material is adhered to the thin-walled member to form a damping layer, it will periodically deform with the vibration of the member to dissipate the vibration energy [5–7].

Nowadays, viscoelastic materials are widely used to suppress the vibration of thin-walled components. A typical passive damping treatment structure consists of unconstrained treatment (Figure 1 (a)), two elastic layers, and a viscoelastic core sandwiched between the constraining layer and the base beam as shown in Figure 1 (b). The viscoelastic core allows the vibration energy to dissipate through a cyclic deformation within it, while the main advantage of the constraining layer is to increase the cyclic shear deformation of the core and withstand high stress, hence directly leading to an increase in the damping energy within the viscoelastic core [8].

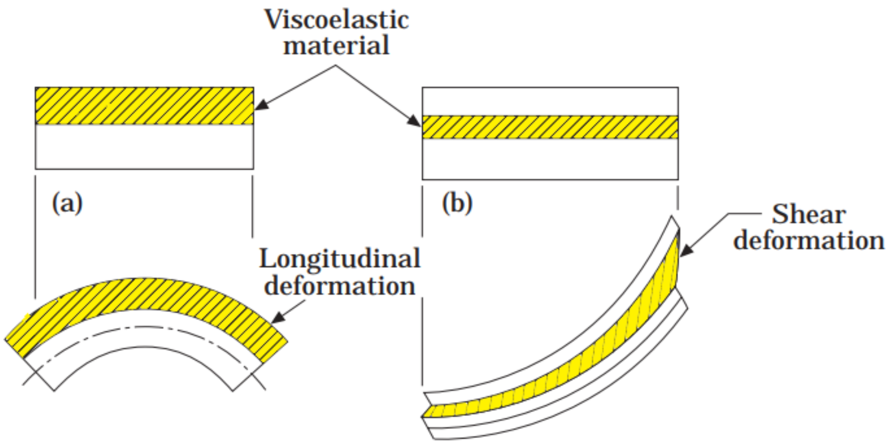


Figure 1: Viscoelastic passive damping treatment in its most basic configuration forms: (a) Unconstrained treatment (b) Constrained treatment [9]

1.2 Theoretical Background of Viscoelastic Material

1.2.1 The Behavior of Viscoelastic Material

The viscoelastic material combines two distinct properties When subjected to an external force, viscous materials deform slowly. The term "elastic" implies that once a deforming force is removed, the material will return to its original shape. Pure viscous fluids, on the other hand, involve deformation followed by a permanent rearrangement of the fluid molecules. The mechanical properties of materials are commonly investigated using stress-strain (or load-deformation) behavior [9]. Loading and unloading "stress versus strain" curves (lines) are superimposed for purely elastic materials. They form what so-called "hysteresis" loop. The area within the loop represents the energy dissipated as heat, as shown in Figure 2. This energy absorption behavior explains why viscoelastic materials are good at damping vibrations.

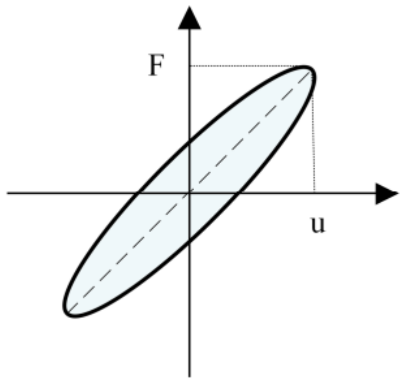


Figure 2: A hysteresis force and deformation loop in viscoelastic material l adapted from [9]

Another important feature of viscoelastic materials is that their mechanical properties are affected by the rate at which they are deformed. Because the molecule's motion gradually kept up with the external force, the dissipated energy decreased, as shown in Figure 3 [10]. As a result, the deformation behavior at different deformation rates is represented by a family of curves rather than a single stress-strain curve. On the other hand, temperature influences viscoelastic behavior because material stiffness varies with temperature so when choosing materials for a certain application, it's crucial to understand what the viscoelastic response will be at a given loading rate and temperature [9]. From Figure 4, it is seen that the storage modulus, loss factor, equivalent stiffness, and equivalent damping decrease significantly by increasing the temperature [11]

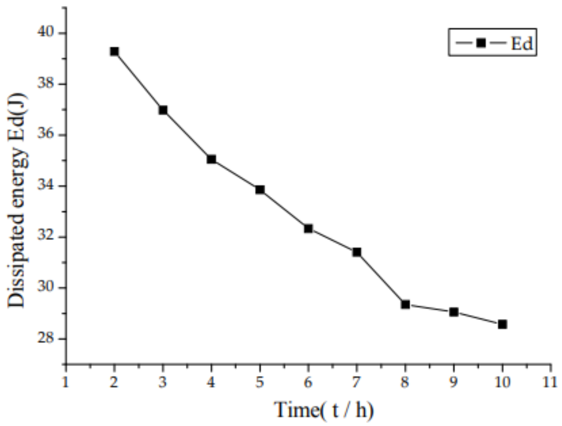


Figure 3: Dissipated energy damping characteristics variation [10]

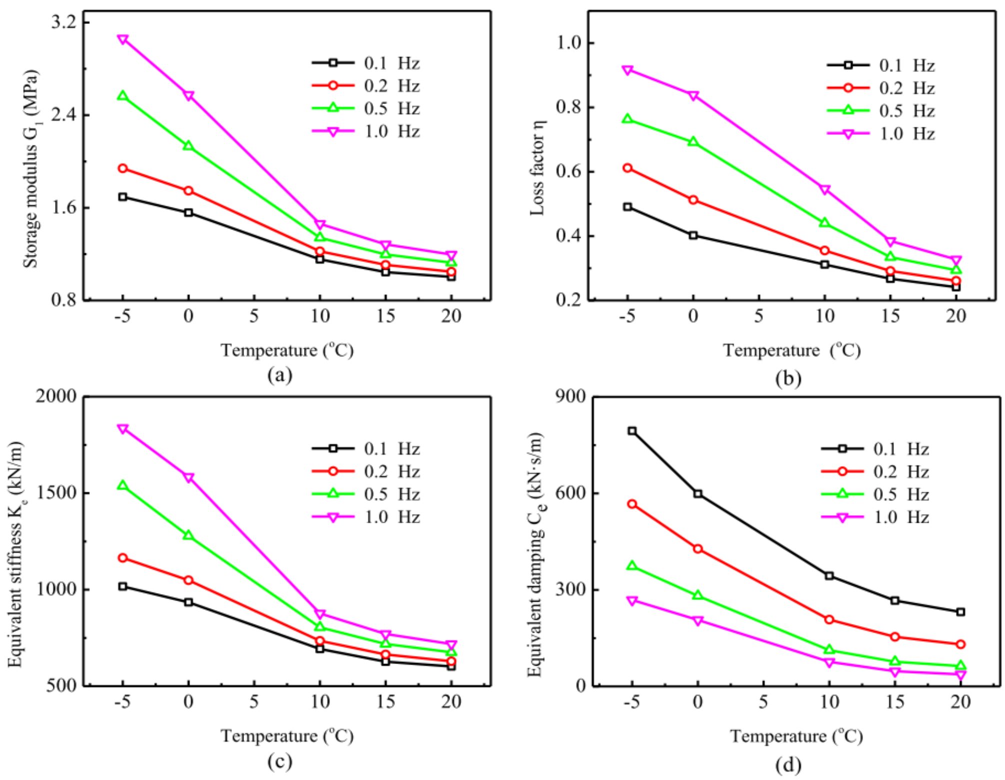


Figure 4: The viscoelastic parameters with a displacement of 1.0 mm are as follows: (a) storage modulus; (b) loss factor; (c) equivalent stiffness; (d) equivalent damping. [11]

1.3 Small-Strain Linear Viscoelasticity and VE Models

Small-strain linear viscoelasticity theory refers to the linear relationship between stress and strain at any time. Based on this theory many constitutive models have been developed, which are divided into classical models and complex models. The relationship between the stress-strain of viscoelastic materials is called the constitutive relationship, and the equation which describes this behavior is the constative equation [12]. for this purpose, researchers have proposed many mathematical models called constitutive models of viscoelastic material. According to different forms of establishment, the constitutive model of a viscoelastic material can be divided into a differential type and an integral type. The differential type is based on the constitutive equations established by the spring-damping system and these equations are in differential form. On the other hand, the integral models are based on the measured creep or relaxation function and these equations are in integral form.

1.3.1 Differential Constitutive Models (Classical Models)

The classic constitutive mechanical models consist of spring and damper elements in series and parallel. The spring-dashpot system employs the elastic behavior of the viscoelastic material which is represented by the spring element while the viscoelastic behavior is represented by the damper element [13]. Typical models include the Maxwell model, Kelvin-Voigt model, and Zener model are shown in Figure 5 [14].

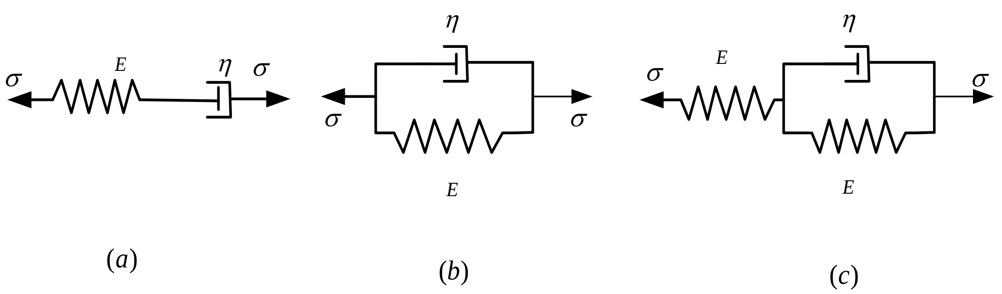


Figure 5: Classical viscoelastic models (a) Maxwell model (b) Kelvin-Voigt model (c) Zener model adapted from [14]

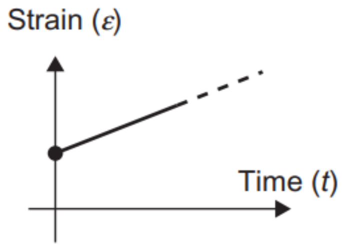
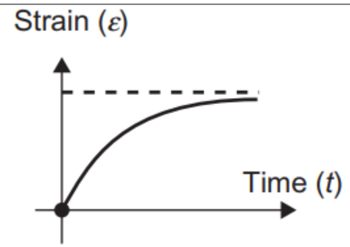
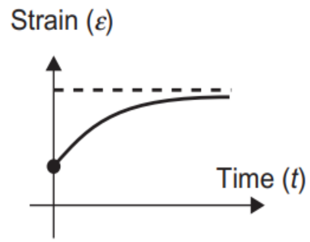
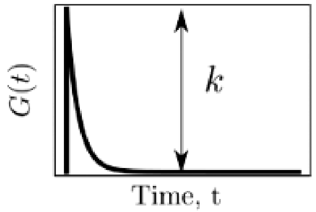
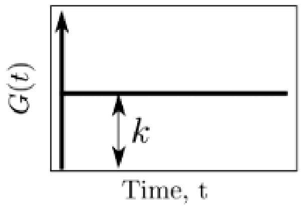
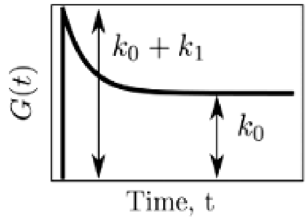
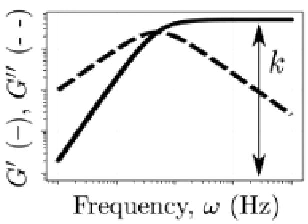
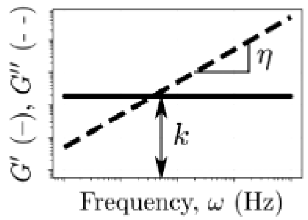
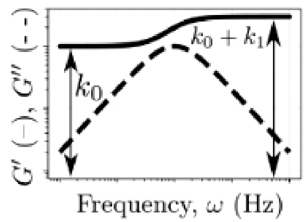
Although these classical models are simple, they cannot simulate the actual mechanical behavior of the viscoelastic material [15]. For instance, in the Maxwell model, there is no anelastic recovery which means that the creep response prediction is poor, which is inconsistent with reality while the relaxation response of the Kelvin-Voigt model does not change with time [16]. In addition, the creep and relaxation of the viscoelastic material have been improved but the main disadvantage is that it cannot accurately describe the frequency characteristics such as the storage moduli and loss factor [17].

Many scholars have improved the previous classical models such as the standard mechanical model which can accurately describe the relaxation response in the frequency domain [17,18]. The main disadvantage is that it needs to obtain the parameters of viscoelastic material in a wide frequency range [19]. Bagley et al. [20] proposed a fractional derivative model that can accurately describe the mechanical properties of the viscoelastic material in a wide frequency range. The disadvantage of this model is that it is difficult to transform into the time domain, and the amount of calculation is immense when the vibration analysis of the structure is performed, which limits the application of the model [21]. Those classical constitutive models are not compatible with the finite element method and are difficult to realize in engineering applications. The following summary can be detected from Table 1 as follow:

- Under static load conditions, the storage modulus E of the Maxwell model is zero while the loss factor η approaches infinity. In addition, at high frequencies value the loss factor η is approaching zero.
- Storage modulus E of Kelvin-Voigt is a constant value and equal to the stiffness of the spring K, while η is zero and unbound at high frequencies.

- The Zener model is a more realistic model than the Maxwell and Kelvin-Voigt in terms of material characteristics.

Table 1: Different classical viscoelastic models with illustrated various properties of behavior

Model	Maxwell	Kelvin-Voigt	Zener
Constitutive Equation	$\sigma = \frac{\eta}{E}\dot{\sigma} = \eta\dot{\varepsilon}$	$\sigma = E\varepsilon + \eta\dot{\varepsilon}$	$\sigma + \frac{\eta}{E_1 + E_2}\dot{\sigma} = \frac{E_1 E_2}{E_1 + E_2}\varepsilon + \frac{E_1 \eta}{E_1 + E_2}\dot{\varepsilon}$
Creep Response [22]			
Relaxation Response [23]			
Storge and loss moduli [23]			

1.3.2 Integral Constitutive Models (Complex Models)

The integral constitutive model is a complex modulus model derived from the creep and relaxation functions of viscoelastic material which are two very important mechanical behaviors. The main standard experiment for viscoelastic material is to obtain those mechanical behaviors and the determination of these complex models is based on experimental work, thus they can reflect the mechanical properties of viscoelastic materials [24]. Furthermore, they simulate the frequency-dependent characteristics of the viscoelastic material because these characteristics are easier to measure in the frequency domain. The stress-strain relationship in the integral form can be expressed as [25]

$$\sigma(t) = G(t)\,\varepsilon(0) + \int_0^t G(t-\tau)\frac{\partial\varepsilon(\tau)}{\partial\tau}\,d\tau \qquad (1)$$

The linear viscoelastic material satisfies the Boltzmann Superposition Principle where $\sigma(t)$ and $\varepsilon\,(t)$ are the stress and strain respectively and $G(t)$ is the shear stress relaxation function, which is a decreasing function representing the energy loss of the viscoelastic material [26]. The constitutive stress-strain relationship consists of two parts: initial stress and stress relaxation, and the constitutive equation can be obtained in the frequency domain as

$$\sigma(s) = G^*(s)\,\varepsilon(s) \qquad (2)$$

Where $G^*(s) = sG(s)$ is the complex shear stress modulus function of the viscoelastic material in the Laplace domain.

1.3.2.1 Complex Constant Modulus Model

From equation (2), substitute *s*=*jw*, we have G^*:

$$\sigma = G\varepsilon \qquad (3)$$

The complex constant modulus of viscoelastic material G^* can be expressed as:

$$G^* = G' + jG'' = G(1 + j\eta) \tag{4}$$

Where G' is the real part, also called the storage modulus, and G'' is the imaginary part which is called the energy dissipation modulus. $\eta = \frac{G''}{G'}$ is the VE loss factor, which is the ratio of the energy dissipation modulus to the storage energy modulus. The loss factor physically represents the ratio of energy consumed from the VE material to the total energy [27].

Many researchers have used the shear complex modulus, since the frequency variation is not considered, they have obtained a simple dynamic equation by applying a simple harmonic excitation force [28–30]. It should be pointed out that the complex modulus of viscoelastic material is not constant and it is related to the excitation frequency [31].

1.3.2.2 Biot Model

Biot [32] proposed a complex model that connects a series of micro-vibrators in parallel where these micro-vibrators are parallel with a spring. micro-vibrators elements are coupled to the global coordinates of the system by the auxiliary local coordinate to simulate the stress-strain behavior of viscoelastic material [33]. The Biot [34] first-order shear relaxation function can be expressed in the frequency domain as :

$$G^*(s) = G^\infty \left[1 + \sum_{i=1}^{N} \frac{a_i s}{s + b_i}\right] \tag{5}$$

where, G^∞ is the equilibrium value of shear stress modulus relaxation of the viscoelastic material, s is the Laplace operator, N is the number of micro-vibrators, and if N mini vibrator is selected 3N+1 must be determined. [a_i, b_i] are positive model parameters determined by the nonlinear curve fitting method using the experimental data of the viscoelastic material

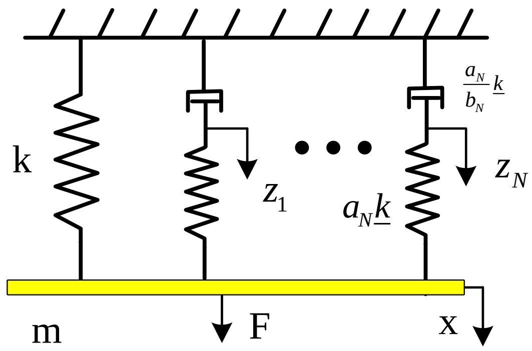


Figure 6: Biot model

Many scholars have used the Biot model to study the dynamic behavior of many applications such as composite viscoelastic structures. For example, in 2006 Zhicheng et al.[35] Developed a finite element model of sandwich beam with two elastic layers using the Biot model to study the damping mechanism of the viscoelastic composite beam. Guo et al. [36] utilized the Biot model to simulate the viscoelastic material behavior in the truss damper of the truss-cored sandwich structure. In 2010 Lin [37] proposed a design mythology for multi-layered damping beam taking into consideration the Vibro-acoustics using the Biot model. Similarly, Leisieutre,[38] presents the categories of different typical constitutive models of viscoelastic. They are like the Biot model in that they rely on auxiliary coordinates to accurately simulate the frequency dependence of VE material.

1.4 Problem Statement

Damping vibration control has become significant in achieving the high mechanical performance of thin-walled structures that are extensively used in many modern aerospace and automobile applications. For this purpose, various studies have been done to point out the damping mechanisms of the sandwich VE structures but, the methods of most scholars are limited to the extraction of the fundamental damping mechanisms of the structure such as natural frequency and loss factor. In addition, there is still a lack of in-depth research on the FEM assessment of forced-harmonic response analysis, as well as there is still considerable debate regarding whether the longitudinal shear deformation or transverse compression deformation of the viscoelastic layer is the damping dissipating energy mechanism of the viscoelastic layer.

1.5 Aim of The Thesis

This work aims to achieve an appropriate design of both viscoelastic beam and plate with integrated Aluminum sheets by investigating their damping characteristics behavior, including the optimization of finite element mesh parameters to enhance the computational efficiency of the simulation. The numerical FE simulation will be based on the assumption of shear deformation theory since the viscoelastic layer will demonstrate a considerable shear deformation under the load, the results will be compared against the literature. A variety of FE modeling methods for the VE sandwich beam/plate structures are proposed to reveal their shear-damping mechanism.

1.6 Objectives of This Study

1. Developing 2D integral finite element shear model based on Hamiltion principle and FSDT.
2. Implying the 2D integral finite element shear model using the finite element model C3D20H®, CPSR® were built using the FE solver interference Abaqus®, and their accuracy and efficiency were compared and studied.
3. Interpreting the MEM and MPF results of beam/plate which shows the directions in the structure has the most harmonic harmful effect.

4. Study the linear dynamic characteristics for VE sandwich beam/plate structures under different boundary conditions and structural parameters respectively
5. Investigate the effect of the loss factor of the VE material on the vibrational behavior of the VE sandwich beam.

Chapter 2

RELATED WORKS

2.1 Studies Reported On VE Sandwiched Structures: Theories And Modeling

Nowadays, the modeling of the VE sandwich structure is divided into two main categories according to basic assumptions of damping mechanisms: the shear and compressional models. Most of the researchers consider the damping energy to be a shear model, thus the longitudinal shear deformation of the viscoelastic core while in the compressional model, the compressive transverse deformation is the only contributed one [39].

Most of the researchers took on the shear model as an assumption of the damping mechanism, in 1958, Kerwin [40] assigned the first analytical model of VE sandwich structure consisting of three layers to predict the loss factor as a function of frequency, temperature, the thickness of the damping layer, and damped bar geometry. Also, Ditaranto developed Kerwin's model, later on, a few researchers such as Blasingame [41] and Reo [42] altered the sixth-order Kerwin's differential equations of motion to study the free vibration of VE sandwich structure with various boundary conditions. They proposed that the concentrating layer and base beam be treated according to the Euler beam so that torsional stresses are ignored. Also, they showed that the viscoelastic layer is treated as a Timoshenko beam and the longitudinal displacement can be expressed in terms of the longitudinal displacement of the constrained layer and base layer, its direction changes linearly within the thickness of the core.

Although the assumptions based on the shear model have been widely accepted, few researchers adopted the compressional assumption of the dissipating energy while ignoring the shear deformation of the viscoelastic layer. On the other hand, Douglas and Yang [43] verified the existence of compressional damping experimentally which can be dominant in narrow bandwidth natural frequencies. Sisemore and Ddacarvennes [44] suggested an accurate analytical model for predicting the natural frequencies of VE sandwich structure and their model has been proved experimentally.

The accurate modeling of the VE sandwich structures is the main key to its vibration analysis. Therefore, a lot of researchers have done a lot of work related to this problem. Nowadays, the modeling of EVEC structure is mainly divided into three categories: analytical, finite element method, and mixed method. The mixed method proposed by some scholars is semi-analytical and semi-numerical.

2.2 Analytical Models

In 1959, Kerwin [45] developed the first analytical model that defines the relative motion between the three layers of the VE sandwich beam structure. Later, based on Kerwin's works, DiTaranto [46] proposed a sixth-order, homogenous differential equation to study the free vibration of a three-layer beam. The results show that the damping loss factor does not depend on the boundary conditions. Mead and Markus [47] utilized DiTaranto's model to develop a model that can be applied to various boundary conditions. The Model considered the constrained layer and the base beam as Euler-Bernoulli beam, neglecting the moment of inertia of the viscoelastic layer.

According to the Mead-Markus model, the three layers have the same lateral displacement, while the longitudinal displacement of the viscoelastic layer can be expressed in terms of the longitudinal displacement of the constrained and base layers. In 2005, Ghinet [48] established a method based on wave theory to analyze the vibration of curved laminate and sandwich composite panels.

The previous analytical models assume that damping vibration is only due to the shear deformation assumption of the viscoelastic layer. as shown in Figure 7. Although this assumption has been widely accepted, some researchers have adapted the compression deformation of the viscoelastic layer to be the damping vibration mechanis. In 1997, Douglas and Yang [43] established an analytical model and then proved experimentally the existence of the transverse compression damping mechanism in a VE sandwich beam in a narrow frequency band centered on the compression resonance frequency of the viscoelastic layer. Few scholars like Sylwan [49] established a composite analytical model based on the shear and compression dissipation energy assumption but the model showed an overestimation of the loss factor of the structure in a wide frequency band.

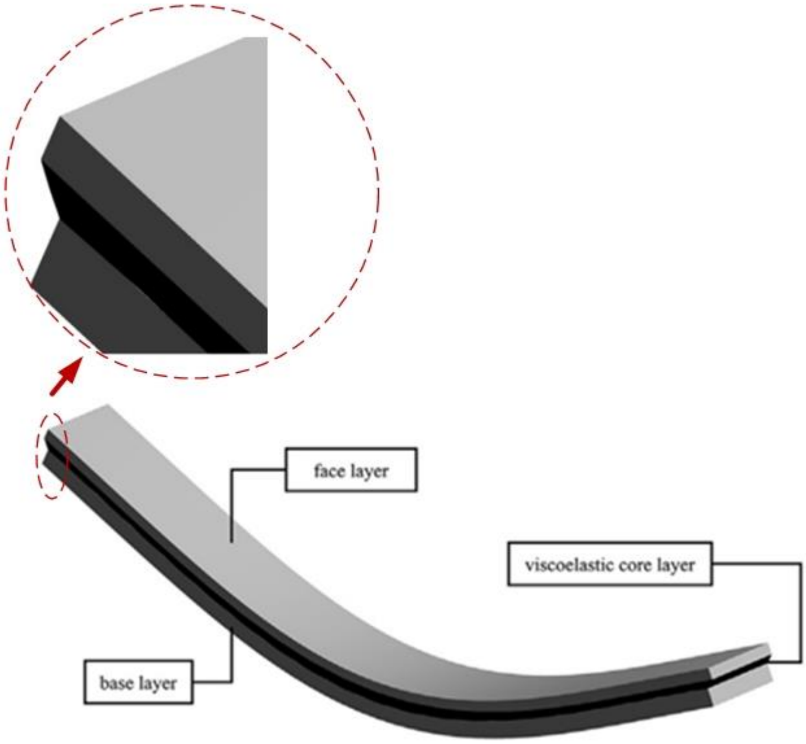


Figure 7: Schematic representation of shear deformation mechanism in VE sandwich beam. Adapted from [50]

2.3 FEM Models

In engineering practice, VE sandwich structures have complex geometric shapes and boundary conditions. At this time, such problems cannot be solved effectively by the analytical process hence the finite element approach is commonly used. There is two main finite element construction for modeling VE sandwich structure, Layered FEM and Direct FEM. The Layered method considers different units for each layer Figure 8 (a) while Global Direct FE. The Layered FEM was used in early commercial software, and it is less efficient. On the other hand, the advantage of Global Direct FE is that it provides less degree of freedom with high accuracy [8]. Based on the shear assumption of damping energy, many researchers have developed FE models such as Johnson [51] and Plouin et al. [10] who used triangular shell elements (QUAD4) to discrete the elastic faces and solid elements (HEXA8) for the viscoelastic core implemented in NASTRAN commercial software. The traditional shell elements and solid elements have been used to model the core layer. Also, Ma et al. [52] modeled the elastic surface layer and viscoelastic layer using shell elements and fully (HEXA8) compatible solid elements.

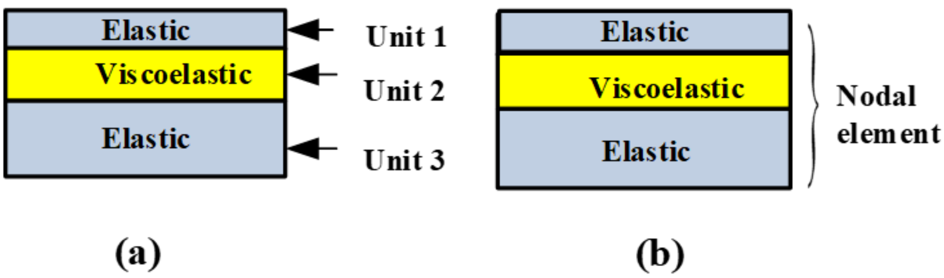


Figure 8: Two typical FEM construction methods (a) Layered method and (b) Global element construction method

Arvin et al. [53] reported a numerical FE investigation on the free and forced vibrational response of a composite sandwich beam with a viscoelastic core (as shown in Figure 9) using ‘‘Mead & Markus” theory. The considered theory was modified by taking into consideration the effects of Young modulus, rotational inertia, and core kinetic energy. Also, a parametric study on the effects of several impressive parameters involving fiber angle, the thickness of faces, and core thickness was investigated.

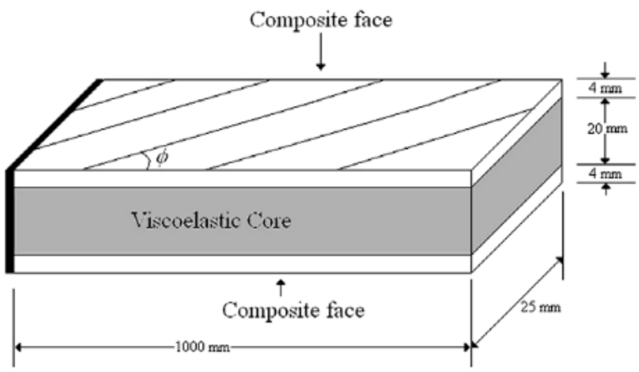


Figure 9: Illustrates the schematic of the VE sandwich composite beam [53]

Bilasse et al. [54] reported a study on the investigation of the VE sandwich beam's Linear and nonlinear vibration response. Moita et al. [54] proposed FE model for the vibrational response of active-passive damped multi-layered plates consisting of VE and piezoelectric layers. later on, Barbosa and Farage [55] proposed an FE modeling formulation of EVE's sandwich beam. The FE element (shown in Figure 10) consists of seven elements: one four-node plate quadrilateral linear viscoelastic element; two-node elastic frame elements; and four rigid connection elements. The proposed model element has 24 physical degrees of freedom besides five dissipative degrees of freedom of the plane stress VE element resulting in a super-element model with 29 degrees of freedom (DOF).

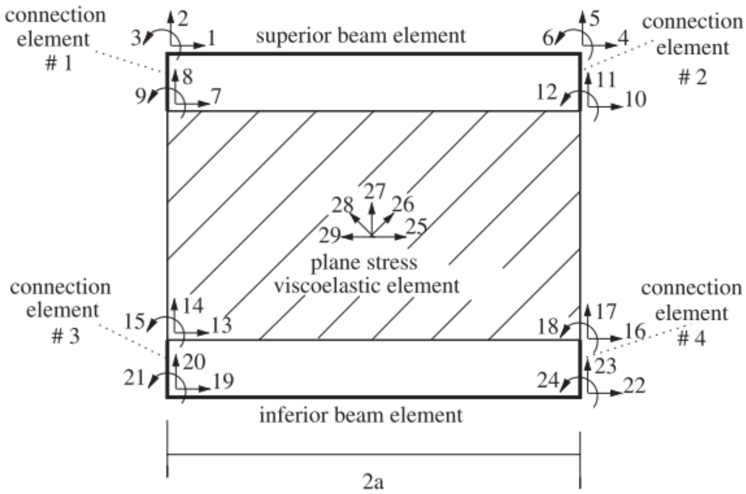


Figure 10: The FE model schematic illustrated super element DOF [55]

Wang and Inman [55] developed a two-dimension FE model to investigate the dynamic vibration of a composite beam integrated with a VE damping layer. The 10 nodal degrees of freedom (DOF) of the composite element are shown in Figure 11. The FE model was assembled based on the Hamilton principle and the VE frequency-dependent material was incorporated into the equation of motion of the composite beam through an auxiliary coordinate using the Golla–Hughes method (GHM) model. Lin and Rao [56] performed a vibration analysis study of Multiple-Layered Structures with a VE-Damping layer. The non-linear behavior of the VE layer was simulated with the Biot damping model. The model showed high accuracy in predicting the time-dependent dissipative energy.

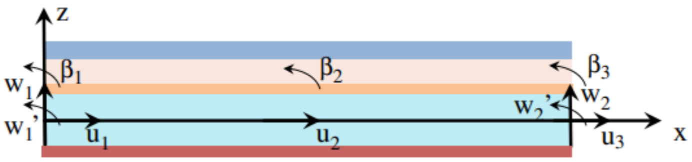


Figure 11: Schematic of the five-layer composite nodal degree layered [55]

2.4 Mixed Models

The analytical method generally involves the solution of higher-order partial differential equations and has limited applicable boundary conditions, which limits its Engineering application[57]. Furthermore., the finite element method has been widely used in the dynamic modeling of complex structures, but some problems are difficult to implement and computationally expensive. Therefore, some researchers try to ensure accurate modeling based on looking for ways to improve computational efficiency, some models between the analytical method and the finite element method are proposed type, called semi-analytical semi-numerical method [58]. Daya et. al [59] used the homotopy algorithm and the asymptotic method to solve the nonlinear eigenvalue problem of composite structure with a viscoelastic layer and obtain the natural frequency and loss factor. This method can greatly simplify the solution process. Bostrom et.al [60] derived the flexural vibration equations for homogenous, elastic, isotropic pates using displacement field expansion series Koutsawa [61] et al. studied the non-linear behavior of laminated glass beam by applying the asymptotic numerical technique to establish a diamond toolbox. The performance of the diamond toolbox was demonstrated numerically and experimentally Komlanvi et al [62] performed sensitivity studies on the non-linear mechanical behavior of laminated glass beams.

Table 2: Summary of reported studies on VE sandwich composite structures using an analytical solution

Author	Theories considered	Structure(s)	Equation formulation	Assumption	Contribution
Ungar [63]	-Euler Beam Theory	-Beam -Plate	-Energy Method	- Shear deformation	- Define the relative displacement motion between the layers of axially uniform linear composite structures. - General expression of loss factor for different configurations.
Yan and Dowell [64]	-Timoshenko Beam Theory	-Beam -Plate	-Linear theory of elasticity. -Linear viscoelasticity -Virtual work principle	- Shear deformation	- Obtain the 4^{th} linear differential equation that governs the damping vibration of sandwich plates and beams.
Rao [42,65]	-Timoshenko Beam Theory	-Beam	-Hamilton's principle	- Shear deformation	- Deduced the equation of motion of the unsymmetric short sandwich beam. - The vibration governs equations have been solved Obtain the natural frequencies and loss factors with various boundary conditions.
Durocher et al. [66]	-Timoshenko Beam Theory	-Plate	-Virtual work principle	- Shear deformation	- Derived a 4^{th} order equation to analyze vibration of EVE three-layered plates.
Wang [48]	-First-order shear deformation	-Plate	-Galerkin principle -Hamilton's principle	- Shear deformation	- Established the dynamic equation of the EVE plate to study the flexural vibration. -Incorporated the frequency-dependent viscoelastic model GHM into the dynamic equation of the EVES plate
Sisemore and Darvennes [44]	-Euler Beam Theory	-Beam	-Hamilton's principle	- Compression deformation	- Established a new analytical model that predicts the natural frequency of structure very well. - Their experiment proved the analytical model as well, but it showed a poor prediction of the loss factor.
Lee and Kim [67]	Finite difference method (FDM)	-Plate	Analytical equation formulation	- Shear deformation -Compression deformation	- Developed an analytic formulation to study the vibration of a square sandwich plate under clamped conditions.
He and Ma [68]	-Timoshenko Beam Theory	-Unsymmetrical plate	-Hamilton's principle -asymptotic technique	- Shear deformation	-Considered the simplified governing equations for flexural vibration of viscoelastic damped layer unsymmetrical sandwich plate with corresponding boundary condition -The loss factor of the viscoelastic material of the core was used as a parameter in an asymptotic solution of the simplified governing equations.
Levy and Chen [69]	-First-order shear deformation	-Beam	-Hamilton's principle	- Shear deformation	- The differential equation of motion is derived according to Hamilton's principle formulation. that improved the Rao vibration governs equations [42,65] - performed a parametric analysis to study the influence of various parameters on the vibration characteristics of partially double EVE beam -Provided a compression between the double and single sandwich beams that indicate double types have better damping than single types.

Table 3: Summary of reported studies on VE sandwich composite structures using FEM solution

Author	Theories/methods	Structure(s)	VE model(s)	Equation formulation	Kinematic formulation's	Solution technique	Assumption/methodology/Contribution
Johnson and David [51]	- Euler Beam Theory (Elastic Layer). -Timoshenko Beam Theory (VE Layer)	- Beam - Plate - Ringreg	-Empirical correction to simulate the frequency-dependent properties	- MSE implemented in NASTRAN	-First-order shear deformation (FSDT)	-Model strain energy -Complex eigenvalue	-Establish a discretized FE model that describes the model damping using the MSE method implemented in NASTRAN for different configurations. -QUAD4 plate element with offset for the elastic layer with HEXA solid element for VE core is adapted for modeling EVE's sandwich structure. -High Aspect ratio for a thin viscoelastic core has been taken into consideration.
Baber et.al. [70]	-Euler Beam Theory -Timoshenko Beam Theory	- Beam - Elastic Layer - VE Layer	-Constant complex Modulus	- Hamilton's principle	-First-order shear deformation (FSDT)	-Plane strain conditions	-Developed a FE model of EVE's beam under harmonic excitation with thin/thick VE cores. -Their model shows high accuracy prediction of the displacement response at resonant frequencies - The VE Layer was based using the Timoshenko beam model
Zape et.al. [51]	- Plane strain conditions	- Five composite beam	- Constant complex Modulus	- Strain Energy	-Linear/Quadratic interpolation function	-Model strain energy (MSEM)	-Established a discrete FE model integrated with the VE damping layer to estimate the model -damping characteristics using the model strain energy approach. - To avoid shear locking, the discretization of the element is performed using continuous quadratic polynomial for the transverse displacement while continuous linear polynomial for the longitudinal displacement -Five composite beam layers have been tested experimentally to obtain the
Sainsbury and Zhang [71]	-Euler Beam Theory (Elastic Layer). -Timoshenko Beam Theory (VE Layer)	- Beam	- Constant complex Modulus	- Galerkin principle	-Kronecker condition	-Polynomial shape functions With Galerkin orthogonal functions -	-All layers have the same transverse displacement. -The displacement between the layer intersection is unformed to maintain the appropriate element. -Investigate the dynamics vibration of a damped clamped free beam using a numerical approximation method based on the Galerkin approach which required less number of elements.
Daya and Potier-Ferrry[59]	-Euler Beam Theory (Elastic Layer). -Timoshenko Beam Theory (VE Layer) - Discrete Kirchhoff (DKT) theory	- Shell - Beam - Plate	- Constant complex modulus - Frequency-dependent Modulus	- Virtual work principle	- First-order shear deformation (FSDT)	- Model strain energy (MSEM) - Asymptotic-numerical method (ANM)	-The displacement in the elastic layer modeled using classical plate theory - The longitudinal shear deformation is taken into consideration -All layers have the same transverse displacement. -Proposed a shell FE model with eight degrees of freedom for damped EVE's sandwich structure

Amichi and Atalla[72]	-Euler Beam Theory (Elastic Layer). -Timoshenko Beam Theory (VE Layer)	- Beam - Sandwich Ring - Symmetrical configurations Unsymmetrical configurations	- Frequency-dependent Modulus	- Strain Energy	- Lagrange shape function - Hermite shape function	- Complex mode approach	-Based on a discrete displacement approach the FE model developed with eight degrees of a freedom beam element -Obtained the vibration response, natural frequencies, and the model damping loss factor for different boundary conditions.
Galucio[73]	-Euler Beam Theory (Elastic Layer). -Timoshenko Beam Theory (VE Layer)	-Beam	-four-parameter fractional derivative model to simulate the frequency-dependent properties	- Hamilton's principle	-First-order shear deformation (FSDT)	- Direct time integration method - Grunwald approximation	-The four-parameter fractional derivative model is fitted with master curves to calibrate the frequency-dependent behaver of the VE layer. -Preformed a forced vibration transient analysis based on the direct time integration method. -The moment of inertia of the VE core was taken into consideration
Moreira and Rodrigues [74]	- Zig-Zag theory - Mindlin thick plate	-Plate	-3M™ Vibration Damping Tapes Frequency/Temperature-dependent Modulus	- Hamilton's principle	- Isoperimetric quadrilateral finite element - Bilinear shape functions	- Direct frequency response	-Established A layer-wise FE model to study the dynamics of EVE's sandwich plate. -Comparison analysis study of the proposed model with the conventical model that uses a standard plate with solid elements. - The proposed FE model has been validated experimentally using two specimens one with a single VE core and the other with multiple VE cores.
Moita et al [75]	-Classical plate theory (CPTs) -Mindlin thick plate	-Multilayer's passive-active sandwich plate	- Frequency-dependent Modulus	- Hamilton's principle	- First-order shear deformation (FSDT)	-Complex eigenvalue -Steady-state harmonic response	-The active damping layer and the elastic layer are modeled using CPT theory, while the passive damping layer is modeled with FSDT theory. - Any point through the thickness of the sandwich plater is considered to have the same transverse displacement. -No relative movement between the layer interfaces -Proposes a triangular FE model with three nodes and eight-degree of freedom to study the vibration of Multilayer's passive-active sandwich plate.

Table 4: Summary of reported studies on VE sandwich structures using mixed methods

Author	Theories considered	Structure(s) description	VE model(s)	Kinematic formulation	Equation formulation	Solution technique	Assumption/methodology/Contribution
Lewandowski et al. [76]	- Refined Zig-Zag theory	- Composite beams - Sandwich beam - Multi-layered 2D - Elastic and VE layers	- Fractional Zener model	-	- Principle of virtual work	- Lagrange polynomials - Orthogonal Gram–Schmidt polynomials - Hierarchical FE - Laplace transformation	- Proposed a new method for multi-layered beams' dynamic behaviors - Implementation of displacements based on refined zig-zag theory on VE layered beams - Mechanical properties were based on the fractional Zener model - Compared to other models, the proposed model contains fewer degrees of freedom - VE physical properties were based on the linear constitutive relations - Investigated the dynamic behaviors of the proposed beams - The beam's dynamic behaviors were obtained using hierarchical FE and Laplace transformation techniques - Numerical solutions were obtained using the continuation method
Huang [77]	-Timoshenko Beam Theory -Euler Beam Theory	-Beam	-Biot model	-First-order shear deformation (FSDT)	-Hamilton principle	-Numerical integration -Model reduction.	-Investigated the non-linear vibration of EVEs. -Developed a finite element model considering the compression vibratory energy of the frequency-dependent viscoelastic material. -Studied the transverse damping mechanism of EVEs experimentally as a broadband existence. -Their results showed that predicting the damping characteristic are related to the thickness of EVE's layers.
F. Abdoun et al. [78]	- Kirchhoff love (CPT) - Mindlin's	- Beam - Plate	- Constant complex modulus	- Virtual work principle	- not mention	-Asymptotic Numerical - Power series expansions - Padé approximants	- Free and force harmonic vibration of viscoelastic sandwich structure - Considered several undamped viscoelastic sandwich models and structures - Constant and frequency-dependent viscoelastic models of Maxwell type were considered - Obtain the linear frequency and the damping characteristics for a clamped beam with different viscoelastic loss factors.
Duigou et al. [79]	-Timoshenko Beam Theory -Euler Beam Theory	- Beam - Plate	- Constant complex modulus - Frequency-dependent Modulus	-First-order shear deformation (FSDT)	- Hamilton's principle	-asymptotic numerical techniques -Pade approximants	- Developed a shell FE sandwich element with two nodes, eight- degrees of freedom per each (DOF) for a damped EVE. - Proposed two iterative methods based on Homotopy algorism and asymptotic numerical method to solve

			(generalized Maxwell model)			-Lanczos algorithm	the non-linear complex eigenvalue problem for damped EVEs.
Alvelid et al. [80]	-Displacement field expansion series	-Beam -Plate	- Fractional order viscoelastic model	- Virtual work principle- Von Karman's theory which	-Interpolation function	- series expansions	-developed an interface FE model for a VE layer in EVE sandwich structure. -The interface element is based on a series expansion of displacement field in the thickness direction. -This method makes it possible to achieve higher resolution without the need of increasing DOF. -Investigated The steady-state response of EVE sandwich plate experimentally and numerically using FE solver then, compared with analytical frequency response.
Bilasse et al. [81]	-Timoshenko Beam Theory -Euler Beam Theory	-Beam	-Constant complex modulus - Frequency-dependent	- Von Karman's theory	- Galerkin principle - Principle of virtual work	-asymptotic numerical techniques	-Developed FE models to investigate the linear and non-linear vibration behaviors of EVE beams with different VE models as well, as various boundary conditions. -Considered the VE frequency-dependent models -Investigated the frequency response using the numerical complex eigenmodes that are based on the Galerkin principle.
C. Cai et al. [82]	-Timoshenko Beam Theory -Euler Beam Theory	-PCLD patch	- Constant complex modulus		-energy approach	- Lagrange equation	-Developed an analytical model to study the steady-state vibration response of passive constraining layer damping (PCLD) patch. -Their proposed model shows high efficiency in predicting the vibration response of PCLD patch in the frequency of interest for common VE material taken into consideration different BC. -The proposed model shows high accuracy for extremely hard VE material other than hard or soft VE material.

Chapter 3

RESEARCH METHOD

This chapter proposed a research methodology for dynamic analysis of VE sandwich beam/plate. In the pre-processing step, the integral FE shear beam element developed based on the shear deformation assumption and the FE model implemented in Abaqus®. The steady-state frequency response combined with MATLAB and FE solver Abaqus® is shown in Figure 13.

3.1 Finite Element Modelling

A FE shear model is introduced based on shear energy dissipation. The configurations of a VE sandwich beam element are shown in Figure 14 with a 2-node 8-degree of freedom (DOF). The element is composed of a VE core sandwiched between two isotropic elastic layers. Most of the former assumes that the energy dissipating throws the longitudinal shear deformation.

3.2 Kinematics Relationships

The geometry configurations and deformation of typical VE sandwich beams are shown in Figure 12 t_1, t_2, and t_3 are the thickness of the constraining layer, the core layer, and the base layer respectively. γ and θ are the shear strain and rotation respectively. The element's local coordinate shown in Figure 12 needs to derive the centered relative displacement of the layer.

$$\gamma = \theta + \varphi \tag{6}$$

where φ is the shear angle of the VE core since the two elastic layers are assumed to be treated as Euler-Bernoulli beams with neglecting the shear deformation

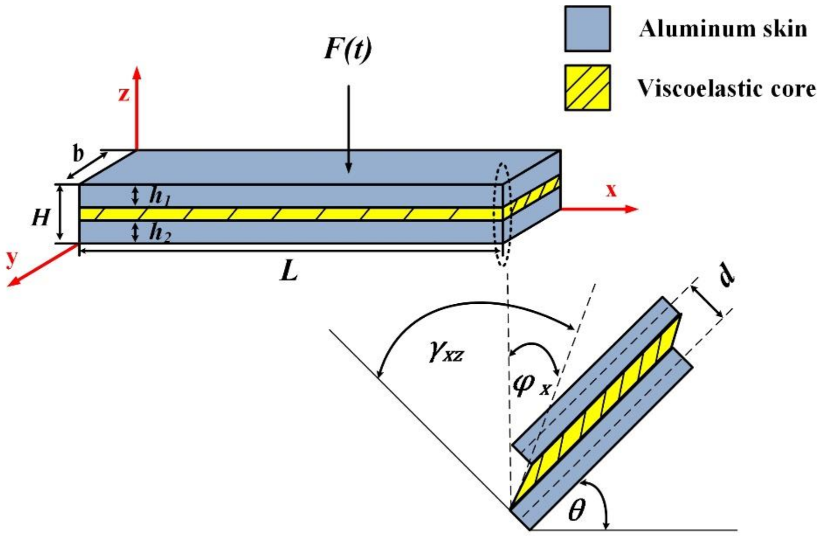


Figure 12: The configurations of the VE sandwich geometry of deformed VE sandwich beam element

the axial and transverse displacements at any point within the elastic layers can be expressed as [83]:

$$\left.\begin{aligned} u^{(i)}(x,z,t) &= u_i(x,t) - z_i\frac{\partial w_i(x,t)}{\partial x} \\ w^{(i)}(x,z,t) &= w_i(x,t) \end{aligned}\right\} i = c, b \tag{7}$$

where $u_i(x,t)$ and $w_i\ (x,t)$ are the axial and transverse displacements of the centroid ith layer respectively, the following kinematics corresponding to the viscoelastic core γ and u_v deriving following the first-order shear deformation theory [84]:

$$u_v = \frac{1}{2}(u_c + u_b) + \frac{1}{4}(t_1 - t_3)\frac{\partial w}{\partial x} \tag{8}$$

$$\gamma = \frac{1}{t_2}\left[(u_c - u_b) + \left(\frac{t_1 + t_3}{2} + t_2\right)\frac{\partial w}{\partial x}\right] \tag{9}$$

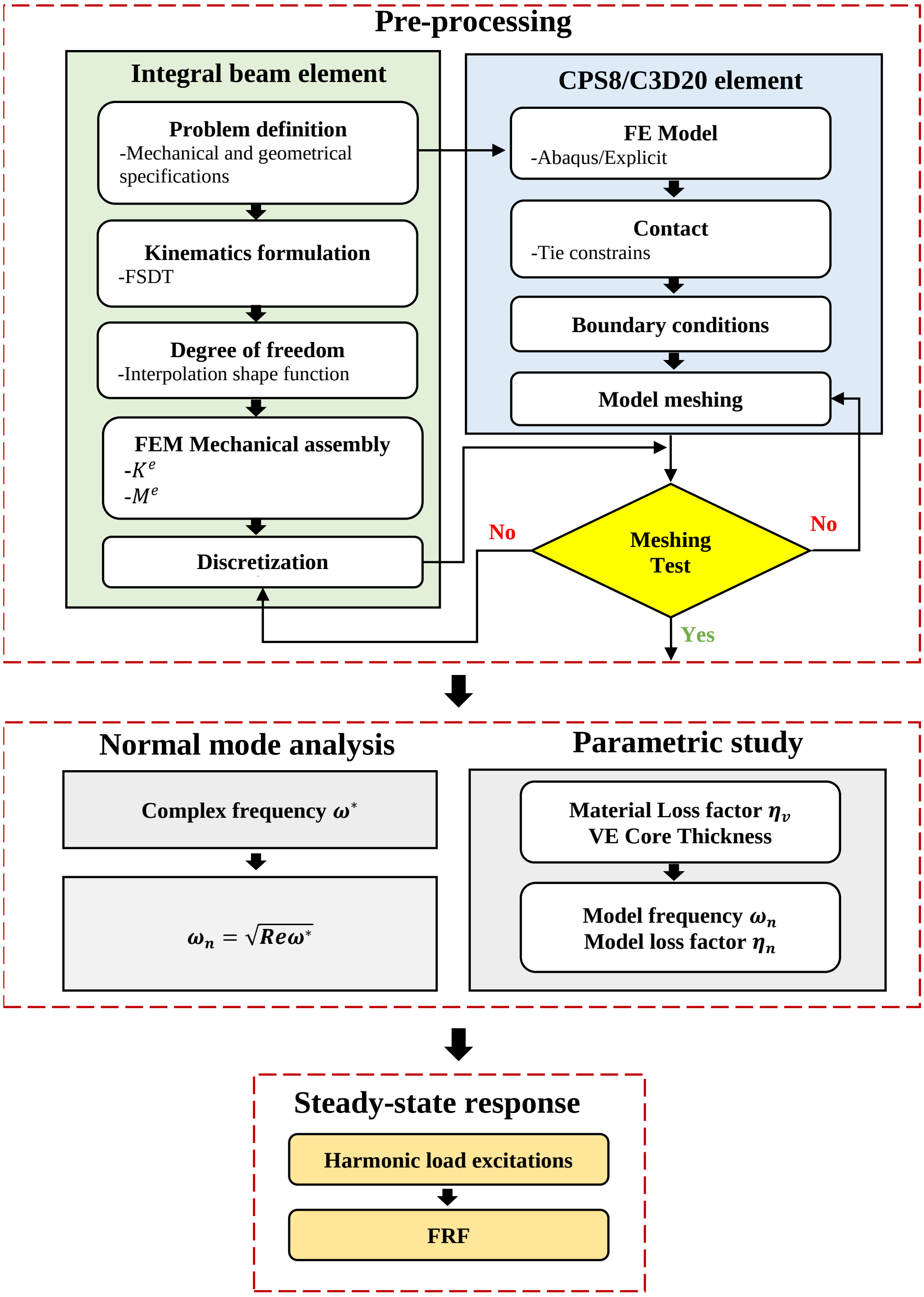


Figure 13: Flow chart for proposed dynamic analysis methodology combined with MATLAB and FE solver Abaqus®

3.4 Degree of Freedom and Shape Functions

The integral FE model of the EVE sandwich beam is shown in Figure 14 with two nodes i and j. Each node has 4 four DOF - lateral displacement w in the z-direction, the rotation θ about the x-axis, and the axial displacement of the elastic layer u_1, u_3. The total set of the nodal displacements $\{d^e\}$ is given by

$$\{d^e\} = \begin{Bmatrix} d_i \\ d_j \end{Bmatrix} = \{w_i \quad \theta_i \quad u_{bi} \quad u_{ci} \quad w_j \quad \theta_j \quad u_{bj} \quad u_{cj}\}^T \tag{10}$$

the element displacement $d = Nd^e$ can be written in matrix form as:

$$\begin{Bmatrix} w \\ \theta \\ u_i \\ u_j \end{Bmatrix} = \begin{bmatrix} N_x \\ N_x{}' \\ N_1 \\ N_2 \end{bmatrix} \{d^e\} \tag{11}$$

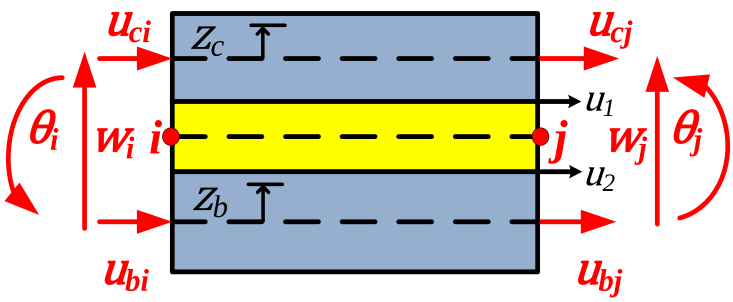


Figure 14: Integral shear beam element

The transverse displacement field $w(x)$, $\theta(x)$rotation field, and axial displacements of elastic layers $u_b$and u_c can be expressed as a cubic polynomial function where s = x/L_e, L_e is element length

$$w(x) = c_0 + c_1 s + c_2 s^2 + c_3 s^3 \tag{12}$$

$$\theta(x) = \frac{\partial w}{\partial x} = c_1 + 2c_2 s + 3c_3 s^2 \tag{13}$$

$$u_b = c_4 + c_5 s, \; u_c = c_6 + c_7 s \tag{14}$$

$$Nd^e = TC = \begin{bmatrix} 1 & s & s^2 & s^3 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_e} & \frac{2s}{l_e} & \frac{3s^2}{l_e} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & s & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & s \end{bmatrix} \begin{Bmatrix} c_0 \\ c_1 \\ c_3 \\ \vdots \\ c_7 \end{Bmatrix} = [w \quad \theta \quad u_b \quad u_c]^T \tag{15}$$

where N is the shape function matrix corresponding to the four displacement components of each node in the form of 4 × 8 and C is the 8× 8 matrix that depends on the applying boundary condition. After applying boundary conditions to Eqs. (8) and (9) the following system of Equations. obtain in a matrix form C as

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_e} & \frac{2}{l_e} & \frac{3}{l_e} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_e} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

the shape function then can obtain easily after the inversion operation of C in the matrix form and decoupling of Eqn.16 as follow:

$$N = TC^{-1} = [N_x \quad N_x{}' \quad N_1 \quad N_2]^T \tag{16}$$

$$C^{-1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{l_e} & \frac{2}{l_e} & \frac{3}{l_e} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{l_e} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The generalized shape function matrix obtains as a colums matrix:

$$N_x = \begin{bmatrix} 1-3s^2+2s^3 \\ (s-2s^2+s^3)\,L_e \\ 0 \\ 0 \\ 3s^2-2s^3 \\ (-s^2+s^3)\,L_e \\ 0 \\ 0 \end{bmatrix}^T, N_x{}' = \begin{bmatrix} \frac{6s(s-1)}{L_e} \\ (s-2s^2+s^3)L_e \\ 1-4s+3s^2 \\ 0 \\ 0 \\ -2s+3s^2 \\ 0 \\ 0 \end{bmatrix}^T, N_1 = \begin{bmatrix} 0 \\ 0 \\ 1-s \\ 0 \\ 0 \\ 0 \\ s \\ 0 \end{bmatrix}^T, N_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1-s \\ 0 \\ 0 \\ 0 \\ s \end{bmatrix}^T$$

the displacement of any point within the element can be determined by the nodal displacement of the element through shape function interpolation:

$$w = N_x d^e, \theta = N_x{}' d^e, u_b = N_1 d^e\,,\; u_c = N_2 d^e, u_v = N_3 d^e, \gamma = N_4 d^e \tag{17}$$

N_3 and N_4 are derived from the geometric relation in Eqs.9 and Eqs10 as follows:

$$N_3 = \frac{1}{2}\left[(N_1 + N_2) + \left(\frac{t_1 + t_3}{2}\right) N_2\right] \quad (18)$$

$$N_4 = \frac{1}{h_v}\left[N_1 - N_2 + \left(\frac{t_1 + t_3}{2} + h_v\right) N_2\right] \quad (19)$$

3.5 Element Stiffens Matrix

To obtain the element stiffness matrix, the principle of strain energy is applied. The strain energy is:

$$U = \int \frac{\sigma^2}{2E} dv \quad (20)$$

3.5.1 Strain Energy Due To Axial Loading

The stress due to axial loading is:

$$U_e = \frac{EA}{2} \int \varepsilon^2 dx \quad (21)$$

The definition of the axial strain is defined as the rate change of the displacement which can be expressed as:

$$\varepsilon = \frac{\partial u}{\partial x} \quad (22)$$

Substitute Eqs.21 into Eqs.20 we get:

$$U_e = \frac{1}{2} E_i A_i \int_0^{l_e} \left(\frac{\partial u_i}{\partial x}\right)^2 dx = \frac{1}{2} d^{e^T} K_{ei}^e d^e \quad (23)$$

$$U_{ec} = \frac{1}{2} E_c A_c \int_0^{l_e} \left(\frac{\partial u_c}{\partial x}\right)^2 dx = \frac{1}{2} d^{e^T} K_{ec}^e d^e \quad (24)$$

$$U_{eb} = \frac{1}{2} E_b A_b \int_0^{l_e} \left(\frac{\partial u_b}{\partial x}\right)^2 dx = \frac{1}{2} d^{e^T} K_{eb}^e d^e \quad (25)$$

where $K_{ec}^{e\,I,j}$, $K_{eb}^{e\,I,j}$ are the nodal element stiffness matrix of the constraining and base layer corresponding to the axial strain energy so that:

$$K_{eb}^e = E_b A_b \int_0^{l_e} \left[\frac{\partial N_1}{\partial x}\right]^T \left[\frac{\partial N_1}{\partial x}\right] dx \quad (26)$$

$$K_{ec}^e = E_c A_c \int_0^{l_e} \left[\frac{\partial N_2}{\partial x}\right]^T \left[\frac{\partial N_2}{\partial x}\right] dx \quad (27)$$

$$K_{ec}^{e\ i,j} = \begin{bmatrix} 2.983e+6\ +3.8e+2i & & & \\ 8.832e+3\ +1.9e-1i & 34.9+0.0015i & & \\ 5.696e+3\ +\ 5.7e+2i & -5.626\ +0.6i & 2.254e+7+1.36e+3i & \\ -5.696e+3-5.7e+2i & 5.626\ +0.6i & -1.363e+04-1.4e+3i & 2.255e+7\ +1.4e+3i \end{bmatrix}$$

$$K_{eb}^{e\ i,j} = \begin{bmatrix} 2.983e+6\ +3.8e+2i & & & \\ -8.832e+3\ +1.9e-1i & 34.9+0.0015i & & \\ -5.696e+3\ +\ 5.7e+2i & 5.626\ +0.6i & -2.254e+7+1.36e+3i & \\ 5.696e+3-5.7e+2i & -5.626\ +0.6i & -6.816e+3-\ 6.81e+2i & -2.25e+7\ +1.4e+3i \end{bmatrix}$$

3.5.2 Strain Energy Due To Bending Loading

$$U_b = \int \frac{M(x)^2 y^2}{I^2} A\, dx \tag{28}$$

The flexural bending of the element is given by:

$$\frac{M(x)}{EI} = \frac{\partial^2 w}{\partial x^2} \tag{29}$$

$$U_b = \frac{E}{2}\iint \left(\frac{\partial^2 w}{\partial x^2}\right)^2 y^2 dA\, dx \tag{30}$$

$$U_b = \frac{1}{2}EI\int \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx = \frac{1}{2}d^{eT}K_b^e d^e \tag{31}$$

$$U_{bc} = \frac{1}{2}E_c A_c \int_0^{l_e} \left(\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right)^2 dx = \frac{1}{2}d^{eT}K_{bc}^e d^e \tag{32}$$

$$U_{bb} = \frac{1}{2}E_b A_b \int_0^{l_e} \left(\left(\frac{\partial^2 w}{\partial x^2}\right)^2\right)^2 dx = \frac{1}{2}d^{eT}K_{bb}^e d^e \tag{33}$$

where $K_{b\ c}^{e\ i,j}$ and $K_{b\ b}^{e\ i,j}$ are the nodal element stiffness matrix of the constraining and base layer corresponding to the bending strain energy so that:

$$K_{bb}^e = E_b A_b \int_0^{l_e} \left[\frac{\partial N_x}{\partial x}\right]^T \left[\frac{\partial N_x}{\partial x}\right] dx \tag{34}$$

$$K_{bc}^e = E_c A_c \int_0^{l_e} \left[\frac{\partial N_2}{\partial x}\right]^T \left[\frac{\partial N_2}{\partial x}\right] dx \tag{35}$$

$$K_{b\ c}^{e\ i,j} = \begin{bmatrix} -2.983e+6\ +3.8e+2i & & & \\ 8.832e+3\ +1.9e-1i & 17.44+0.0004i & & \\ -5.696e+3\ +\ 5.7e+2i & 5.626\ +0.6i & -2.254e+7+1.36e+3i & \\ 5.696e+3-5.7e+2i & -5.626\ +0.6i & -6.816e+3-\ 6.81e+2i & -2.25e+7\ +1.4e+3i \end{bmatrix}$$

$$K_{b\ b}^{e\ i,j} = \begin{bmatrix} -2.983e+6\ +3.8e+2i & & & \\ 8.832e+3\ +1.9e-1i & 17.44+0.0004i & & \\ 5.696e+3\ +\ 5.7e+2i & 5.626\ +0.6i & -2.254e+7+1.36e+3i & \\ -5.696e+3-5.7e+2i & -5.626\ +0.6i & -6.816e+3-\ 6.81e+2i & -2.25e+7\ +1.4e+3i \end{bmatrix}$$

3.5.3 Strain Energy Due To Shear Loading

$$U_{shear} = \int \frac{\tau^2}{2G} dv = \frac{G\gamma^2}{2} \int dv \tag{36}$$

$$U_{shear} = \frac{G^*\gamma^2}{2} \iint dA\, dx \tag{37}$$

$$U_{shear} = \frac{1}{2} G^* A_v \int_0^{l_e} \gamma^2\, dx = \frac{1}{2} d^{e^T} K_{shear}^e d^e \tag{38}$$

where and K_v^e are the shear element stiffness matrix associated with the shear strain energy so that:

$$K_S^e = G^* A_v \int_0^{l_e} N_6{}^T N_6 dx \tag{39}$$

the elastic element stiffness matrix of the elastic layers of the VE sandwich beam is

$$K_e^e = K_{eb}^{e\,i,j} + K_{ec}^{e\,i,j} + K_{b\,b}^{e\,i,j} + K_{b\,c}^{e\,i,j} \tag{40}$$

$$\underbrace{K^e}_{[8\times8]} = \begin{bmatrix} [K_{eb}^e] & [K_{bb}^e] \\ [K_{bc}^e] & [K_{ec}^e] \end{bmatrix} + \underbrace{K_S^e}_{[8\times8]} \tag{41}$$

K^e is the total element stiffness matrix obtained as the summation of the elastic element stiffness K_e^e matrix and shear element stiffness matrix K_S^e of EVE sandwich beam.

3.6 Element Mass Matrix

A similar principle should be to obtain the element mass matrix, the principle of kinetic energy applied. With neglecting the axial and rotation loading associated with kinetic energy The kinetic energy of the three-element layer T is:

$$T = \frac{1}{2}(\rho_c A_c + \rho_v A_v + \rho_b A_b) \int \dot{w}^2\, dx + \frac{1}{2}(\rho_c A_c + \rho_b A_b) \int \dot{u}^2\, dx \tag{42}$$

$$T_{extension\ c} = \frac{1}{2}\rho_c A_c \int_0^{l_e} \left(\frac{\partial u_c}{\partial t}\right)^2 dx = \frac{1}{2} \dot{d}^{e^T} M_{ec}^e \dot{d}^e \tag{43}$$

$$T_{bending\ c} = \frac{1}{2}\rho_c A_c \int_0^{l_e} \left(\frac{\partial w}{\partial t}\right)^2 dx = \frac{1}{2} \dot{d}^{e^T} M_{bc}^e \dot{d}^e \tag{44}$$

$$T_{extension\ b} = \frac{1}{2}\rho_b A_b \int_0^{l_e} \left(\frac{\partial u_b}{\partial t}\right)^2 dx = \frac{1}{2} \dot{d}^{e^T} M_{eb}^e \dot{d}^e \tag{45}$$

$$T_{bending\ b} = \frac{1}{2}\rho_b A_b \int_0^{l_e} \left(\frac{\partial w}{\partial t}\right)^2 dx = \frac{1}{2}\dot{d}^{e^T} M_{bb}^e \dot{d}^e \tag{46}$$

$T_{extension\ c}$ $T_{extension\ b}$ and are the kinetic energies due to extension corresponding to the constraining, base, and viscoelastic layer respectively,.where $T_{bending\ c}$, $T_{bending\ b}$ and are the kinetic energies due to bending corresponding to the constraining, base, and respectively. M_{ec}^e, M_{bc}^e, M_{eb}^e, M_{bb}^e, as are the element mass matrix associated with layers.

$$M_{ec}^e = \rho_c A_c \int_0^{l_e} N_1^T N_1 dx, \quad M_{bc}^e = \rho_c A_c \int_0^{l_e} N_2^T N_2\, dx, \quad M_{eb}^e = \rho_b A_b \int_0^{l_e} N_1^T N_1\, dx \tag{47}$$

$$M_{bb}^e = \rho_b A_b \int_0^{l_e} N_2^T N_2\, dx, \quad M_{ev}^e = \rho_v A_v \int_0^{l_e} N_3^T N_3\, dx, \quad M_{bv}^e = \rho_v A_v \int_0^{l_e} N_x^T N_x dx \tag{48}$$

The element mass matrix obtained as the summation of each layer of the VE sandwich beam element is

$$M^e = M_{extension\ c}^e + M_{bending\ c}^e + M_{extension\ b}^e + M_{bending\ b}^e \tag{49}$$

3.7 Finite Element Equation of Motion

After assembling the stiffness and mass matrix of the element then following the Hamilton variation principle:

$$\int_{t_1}^{t_2} \delta(T - U)\, dt + \int_{t_1}^{t_2} \delta w\, dt = 0 \tag{50}$$

The governing equation of the integral FE element of VE sandwich beam can be expressed as:

$$M^e \ddot{d}^e + K_e^e d^e + K_v^e d^e = F^e \tag{51}$$

3.8 Linear Vibration Eigenvalue Problem (Constant Complex Modulus)

The complex constant Young's modulus of viscoelastic material E can be expressed as:

$$E = E' + jE'' = E' \tag{52}$$

Where E' is the real part, also called the delayed Young's modulus of elasticity, and E'' is the imaginary part which is called the energy dissipation modulus. $\eta = \frac{E''}{E'}$ is the loss factor, which is the ratio of the energy dissipation modulus to storage energy modulus The loss factor physically represents the ratio of energy consumed from the viscoelastic material to the total energy [27]. Many researchers have used the shear complex modulus, since the frequency variation is not considered, they have obtained a simple dynamic equation by applying a simple harmonic excitation force [30,85].

The natural frequency and loss factor of the VE sandwich structure is obtained by solving the eigenvalue problem of the VE sandwich structure. The constitutive model of a viscoelastic material is taken as the complex constant modulus model and the frequency dependence of viscoelastic materials is not considered so the vibration of the VE sandwich beam is linear. The dynamic equation can be expressed as:

$$M^e \ddot{d}^e + d^e K^e = 0 \tag{53}$$

where $\mathrm{K^e}$ is the total stiffness matrix of the VE sandwich beam and $\mathrm{M^e}$ is the total mass matrix of the VE sandwich beam, then the eigenvalue problem can be expressed as [86]:

$$\left(K^e - \omega^{*2} M^e\right) X = 0 \tag{54}$$

where $\omega^*$is the complex frequency of the EVE sandwich beam, and X is the complex eigenmodes of the three-layer VE sandwich beam. The natural frequency and loss factors are calculated as follows

$$f_n = \sqrt{Re\omega^*} \tag{55}$$

Chapter 4

NUMERICAL RESULTS AND VALIDATION

4.1 Normal Mode Analysis (Free Vibration)

Normal modal analysis is an essential step in modeling the dynamic behavior of the VE sandwich structures. It is a semi-final step that gives theoretical values for eigenvalues and eigenvectors that have no physical meaning unless the normal model analysis is paired with FRF or time response analysis.

4.1.1 Case Study #1: Clamped-Free VE Sandwich Beam

To study the linear vibration of the VE sandwich structure, a cantilever beam was discretized into 35 elements. In this case, the viscoelastic is considered independent of Young's modulus E. The mechanical and geometrical properties of the elastic layer and core layer of the VE sandwich cantilever beam are shown in Table 5. A MATLAB® code was developed to solve the eigenvalues problem. In addition, 2D and 3D finite element models were built using FE software Abaqus® to show the efficiency of the numerical approach.

The equivalent linear natural frequencies and loss factors of the VE sandwich beam correspond to the first six modes as shown in Table 6 The presented results show that the natural frequencies obtained with the present FE model and Abaqus® simulation with various VE core loss factors.

Table 5: Mechanical and geometrical specifications of the sandwich with a frequency-independent VE core

	Aluminum layers	**Viscoelastic core layer**
Young modulus (MPa)	6.9×10^4	1.794×(1+0.3i)
Poisson's ration	0.3	0.3
Loss factor	-	0.1,0.6,1,1.5
Density (kg / m^3)	2766	968.1
Thickness (mm)	1.52	0.127
Length (mm)	177.8	177.8
Width (mm)	12.7	12.7

For the sake of comparison, the first six natural frequencies and loss factors of VE sandwich beam structures were calculated using five different methods when the loss factors of viscoelastic materials were 0.1, 0.6, 1, and 1.5, respectively. These five methods are based on the assumption of shear energy consumption, and they are the finite element shear models implemented in MATLAB® and Abaqus®, Table 6 and Table 7 show that the finite element shear models, RM, and ANM methods all have good accuracy in calculating the first six natural frequencies of the cantilever VE sandwich beam as well as loss factor with the clamped-free boundary condition and the relative error with the analytical solution.

In comparison, the accuracy of the FE shear model is the highest. Compared with the analytical method [87], its error range is 0-0.15% and The C3D20RH® is second, with an error range of 0-5.918 percent and an average error of 0.2 %; the CPS8R® is third, with an error range of 0-6.565% and an average error of 0.858 %; and the RM method has the lowest accuracy, with the lowest error value of 0 % and the highest error value of 8.892 % and an average error of 0.97 %. Although the ANM approach has a little higher average error than the RM method, the ANM method cannot be deemed accurate. Because the natural frequency calculated using the RM approach has nothing to do with the VE material's loss factor, which contradicts reality.

Although the prediction result of the natural frequency by the RM method is close to the analytical solution when the loss factor of the VE material is small, when the loss factor of the viscoelastic material is increased to 1.5, the prediction error of the natural frequency reaches 8.892 %. The prediction error of ANM to the natural frequency, on the other hand, is very consistent and does not fluctuate much, thus the calculated loss factor of ANM to natural frequency is generally accurate. As a result, the prediction accuracy of the five numerical approaches for the natural frequency of the cantilever VE sandwich beam in this work is ranked as follows: Integral shear element> C3D20RH® > CPS8R® > ANM> RM.

Table 7 shows that the accuracy of the finite element shear element models implemented in MATLAB® and Abaqus® are very high in predicting the loss factor corresponding to the first six modes of the VE sandwich beam structure. The finite element shear model has the highest accuracy, with the lowest and maximum errors of 0 % and 2.56%, and an average error of 0.43%. With an error range of 0 to 11.45 % and an average of 2.38%, the second-ranked ANM [78] element is the most accurate. The C3D20RH® element has an error range of 0-14.7%, with an average of 4.19 %, which places it third. The RM [54] element, which has a minimum error of 0 % and a maximum error of %, has the lowest accuracy.

The results show a significant improvement in the overall damping performance of the structure as the damping factor of the VE core increases. When low frequencies are close to loading frequencies, they can generate huge amplitudes, causing what is known as resonance.

Table 6: Linear frequencies of the C-F VE sandwich composite beam for various core loss factors

η_v	Analytical [34]	CPS8	FE Model	RM [35]	C3D20	ANM [36]
	$\omega_n(Hz)$	$\omega_n(Hz)$	$\omega_n(Hz)$	$\omega_n(Hz)$	$\omega_n(Hz)$	$\omega_n(Hz)$
0.1	64.1	64.1	64.1	64.1	64.5	64.5
	296.4	296.5	296.3	296.6	298.6	298.9
	743.7	743.6	743.4	744.3	750.1	746.5
	1393.9	1392.6	1393.4	1395.2	1408.3	1407.7
	2261.1	2256.7	2260.2	2263.4	2288.4	2286.2
	3343.6	3332.8	3342.3	3347.3	3388.7	3385.7
0.6	65.5	64.3	65.4	64.1	64.7	65.9
	298.9	297.4	298.8	296.6	299.5	303.1
	745.5	744.5	745.2	744.3	751	752.3
	1394.9	1393.1	1394.3	1395.2	1409	1412.7
	2261.7	2257	2260.8	2263.4	2289	2290.6
	3344	3333	3342.6	3374.3	3389	3389.5
1	67.4	64.7	67.4	64.1	65.1	67.8
	302.8	299.1	302.7	296.6	301	309.1
	748.6	746.1	748.4	744.3	753	761.1
	1396.6	1394	1396	1395.2	1410	1420.6
	2262.8	2257.6	2261.9	2263.4	2289	2297.9
	3345	3333.3	3343.4	3374.3	3389	3395.9
1.5	69.8	65.5	69.9	64.1	65.9	67.8
	308.8	302.3	308.7	296.6	304	309.1
	754	749	754.2	744.3	756	761.1
	1399.7	1395.8	1399.1	1395.2	1412	1420.6
	2265	2258.7	2264.1	2263.4	2290	2297.9
	3346	3333.9	3344.7	3374.3	3390	3395.9
	Average Error (%) ≈	**0.858**	**0.03**	**0.94**	**0.2**	**0.97**

Table 7: Associated loss factor of the C-F VE sandwich for various core loss factors

η_v	Analytical [34]	CPS8	RM [35]	C3D20	ANM [36]
	η_n/η_v	η_n/η_v	η_n/η_v	η_n/η_v	η_n/η_v
0.1	0.281	0.283	0.283	0.282	0.281
	0.242	0.242	0.243	0.241	0.242
	0.154	0.154	0.154	0.153	0.154
	0.088	0.089	0.089	0.088	0.089

	0.057	0.057	0.057	0.056	0.057
	0.039	0.039	0.039	0.038	0.039
0.6	0.246	0.281	0.283	0.280	0.247
	0.232	0.241	0.243	0.239	0.224
	0.152	0.154	0.154	0.152	0.150
	0.088	0.089	0.089	0.088	0.088
	0.057	0.057	0.057	0.056	0.057
	0.039	0.039	0.039	0.038	0.039
1	0.202	0.277	0.283	0.276	0.204
	0.217	0.238	0.243	0.236	0.201
	0.150	0.153	0.154	0.152	0.142
	0.088	0.089	0.089	0.088	0.086
	0.057	0.057	0.057	0.056	0.057
	0.038	0.039	0.039	0.038	0.037
1.5	0.153	0.271	0.283	0.270	0.155
	0.197	0.232	0.243	0.231	0.176
	0.146	0.152	0.154	0.150	0.131
	0.087	0.088	0.089	0.087	0.083
	0.056	0.057	0.057	0.056	0.056
	0.038	0.039	0.039	0.038	0.039
	Average Error (%) ≈	**5.42**	**6.02**	**4.19**	**2.38**

Figure 15 shows the relative change in normalized deflection amplitudes for the first three eigenmodes of normalized clamped-free VE sandwich beam with a different damping factor of the viscoelastic core η_v= [0.1,0.6,1.5]. The results were obtained using 2D plan stress element CPS8R implemented in Abaqus®. It can be noticed from Figure 15 that, both the real and imaginary part has different amplitudes but the same mode shape. For the high damping factor of the VE core, the imaginary part of the complex eigenmodes is significant which affects the whole structure's damping performance.

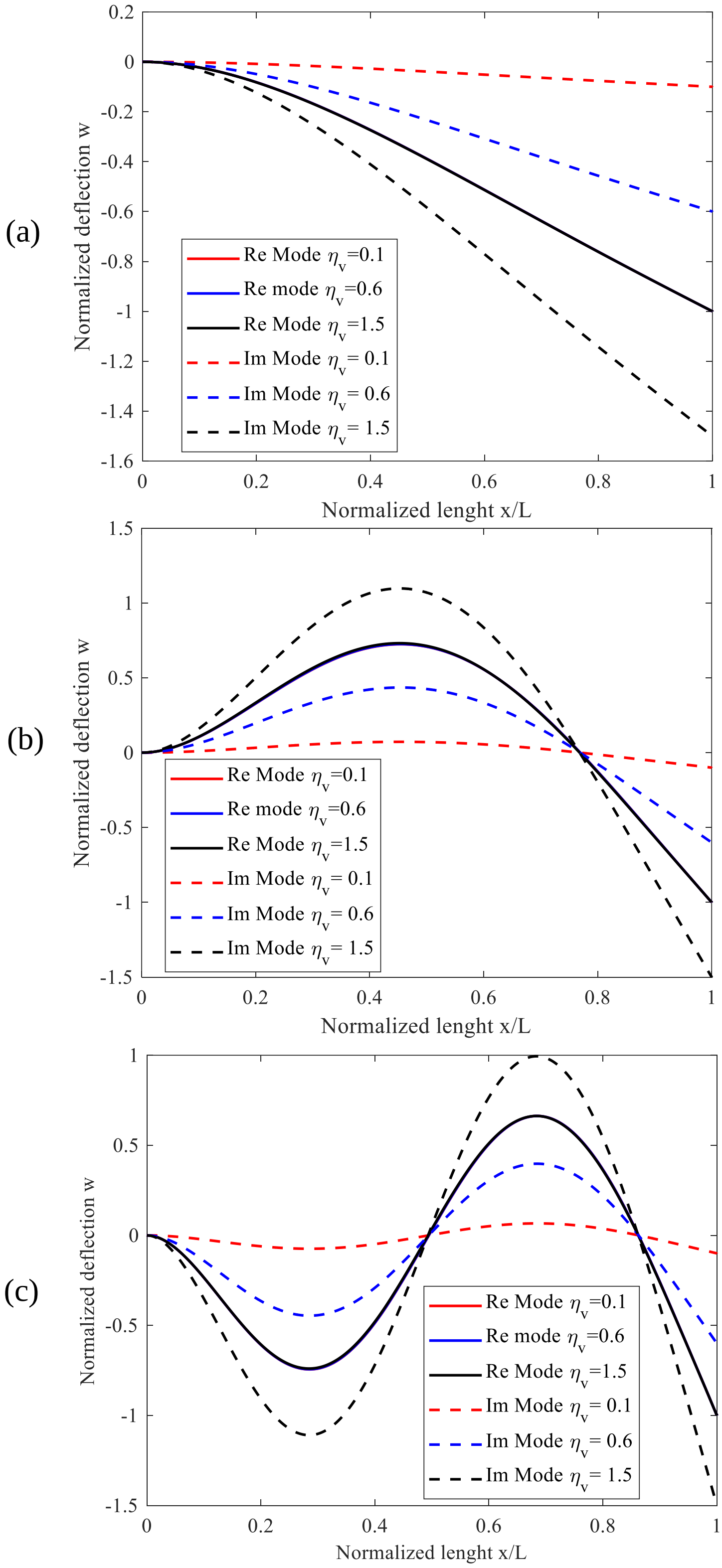
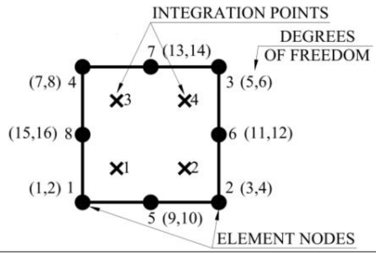
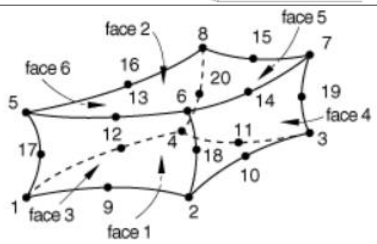


Figure 15: The complex eigenmodes normalized deflection of the C-F VE sandwich beam at x =1 with different loss factor of the VE (a) complex eignmode 1;(b) complex eignmode 2 (c) complex eignmode 3

4.1.2 Case Study #2: S-S VE Sandwich Beam

For the same Mechanical and geometrical properties shown in Table 5, the linear vibration characterizations of simply supported-simply supported boundary VE sandwich beam are considered. Based on the analytical solution of Rao [42] the prediction accuracy of the natural frequency and loss factor of the VE sandwich beam structure of the three finite element models is compared. For beam discretization, 35 elements were used for the shear element model implemented in MATLAB® while 2D and 3D finite element models were built using FE software Abaqus®, and the discretization of the models is described in Table 8.

Table 8: Mesh specifications of shear element implemented in Abaqus®

Element Type	Description	Schematic	Mesh density
CPS8R® [88]	- Continuum biquadratic. - 8-nodes. - Plane stress. - reduced integration.		- core 5080 - Skin 55880
C3D20RH® [89]	- Continuum quadratic brick. - 20-nodes. - Reduced integration. - a hybrid with linear pressure.		- core 4450 - Skin 8900

It can be seen from Table 9 and Table 10 that all the finite elements shear models implemented in MATLAB® and Abaqus®, have good accuracy in calculating the first six natural frequencies of the VE sandwich beam compared with the analytical solution [42] with the S-S boundary condition and the relative error. In comparison, the shear element model implemented in MATLAB® has the highest accuracy with 0.23% average relative error, CPS8R® has 0.59 % average relative error while C3D20H®

has the lowest accuracy with 3.6 %**.** On the other hand, from Table 10 it can be noticed that all shear models have relatively good accuracy in predicting the first six loss factor of the S-S VE sandwich beam compared with the analytical solution [42]. As a result, the prediction accuracy of the three numerical approaches for the natural frequency of the S-S VE sandwich beam in this work is ranked as follows: MATLAB® > CPS8R®> C3D20H®.

The dynamic characteristics of the VE sandwich beam are studied, and the natural frequency of the VE sandwich beam system is larger under S-S boundary conditions. In terms of loss factors, the fundamental loss factor for VE sandwich beams with C-F boundary conditions is higher than that for VE sandwich beams with S-S boundary conditions, but the loss factors for other orders are lower. The results show that different boundary conditions have a great influence on the vibration characteristics of the VE sandwich beam.. Complex frequency analysis is necessary to investigate complex vibration modes which affect the damping capacity of the structure. For the high loss factor of the VE material, the imaginary part of the stiffness matrix is considered which implies the existence not only of the storge modulus but, also the loss modulus. As seen above in Figure 16 the first normalized fundamental complex vibration modes of the simply supported VE sandwich beam with a different damping factor of the viscoelastic core η_v= [0.1,0.6,1.5] using 2D plan stress element CPS8R implemented in Abaqus®. The results show that the relative amplitude difference between the real and imaginary modes is increasing as the loss factor of VE material increases so that the imaginary part of the complex fundamental vibration modes is significant and cannot be neglected.

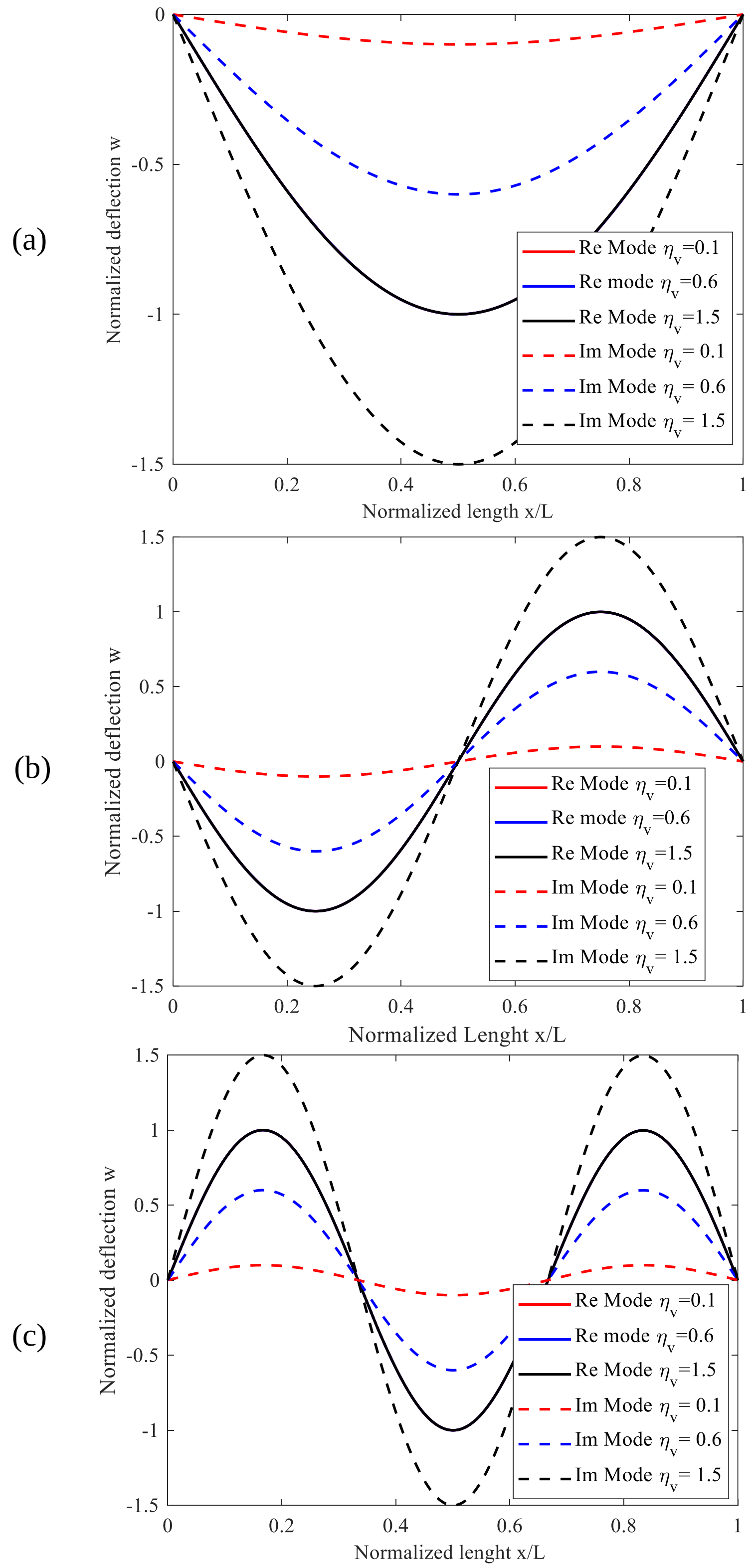


Figure 16: The complex eigenmodes deflection of S-S the VE sandwich beam with a different loss factor of the VE (a) complex eignmode 1;(b) complex eignmode 2; (c) complex eignmode

Table 9: Linear frequencies of the simply supported-simply supported boundary VE sandwich beam for various core loss factors

η_v	**Analytical** [37]	**CPS8 element**	**C3D20 element**
	$\omega_n(Hz)$	$\omega_n(Hz)$	$\omega_n(Hz)$
	148.51	148.44	148.55
	488.47	488.19	489.797
0.1	1034.69	1033.5	1040.57
	1795.13	1791.56	1408.3
	2771.49	2763	2288.4
	3964.28	3946.95	3388.7
	150.71	149.23	149.34
	489.75	489.01	490.61
0.6	1035.38	1034.01	1041.08
	1795.54	1791.89	1811.05
	2771.76	2763.23	2802.92
	3964.47	3947.11	4016.7
	154.42	150.62	150.73
	492.06	490.48	492.08
1	1036.63	1034.95	1042.02
	1796.3	1792.5	1811.65
	2772.27	2763.64	2803.33
	3964.83	3947.41	4016.99
	160.72	153.16	153.27
	496.49	493.29	494.89
1.5	1039.07	1036.78	1043.84
	1797.78	1793.68	1812.82
	2773.25	2764.45	2804.12
	3965.52	3947.99	4017.55
	Average Error (%) ≈	**0.59**	**3.60**

Table 10: Associated normalized loss factor of the simply supported-simply supported boundary VE sandwich beam for various core's loss factors

η_v	**Analytical [37]**	**CPS8 element**	**C3D20 element**
	η_n/η_v	η_n/η_v	$\eta_n/$
	0.107	0.1069	0.10[illegible]
	0.065	0.0651	0.0644
0.1	0.043	0.0433	0.0426
	0.031	0.0307	0.03
	0.3328	0.3468	0.3465
	0.1943	0.195	0.1945
	0.1068	0.1068	0.1062
	0.0652	0.0651	0.0644
0.6	0.0433	0.0433	0.0426
	0.0308	0.0307	0.03
	0.305	0.3405	0.3402
	0.192	0.1938	0.1933
	0.106	0.1066	0.106
	0.065	0.0651	0.0644
	0.043	0.0433	0.0426
	0.031	0.0307	0.03
	0.2626	0.3292	0.27
1	0.1871	0.1916	0.231
	0.1059	0.1063	0.15
1.5	0.065	0.065	0.087
	0.0433	0.0432	0.056
	0.0308	0.0307	0.038

0.0410	0.0432	0.0510
3346	3333.9	3390
Average Error (%) ≈	**1.8**	**6.28**

4.1.3 Case Study #3: Four-Sided SSSS VE Sandwich Plate

Consider the material and geometrical specifications of a four-sided simply supported VE plate structure shown in Table 11. The natural frequencies and loss factors of the structure were calculated using the present finite element model implemented in Abaqus® and compared with the results obtained by Johnson [90], who used the modal strain energy method (MSE) in the NASTRAN commercial software to calculate the structure's natural frequency and loss factor. The composite plate structure is divided into 2200×120 and 1050×255 using the C3D20H® element along with the length and width directions during the calculation. Tables 4.5 and 4.6 show the results of the natural frequency and loss factor calculations, respectively.

Table 11: Mechanical and geometrical specifications of the VE Four-Sided SS VE plate with a frequency-independent VE core

	Elastic layer	VE core layer
Young modulus (Pa)	6.89×10^{10}	1.794×10^{6}
Poisson's ration	0.3	0.49
Density (kg / m^3)	2740	999
Loss factor	-	0.5
Thickness (mm)	1.52	0.127
Length (mm)	348	348
Width (mm)	304.8	304.8

The mode shapes of the structure associated with the first fundamental natural frequency obtained using C3D20H® element with different mesh sizes are shown in Figure 17. The sweet spot can be visualized and shown as the maximum deformation location points. It is clear from the Table that the C3D20H® element has good accuracy in predicting the damping characteristics of the VE sandwich SSSS plate with a range of average error in predicting the fundamental natural frequency of less than 0.4 % and an average error of less than 6.5 % in predicting the damping loss factor.

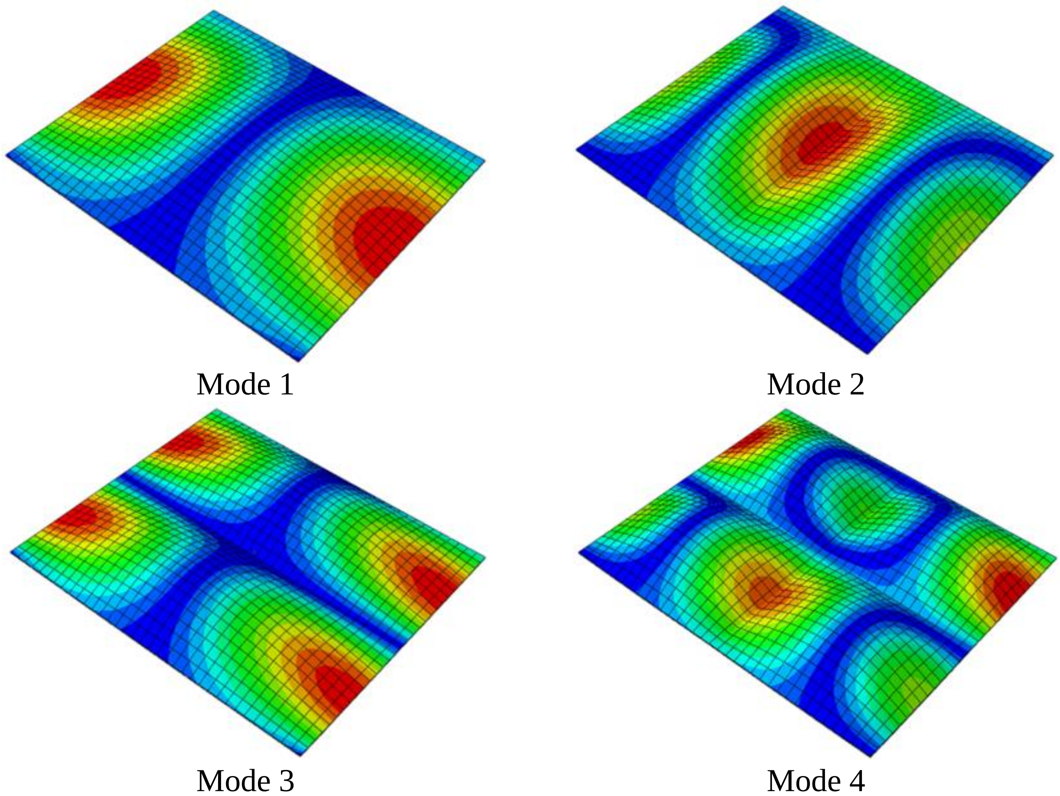


Figure 17: Fundamental eigenmodes shape of viscoelastic sandwich SSSS plate η_v=0.5 (a) Mode 1 (b) Mode 2 (c) Mode 3 (d) Mode 4

It is clear from Table 12 and Table 13 that the C3D20H® element has good accuracy in predicting the damping characteristics of the VE sandwich SSSS plate with a range of average error in predicting the fundamental natural frequency of less than 0.4 % and an average error of less than 6.5 % in predicting the damping loss factor.

Table 12: Linear complex natural frequencies of VE sandwich plate obtained by the different mesh sizes of C3D20H®

Mode	**MSE method** [90]	**C3D20H®2200×120**		**C3D20H® 1050×255**	
	Natural frequency (Hz) f_n	**Natural frequency (Hz)** f_n	**Error (%)**	**Natural frequency (Hz)** f_n	**Error (%)**
1	57.4	57.3	0.174	57.2	0.348
2	113.2	113.5	0.265	113.3	0.088

3	129.3	129.5	0.155	129.1	0.155
4	179.3	177.7	0.892	177.1	1.227

Table 13: Associated loss factor of VE sandwich plate obtained by the different mesh sizes of C3D20H®

544	**MSE method** [90]	**C3D20H® 2200×120**		**C3D20H® 1050×255**	
	Loss factor η_n/η_v	**Loss factor η_n/η_v**	**Error (%)**	**Loss factor η_n/η_v**	**Error (%)**
1	0.352	0.366	3.98	0.366	3.963
2	0.376	0.394	4.79	0.396	5.342
3	0.376	0.390	3.72	0.391	4.038
4	0.306	0.346	13.07	0.348	13.835

4.2 Parametric Studies

It is required to study the effect of various parameters on the vibration characteristics such as the natural frequency of the system and loss factor. A normal mode study performed an investigation on the dynamic characteristics of VE sandwich beam structure using commercial software Abaqus® and MATLAB®. The thickness of the core, boundary conditions, and length are varying which affects the natural frequency and mode shape.

4.2.1 Effect of VE Material Loss Factor on Dynamic Characteristics of VE Sandwich Beam

To study the influence of the loss factor η_v on the dynamic characteristics of natural frequency f_n and loss factor, the loss factor η_v varied from 0.1 to 1.5 and the change trends of the natural frequency f_n and loss factor corresponding to the fundamental modes of the C-F VE sandwich beam is illustrated in Figure 18. The fundamental natural frequencies of VE sandwich beams only increase minimally when the loss factor of viscoelastic materials is increased from 0.1 to 1.5, as shown in Figure 18, indicating that the loss factor of viscoelastic materials seems to have little effect on the natural frequencies of VE sandwich beams.

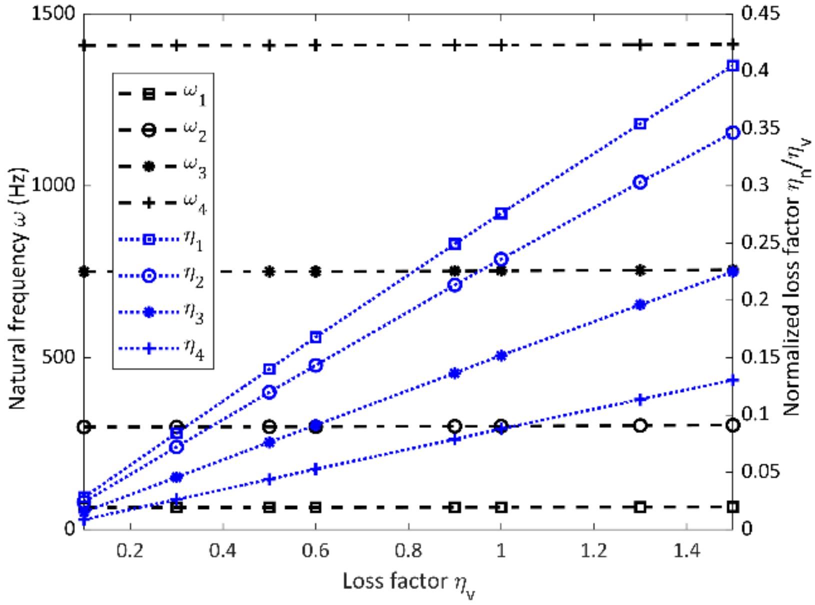


Figure 18: The effect of the loss factor η_v on the natural frequency f_n and loss factor η of the C-F VE sandwich beam

The constant complex module $E'(1 + j\eta)$, shows that the complex elastic modulus E of the viscoelastic material increases as the loss factor of the VE η_v increases. In addition, the stiffness of the viscoelastic layer increases, and the stiffness of the entire VE sandwich beam increase, since the mass of the VE sandwich beam does not vary, the fundamental frequency of the beam will increase. However, as shown in Table 5, the viscoelastic layer's elastic modulus is 4 times lower than the elastic modulus of elastic layers, thus the increase in stiffness induced by the viscoelastic layer's loss factor can only make the VE sandwich C-F beam stiffer. Under the condition of constant mass, the fundamental frequencies of the VE sandwich C-F beam will only increase slightly with a small increase in stiffness.

On the other hand, when the loss factor of the viscoelastic material increases, the loss factor of the VE sandwich beam increases linearly as shown in Figure 18, implying that the viscoelastic material's loss factor has a significant impact on the VE sandwich beam's loss factor. The viscoelastic material's energy dissipation capacity is characterized by the loss factor. The higher the viscoelastic material's loss factor, the more vibration energy it can dissipate and hence, better noise suppression performance.

4.2.2 Effect of VE Core Layer Thickness on Dynamic Characteristics of VE Sandwich Beam

The core thickness varied from 0.5 to 6.5 mm to study its effect on the dynamic characteristics f_n and η_v. Indeed, the natural frequency of the system decreases as the thickness of the VE layer increases, as seen in Figure 19. so does its stiffness and mass increase, and hence the stiffness and mass of the C-F VE sandwich beam. The increase in the stiffness of the VE sandwich beam system generated by increasing the thickness of the viscoelastic layer is much smaller than the increase in the whole C-F VE sandwich beam stiffness since the elastic modulus of the VE layer is 4 times lower than the elastic modulus of elastic layers and the mass of the core is about 30% of the elastic layers. The changing of the first fundamental loss factors η for the C-F beam is more significant than the changing trend of the natural frequency, which is related to the vibration.

4.3 Frequancy Response Anaylsis

Harmonic analysis is required to ensure that the design will successfully overcome resonance and harmful effects. The solution method is mode superposition which uses linear iterations of the mode shapes to coverages to the frequency response.

4.3.1 Steady-State Dynamics of VE Sandwich Beam/Plate

To excite the first three fundamental mode shapes, a harmonic concentrated force is applied at point p = (59.27,6.3) m with magnetite of F_0 = 100 N. The system linear frequency response (FRF) is shown in Figure 20 with different loss factors of η_{v}= 0.1,0.3,0.6. It can be noticed from Figure 20 that the effect of the VE core's loss factor

η_v is significant as the system has a relatively minimum amplitude at the core's loss factors of η_v= 0.6 and the maximum response of the system is within the range of the first fundamental natural frequency that has the highest value of MEM in the direction of the applied load

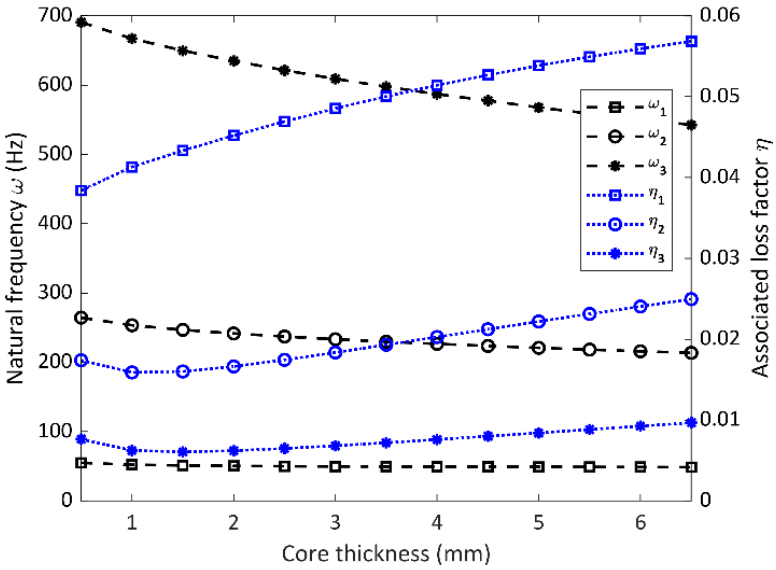


Figure 19: The effect of the core thickness on the natural frequency f_n and loss factor η of the C-F VE sandwich beam

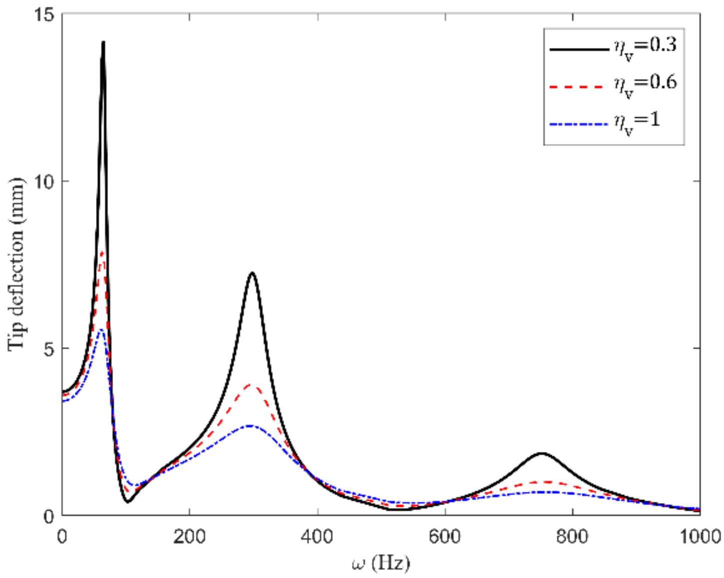


Figure 20: Linear frequency response curve of the fundamental complex vibration modes of C-F VE sandwich beam/plate for various core's loss factor η_v= 0.3,0.6,1

Chapter 5

CONCLUSION

In this work, dynamic analysis of VE sandwiched beam/plate structures was examined to evaluate the damping characteristics of natural frequency f_n, and loss factor η_v based on the longitudinal shear deformation assumption. The numerical results are compared with analytical values to validate the FE models. The numerical simulation is computed using commercial software MATLAB® and Abaqus®, the results show that all numerical FE models are applicable for approximating the damping characteristics of the VE sandwiched beam structure. The FE element model developed using FE formulation has the lowest error range against the other models. In addition, linear complex natural frequencies of VE sandwich plate were obtained by the different mesh sizes of C3D20H®. The model shows accuracy in prediction of damping characteristics of the VE sandwiched plate structure Finally, a parametric study is performed to study the effect of various parameters on the dynamic characteristic of the VE sandwiched beam. The following conclusion is summarized as follows:

- Increasing the viscoelastic layer thickness will decrease the natural frequency as well as increase the loss factor of the viscoelastic layer.
- The loss factor of the viscoelastic layer η_v has a significant effect on the overall loss factor of the structure, whereas it has not a significant effect on the natural frequency.

- The calculated resonance frequencies for both beam and plate are in the range of the modal natural frequency.

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