

Vaccine Inventory Management in Pandemic

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ABSTRACT

The countries are the units that procure the vaccines during the Coronavirus disease 2019 (COVID-19) pandemic. The delivered quantities are huge. Countries must bear inventory holding costs (IHC) based on stock quantity variations. This cost depends on the speed of the vaccination in the country. This speed is time-dependent. The COVID-19 pandemic showed that stocking and procuring vaccines are different from the inventory holding (IH) problems of production. The vaccinated portion of the population can be approximated by the cumulative distribution function of the Cauchy distribution. Firstly, a model is provided for determining the minimal-cost inventory policy (IP) and its optimality conditions are provided. The model is solved for 20 countries for different numbers of procurements. The results reveal the individual behavior of each country. Secondly, the Hungarian inventory model is applied to determine the initial stock if the probability of the non-shortage is given. This initial stock is the function of the procurements. The problem is non-linear as the willingness to be vaccinated is a sigmoid function of the time. The stocking of vaccines is a single-period inventory problem. The cases of three countries are simulated. The countries are Denmark, Hungary, and Mexico. They have different characteristics of willingness to be vaccinated. The numerical results of the simulation show that the prescribed probability of non-shortage can be achieved.

Keywords: COVID-19, Pandemic, Vaccination, Inventory Policy, Inventory Holding Cost, Bounded Total Demand; Single-Period Inventory Problem, Hungarian Inventory Model, Cauchy Distribution, Sigmoid Function.

ÖZ

Ülkeler, COVID-19 salgını sırasında aşıları tedarik eden birimlerdir ve teslim edilen miktarlar çok büyüktür. Ülkeler, stok miktarlarının değişimine göre stok maliyetine katlanmak zorundadır. Bu maliyet, ülkedeki aşılama hızına bağlıdır. Bu hız ise zamana bağlıdır. COVID-19 salgını, aşıların stoklanması ve tedarik edilme sürecinin; üretilen miktarların stokta tutma sürecinden farklı olduğunu göstermektedir. Nüfusun aşılanmış kısmı, Cauchy dağılımının kümülatif dağılım fonksiyonu ile tahmin edilebilmektedir. İlk olarak, minimum maliyetli stok politikasının belirlenmesi için optimallik koşulları sağlanarak bir model geliştirilmektedir. Model, farklı satın alımlar olmak üzere; 20 ülke için uygulanmaktadır. Sonuçlar, her ülkenin bireysel davranışını ortaya koymaktadır. İkinci olarak, kıtlık olmama olasılığı verilme durumunda, başlangıç stokunu belirlemek için Macar envanter modeli uygulanmaktadır. Bu ilk stok, tedariklerin işlevidir. Aşı olma isteği zamanın sigmoid bir fonksiyonu olduğundan, problem doğrusal değildir. Aşıların stoklanması tek dönemlik bir envanter problemidir. Danimarka, Macaristan ve Meksika olmak üzere üç ülkenin vakaları simüle edilmiştir. Bu ülkeler, aşılarmaya istekli olma konusunda farklı özelliklere sahiptir. Simülasyonun sayısal sonuçları, öngörülen kıtlık olmama olasılığının elde edilebileceğini göstermektedir.

Anahtar kelimeler: COVID-19, Salgın, Aşılama, Envanter Politikası, Envanter Tutma Maliyeti, Sınırlı Toplam Talep, Tek Dönemlik Envanter Problemi, Macar Envanter Modeli, Cauchy Dağılımı, Sigmoid Fonksiyonu.

DEDICATION

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This doctoral dissertation is proudly dedicated to:

My lovely family

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LIST OF ABBREVIATIONS

CDF	Cumulative Distribution Function
COVID-19	Coronavirus Disease 2019
EOQ	Economic Order Quantity
IH	Inventory Holding
IHC	Inventory Holding Cost
IM	Inventory Management
IP	Inventory Policy
PD	Probability Distribution
PDF	Probability Density Function
QE	Quadratic Error
SC	Supply Chain
TQE	Total Quadratic Error
UK	United Kingdom
USA	United States of America
WHO	World Health Organization

Chapter 1

INTRODUCTION

1.1 Background

It is known that pandemics had a great influence on mankind. The most important case is the plague (Wikipedia, n.d.-e). It caused approximately 50 million deaths in Europe during the 14th century (World Health Organization, n.d.-a). The pandemic that devastated Byzantium in 541 and 542, which killed 200,000 people, is also thought to be a plague (Wikipedia, n.d.-d). The disease still exists. The greatest number of cases occur in the Democratic Republic of Congo, Madagascar, and Peru (World Health Organization, n.d.-a). Another notable case in history is an unidentified epidemic in the ancient Hittite Empire. The plague reduced the population and weakened the empire, which contributed to its destruction around 1180 BC. Smallpox is also a serious disease caused by an orthopoxvirus. Throughout history, smallpox also killed several hundreds of millions. The death toll just in the 20th Century was 300 million (Wikipedia, n.d.-f). The disease was stopped by well-organized vaccination. The method was found by Edward Jenner in 1796. Regular vaccination was applied until 1980 when WHO (World Health Organization) declared the global eradication of the disease. The words *vaccine* and *vaccination* are connected to smallpox. Jenner extracted weak viruses from cows that had similar but not identical diseases, although viruses and bacteria were not known at that time. Infection with weak viruses enables the body to fight strong viruses. The language Latin was used in

science in the times of Jenner. The name of a cow in Latin is *vacca*. The plural is *vaccae*. If something belongs to the cow, then the proper adjective is *vaccinus*.

The greatest epidemic close to the present age was the Spanish flu. It started in 1918 just after the Great War. The peak was in 1918 and 1919. However, it lasted several years (Wikipedia, n.d.-g). For example, Charles I, the former emperor of Austria died in 1922 of Spanish flu.¹ The total death toll is estimated between 17 and 50 million. Spanish flu is especially important from the point of view of this thesis as it is a similar disease to COVID-19.

Large-scale pandemics may occur in the future as well. One of the most important phenomena in this regard is that due to global warming, part of the permafrost, will surely thaw or is already thawing. There are many bacteria and viruses in the permafrost (Zhong et al., n.d.). Many of them belong to undescribed species. It is not known how they can affect the human body. There are likely species that can cause epidemics.

The COVID-19 pandemic gives a good example of how individuals and countries behave during an epidemic. It helps to prepare the world for these future events. The main purpose of this thesis is to describe these behaviors.

COVID-19 is the first pandemic in the history of mankind detailed data are available on the number of cases and the used vaccines in every country of the world (Our World in Data, n.d.). This fact gives the unique opportunity to determine the theoretical bases

¹ Charles was a Hungarian King under the name Charles IV at the same time. The English Wikipedia (Wikipedia, n.d.-b) and the Hungarian one (Wikipedia, n.d.-a) contradict the reason for his death.

of the logistics of future pandemics. The COVID-19 pandemic raised many questions which can be solved only by scientific methods. Many answers can be used only in the case of future pandemics. The reason is that the correct understanding of the problem and the elaboration of the solution takes time. The economic effects of the pandemic belong to that category. In this thesis, an inventory strategy is elaborated for the procurement of vaccines from the point of view of individual states. The strategy finds a balance between safety and costs.

The rich individual countries purchased high initial stocks to make sure that there would be no shortage of the vaccine. The stocks could not be used in some countries as the willingness to be vaccinated is limited in every country. Later, the countries presented superfluous quantities of vaccines to poor countries or sold them at a low price. This behavior is bad for every agent of the market. The countries that purchased too many vaccines lost money. The pharmaceutical companies could not meet all the needs. They may lose markets in the future. The countries that did not get the vaccine in time had a more serious pandemic. Many people died unnecessarily. Finally, the world as a single closed system had and still has the pandemic for a longer period because of the missing vaccines as people from low-income countries can infect people from high-income countries.

1.2 Aim of the Study

The Hungarian inventory model is applied to obtain optimal inventory policy (IP) for vaccines. This model keeps both the probability of shortage and the initial stock low in general. Most of the inventory models assume that the time horizon is infinitely long. The Hungarian inventory model can be applied both in a multi-period and a single-period environment. The latter one is important concerning the pandemic.

Here, we ensure an IP for the countries during the pandemic time. This thesis proposes a deterministic vaccine model with country-specific demand rates that vary over time. It intends to create an analytical model for minimizing holding costs and to establish IPs for usage with a variety of perishable products such as vaccinations.

1.3 Research Questions

In order to achieve the study's aims, the research questions that have been addressed are as follows:

1. What is the distribution of COVID-19 cases worldwide and how should they be analyzed?
2. What is the importance of distribution and stocking policies for vaccines?
3. How inventory management (IM) strategies have developed in countries?
4. How have vaccine applications evolved in the countries?
5. Will it be allowed for COVID-19 vaccines to be available with equity for the entire population of the world?
6. What measures can be taken to improve IM for the next pandemic?

1.4 Significance of the Study

The results of this study are expected to draw the attention of legislators, policy-makers, and development practitioners in the field of IM to review their policies and plans in order to enhance the effectiveness of IP. This can be considered as lighting for the rapid response from the perspective of management of vaccine sources in the countries for possible future pandemics. Accessibility of the existing vaccine sources is not discussed in this dissertation.

1.5 Data Sources

Data from every country reported in the world who have been vaccinated with COVID-19 vaccines are available (Africa Data Hub, n.d.). Data was provided

regarding total cases that had occurred up to February 15, 2023. Vaccination status was categorized as total vaccinations, people vaccinated, and people fully vaccinated.

1.6 Outline of the Study

This doctoral dissertation has 6 chapters. The study's description and background information are included in Chapter 1, as well as an overview of the study's key research aims, questions, and contributions. Chapter 2 is devoted to reviewing related IM and vaccination procedures which is relevant and necessary for conducting this study. It also explains the fundamental terms and key aspects of IM. In Chapter 3, the methodology is introduced. Chapter 4 provides a regression analysis of COVID-19 data and an approximation of the vaccination curve of the countries has been discussed. In Chapter 5, the stocking process is explained and a deterministic model for IP has been developed. Moreover, discusses the evaluation of the numerical results and the results of numerical simulations. The thesis is finished with conclusions by giving also future aspects of the topic.

Chapter 2

LITERATURE REVIEW

In this particular chapter, a presentation of the considered literature material is given. The first section mentions the literature relevant to the last pandemic all over the world. The second section provides explanations of the principles of IM and discusses the key terms and concepts that are used throughout the study to ensure their clarity and proper application. The third section describes IM of perishable products which focuses on more vaccines for his research. The fourth section studies the stocking process of vaccines and gives strategies from current applications.

2.1 Literature of Pandemic

Yellow fever, caused by Flavivirus, a filterable agent, was reported as the first human-infected virus in 1901. Afterward, the 1918–1920 pandemic, Spanish flu (H1N1), is regarded as the greatest medical disease of the time, affecting millions of people worldwide. Furthermore, in 1997, the first human cases of a new and extremely dangerous avian influenza virus—the H5N1 strain—were discovered in Hong Kong. Recently, in late 2019, the outbreak of COVID-19 is declared in Wuhan city of China (Kinlaw et al., 2009; Pradhan et al., 2020)

The breakout of the 2019 novel COVID-19 has caused challenges in various parts of the world (Dwyer et al., 2020; Gabster et al., 2020; Mamun & Ullah, 2020; Wollina, 2020; C. Zhou et al., 2020). (World Health Organization, n.d.-b). It is a pandemic viral disease that has infected over 620 million individuals and killed over 6,540,000 people

over the world as of October 12, 2022. Vaccination commenced in the end of 2020, with Pfizer receiving complete approval from the United States Food and Drug Administration (FDA). Vaccination requires vaccine supplies, workers to implement them, and people to be vaccinated (Mills & Salisbury, 2021; U.S. Food and Drug Administration, n.d.)

Other means of prevention, such as wearing a mask, washing hands, and following the social distance rule, help to lower the danger of transmission within the community (Güner et al., 2020). However, it has been revealed that the vaccine must have an efficacy of at least 70% to avoid an epidemic, highlighting the vaccine's significance on epidemics (Bartsch et al., 2020).

2.2 Essentials of Inventory Management

IM is a critical aspect of any business that involves handling and tracking goods, materials, or products. All organization keeps inventory. “Inventory” includes a company’s raw material, work-in-process, suppliers used in operations, and finished goods. Is it really necessary to keep inventory? In a just-in-time manufacturing environment, inventory is considered waste. However, inventory plays a crucial role in situations where a company has inadequate cash flow or weak control over the following: (1) lead times; (2) quality of commodities received; and (3) electronic information sharing among all departments and significant suppliers (Essentials of Inventory Management - Max Muller - Google Kitaplar, n.d.).

Describing IM involves explaining how businesses systematically oversee the procurement, storage, tracking, and distribution of goods to ensure they have the right products, in the right quantities, at the right time, and at the right cost. It includes a range of tactics, methods, and tools designed to maximize effectiveness, reduce

expenses, and optimize inventory levels across the supply chain (SC). A corporation that practices effective management can minimize the costs associated with stockouts and overstocking while maintaining adequate stock levels to fulfill consumer demand. For effective IM, it has to be provided such as a combination of accurate data, robust processes, and proactive decision-making to meet customer demands while achieving business objectives.

Determining the economic lot size has always been one of the most significant aspects of production planning. This subject has long piqued the interest of researchers, and numerous models have been devised to meet criteria at a low cost. The economic order quantity (EOQ) model is among the oldest in the inventory analysis literature. Ford Whitman Harris was the first to address the issue of determining economic lot sizes in production processes (Andriolo et al., 2014). The basic model developed by Harris in 1913 (Harris, 1990a). Later, the causes for the mismatch between inventory modeling theory and practice have been investigated, and various variables that impede implementation have been identified by Hungarian Academy of Sciences (Barancsi et al., 1983). We need to look ahead and forecast future demand. (Axsäter, 2015), introduce the reasons why an inventory control system needs to order items some time before customers demand them.

2.3 Inventory Management of Perishable Products

Perishable products are goods that have a limited shelf life and are susceptible to deterioration, spoilage, or decay over time. These products typically require special handling and storage conditions to maintain their freshness and quality. Examples of perishable products include fresh produce, dairy products, meats, seafood, baked goods, certain pharmaceuticals, and other products with a short lifespan, such as blood,

drugs, and concrete (L. Liu & Liu, 2020). Non-storable perishable commodities or services, such as aircraft tickets and hotel rooms, are subjected to purchase restrictions, such as the booking requirement employed by airlines and hotels. When the availability reaches zero, you cannot sell it (M. Z. F. Li, 2001). In the case of newspapers, they are highly perishable because they strive to deliver the most up-to-date news, and new issues replace older ones on a daily basis (Shaabani, 2022).

It is widely believed that the quality of perishable products is determined by three factors: the product, the user, and the market condition. For this reason, defining quality and establishing control over it are challenging tasks. A product's acceptance and ascribed quality can be distinguished by the breakdown of these components' influence on quality (Wilkinson et al., 1996).

Designing networks for perishable products becomes more challenging and crucial. Perishable products, such as pharmaceuticals have a limited shelf life, and if not consumed or utilized before their expiration date, they are disposed of, generating waste. This causes economic losses for businesses, resource waste, environmental difficulties, and the possibility of vital drug shortages. As a result, optimizing SCs for perishable pharmaceutical products is an important method for increasing the efficiency and robustness of these networks (Alexandra & Bule De Carvalho, n.d.).

In inventory problems, stockouts and overstock can be addressed to satisfy the demands of dynamic customers (Shekhawat et al., 2016). Both dimensions are costly scenarios for businesses, and the risks associated with them are manageable. Improving ordering policies to deal with stockouts (Antic et al., 2022). Perishable

items, such as vaccines, are difficult to transport and store, necessitating the use of reliable inventory control models. To avoid inventory holding (IH), inventory optimization solutions must strike the correct balance between consumer demand and inventory levels (O'Neill & Sanni, 2018). This is especially true in extremely uncertain and volatile marketplaces, like those generated during the epidemic period. It is crucial today because of the huge demand for vaccination caused by social demands during the current COVID-19 pandemic, as well as the fact that it is subject to public procurement with an urgent timeline (Patriarca et al., 2020a). We presume that there are enough vaccines to cover a considerable portion of the population. To lessen disease-related difficulties, decision-makers are playing an increasingly important role in vaccine allocation. Certain outcomes can be used to assess the effectiveness of distributing a restricted vaccination inventory to a specific group. After implementing a good distribution model, it is expected that the total number of cases and deaths will be reduced. However, distributing the vaccine to people entails not only transporting a specific amount of vaccine, but also keeping it at stock sites. In this scenario, some IPs should be used to give a yield of reducing the cost of IH. It is vital to provide vaccines continuously and to develop a vaccination policy for this aim. The properties of perishable products should also be considered. On the other hand, decisions are influenced by the population's willingness to get vaccinated in society, which has a direct impact on the number of standards that countries must meet. The thesis proposes a new vaccine procurement policy for specific countries. The policy enables governments to operate logically and economically, ensuring that every citizen who volunteers for the vaccine receives it.

Relevant reviews in IM and policy into consideration of perishable products literature have been evaluated in Table 1.

Table 1: Analysis of earlier studies about inventory management

Author	Title	Problem of interest	Application studied	Inventory aspect
(A. Liu et al., 2021)	Sustainable SC management for perishable products in emerging markets: An integrated location-inventory-routing model	Multi-objective planning model on perishable products	YALMIP (MATLAB) toolbox	Location-inventory-routing
(Mallidis et al., 2020)	Creation of a single-period inventory planning methodology for perishable product redistribution	Optimizes the replenishment policy of perishable products	Quantitative modeling methodology	Single period
(Patriarca et al., 2020b)	EOQ inventory model for perishable products with uncertainty	Definition inventory functions on perishable products	Monte Carlo simulation	EOQ
(H. Zhou et al., 2023)	Two-period pricing and inventory decisions of perishable products with partial lost sales	Dynamic periodic-review inventory system for perishable products	Dynamic programming	Two-period
(Zhang & Wang, 2020)	Joint Ordering, Pricing, and Freshness-Keeping Policy for Perishable	Joint optimization on perishable products	Kuhn–Tucker condition	Single period

	Products: Single-Period Deterministic Case			
(K. Chen et al., 2021)	Inventory strategies for perishable products with two-period shelf-life and lost sales	Developing inventory strategies for perishable products	Regular order policy and the mid-cycle inventory adjustment policy	Two-period
(Díaz et al., 2020)	EOQ for perishables with decreasing willingness to purchase during their life cycle	Minimization of the expected management cost for perishable products	Arena Rockwell 14.0	EOQ
(Stephan Onggo et al., 2019)	Agri-food SCs with stochastic demands: A multi-period inventory routing problem with perishable products	Minimization of inventory costs on perishable products	Mixed integer program	Multi-period
(Wu et al., 2015)	A multi-period location model with transportation economies-of-scale and perishable inventory	Minimization of total cost and a multi-period location model for single perishable product	Mixed integer nonlinear programming	Multi-period
(Saracoglu et al., 2014)	A genetic algorithm approach for multi-product multi-period continuous review inventory models	Calculating the optimal order quantity and optimal reorder point	Integer linear programming and genetic algorithm	Multi-period

(Sazvar et al., 2016)	A novel mathematical model for a multi-period, multi-product optimal ordering problem considering expiry dates in a FEFO system	Determination the order quantities of different types of products	Genetic Algorithm and Particle Swarm Optimization	Multi-period
(Dobson et al., 2017)	An EOQ model for perishable goods with age-dependent demand rate	Obtainment the optimal cycle length	Deterministic EOQ style model	EOQ

2.4 The Stocking Process of Vaccines And Some Lessons of the COVID-19 Pandemic

There are few pharmaceutical companies that are able to produce a certain vaccine. Production is a long-term process. Another problem is that if the production process stops for any reason, then it is time-consuming to restart the process. One important lesson of the COVID-19 pandemic was that a company must satisfy the demands of many countries if a new disease occurs. There was a shortage of COVID-19 vaccines (“Unacceptable” Vaccine Delays Cause Frustration across European Union | World News | Sky News, 2021). This fact cannot be explained only by the high demand. This type of vaccine is a new product causing some technical difficulties. However, the shortage can occur even from regular vaccines such as diphtheria (U.N. Reports Shortage Of Diphtheria Vaccine - The New York Times, 1994) even in countries like the USA (Diphtheria-Tetanus-Pertussis Vaccine Shortage -- United States, 1984) and European Union (April - Shortage of Essential Diphtheria Treatment Drugs Needs International Action, Experts Warn - University of Exeter, 2017). Also, it is observed

that prioritized available doses were applied due to the limited initial supply of vaccines during the COVID-19 pandemic (Bubar et al., 2021).

One of the difficulties to the success of immunization programs is vaccine shortages at health facilities, which could be caused by poor vaccine stock management (Iwu et al., 2019). Distributional issues and delays result in continual infections, allowing viral variations to evolve (Kim et al., 123 C.E.). The lack of quality data and end-to-end visibility of vaccinations makes it difficult for decision makers to efficiently manage the SC and avoid stockouts and waste. UNICEF and WHO created the next-generation Stock Management Tool (SMT) to help countries transition to a digital future. The SMT is used to record day-to-day operations in vaccine stores, improving stock visibility across levels and providing an effective method of monitoring supplies and cold chain equipment (Introducing the Vaccine Stock Management Tool (SMT) | UNICEF Supply Division, n.d.). Vaccine stock management (Vaccine Stock Management: Guidelines on Stock Records for Immunization Programme and Vaccines Store Managers, n.d.) involves several key components to ensure the efficient distribution and administration of vaccines. On the other hand, vaccine storage and handling resources is an important issue that need to provide certain instructions (Vaccine Storage and Handling Resources | CDC, n.d.).

Relevant reviews in vaccines into consideration of both the COVID-19 period and previous literature have been evaluated in Table 2. This shows us there are many research has occurred after the COVID-19 pandemic about both ways of vaccine effectiveness and inventory management.

Table 2: Analysis of earlier studies about vaccine inventory

Author (Year)	COVI D-19	Public Health	Vaccine Effectiveness	Qualitative Analysis	Quantitative Analysis	Vaccine Inventory
(Rodrigues & Plotkin, 2020)		✓	✓	✓		
(Nahofti Kohneh et al., 2023)	✓	✓	✓		✓	✓
(Rastegar et al., 2021)	✓	✓			✓	✓
(Kashte et al., 2021)	✓	✓	✓	✓		
(Park et al., 2021)	✓	✓	✓	✓		
(Forni & Mantovani, 2021)	✓	✓	✓	✓		
(Lopez Bernal et al., 2021)	✓	✓	✓		✓	
(Su et al., n.d.)	✓	✓	✓	✓		
(Sinha et al., 2023)	✓				✓	✓
(Georgiadis & Georgiadis, 2021)	✓				✓	✓
(S.-I. Chen, 2017)		✓			✓	✓
(Alghamdi et al., 2015)		✓	✓	✓		
(Z. Li et al., n.d.)		✓			✓	✓
(Duijzer et al., 2018)		✓		✓		✓

3 steps have been implemented in the study (Figure 1). In the beginning, the definition of the problem is given. According to the research questions, the data set has been established (Africa Data Hub, n.d.). The literature review is examined, and the statistical analysis is provided in the methodology. Also, the simulation of the analysis is completed. The results can be found in the conclusion part with future recommendations.

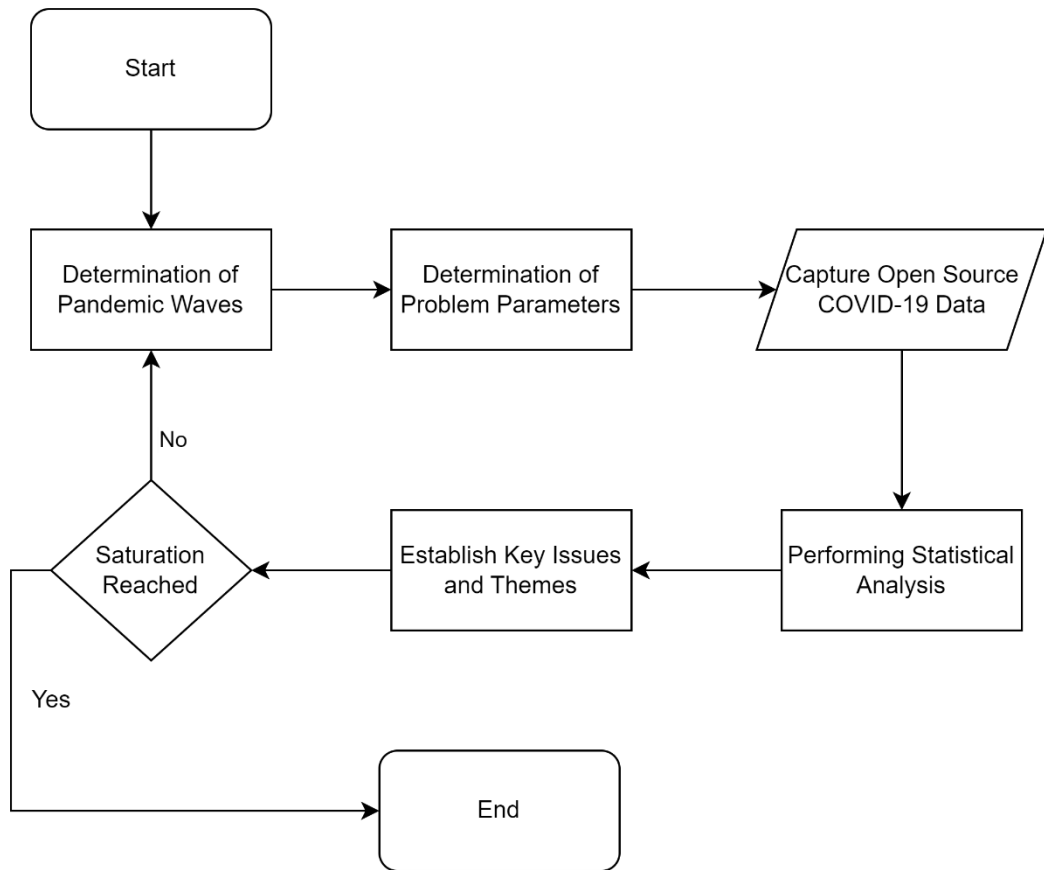


Figure 1: The flowchart of the study

Chapter 3

METHODOLOGY

3.1 Saturation Process

3.1.1 The Life Cycle of a Product

The life cycle of a product refers to the stages a product goes through from its conception to its eventual decline and removal from the market. At each stage of the product life cycle, a product faces different challenges and opportunities in the market.

The product life cycle typically consists of the following stages:

1. **Development/Introduction:** This is the initial stage where the product is introduced to the market and has limited awareness among consumers. The focus during this stage is on building brand recognition and generating interest in the product. The product is new to the market and market acceptance is uncertain. Market research and advertising are necessary to build product awareness. Sales are low and the product incurs high production and marketing costs.
2. **Growth:** Sales start to increase as the product gains acceptance. During the growth stage, sales of the product increase rapidly as more and more consumers become aware of the product and its benefits. The company focuses on expanding distribution and promoting the product to reach a wider audience. Market competition may increase and price wars may develop. Production expenses decline as sales increase.
3. **Maturity:** The maturity stage is characterized by a slowing of sales growth and a decline in the rate of market penetration. At this stage, the market for the product is becoming saturated, and competition is increasing. Companies may focus on

maintaining market share by differentiating their product or lowering prices. Competition becomes intense and companies focus on maintaining market share through advertising and improving product features.

4. Saturation: The saturation stage is reached when the market for the product is fully developed and there is limited opportunity for further growth. Sales growth has slowed to a point where it is difficult to achieve significant increases. It typically happens in the maturity stage of the product life cycle, where the market becomes saturated with similar products and it becomes increasingly difficult to increase sales. Companies may focus on maintaining market share through price competition, promotions, and product differentiation to stay competitive. Saturation may also indicate that the market is reaching its peak and the decline stage may soon follow.

5. Decline: In the decline stage, sales of the product start to fall due to increased competition, changes in consumer preferences, or technological advancements that make the product obsolete. Companies may exit the market, reduce their investment in the product, or reposition it to target a different segment.

6. Discontinuation/Phase-out: The product is no longer profitable and is phased out of the market.

The organization's functional areas such as research & development, production, and SC manage various components of the product life cycle. All of these parties are guided by strategic decisions made on capital expenditure allocation, growth and marketing objectives, product and technology portfolio management, and SC design choices (Alexandra & Bule De Carvalho, n.d.).

3.1.2 Some Market Related Notions in Economics

Substitute products are goods or services that can be used instead of one another, offering similar utility to the consumer but with slight differences in features, quality, price, or other characteristics. Substitutes can be direct or indirect, and they can vary from close replacements to distant alternatives, depending on how closely they meet the needs and preferences of the consumers.

Some examples of substitute products are as follows:

- Soft drinks and juices as substitutes for each other.
- Gasoline-powered cars and electric cars as substitutes for personal transportation.
- Butter and margarine as substitutes for each other in cooking and baking.
- Public transportation and taxis as substitutes for personal transportation.
- Landline phones and cell phones as substitutes for telecommunication services.
- Laptops and desktops as substitutes for personal computers.

The relationship between substitute products in the market can have a significant impact on the demand and price of each product. A few key points to understand this relationship are:

Competition: The presence of substitutes in the market can increase competition and put downward pressure on prices, as consumers have alternative options to choose from.

Elasticity of demand: The degree to which demand for a product changes with a price change is known as price elasticity of demand. If the demand for a product is elastic, a small price change can result in a significant change in the quantity demanded, as consumers switch to a substitute product.

Market share: The market share of a product can be impacted by the availability and popularity of substitute products. If a substitute product offers a better value proposition, it can attract consumers away from the original product, reducing its market share.

Innovation: The threat of substitute products can drive innovation, as companies strive to differentiate their products and stay ahead of the competition. This can result in the development of new and improved products that better meet the needs and preferences of consumers.

In summary, the relationship between substitute products can have a profound impact on the market, shaping consumer behavior, driving innovation, and determining the success of each product in the market.

To observe the movement of substitute products in the market, one can use the following methods:

- Market Surveys: Conducting market surveys or consumer research to understand consumer preferences, purchasing behavior, and the factors that influence their decision-making process.

- Sales Data Analysis: Analyzing sales data of substitute products to identify trends and patterns in consumer demand.

- Competitor Analysis: Keeping track of competitor offerings and promotions to understand how they may impact the demand for substitute products.

- Social Media Monitoring: Monitoring social media platforms to get real-time insights into consumer opinions and preferences for substitute products.

- Price Tracking: Monitoring the prices of substitute products to understand how price changes affect consumer demand and market competition.

By using a combination of these methods, one can get a comprehensive understanding of the movement of substitute products in the market and make informed business decisions.

3.2 Saturation Function/Sigmoid Function

The product's life cycle best represents the saturation process. The life cycle of a product begins when it enters the market. Because the new product is unknown to many individuals, consumption is slow to increase. As time passes, more individuals learn about the product. A fairly steady consumption is established. Other, alternative, and substitute items will eventually enter the market. Therefore, the consumption of the product being investigated slows and eventually ends. The first obvious conclusion is that an unlimited amount of the product cannot be produced. The market is saturated, and the overall consumption curve fails to reach a specific barrier. If we look at the curve that indicates the volume of all products sold from their release until a specific point in time, it will resemble an elongated letter S. This particular curve is constantly increasing. If the events described above occur, the curve will have at least one inflection point. The event curve for particularly famous products, such as the publication of a new volume from a well-known book series or the beginning of sales of a new mobile phone from a recognized brand, may alter depending on whether eager buyers line up for the book or the phone. Specifically, consumption is relatively high in the first period and then drops subsequently. As a result, the overall sales curve rises sharply before falling. The curve has no inflection point. In that situation, the curve itself resembles a concave function.

A saturation function is a mathematical function that models a phenomenon where the increase in one variable has diminishing returns as the variable approaches a saturation

value. Mathematically, a saturation function is typically defined as a sigmoidal or S-shaped curve that approaches an asymptote at the saturation value if the independent variable is approaching $+\infty$. It is also called a logistic function. Logistic function is an example of saturation or sigmoid function. It is defined as:

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

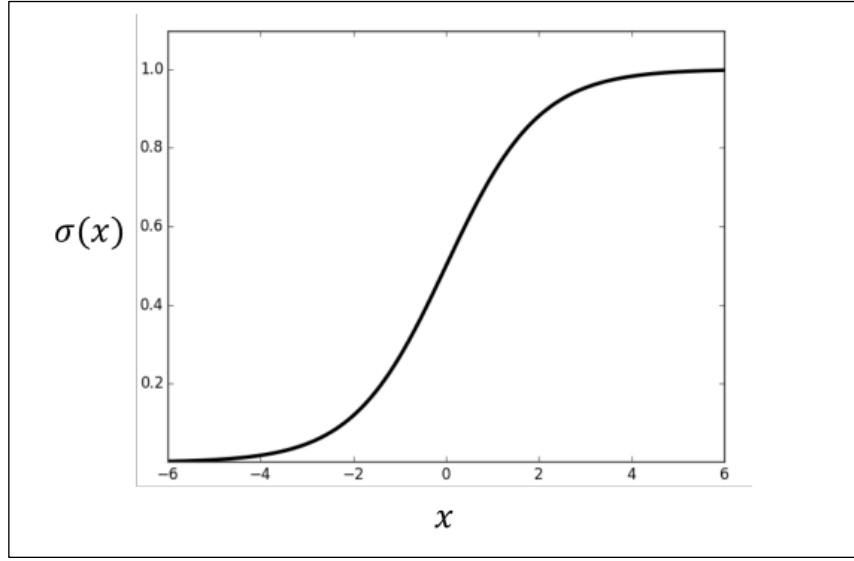


Figure 2: The function of total sales in time

A saturation function is mathematically defined as the following:

Definition: A function $f(x)$ is called saturation function if $f(x)$ is a real-valued function on the $[-\infty, \infty]$ interval such that (i) it is monotone increasing, (ii) $\lim_{x \rightarrow -\infty} f(x) = 0$, (iii) there is a positive number L such that $\lim_{x \rightarrow +\infty} f(x) = L$.

It is obvious that the saturation function is very similar to the cumulative distribution functions of probabilistic distributions. The main distinction is that the limit at $+\infty$ is not always 1, but can be any positive integer. Figure 2 depicts a saturation function

and a cumulative distribution function at the same time. (It represents the cumulative distribution function of the Cauchy distribution).

In mathematics, the sigmoid function is remarkably similar to the saturation function. The sigmoid function must have only one inflection point. There is no similar claim for the saturation function. In contrast, the sigmoid function can converge at $-\infty$ to any value. There are numerous functions that are both sigmoid and saturation. The term saturation more accurately describes the role of the function in applications.

A saturation function is the behavior of a system or process as it approaches a limiting value or “saturation” point. It is commonly used in control theory, electrical engineering, and other fields where the behavior of a system changes as it reaches a certain level of capacity or saturation. Saturation functions typically have an “S” shaped curve, with a slow start, an accelerated increase, and then a slow approach to the saturation level, where the rate of change becomes the increase converges to zero. The specific form of a saturation function depends on the application, but common examples include the logistic function and the hyperbolic tangent function.

The sigmoid function is a mathematical function that is widely used in machine learning and artificial intelligence. As an S-shaped curve that maps any input value to an output value in the range of 0 and 1, making it useful for representing probabilities or modeling binary classification problems. The sigmoid function is commonly used in logistic regression and neural networks for prediction and decision-making.

One common use of the sigmoid function is in binary classification problems. Given a set of features for an observation, the sigmoid function can be used to calculate the probability that the observation belongs to a positive class (e.g., 1) versus a negative class (e.g., 0). This is often done by first computing a linear combination of the features, often referred to as the “logits,” and then passing the logits through the sigmoid function to obtain probabilities. The probabilities can then be compared to a threshold (often 0.5) to make a binary prediction.

Another example of the use of the sigmoid function is in modeling logistic growth in populations. The sigmoid function can be used to model the growth of a population over time, where the rate of growth starts slow, accelerates, and eventually levels off as the population approaches carrying capacity. This kind of relationship is often modeled with the logistic differential equation, which can be solved to obtain a logistic curve.

3.3 Sigmoid Functions in Vaccination Case

The vaccination process is remarkably similar to the life cycle of a product. It is impossible to vaccinate more people than the entire population. The actual amount is considerably lower because some persons cannot be vaccinated due to medical reasons, while others refuse or are unavailable. Thus, the vaccination is a product with a limited total demand.

A pandemic can show the same characteristics of the life cycle of some diseases in history such as smallpox. Vaccination is the most effective solution for these situations. Everybody needs a vaccine. At the beginning of the pandemic, there is no vaccine. The total number of vaccinated people as a function of time has the

phenomenon of saturation. It starts with a low number of daily vaccinations which has been increasing for a while. In this research $\arctan(x)$ function is used as a saturation function as the cumulative distribution function of the Cauchy distribution function. Due to the different functions that have been taken into account, Figure 3 differs from Figure 2.

The Cauchy distribution is a continuous probability distribution (PD) that is heavy-tailed. It is defined by its cumulative distribution function (CDF), which is given by:

$$F(x) = \frac{1}{2} + \frac{1}{\pi} * \arctan(x)$$

where x is the random variable and $\arctan$ is the inverse tangent function. The CDF gives the probability that a random variable X with a Cauchy distribution is less than or equal to x . The CDF is an increasing function that ranges from 0 to 1 as x ranges from negative infinity to positive infinity.

The Cauchy distribution is also defined by its probability density function (PDF), which is given by:

$$f(x) = \frac{1}{(\pi * (1 + x^2))}$$

where x is the random variable and π is the mathematical constant pi. The PDF gives the density of the distribution at a given point. The PDF has a peak at $x = 0$, but slowly it does not approach zero as x moves away from 0, so it has heavy tails that extend to infinity.

The Cauchy distribution has several notable features:

-No Mean: The Cauchy distribution does not have a mean because it does not converge. This means that the average of an infinite number of observations from a Cauchy distribution does not exist.

-No Variance: The Cauchy distribution does not have a variance for the same reason that it does not have a mean. This means that the spread of the distribution cannot be quantified using variance.

-No Higher Moments: The Cauchy distribution does not have any higher moments, such as skewness or kurtosis, because it does not have a mean or variance.

-Heavy Tails: The Cauchy distribution has heavy tails that extend to infinity, which means that it is highly sensitive to outliers. This property makes the Cauchy distribution useful for modeling real-world phenomena that exhibit heavy tails, such as extreme values in financial data or the distribution of distances between two random points in a plane.

It can be said that the Cauchy distribution is a unique and important distribution in probability and statistics. Despite its lack of mean, variance, and higher moments, it has a heavy-tailed shape that makes it useful for modeling real-world phenomena with extreme values.

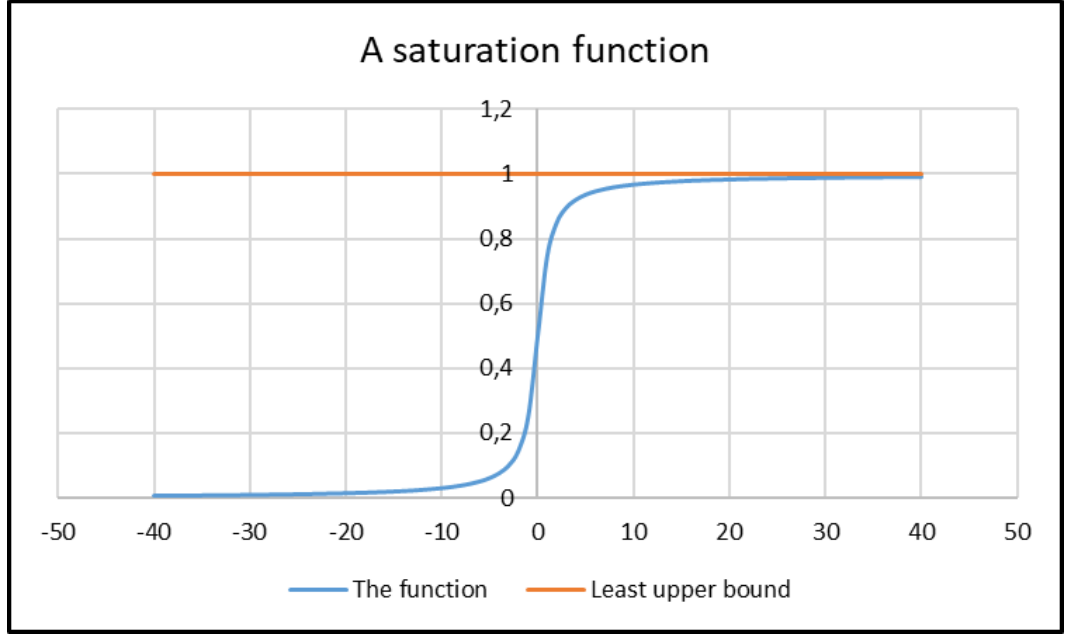


Figure 3: The function $\frac{\arctan(x)}{\pi} + 0.5$.

The arctan function is also known a sigmoid function. Compared to the function in Figure 2, the arctan function in Figure 3 produces superior results.

3.4 Curve Fitting of Regression

3.4.1 The Fitted Curve

There is a class of real-valued functions such that it consists of functions being especially suitable for modeling smooth saturation processes. The name of the class is sigmoid functions. One of the members is the inverse of the tangent function called *arcus tangent* function. The domain of the function is the set of all real numbers. The range of the values are the real numbers in the open interval $(-\pi/2, \pi/2)$. A cumulative distribution function can be obtained from the *arcus tangent* function by a vertical contraction and a vertical shifting as follows:

$$\frac{\arctan(x)}{\pi} + \frac{1}{2}.$$

This function is the cumulative distribution function of the so-called Cauchy distribution. The multiplier $1/\pi$ is the parameter of the vertical contraction and the

additional $1/2$ is the vertical shifting. In the case of general curve fitting, horizontal contraction and horizontal shifting are also necessary. Thus, the general form of the fitted function is

$$\frac{a \times \arctan (b \times (x - c))}{\pi} + d.$$

The meaning of the four parameters is given in Table 3. The two operators of the multiplication are omitted for notational convenience. Then the form of the function denoted by $f(x, a, b, c, d)$ is

$$f(x, a, b, c, d) = \frac{a \arctan (b (x - c))}{\pi} + d.$$

Table 3: The parameters of the fitted function

Parameter	Description
a	Vertical contraction
b	Horizontal contraction
c	Horizontal shift
d	Vertical shift

In general, curve fitting is applied to measured data which are time series in many cases. The independent variable of the time series is time. It is the day in case of the COVID-19 data. The dependent variable is the measured value, which is one of the above-mentioned data, *i.e.*, either the total number of cases, the total number of vaccinations, the number of vaccinated persons, or the number of fully vaccinated persons.

3.4.2 Curve Fitting in the Case of a Single-wave

Two more notations are necessary for the discussion of the optimization problem of the curve fitting. One of them is the length of the time series, i.e. on how many days is available the measured data. The other one is the values of the measured data. They are given in Table 4.

Table 4: Parameters of the time series

Parameter	Description
T	The length of the time series
y_t	The measured value in time period t

The quadratic error (QE) of the fitted function at time period t is the square of the difference of the measured value and the fitted value, *i.e.*

$$(y_t - f(t, a, b, c, d))^2.$$

The sum of these kinds of terms is the total quadratic error (TQE) which is the objective function to be minimized. Thus, the unconstrained version of the optimization problem of curve fitting is

$$\min_{a,b,c,d} \sum_{t=1}^T (y_t - f(t, a, b, c, d))^2. \quad (\text{UC})$$

The problem has only four variables which are the four parameters of the fitted function. It is a nonlinear optimization problem not only because the objective function is quadratic. Function f is nonlinear as well. Problem (UC) is a so-called global optimization problem, i.e. the problem has many local optimum points.

The minimal TQE is obtained by solving problem (UC). Let (a^*, b^*, c^*, d^*) be the optimal solution. It defines an artificial time series denoted by

$\{f(1, a^*, b^*, c^*, d^*), f(2, a^*, b^*, c^*, d^*), \dots, f(T, a^*, b^*, c^*, d^*)\}$. The artificial times series always has a high correlation with the measured time series. However, the artificial time series may have some drawbacks considering the particular application in vaccination. The most frequent one is that some values are negative at the beginning. It can be corrected by adding the constraint

$$f(1, a, b, c, d) \geq 0$$

to problem (UC). Then, it is not unconstrained anymore. By adding the constraint, a more realistic solution is obtained on the expense of an increase in the objective function. This issue is discussed further on in subsection 3.4.3.

3.4.3 Curve Fitting in the Case of Multiple-waves

If there are several waves, then the function cannot be characterized by a single *arcus tangent* function with rare exceptions. Instead, it can be approximated by joining as many *arcus tangent* functions as many waves exist. The mathematical formulation is similar but more complicated.

Assume that the number of waves is n . Let t_k^b and t_k^e be the k -th wave first and last days, respectively. Thus, the time series of the measured data is divided into n time intervals

$$[t_1^b, t_1^e], [t_2^b, t_2^e], \dots, [t_n^b, t_n^e],$$

where

$$t_1^b = 1$$

$$t_k^e + 1 = t_{k+1}^b \quad k = 1, 2, \dots, n - 1$$

$$t_n^e = T$$

To obtain the proper optimization function, a separate parameter set must be introduced for each wave. The parameter set of the k -th wave is $\{a_k, b_k, c_k, d_k\}$. The TQE is to be calculated in every wave. The objective function is the sum of TQE.

If no constraint added to the problem, then the optimal solution is n independently fitted function. However, it may occur that these functions are not joining well. The k -th and $k+1$ -st functions join correctly, if (i) the last value of the previous wave does not exceed the starting value of the new wave and (ii) the difference between these two values are not too high. The mathematical formulation of the two criteria is as follows:

$$f(t_{k+1}^b, a_{k+1}, b_{k+1}, c_{k+1}, d_{k+1}) \geq f(t_k^e, a_k, b_k, c_k, d_k)$$

and

$$\varepsilon \geq f(t_{k+1}^b, a_{k+1}, b_{k+1}, c_{k+1}, d_{k+1}) - f(t_k^e, a_k, b_k, c_k, d_k)$$

where $\varepsilon > 0$ is constant. The average of the daily new COVID-19 cases or a somewhat greater value is a good choice for ε . The relaxation of the two inequalities is that the absolute value of the difference must be small, i.e.

$$\varepsilon \geq |f(t_{k+1}^b, a_{k+1}, b_{k+1}, c_{k+1}, d_{k+1}) - f(t_k^e, a_k, b_k, c_k, d_k)|.$$

Actually, this inequality can be substituted by two linear inequalities. Other constraints can be useful as well. For example, the difference of the measured value and the approximation must be small. Assume that the difference of the measured value and the approximation is too high at the end of the k -th wave. Then the corresponding version of the last inequality is

$$\varepsilon \geq |y_{t_k^e} - f(t_k^e, a_k, b_k, c_k, d_k)|.$$

These types of constraints are called *smoothing constraints*. It depends on the particular problem which ones should be introduced.

Thus, the optimization problem in case of multiple-waves is as follows:

$$\min \sum_{k=1}^n \sum_{l=t_k^b}^{t_k^e} (y_l - f(l, a_k, b_k, c_k, d_k))^2$$

s.t.

The particular smoothing constraints are satisfied.

In general, it is true that the stricter the smoothing constraints are the higher is the loss in the value of the correlation.

Chapter 4

RESULTS OF REGRESSION ANALYSIS ON COVID-19

DATA

4.1 The Approximation of the Total Number of Cases in a Single-wave

Regression analysis, also called curve fitting, is used to investigate the individual time series. The calculations are carried out in Excel. The optimizer function of Excel called Solver was used to minimize the TQE of the fitted function. The parameters of the function are variables in optimization. The goodness/accuracy of the fitted function is measured by the correlation between the original time series and the fitted function.

As a regression analysis, the same curve-fitting method was applied to the total number of used vaccines, the number of vaccinated people, and the number of fully vaccinated people. This saturation process shows that the same arctan function can be applied with different parameters.

The total percentage of vaccinated persons in a country divided by 100 yields a value between 0 and 1 (Our World in Data, n.d.). This total number is an increasing function of time. Thus, it can be approximated by the cumulative distribution functions of a PD. There are numerous known functions that represent the cumulative distribution function of commonly used PDs. The function in Figure 3 is based on the inverse of the tangent function and its formula, which is discussed in Chapter 3 is:

$$\frac{\arctan(x)}{\pi} + 0.5$$

It is both a sigmoid and saturation function. The function is not suitable for regression in this pure form. Four parameters must be introduced. The vertical stretch one is denoted by a . The position of the inflection point is denoted by c . The next one determines its steepness at the inflection point. It is b . The last one is the height of the shift vertically such that at the beginning the value of the function is d . The general formula of the function is:

$$\frac{a \cdot \arctan(b(x-c))}{\pi} + d. \quad (1)$$

Many saturation processes have the feature that the saturating attribute can be uncovered earlier than the process reaches the inflection point. This also applies to vaccinations. It is illustrated in the example of Denmark. The country's time series consists of 319 days. Denmark did not reach yet the 80 percent vaccination rate until the end of the period. The inflection point is approximately the 160th day. Regression occurs after 32, 64, 96, and 300 days. Figure 4 depicts the ultimate result of the regression after 300 days. The two curves, the original time series and the regression function, are quite close together. Figure 5 shows a comparison of the first three curves to the last. Table 5 provides the basic information. After 32 days, saturation is still not acknowledged. The regression function in Figure 5(a) is nearly linear. The process is still in its early stages. The realized vaccination rate is less than 3%. The saturation is revealed after 64 days. Figure 5(b) demonstrates that the curve's steepness is somewhat overestimated. However, the regression function is quite near after 96 days, and the inflection point has not yet been reached, indicating that the process is still in its first half in Figure 5(c).

Table 5: Time length of the regression period and vaccination percentage.

Day	Achieved percentage of vaccination
32	2.95
64	6.94
96	12.48
300	76.86

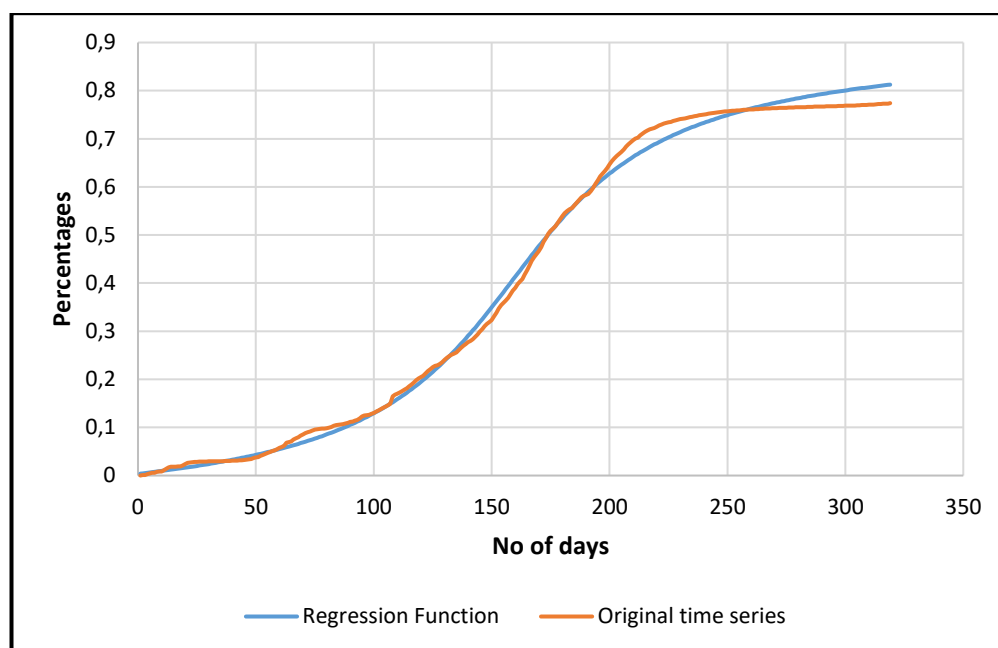


Figure 4: The final result of the regression after 300 days.

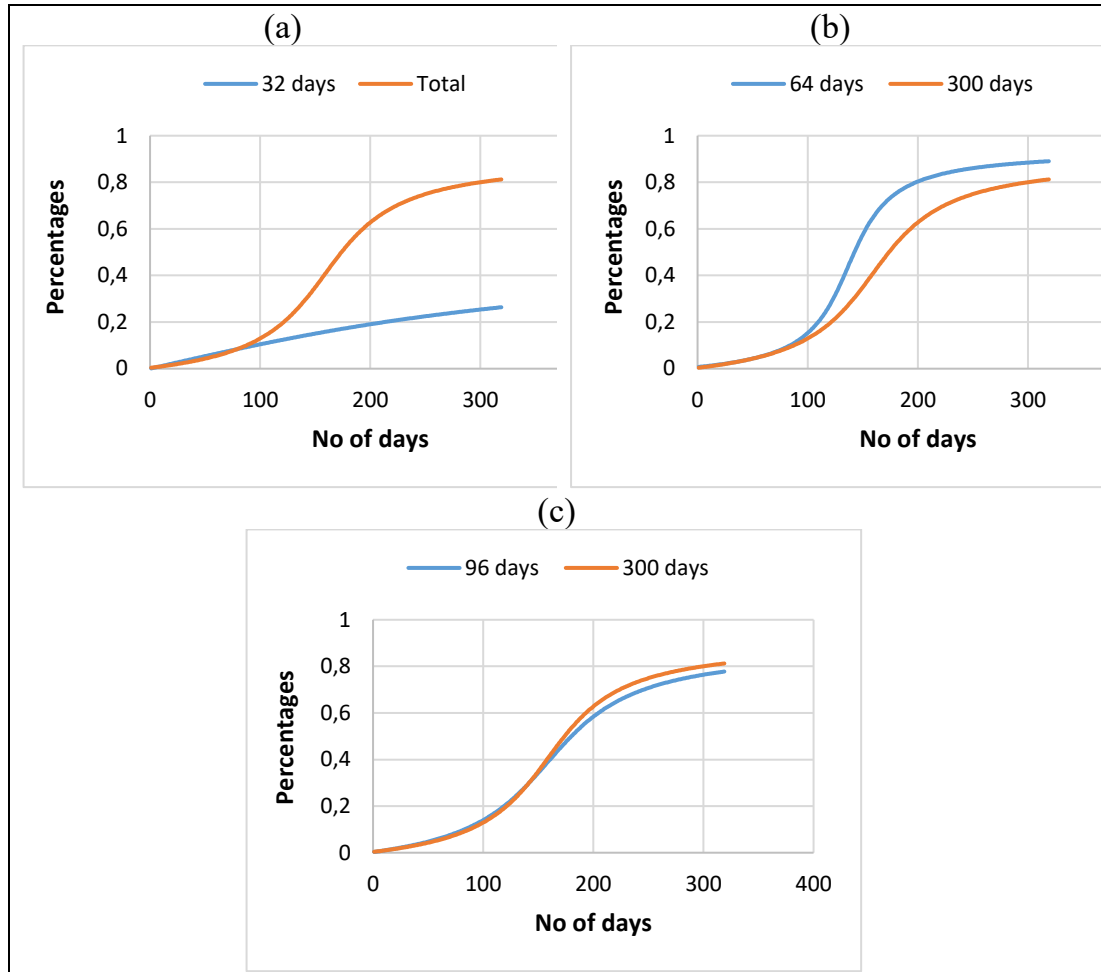


Figure 5: The results of regression for after (a) 32 days; (b) 64 days; (c) 96 days

When we look at the cases of countries, at least 4 waves can be observed in the case of the COVID-19 pandemic for most of the countries.

4.2 The Case of Multiple-Waves

COVID-19 is the first pandemic in history such that daily data are available for every country on the number of cases, death, vaccination, partially and fully vaccinated people, and many other data and statistics. The main source of the data is (Africa Data Hub, n.d.).

The most important phenomena are the speed of the spreading of the disaster, i.e., the number of cases, and the characteristics of the defense, i.e., the vaccination. Thus, four

data are analyzed as follows: the total number of cases, the total number of vaccinations which describes the demand for vaccines, and the partially and fully vaccinated people which characterize the self-defense of people and society.

COVID-19 attacked in several waves. Most authors mention the four waves. However, some countries have more and even microwaves occur. Vaccination has waves in most countries. These waves are not necessarily parallel to the waves of the pandemic. Vaccines did not exist at the beginning of the pandemic. Thus, no parallel waves were possible at the beginning. The shapes of the time series show that vaccination was accepted by a significant proportion of the population. Therefore, the vaccination process could continue at the same speed even when the rate of infection subsided. However, a serious wave in the pandemic could accelerate the vaccination process, too.

It has been determined that a new wave has begun for the country as follows:

A certain period of time after the cases occur, the number of daily cases decreases. If a new wave is coming, the number of daily cases increases. It is the way to know that a new wave starts.

Figure 6 shows five waves in Hungary. Each wave is saturated independently from the other ones. There is one common property. If the measured value is the total number of a factor, then the starting point of the next wave is where the end point of the previous wave is. It means that later waves have higher vertical shifts than the previous ones. The property is also important if a curve is fitted to the complete time series.

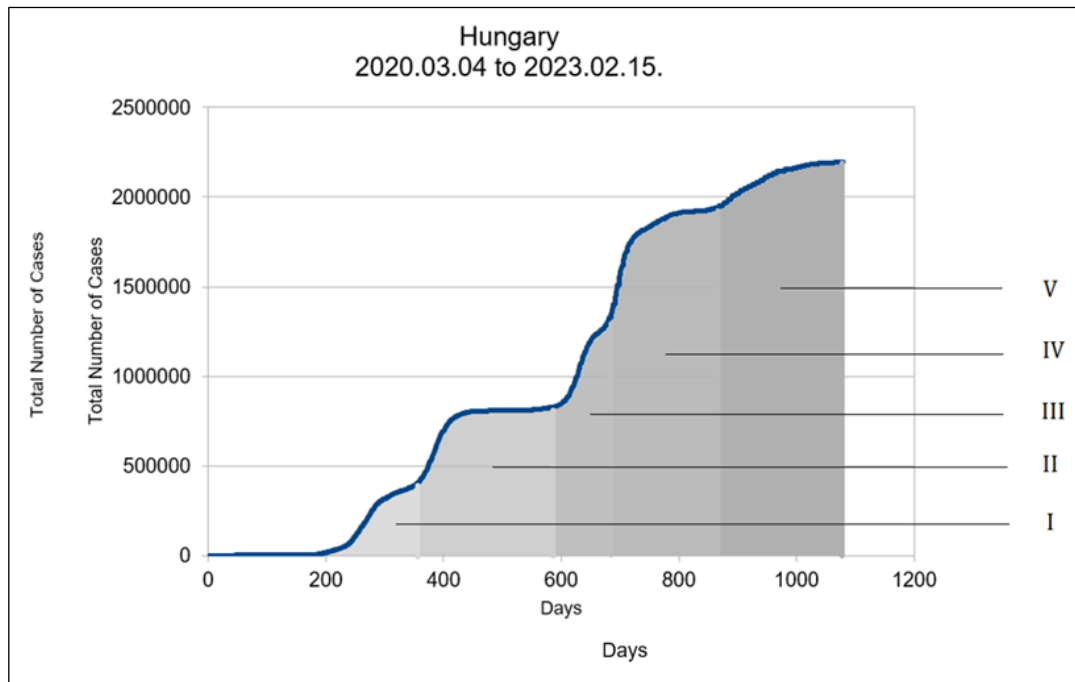


Figure 6: Five waves of COVID-19 in Hungary.

In Figure 7, it can be seen that the total number of cases reaches almost 180,000 for Afghanistan. And, in Figure 8, 4 waves of this country can be seen.

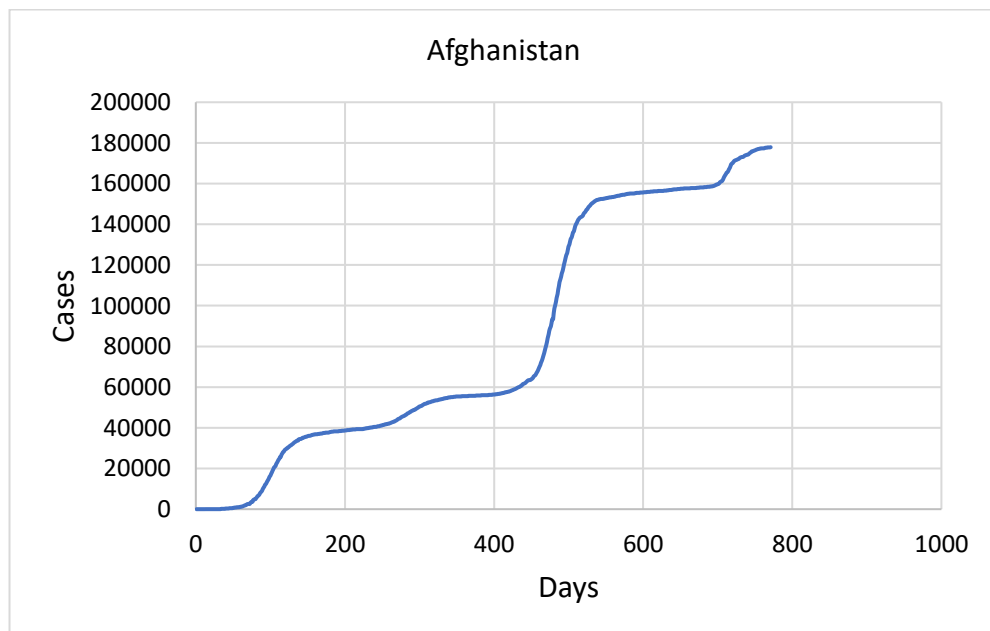


Figure 7: Total cases for Afghanistan

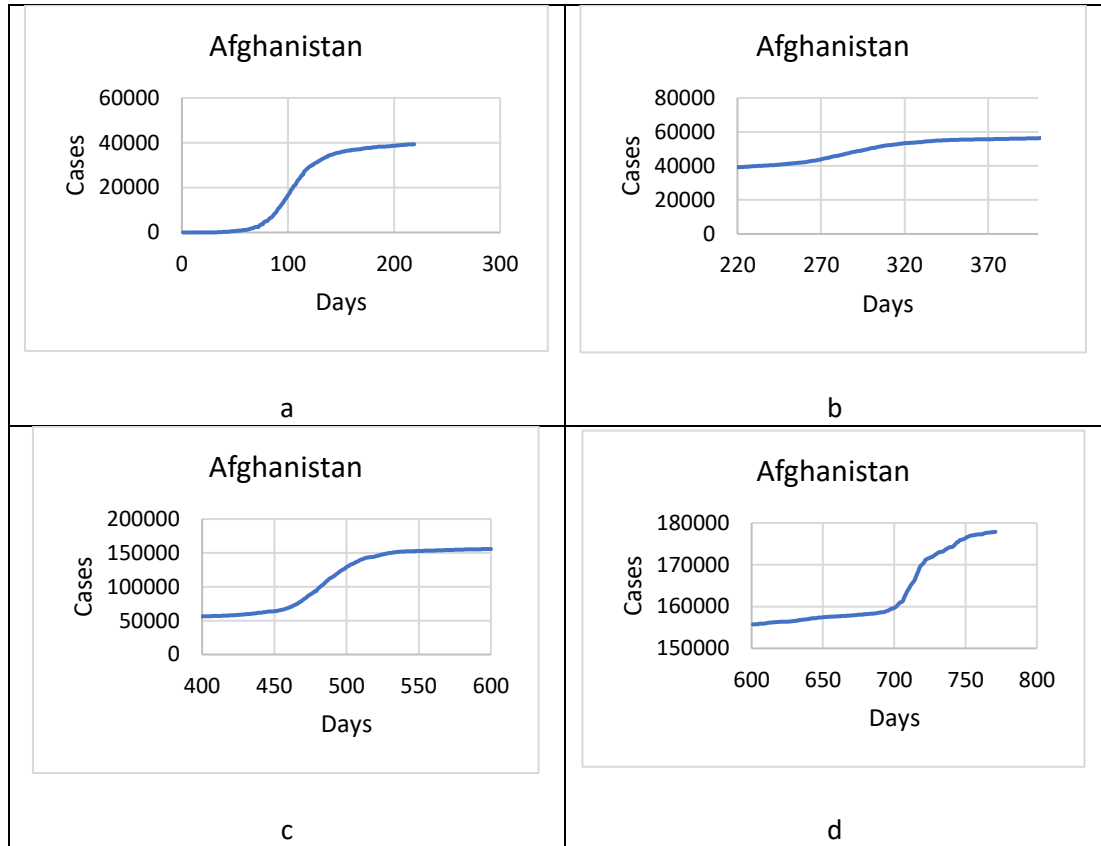


Figure 8: (a) First wave, (b) Second wave, (c) Third wave, (d) Fourth wave

Table 6 shows, the information of 4 waves for Afghanistan. For all waves, 4 parameters, correlation, and TQE are given. By the calculation, one of the aims is to reduce the TQE. Moreover, we need to get a high correlation to show how the curve fitting is successful. As can be seen in Table 6, there is an extremely high correlation for Afghanistan. Appendix A is a summary of the results for a selected set of countries.

Table 6: Information of waves of Afghanistan for 4 parameters

Waves	Interval of Days	Parameters				Correlation
		a	b	c	d	
1	1-214	0.056727	103.524766	0.422193904	1.1313986	0.9995740
2	215-401	0.035278	71.65224	0.37584109	1.24862387	0.999879
3	401-600	0.055151	84.88858	0.424211334	1.145997289	0.9997438
4	601-771	0.076850	116.271979	0.489571723	1.023061479	0.9980335

The curve fitting can be solved by using a solver in Microsoft Excel in a minute. The curve-fitting results can be seen in Figure 9 for Afghanistan and for other countries (See Appendix B).

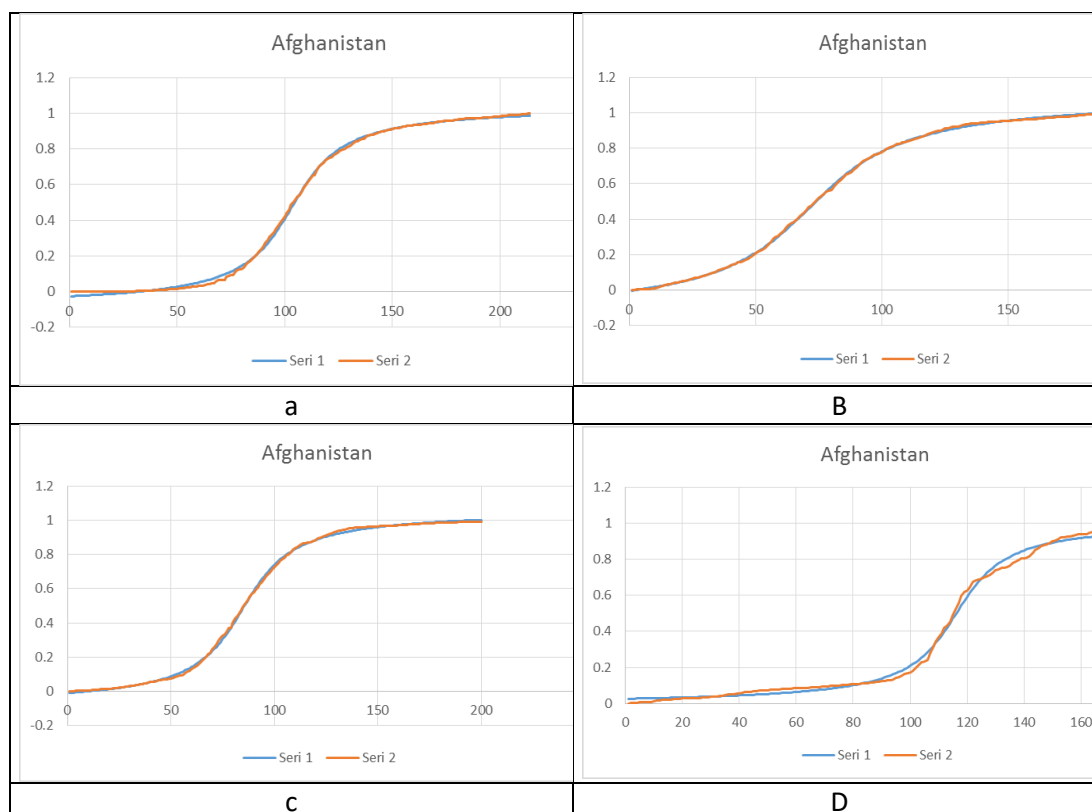


Figure 9: Curve fitting for Afghanistan (a) First wave, (b) Second wave, (c) Third wave, (d) Fourth wave

Figure 10 shows France. Part (a) is the process of the pandemic and part (b) is the vaccination. The first case was detected on 2020.01.24. The last day on the figure is 2022.04.03. There are several waves until day 660. A new and very serious wave starts after day 660. Notice the decrease on day 483 which indicates an error in reporting data. In the case of vaccination, it has only two waves. The second wave starts after critical day 660.

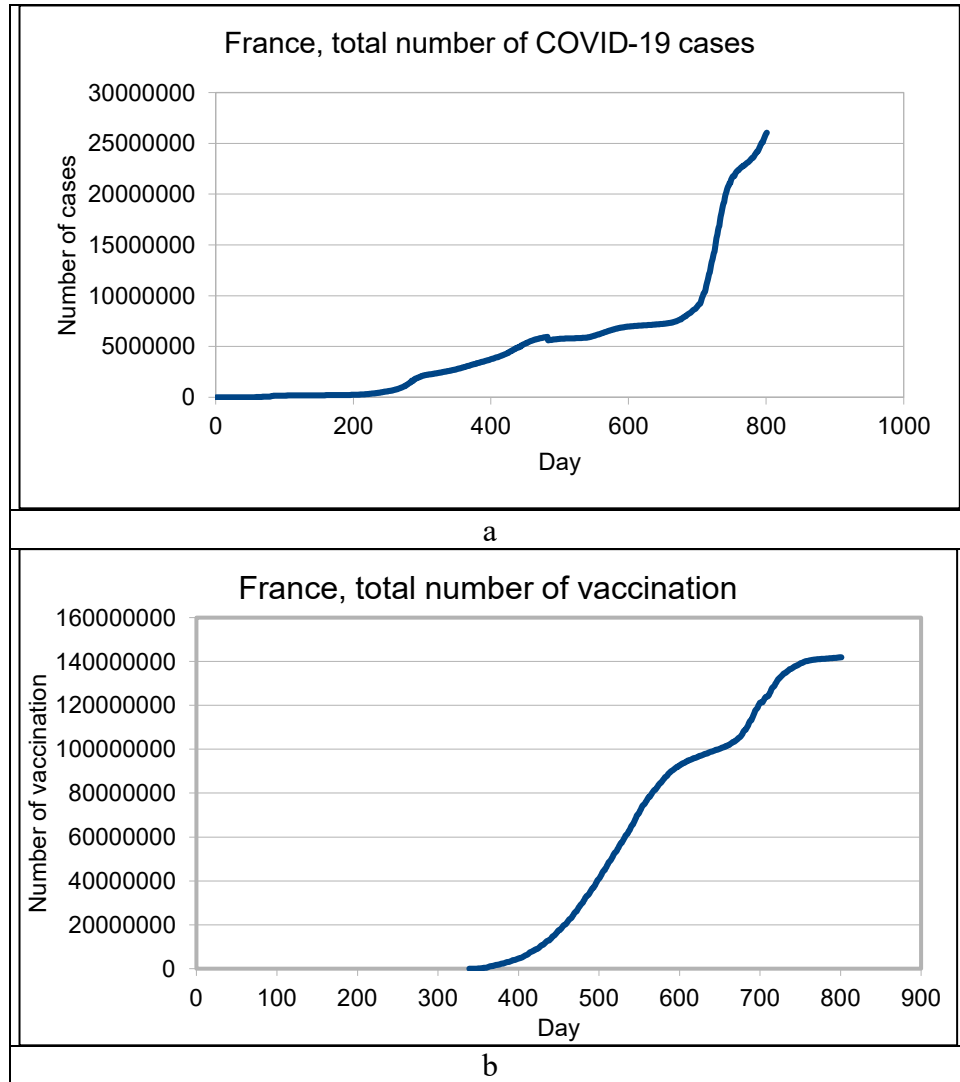


Figure 10: The case of France (a) The total number of COVID-19 cases in France. (b) The process of the vaccination in the same time interval as the total number of the vaccination.

4.3 A Selected Country: Argentina

Argentina was chosen for the purpose of presenting the calculations. The total number of cases in Argentina can be seen in Figure 11. The lengths of the waves are different showing that random events are influencing the process.

Curve fitting was applied to several waves for Argentina. It was necessary to take into consideration waves joining well with each other. For this purpose, constraints were

used to ensure smooth transitions of the waves. In the process of determining them the first wave can be higher or the following wave can be lower.

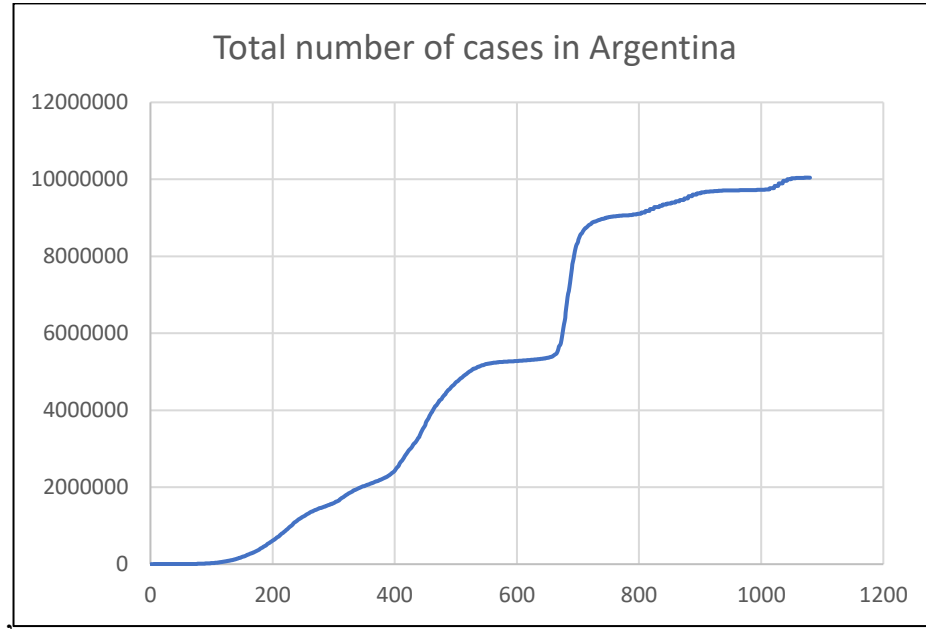


Figure 11: The first day is 2020-03-03. The last day is 2023-02-15.

At least six waves can be observed in Figure 11. There is not too much characteristic second wave. The time series is closed with a short new wave. The time intervals of the waves are in Table 7. The figure of the curve of the total number of cases helps to determine the boundaries of the waves. A wave is finished if it is saturating and the saturation is over. In the language of mathematics, it means as follows: every wave starts with a part where the function is convex. The saturation starts when it changes from a convex to a concave function. The wave is finished if a new convex part starts.

The function is convex (concave) if its difference is increasing (decreasing). The mathematical difference is identical to the number of daily new cases. Thus, the boundary of two waves is at a minimal point of the daily new cases in principle. In practice, some other factors must be taken into consideration. A value may be a local

minimum because of the weekly periodicity of the process. If the data are not reported every day, the database may contain 0 on the missing day which is automatically a local minimum. Thus, the final determination of the boundaries needs some consideration.

Table 7: The six waves of Argentina.

Number	First day	First day in number	Last day	Last day in number	Remark
1	2020-03-16	14	2020-12-26	299	No data before day 14.
2	2020-12-27	300	2021-03-07	370	
3	2021-03-08	371	2021-10-10	587	
4	2021-10-11	588	2022-04-18	777	
5	2022-04-19	778	2022-10-30	972	Weekly data only
6	2022-10-31	973	2023-02-15	1080	Weekly data only

The first investigation is to fit a single function to the curve. It is *a posteori* investigation as curve fitting is possible only if the values of the elements of the time series are known. However, it is shown in (Pınarbaşı & Vizvári, 2022a) that it is possible to obtain information for the final shape of the function by fitting the *arcus tangent* function before the end of the process. Furthermore, another epidemic may have other infectious characteristics. Still, the results can give an idea to the reader what to expect in the event of a new pandemic. The result can be seen in Figure 12. The parameter values of the fitted function are as follows: $a = 14169554$, $b = 0.0041127$, $c = 562.360$, and $d = 5243850$. More importantly, the correlation between the time series and the fitted curve is 0.9918.

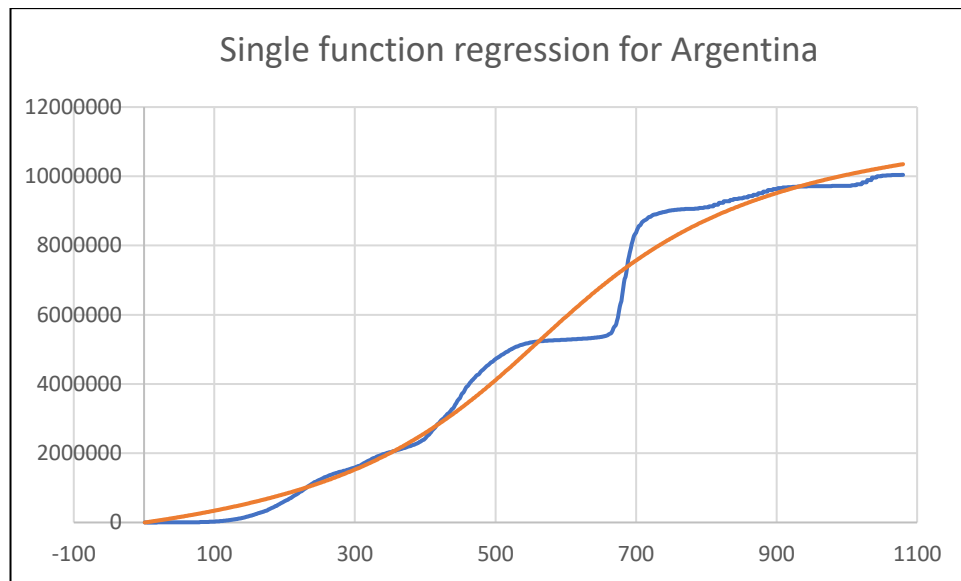


Figure 12: The fitted single curve to the time series of Argentina.

Next, each wave can be analyzed separately. One can expect an even higher correlation in the case of individual waves. It is what happens. The original and the regression functions cover one another almost perfectly. The relative absolute errors are small, i.e. not greater than 2.3 percent, apart from the beginning of the first wave where the values are still small. For Argentina, 6 waves can be seen in Figure 13.

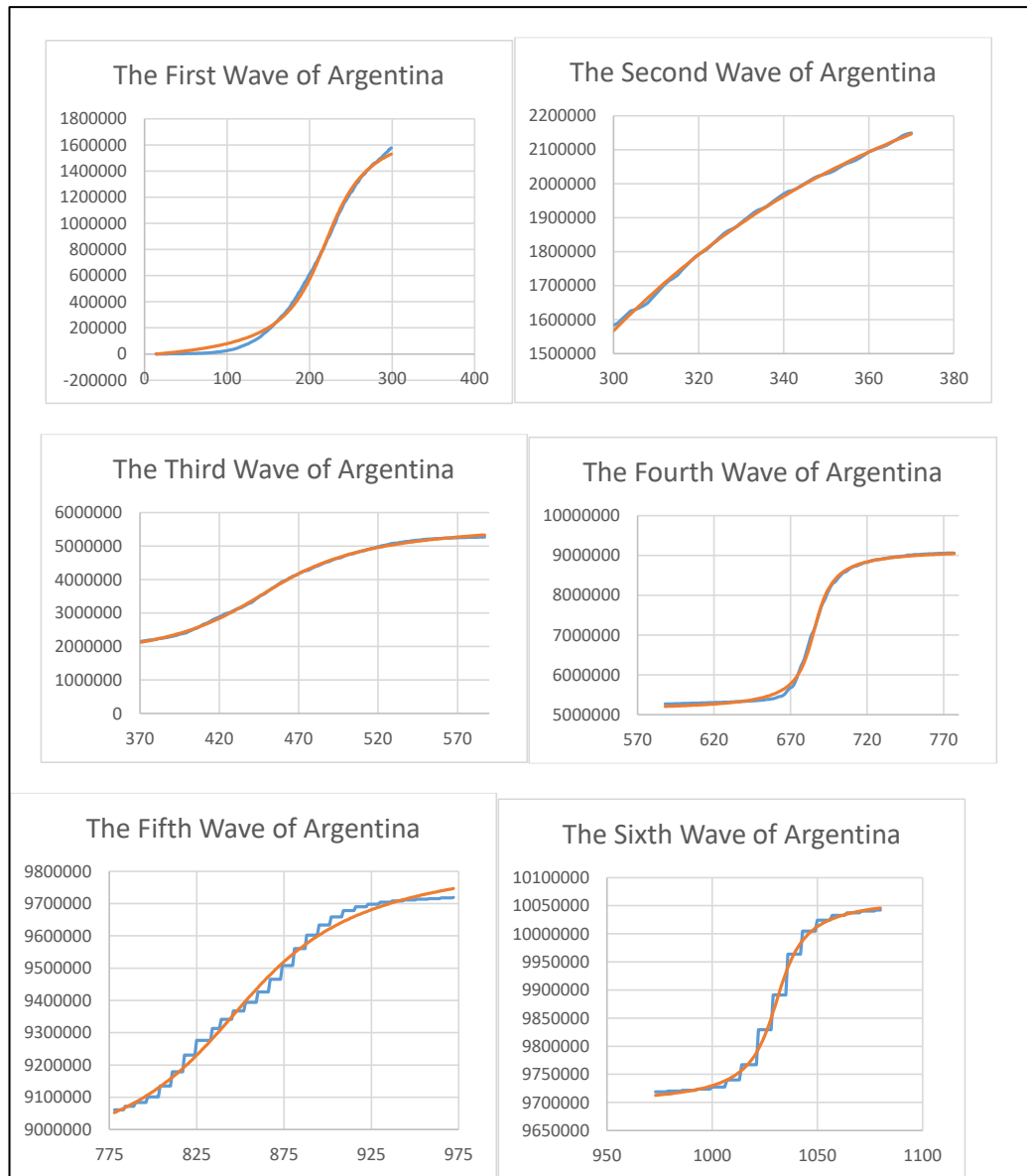


Figure 13: The six waves of Argentina

A good approximation can be obtained for the complete function of the total number of cases if the model discussed in Section 3.4.3 is applied. *Spline* is a notion in numerical mathematics. It means the approximation of a function by polynomials in a piecewise way such that the polynomials are joined in smooth ways. Something similar can be done here, although the *arcus tangent* function is not a polynomial. Instead of smooth joining, it is enough to claim that the end point of the approximation of one wave and the starting point of the approximation of the next wave must be close to

each other. Only two such constraints had to be introduced in the case of Argentina. The first one is that the approximation must have a nonnegative value at the beginning. The second one is that the approximation value at the end of the third wave is less than the approximation value at the beginning of the fourth wave. The correlation between the original function and the approximation is 0.999952. The approximation of the complete function for Argentina can be seen in Figure 14.

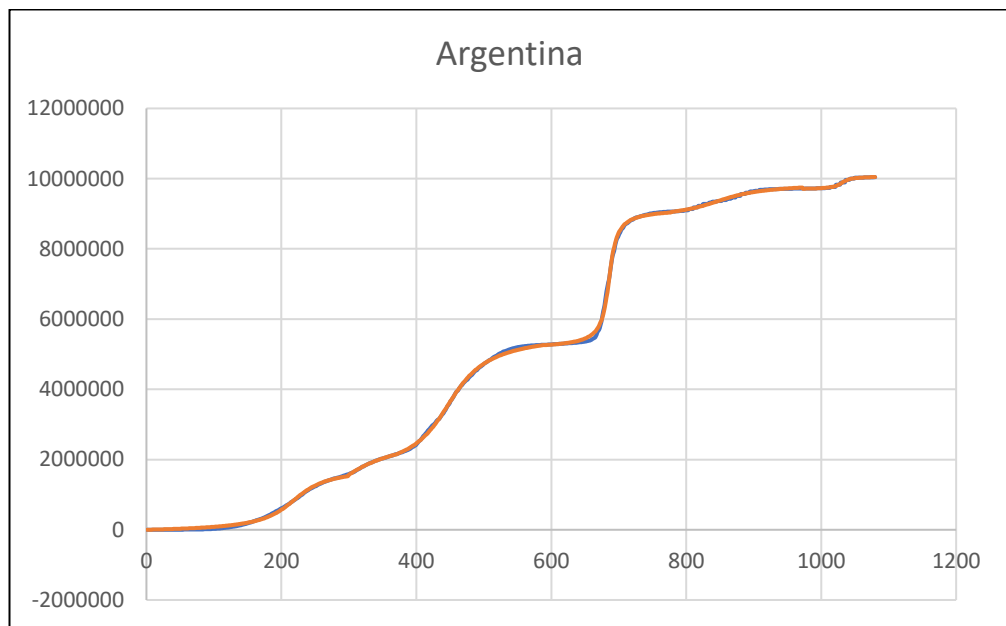


Figure 14: The approximation of the complete function of the total number of cases by piecewise “arcus tangent” functions.

The vaccination started on day 302 which is 2020-12-29. There are three vaccine-related parameters. The greatest value is the number of used vaccines which is called total vaccination in the database. The second greatest value is the number of people who got at least one vaccine. The smallest value is the number of people who are fully vaccinated as they had to get enough vaccines. The three functions can be seen in Figure 15.

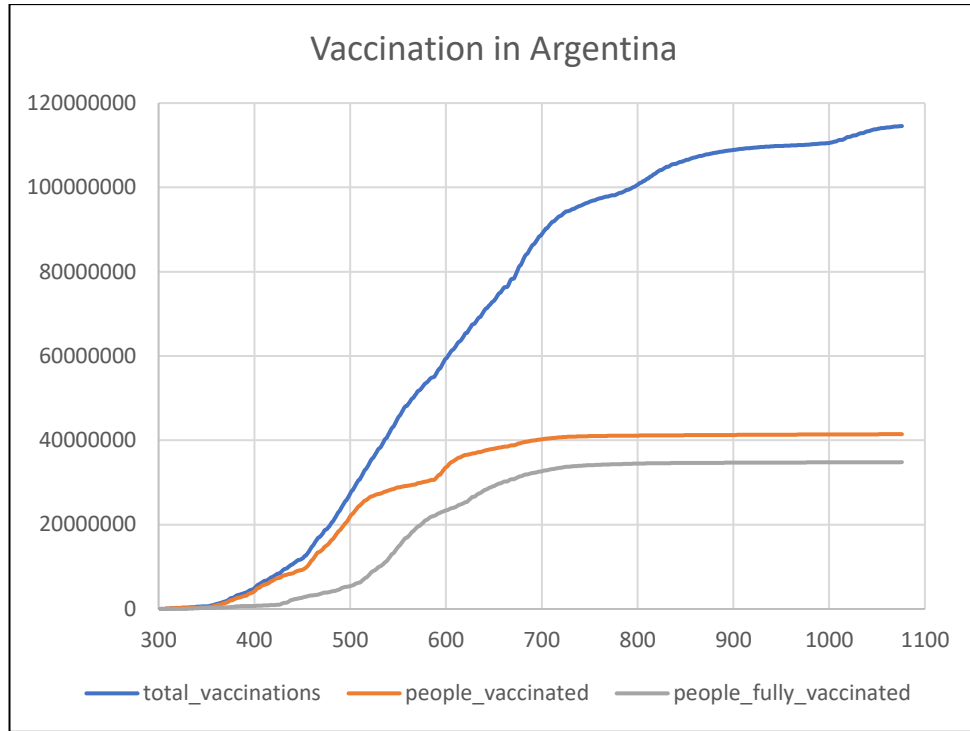


Figure 15: The vaccination process of Argentina.

In Argentina, the vaccination process has unique features. The total number of vaccinations has one main wave with some microwaves. The numbers of vaccinated and fully vaccinated people have two waves. However, the second wave has limited effects. Thus, each function can be approximated by a single *arcus tangent* function. The numerical results can be found in Table 8.

Table 8: Numerical results for the vaccination in Argentina.

	a	b	c	d	Correlation
Total vaccination	144016675	0.0081032	581.616	52981710	0.9991
People vaccinated	49652018	0.0142526	494.824	19320330	0.9980
Fully vaccinated	39148228	0.0188857	565.101	17107966	0.9991

4.4 The Numerical Results

For the selection of the countries, the main concept was every content must be represented. Regarding this perspective, countries were selected.

4.4.1 The First Wave

There are at least two important factors why COVID-19 had several waves. Even though COVID-19 is an infectious disease, it was still possible to get it again after a previous infection from which the patient recovered. The second factor is the globalization of the world including transportation. The countries could not close their border completely. So, it was always an opportunity for the emergence of newer variants of the virus.

The first wave in a country is typical for the behavior of a country in the case of a pandemic such that the recovery from infection gives protection forever. It is the motivation for investigating the first wave even separately. The further waves are based on more specific properties and are less typical. The parameters for the first wave of the countries are shown in Table 9.

Table 9: The first wave of the countries

Country	First day	Last day	a	b	c	d	R
Canada	2020-01-23	2020-08-16	132015	0.044363	101.271	56744	0.99669
Denmark	2020-02-27	2021-01-15	274307	0.031283	298.173	132200	0.99861
Ethiopia	2020-03-13	2021-01-25	154027	0.025815	197.924	67525	0.99330
Hungary	2020-03-04	2021-01-26	407721	0.048791	266.926	189357	0.99904
Japan	2020-01-22	2020-06-27	18798	0.104201	85.308	8725	0.99938
Kazakhstan	2020-03-13	2020-10-27	164403	0.047255	131.054	73798	0.99868
Mexico	2020-02-28	2021-06-28	3585522	0.008639	293.268	1363072	0.99778
Morocco	2020-03-02	2021-03-02	569480	0.027440	275.242	258656	0.99950
Nigeria	2020-02-28	2020-10-24	73205	0.030022	134.197	30908	0.99902
Peru	2020-03-06	2020-12-22	1352157	0.015800	159.605	512928	0.99871
Portugal	2020-03-02	2020-12-26	497196	0.035468	265.997	250916	0.99630
Turkey	2020-03-11	2021-03-01	2515730	0.105005	275.540	1388624	0.99541
UK (United Kingdom)	2020-01-31	2021-05-19	5133530	0.025168	329.372	2427593	0.99848
Ukraine	2020-03-03	2021-03-03	1622507	0.026520	272.814	740437	0.99980
USA	2020-01-22	2021-03-21	44125425	0.013406	339.529	19074858	0.99803
Vietnam	2020-01-23	2021-10-23	929104	0.065668	586.648	457096	0.99622

4.4.2 The Course of COVID-19 in Different Countries

It is possible to approximate the course of the COVID-19 pandemic with a single instance of the *arcus tangent* function. It gives some ideas about what can be expected in case of a similar multi-wave pandemic. The correlation is lower than the correlation obtained by a spline-like approximation. However, they are still high.

The parameters for regression of the complete course of the pandemic with a single function can be seen in Table 10.

Table 10: The regression of the complete course of the pandemic with a single function.

Country	a	b	C	d	R
Canada	6441899	0.0042109	706.476	2550727	0.9937
Denmark	3369625	0.0387160	703.680	1743604	0.9480
Ethiopia	1577632	0.0012144	256.294	151049	0.9874
Hungary	3206325.4	0.0038463	623.834	1200185	0.9897
Japan	39130609	0.0103513	964.264	18329572	0.9953
Kazakhstan	1779704	0.0072435	532.411	773251	0.9975
Mexico	13047875	0.0024708	637.094	4172514	0.9965
Morocco	1891955	0.0036882	489.000	627823	0.9911
Nigeria	460000	0.0030383	214.262	80402	0.9906
Peru	11119604	0.0014305	608.388	2533175	0.9922
Portugal	6262133	0.0113916	729.995	3090569	0.9946
Turkey	23229479	0.0048952	615.437	9246086	0.9951
UK	28571877	0.0083993	688.520	13260736	0.9961
Ukraine	8129459	0.0040384	566.591	2998384	0.9925
USA	159615134	0.0031207	680.016	57433927	0.9935
Vietnam	11887278	0.0464355	777.757	5873292	0.9986

It was possible to observe in the case of Argentina that the original time series of one wave and the approximation of the same wave cover one another almost perfectly. The same is true for all of the countries and all of their waves. The achieved correlation values are provided in Table 11. It also shows information about several constraints which is used to provide smoothness of curves. Each country has a different number of constraints. This constraint is applied in Solver Optimizer in Microsoft Excel.

Table 11: The spline-like regression of the complete course of the pandemic.

Country	First day	Last day	The number of waves	The number of constraints	Regression
Canada	2020-01-23	2023-02-15	7	1	0.999972
Denmark	2020-02-27	2023-02-15	5	0	0.999992
Ethiopia	2020-03-13	2023-02-15	5	4	0.999865
Hungary	2020-03-04	2023-02-15	5	0	0.999941
Japan	2020-01-22	2023-02-15	6	6	0.999897
Kazakhstan	2020-03-13	2023-02-15	5	0	0.999960
Mexico	2020-02-28	2023-02-15	5	0	0.999814
Morocco	2020-03-02	2023-02-15	4	3	0.999942
Nigeria	2020-03-11	2023-02-15	5	0	0.999962
Peru	2020-03-06	2023-02-15	5	0	0.999954
Portugal	2020-03-02	2023-02-15	5	0	0.999958
Turkey	2020-03-11	2023-02-15	5	2	0.999913
UK	2020-01-31	2023-02-15	2	2	0.999489
Ukraine	2020-03-03	2023-02-15	5	0	0.999950
USA	2020-03-19*	2023-02-15	5	0	0.999896
Vietnam	2020-01-23	2023-02-15	5	0	0.99998

*The numbers of cases are unrealistically low on the first 57 days.

4.4.3 Vaccination

The most important vaccination-related parameter is the total vaccination as it is what an IP must serve in case of a future pandemic. In the case of Argentina, it was shown that the function of every vaccination-related parameter can be modeled by the *arcus tangent* function, see Table 8. The regression results of the parameter total vaccinated are discussed here.

The vaccination started to be one year after the pandemic or even later as the vaccine did not exist at the beginning at all. As it is mentioned above, many vaccination data are missing. Japan has the least number of missing data. All are shortly after the start of vaccination.

Table 12: The single function regression of the total vaccination.

Country	First day	a	b	c	d	Regression
Canada	2020-12-14	317378577	0.0020139	327.000	0	0.9855
Denmark	2020-12-08	16531112	0.0102537	516.898	6165741	0.9936
Ethiopia	2021-04-08	53038282	0.0517579	688.000	26754244	0.9763
Hungary	2020-12-28	12322511	0.0302076	414.631	5061086	0.9986
Japan	2021-02-17	708350419	0.0031634	612.828	137077311	0.9897
Kazakhstan	2021-02-11	24422161	0.0139686	511.268	9201267	0.9952
Mexico	2020-12-24	287976471	0.0074349	588.026	103861500	0.9979
Morocco	2021-01-30	73674297	0.0102989	535.604	25161013	0.9961
Nigeria	2021-03-15	163000000	0.0059535	890.000	64954644	0.9971
Peru	2021-02-08	111869005	0.0079922	630.575	41476661	0.9994
Portugal	2021-01-01	47442829	0.0042569	492.331	10131889	0.9945
Turkey	2021-01-14	189076721	0.0114334	507.087	70322797	0.9986
UK	2021-01-10	365854072	0.0038382	406.151	26437003	0.9956
Ukraine	2021-02-24	42359070	0.0149457	602.794	17576902	0.9985
USA	2020-12-13	455476759	0.0217481	432.989	1684789	0.9994
Vietnam	2021-03-08	270496951	0.0170051	680.238	116792432	0.9966

The parameters of the single function regression of the total vaccination can be seen in Table 12.

Most countries have two waves of vaccination. Japan has 4 waves. Canada, Mexico, and Morocco have 3 ones. Nigeria and Peru have only one wave. Ethiopia reported irregularly.

Both the total number of COVID-19 cases and the vaccination are processes showing saturation. The population size is an upper limit. However, there are always people who avoid the disease because they combine genes. Thus, the real smallest upper limit is under the population size. It is true for COVID-19 even though some people got COVID several times. If there are several waves, then each wave is saturated

independently from the other waves. The characteristic properties of the saturation of a wave depend on exogenous factors.

The phenomenon of saturation is discussed in a more detailed way in Chapter 3 (Pınarbaşı & Vizvári, 2022a). It is shown that it can be described by the parametrized version of a cumulative distribution function of a random variable. The cumulative distribution functions have the required properties. They are monotone increasing and converge to 1. Thus, their minimal upper limit is 1.

Saturation exists even in the case of the multi-waves.

4.5 Business and Safety Considerations

Table 11 concerns all types of products not only vaccines. The values for probability levels other than 0.9 are similar to those in Table 11. More precisely, the higher the probability of the non-shortage, the higher the initial stock level is. On the other hand, the initial stock level is a decreasing function of the number of procurements. If the cost is the most important factor, then a good compromise is to be found depending on the order cost.

Safety is the most important factor in the case of vaccines. If a country may purchase a high initial stock, then this country is safe with a high probability. However, there were cases in the COVID-19 pandemic such that a country could not use all the initial stock. The country suffered financial loss even in the case of a part of the superfluous stock has been sold.

Another drawback that a country has too high initial stock is that other countries can have a problem with procurement as not enough vaccines remain on the market. If other countries are unable to vaccinate their citizens, then the infection will continue to spread there, and this will re-infect, to a greater or lesser extent, the citizens of countries that have come to safety. Therefore, at the international level, it is desirable to develop a system in which each country is guaranteed a lot of deliveries evenly distributed on time.

Chapter 5

THE STOCKING PROCESS

5.1 A Deterministic Model for Inventory Policy

The first ever paper on IP is (Harris, 1990a). It was republished in 1990. The paper remained practically unobserved until 1934 when (*A Scientific Routine for Stock Control: Simple Tables, Scientifically ...* - R. H. Wilson - Google Kitaplar, n.d.) emphasized its importance. It gives a formula for the optimal quantity to be ordered which is called EOQ. The underlying assumptions of the Harris formula are as follows: (a) there is a production process on an infinite time horizon, (b) the stocking and order of the raw material have costs, (c) the demand rate is constant in time, (d) the cost elements are also constant in time, (e) the IHC is proportional to the stored quantity, (f) the order cost is a fixed cost what is to be paid at every order, (g) there is no shortage of the raw material at the supplier, and (h) the order arrives immediately, *i.e.* the lead time can be considered zero. The Harris formula minimizes the average cost on the infinite time horizon. The crucial features of the assumptions are that everything is deterministic, and the time horizon is infinite. The latter one is a good approximation of a long and steady economic environment like the times before World War I in the USA.

Many further versions of the inventory problem have been published such that the two features are valid (Chikán, 1984). However, there are cases in economics without

having the two features. See for example the newsboy problem below. They also have inventory problems needing different approaches.

This chapter discusses two kinds of IPs for vaccination. The first one solves a deterministic case. The second one takes into consideration the stochastic nature of the vaccination process.

The methodology for reducing the cost of keeping vaccine inventories on hand is expanded upon in this section. It is predicated on the findings from the preceding section. It was demonstrated that the countries' vaccination schedules may be roughly predicted with great accuracy in a short period of time after the immunization campaign began. The following series of presumptions form the foundation of the model:

1. It is assumed that the vaccination process is deterministic, and the total percentage of the vaccinated population has an a priori course in time.
2. This course is described by a saturation function.
3. The number of procuring of the vaccine is given. Each procurement is made by issuing an order.
4. The lead time is 0. *i.e.* the ordered quantity arrives immediately.
5. The first order is issued at the beginning of the process when still nobody is vaccinated.
6. Any further order is issued when the stock level becomes 0.
7. The process is finished when a certain percentage is achieved. This vaccination level can be different in different countries as the willingness for vaccination is depending on the country.

The IH process is always ahead of the vaccination, i.e. there is no negative stock level. The total vaccinated percentage of the population must be compared to the total arrived vaccine quantity. The latter one is an increasing step function. The first one is a saturation function obtained by regression of the previous section. The two can be seen in Figure 16.

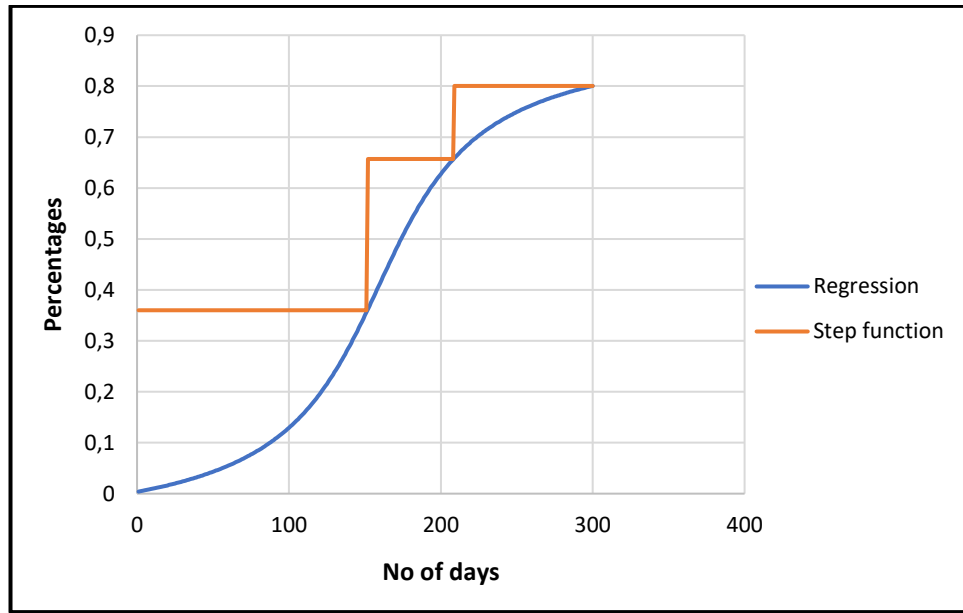


Figure 16: The solution for Denmark if the number of procurements is 3.

The time unit that can be used in real practice is a day. A dummy order is added to the set of orders. It is when the process is finished at the target percentage, *i.e.* a day T . Its ordered quantity is 0. Table 13 summarizes the results for 3 parameters of a selected set of countries. The 3 parameters version is used for the same function instead of 4 parameters since the first multiplier is fixed to 1. High correlations are achieved.

Table 13: The parameters of the countries

Country	Parameters			Correlation	Available Percentage
	a	b	C		
Belgium	0.019440264	142.3695511	0.372352365	0.998409235	78
Canada	0.023610645	141.5326286	0.367132913	0.998222367	79
Cyprus	0.045090581	20.01509258	0.224373146	0.994908854	65
Czecia	0.011369405	132.4762014	0.264628608	0.993386019	62
Denmark	0.020214063	159.1129644	0.407710152	0.998316487	81
Finland	0.021794171	94.2677651	0.367909632	0.999739483	77
France	0.016971981	149.3836653	0.381809048	0.998825287	77
Hungary	0.018408277	73.06371651	0.252787089	0.995603327	64
Iceland	0.063514378	54.17233745	0.434359993	0.99793347	84
Ireland	0.018075924	150.5716412	0.389356595	0.998955126	78
Italy	0.018014458	151.3257418	0.377070492	0.999624445	77
Malta	0.022652701	95.41681316	0.39760535	0.999183087	82
Mexico	0.010377526	188.0011619	0.32312548	0.998082526	56
Peru	0.012379217	200	0.349426928	0.995727225	58
Portugal	0.025237491	162.0226288	0.461236454	0.99488844	83
South Korea	0.021931008	161.1951162	0.437882706	0.991209179	79
Spain	0.028466217	107.8294504	0.417967064	0.998390744	82
Turkey	0.012053373	140.355595	0.368700663	0.993605162	68
UK	0.015992445	55.39616283	0.303422349	0.995669144	72
USA	0.013378891	91.25265859	0.249549082	0.99807531	65

The total IHC is proportional to the area between the two functions as this is the quantity in the stock. It is what should be minimized primarily. However, the area under the saturation function is fixed. See a more detailed mathematical explanation below. Thus, the area between the two functions depends only on the step function. Consequently, it is enough to minimize the area under the step function.

Table 14: To formalize the mathematical model, some notations are used as follows:

$f(t)$	The saturation function, <i>i.e.</i> the function of the total demand
n	The number of orders
t_i	The time of order i ($i = 1, 2, \dots, n + 1$)
T	The time when the target percentage of the vaccination is achieved

The stocking process consists of a first order at time 0, and there are $n - 1$ further reorders. A reorder is issued exactly at the moment when the stock becomes 0. The ordered quantity arrives immediately as the lead time is 0. In other words, the ordered

quantity is enough exactly until the next reorder point. Thus, the total ordered quantity until the time t_i is $f(t_{i+1})$, as it must be exactly enough until t_{i+1} . Hence, the ordered quantity at t_i is $f(t_{i+1}) - f(t_i)$. The latter quantity is the demand in the time interval $[t_i, t_{i+1}]$.

The IHC in the time interval $[t_i, t_{i+1}]$ is proportional to the area between the step function and the saturation function. This area is:

$$(t_{i+1} - t_i)f(t_{i+1}) - \int_{t_i}^{t_{i+1}} f(t)dt.$$

Thus, the total IHC is proportional to

$$\begin{aligned} & \sum_{i=1}^n \left((t_{i+1} - t_i)f(t_{i+1}) - \int_{t_i}^{t_{i+1}} f(t)dt \right) \\ &= \sum_{i=1}^n (t_{i+1} - t_i)f(t_{i+1}) - \sum_{i=1}^n \int_{t_i}^{t_{i+1}} f(t)dt = \sum_{i=1}^n (t_{i+1} - t_i)f(t_{i+1}) \\ & \quad - \int_0^T f(t)dt. \end{aligned}$$

The last term is independent of the selection of the reorder points. Thus, it is enough to minimize the last but one term. Hence, the optimization problem determining the best IP is as follows:

$$\min \sum_{i=1}^n (t_{i+1} - t_i)f(t_{i+1}) \quad (2)$$

$$t_1 = 0 \quad (3)$$

$$t_{i+1} \geq t_i \quad i = 1, \dots, n \quad (4)$$

$$t_{n+1} = T. \quad (5)$$

The results obtained from the model are in Table 15. Also, population information of the countries is provided in same table (Worldometer, n.d.).

Table 15: Direct results obtained from the model.

Countries	The number of procurements			Target percentage	Population
	3	4	5		
Belgium	118,98	113,38	109,91	70	11,589,623
Canada	119,33	114,21	111,02	70	37,742,154
Cyprus	24,43	22,46	21,31	60	1,207,359
Czechia	119,12	111,61	107,10	60	10,708,981
Denmark	158,97	150,83	145,81	80	5,792,202
Finland	81,15	76,68	73,95	70	5,540,720
France	121,81	115,92	112,28	70	65,273,511
Hungary	70,34	65,59	62,76	60	9,660,351
Iceland	50,05	47,76	46,33	80	341,243
Ireland	120,41	114,86	111,42	70	4,937,786
Italy	125,03	119,15	115,51	70	60,461,826
Malta	105,01	98,13	93,99	80	441,543
Mexico	99,22	94,67	91,86	50	128,932,753
Peru	101,73	97,61	95,04	50	32,971,854
Portugal	137,94	132,63	129,28	80	10,196,709
South Korea	114,42	110,18	107,51	70	51,269,185
Spain	105,09	99,73	96,42	80	46,754,778

The optimality condition of (2)-(5)

The Karush-Kuhn-Tucker optimality conditions are first order conditions. The easy way to obtain the same conditions in this particular case is as follows:

Assume that the times of orders $i - 1$ and $i + 1$ are fixed. What is the optimal time of order i ? The area under the step function in the interval $[t_{i-1}, t_{i+1}]$ is

$$g(t_i) = (t_i - t_{i-1})f(t_i) + (t_{i+1} - t_i)f(t_{i+1}).$$

Notice that the only unknown quantity in the formula is t_i . The derivative of $g(t_i)$ must be 0 at the optimal position of t_i . Hence,

$$g'(t_i) = f(t_i) + (t_i - t_{i-1})f'(t_i) - f(t_{i+1}) = 0. \quad (*)$$

Thus, the optimality condition is that (*) holds for $i = 1, 2, \dots, n$.

5.2 Evaluation of the Numerical Results

The optimality condition of problem (2)-(5) can be obtained by the Karush-Kuhn-Tucker Theorem (May, 2020) of optimization theory. However, the same condition can be obtained in a simpler way as well. On the other hand, the problem has a very easy structure. Excel solver got the optimal solutions in all cases within 1 minute. Thus, the optimality condition was not used directly in the research.

The optimal value given in Table 13 has the unit

$$\text{proportion of the population} \times \text{vaccine} \times \text{day}.$$

The result can be transferred $\text{vaccine} \times \text{days}$ if it is multiplied by the population. Still, it is necessary to subtract the area under the approximation curve again multiplied by the population.

To obtain the latter quantity, the function given by formula (1) must be integrated between 0 and the day when the target percentage of the vaccination is reached according to the approximation function. Notice, that the stocking policy is determined far before the vaccination process is finished. Thus, the only approximation can be used in the calculations.

The requested formula of

$$\int_0^T \left(\frac{\arctg(a(t-b))}{\pi} + c \right) dt$$

is based on the well-known equation as follows:

$$\int \arctg(t) dt = t \arctg(t) - \frac{\ln(1+t^2)}{2}.$$

Hence

$$\begin{aligned}
& \int_0^T \left(\frac{\arctg(a(t-b))}{\pi} + c \right) dt \\
&= \left[\frac{(t-b)\arctg(a(t-b))}{\pi} - \frac{\ln(1+(a(t-b))^2)}{2\pi a} + ct \right]_0^T \\
&= \frac{(T-b)\arctg(a(T-b))}{\pi} - \frac{\ln(1+(a(T-b))^2)}{2\pi a} + cT + \frac{b\arctg(-ab)}{\pi} + \frac{\ln(1+a^2b^2)}{2\pi a}. \tag{6}
\end{aligned}$$

If the value of formula (6) is subtracted from the optimal value given in Table 5 and the result is multiplied by the population, then the IHC is obtained in *vaccine* $\times$ *days*. These results are in Table 16. It is converted to money if this quantity is multiplied by the inventory holding unit cost, *i.e.* the cost of storing one unit of vaccine for one day.

The final inventory cost can be obtained if the order costs are added. In the case of the order cost, only the transportation cost can be a non-negotiable amount of money. The decision-maker can solve the model easily for different numbers of procurements. The best solution can be selected after taking into account the order cost.

Table 16: Numerical results for 20 countries for 3, 4, 5 procurements

Countries	a	b	c	Formula 3	4	5	3-F	4-F	5-F	Population (3-F)*P	(4-F)*P	(5-F)*P
Belgium	0.019	142.369	0.372	64.29	90.19	83.57	79.64	25.90	19.28	15.35	11,589,623	300176977. 223523430. 177959987.
Canada	0.023	141.532	0.367	53.24	78.03	71.64	67.86	24.79	18.40	14.62	37,742,154	935753046. 694571977. 551942306.
Cyprus	0.045	20.015	0.224	27.49	33.94	32.43	31.49	6.44	4.93	3.99	1,207,359	7781313.69 5955785.68 4821865.71
Czechia	0.011	132.476	0.264	83.11	112.38	105.19	100.83	29.26	22.08	17.72	10,708,981	313427384 236462617. 189767718.
Denmark	0.020	159.112	0.407	115.04	149.78	141.27	136.08	34.73	26.22	21.03	5,792,202	201164410. 151911823. 121837584.
Finland	0.021	94.267	0.367	58.29	77.20	72.47	69.63	18.90	14.17	11.33	5,540,720	104766475. 78562581.9 62812173.1
France	0.016	149.383	0.381	71.59	98.54	91.66	87.57	26.94	20.06	15.97	65,273,511	1758720037 1309740604 1042905563
Hungary	0.018	73.063	0.252	55.34	72.99	68.71	66.10	17.65	13.37	10.75	9,660,351	170548359. 129190329. 103931207.
Iceland	0.063	54.172	0.434	31.90	42.43	39.78	38.19	10.53	7.88	6.28	341,243	3594263.49 2689429.72 2146378.41
Ireland	0.018	150.571	0.389	66.45	92.56	85.84	81.86	26.10	19.39	15.41	4,937,786	128924000. 95750847.8 76124287.3
Italy	0.018	151.325	0.377	68.64	95.90	88.93	84.79	27.25	20.28	16.14	60,461,826	1648131934 1226660638 976366768.
Malta	0.022	95.416	0.397	110.42	135.84	129.81	126.07	25.41	19.38	15.64	441,543	11220621.3 8558142.11 6906738.06
Mexico	0.010	188.001	0.323	49.26	66.27	60.45	57.06	17.00	11.18	7.79	128,932,75	2192072033 1442519793 1004841885
Peru	0.012	200.000	0.349	37.56	59.13	53.34	50.00	21.57	15.78	12.44	32,971,854	711258786. 520501634. 410397780.
Portugal	0.025	162.022	0.461	72.65	99.74	92.69	88.54	27.08	20.03	15.88	10,196,709	276204080. 204263806. 161988862.
S. Korea	0.021	161.195	0.437	52.25	74.99	68.90	65.39	22.73	16.65	13.13	51,269,185	1165717374 853668468. 673328272.
Spain	0.028	107.829	0.417	76.64	99.72	94.03	90.57	23.07	17.38	13.92	46,754,778	1078861448 812826558. 651156182.
Turkey	0.012	140.355	0.368	60.35	80.66	75.43	72.35	20.31	15.08	11.99	84,339,067	1712991067 1272413017 1012001364
UK	0.015	55.396	0.303	117.00	137.44	132.73	129.77	20.43	15.72	12.76	67,886,011	1387211503 1067226602 866687270.
USA	0.013	91.252	0.249	77.73	100.77	95.22	91.82	23.03	17.48	14.08	331,002,65	7626154535 5787765209 4661634801

5.3 Single Period Inventory Problem in the Stochastic Case

The first ever thesis on an inventory problem is (Harris, 1990b) which is the republication of an old paper. It determines the EOQ in a deterministic case such that there are infinitely many time-periods and all time-periods are completely identical. Still, the majority of inventory problems have an infinite time horizon. An important exception is the newsboy problem (R. R. , Chen et al., 2016). The decision of the stock level is made for a single period in this case.

Is vaccination a single-period or a multi-period inventory problem? There is no unique answer to this question as there are many different vaccination problems. Every country applies a routine immunization profile which is a standardized schedule of giving the vaccine mainly to children and preventing the outbreak of certain pandemics such as diphtheria, measles, smallpox, etc. The realization of the profile is repeated every year. Thus, the inventory problem of routine immunization is having an infinite time horizon.

The case of COVID-19 is quite different. It is a new pandemic. Thus, there is no routine immunization against it. The situation was even worse at the beginning as no vaccine existed against this disease. The general idea was that if a vaccine is developed and the population is vaccinated, then the pandemic will stop. If the assumption is correct, then it is a single-period problem. The period is the duration of the pandemic. The same can be expected in the future in the case of still-unknown pandemics. The three main steps will be as follows: (a) developing the vaccine, (b) producing the vaccine in large quantities, and (c) vaccinating the population.

The above-mentioned assumption is applied in this chapter. Thus, there is only one period.

The solution to the inventory problem starts with the determination of the demand. The demand of a country depends on two main characteristics of the country which are the size of the population and the shape of the willingness to be vaccinated. The further steps of the solution are the determination of the number of purchases including the initial one and the purchased quantities. This chapter provides a solution to the mentioned decision structure taking into account the stochastic nature of the market.

5.3.1 Newsboy Problem

"Newsboy" typically refers to a young person, often a boy, who sells newspapers on the streets. Figure 17 represents a newsboy in New York in 1902 who is selling typically 1 kind of newspaper to many customers. Historically, newsboys were a common sight in cities around the world, particularly during the late 19th and early 20th centuries. They would typically shout out headlines or carry signs to attract attention to the newspapers they were selling. The newsboy has one kind of product in a day that is obsolete on the next day. So, the ordered quantity must be determined for a single period.



Figure 17: 1902 illustration of a man in New York purchasing from a newsboy (Wikipedia, n.d.-c)

Single-period decision models are referred to as the Newsboy Problem. The newsboy problem (Kogan & Lou, 2003) is a classic problem in the field of operations research and decision theory that deals with IM. It is a problem of determining the optimal order quantity for a perishable product (Vijayarangam et al., 2020) that has uncertain demand (Lau & Lau, 1988).

The problem is typically framed as follows: a newsboy buys newspapers from a distributor and sells them to customers. The newsboy must decide how many newspapers to buy each day, given that the demand for newspapers is uncertain. If the newsboy orders too many newspapers, he will have excess inventory that will go to waste. On the other hand, if he orders too few newspapers, he will miss out on potential sales and revenue.

The objective of the newsboy is to maximize his expected profit, which is the difference between the revenue from selling newspapers and the cost of purchasing them. The cost of purchasing newspapers is assumed to be known, while the demand for newspapers is uncertain and follows a PD.

The newsboy problem can be formulated as a mathematical optimization problem, and various techniques such as probability theory, statistics, and optimization theory can be used to solve it. Some common approaches include the newsvendor model, the Poisson demand model, and the Bayesian approach.

The newsboy problem can be formulated mathematically as an optimization problem.

The basic formulation of the problem is as follows:

- Let Q be the quantity of newspapers that the newsboy orders from the distributor.
- Let D be the demand for newspapers, which is assumed to be a random variable with a known PD.
- Let C be the cost of purchasing one unit of newspapers from the distributor.
- Let R be the revenue generated by selling one unit of newspapers to a customer.
- Let H be the holding cost per unit of unsold newspapers.

The newsboy's objective is to maximize expected profit, which is given by:

$$E[\text{Profit}] = E[\text{Revenue}] - E[\text{Cost}]$$

where $E[\text{Revenue}]$ is the expected revenue from selling newspapers and $E[\text{Cost}]$ is the expected cost of purchasing and holding newspapers. The expected revenue and cost can be expressed as follows:

$$E[\text{Revenue}] = R * \min(Q, D)$$

$$E[\text{Cost}] = CQ + HE[\max(Q-D,0)]$$

where $\min(Q,D)$ is the minimum of the order quantity Q and the demand D , and $\max(Q-D,0)$ is the maximum of $Q-D$ and zero. The newsboy's decision variable is Q , which represents the order quantity of newspapers. The optimal order quantity, denoted as Q^* , can be obtained by solving the following optimization problem:

$$\text{maximize } Q \ E[\text{Profit}] = R * \min(Q,D) - CQ - HE[\max(Q-D,0)]$$

$$\text{subject to } Q \geq 0$$

The optimal order quantity Q^* is the value of Q that maximizes the expected profit.

The solution depends on the parameters of the problem, such as the demand distribution, the cost of ordering and holding inventory, and the revenue from selling the product.

5.4 The Hungarian Inventory Model

The Hungarian inventory model is another example of a single-period inventory problem (Prékopa, 2006). The model was developed in the early 1960's (Prékopa, 2006). It is also called the Prékopa-Ziermann model. More precisely, it can be used both in single-period and multi-period cases. It is discussed as a multi-period problem first. A raw material is consumed by the production in the model. Its procurement is uncertain. The safety of the production, *i.e.* avoiding unwanted interruption of the production, is more important than the inventory cost.

The main question of the Hungarian inventory model is how large should be the initial stock such that the probability of shortage is not greater than an *a priori* given threshold. In the simplest version of the model, the uncertainty is in the delivery times. The uncertainty is modeled by a stochastic approach.

The basic assumptions and the mathematical formalism of the model are as follows:

- The events take place in the time interval $[0, T]$ where $T > 0$.
- The initial stock is M . It is unknown and is obtained from the optimal solution of the model.
- The demand is deterministic, and its value is D in the time interval.
- There are n deliveries in the time interval where n is a positive integer. This number does not contain the delivery of the initial stock.
- Each delivery transports the same quantity which is D/n .
- The deliveries take place in uniformly distributed and independent times, say at $t_1, t_2, \dots, t_n$. These values are uniformly distributed and independent of the interval $[0, T]$, thus the index order may be different from their increasing order. The history of the deliveries, i.e. when they happen in time order, is given by their order statistics $t_1^* < t_2^* < \dots < t_n^*$ where $\{t_1, t_2, \dots, t_n\}$ and $\{t_1^*, t_2^*, \dots, t_n^*\}$ run over the same set of real numbers
- There is a probability value $p(0 < p < 1)$ given. It is the required probability of the non-shortage.

Two further functions need special attention. Both are the function of time t , where $0 \leq t \leq T$. The first one is the total demand until time t . It is denoted by $F(t)$. It was supposed in the original Hungarian inventory model to be linear, like $F(t) = ct$ where c is a positive constant (Prékopa, 2006). However, it is not linear in the case of vaccination as Figure 3 shows. Concerning the second function, let $W(n, t)$ be the total available quantity of vaccine until time t . It is a random function as the delivery times are random. Its value is M plus D/n multiplied by the number of deliveries until time t . Its mathematical formula is as follows:

$$W(n, t) = M + \frac{kD}{n}, \text{ where } t_k^* \leq t < t_{k+1}^*. \quad (7)$$

The default values $t_0^* = 0$ and $t_{n+1}^* = +\infty$ are applied. Formula (7) concerns the multi-period case. It is important to note that the term M is deterministic and constant on the time interval $[0, T]$. The term $\frac{kD}{n}$ is constant; however, the interval where it is added depends on random factors. The material balance in the $[0, T]$ interval is, that the initial stock level is M and the delivered total quantity is $n(\frac{D}{n}) = D$, i.e. the total available quantity is $M + D$. On the other hand, the used quantity is D . Thus, at the end of the interval the remaining quantity is $(M + D) - D = M$. It is the initial stock of the next time period. Thus, the system is repeating itself. However, if there is only one period as in the case of the pandemic, then the stock level is to be decreased to the end of the period to 0. Thus, the initial stock is M again and the total quantity of the procurement is $D - M$. Hence, the delivered quantity is D/n if at least this quantity is missing from the total demand D . If less but still positive value is missing, then the purchased quantity is the missing quantity and finally, if the available quantity achieved D , then the purchased quantity is 0. Let $Y(n, k)$ be the quantity which is purchased at the k -th delivery. Then

$$Y(n, k) = \begin{cases} D/n & \text{If } D \geq M + kD/n \\ D - M - (k-1)D/n & \text{If } M + \frac{(k-1)D}{n} < D \leq M + kD/n \\ 0 & \text{If } M + (k-1)D/n \geq D \end{cases} \quad (8)$$

It is easy to see that (8) is equivalent to

$$Y(n, k) = \min\{D/n, \max\{0, D - M - (k-1)D/n\}\}. \quad (9)$$

The mathematical condition that there is no shortage at a moment t is that the total arrived quantity is at least as large as the demand, i.e.

$$W(n, t) \geq F(t).$$

Similarly, the condition that there is never a shortage is that the same inequality holds in every moment, *i.e.*

$$\forall t \in [0, T]: W(n, t) \geq F(t). \quad (10)$$

The satisfaction of condition (10) is a random event. Its probability depends on the value of the initial stock M . The probability is an increasing function of M . The Hungarian inventory model claims to determine the minimal value of M such that this probability is at least p . The mathematical formulation of the model is as follows:

$$\min_M \text{Prob}(\forall t \in [0, T]: W(n, t) \geq F(t)) \geq p. \quad (11)$$

Thus, the optimal solution of the model gives the minimal initial stock that the vaccination process can be carried out without interruption with probability p .

Formula (7) allows that the total available quantity, that is the initial stock and the total delivered quantity is higher than the demand in a time period. It is not necessary to purchase that much in the single period case. No shortage is created if at time t_k^* quantity $Y(n, t)$ is delivered. Hence, the total available quantity is:

$$X(n, t) = M + \sum_{l=1}^k Y(n, l), \text{ where } t_k^* \leq t < t_{k+1}^*. \quad (12)$$

$X(n, t)$ is the single period version of $W(n, t)$.

5.4.1 Basic Mathematical Facts From (Birnbaum & Tingey, 1951)

The mathematical bases of (Prékopa, 2006) can be found in (Birnbaum & Tingey, 1951). This thesis introduced the *upper confidence contour*. Similar results can be found in (Bernstein, 1946). The same notations are used as in the previous section. The staircase function $W(n, t)$ is closely related to the *empirical distribution function* of the total delivered quantity. Let $Z(n, t)$ be the empirical distribution function, *i.e.*

$$Z(n, t) = \frac{k}{n}, \text{ if } t_k^* \leq t < t_{k+1}^*, k = 1, \dots, n; Z(n, t) = 0, \text{ if } t \leq 0; Z(n, t) = 1, \text{ if } t \geq T. \quad (13)$$

Hence, $W(n, t) = M + Z(n, t)D$, *i.e.*, the function $W(n, t)$ is obtained if $Z(n, t)$ is multiplied by D and is shifted up by M .

The upper confidence contour is a function which majorizes the empirical distribution function. Let $\varepsilon > 0$ be a real number. The sample size is still n . The upper confidence contour is defined by the equation:

$$Z(n, \varepsilon, t) = \min\{Z(n, t) + \varepsilon, 1\}. \quad (14)$$

There is a simple homogeneous linear relation of the single period available quantity and the upper confidence contour. Assume that $M = \varepsilon D$. Notice that it implies that:

$$X(n, t) = Z(n, t)D. \quad (15)$$

(Birnbaum & Tingey, 1951) introduced a distribution-independent quantity which depends on two factors as follows: (i) the sample size n , and (ii) ε . It is the probability that upper confidence contour is above the distribution function of the sample. It is denoted by $P(n, \varepsilon)$ and its mathematical formula is:

$$P(n, \varepsilon) = \text{Prob} \left(\forall F: \left(\forall t: Z(n, \varepsilon, t) \geq F(t) \right) \right).$$

The main result of (Birnbaum & Tingey, 1951) is the value of $P(n, \varepsilon)$.

Theorem (Birnbaum & Tingey, 1951). For all positive integer n and real number $0 < \varepsilon \leq 1$

$$P(n, \varepsilon) = 1 - \varepsilon \sum_{j=0}^{\lfloor n(1-\varepsilon) \rfloor} \binom{n}{j} \left(1 - \varepsilon - \frac{j}{n}\right)^{n-j} \left(\varepsilon + \frac{j}{n}\right)^{j-1}. \quad (16)$$

■

In what follows, the solution of the equation in ε that (16) is equal to p is denoted by $\varepsilon(n, p)$, *i.e.*

$$P(n, \varepsilon(n, p)) = p.$$

5.5 How to Apply the Theory to The Vaccination Problem?

The application of the Hungarian inventory model is discussed for a single-wave pandemic.

The total willingness to be vaccinated as the function of time is not linear in time (Pınarbaşı & Vizvári, 2022b). It can be approximated by sigmoid functions including even the *arctan* function. The reason is mentioned in section 2.2. It is that each person gets only a restricted number of vaccines. Thus, the total number of used vaccines as a function of time is increasing and has a horizontal asymptote.

The necessary condition for the applicability of the results discussed in the previous section is that the delivery times must be drawn from the arctan function obtained from regression analysis as the arctan function is a cumulative distribution function of the random demand.

The daily data of the total number of vaccinated people in countries is available in (Our World in Data, n.d.). When a particular country is investigated, then a new time series is to be created. The letter t indicating the time means the number of the day. Similarly, the last day of the observations is T and the first day is 1. The new time series is:

$$\frac{\text{the total number of vaccines used until day } t}{\text{the total number of vaccines used until day } T}$$

This value is between 0 and 1. Furthermore, it is monotone increasing. Any decrease indicates an error in the data collection. It happened in the case of France (Our World in Data, n.d.). The delivery times must be drawn from the distribution fitted to the generated time series.

Hence, the necessary initial stock is:

$$\varepsilon(n, p) \times \text{the total number of used vaccines until day } T. \quad (17)$$

However, it is obvious that the “*the total number of vaccines used until day T*” is *a posteriori* information. It can be estimated by the total number of vaccines used in previous pandemics. The size of the population is an upper bound of it as it is supposed that there is only one wave of the pandemic. The number of small children and the number of people suffering from special diseases can be deduced from the size of the total population. All these considerations give an estimation of the total number of vaccinated people giving the initial stock instead of (17) one vaccine per person is enough:

$$\varepsilon(n, p) \times \text{the estimated number of vaccinated people}. \quad (18)$$

The results can be checked by simulation.

It is important to remark that (Pınarbaşı & Vizvári, 2022b) shows that even during the pandemic it is possible to obtain an estimation of good accuracy of used vaccines by curve fitting. Estimation obtained only 64 days after the outbreak of the pandemic has good quality. The error of the estimation obtained 96 days after the outbreak is small. The result is discussed in section 4.1.

5.6 Results of The Numerical Simulation

(Birnbaum & Tingey, 1951) also determined the value of $\varepsilon(n, p)$ for $n = 5, 8, 10, 20, 40, 50, p = 0.9, 0.95, 0.99, 0.999$. The values for $p = 0.9$ are reconstructed in Table 17. The quantity of each procurement is the same. Assume that the achieved total vaccination percentage is known *a priori*. Then the quantity of procurement is:

$$\frac{\text{total population} \times \text{vaccination percentage}}{\text{the number of procurements}}.$$

Without loss of generality, it is assumed in the simulation that the size of the total population of a country is 1. The assumption is based on the homogeneous linear relation (15), *i.e.* it is justified by the fact that both the initial stock and the procurement quantity are proportional to the total population.

Table 17: The relation of the number of procurements and the necessary initial stock to achieve 90 percent probability of non-shortage.

Number of procurements	Relative initial stock
5	0.4470
8	0.3583
10	0.3226
20	0.23155
40	0.16547
50	0.14840

It is recalled that the function $P(n, \varepsilon)$ is distribution-independent. Thus, if the initial stock is exactly the value given in Table 16, then the achieved probability for a particular distribution can be higher than 0.9.

Three countries are involved in the simulation. They are Denmark, Hungary, and Mexico. The required probability of non-shortage is 0.9. The numbers of procurements are 5, 8, and 10. The higher number of procurements is not realistic. The three countries were selected because of their different cultures and size.

The simulation is carried out as follows: The vaccination speeds of the countries are approximated by the fitted *arctan* function. 44.7, 35.83, and 32.26 percentages were the initial stock for the number of procurements 5, 8, and 10, respectively. The

quantities of the procurements are 20, 12.5, and 10 percent, respectively. 10,000 scenarios are generated randomly in each of the nine cases. Each scenario consists of as many delivery days as the number of procurements. Each scenario is checked such that if the procurement quantities arrive on the days of the scenario, then whether or not shortage exists at any time. The ratio of the non-shortage scenarios is the simulated value of the probability. 5, 8, and 10 procurements results of Denmark, Hungary, and Mexico can be seen in Figures 17, 18, and 19 respectively.

Table 18: The case of Denmark is simulated for 5, 8, and 10 procurements. The achieved probabilities of non-shortage in case of different procurement quantities are provided.

Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Prob.
	0,188	0,889		0,113	0,8673		0,088	0,842
100,00%	0,189	0,8911	100,00%	0,114	0,8724	100,00%	0,089	0,8479
	0,19	0,8911		0,115	0,8753		0,09	0,856
Number of procurements	0,191	0,8935	Number of procurements	0,116	0,8791	Number of procurements	0,091	0,8628
	0,192	0,8935		0,117	0,8863		0,092	0,8688
	0,193	0,8973		0,118	0,8876		0,093	0,8793
5	0,194	0,8995	8	0,119	0,8923	10	0,094	0,8816
	0,195	0,8995		0,12	0,8924		0,095	0,8877
Initial stock in percentage of total population	0,196	0,8995	Initial stock in percentage of total population	0,121	0,8974	Initial stock in percentage of total population	0,096	0,8923
	0,197	0,901		0,122	0,8989		0,097	0,896
	0,198	0,901		0,123	0,9001		0,098	0,9022
	0,199	0,903		0,124	0,9034		0,099	0,9065
	0,2	0,906		0,125	0,9062		0,1	0,9096
0.447	0,201	0,906	0.3583	0,126	0,9086	0.3226	0,101	0,912
	0,202	0,9075		0,127	0,9095		0,102	0,9144
	0,203	0,9075		0,128	0,9115		0,103	0,9185
	0,204	0,9075		0,129	0,9128		0,104	0,9206
	0,205	0,909		0,13	0,9136		0,105	0,923
	0,206	0,909		0,131	0,9172		0,106	0,9254
	0,207	0,9102		0,132	0,9178		0,107	0,9296
	0,208	0,9131		0,133	0,9209		0,108	0,9301

Table 19: The case of Hungary is simulated for 5, 8, and 10 procurements. The achieved probabilities of non-shortage in case of different procurement quantities are provided.

Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Prob.
	0,188	0,8992		0,113	0,8701		0,088	0,8362
100,00%	0,189	0,8992	100,00%	0,114	0,8719	100,00%	0,089	0,8514
	0,19	0,9014		0,115	0,879		0,09	0,857
Number of procurements	0,191	0,9014	Number of procurements	0,116	0,879	Number of procurements	0,091	0,8647
	0,192	0,9014		0,117	0,8801		0,092	0,8718
	0,193	0,9014		0,118	0,8873		0,093	0,8774
5	0,194	0,9014	8	0,119	0,8956	10	0,094	0,8774
	0,195	0,9031		0,12	0,8972		0,095	0,8919
Initial stock in percentage of total population	0,196	0,9031	Initial stock in percentage of total population	0,121	0,8976	Initial stock in percentage of total population	0,096	0,8931
	0,197	0,9072		0,122	0,9029		0,097	0,8968
	0,198	0,9072		0,123	0,9029		0,098	0,9022
	0,199	0,9089		0,124	0,9037		0,099	0,9056
	0,2	0,9089		0,125	0,9066		0,1	0,9063
0,447	0,201	0,9089	0,3583	0,126	0,912	0,3226	0,101	0,9148
	0,202	0,9089		0,127	0,912		0,102	0,9153
	0,203	0,9089		0,128	0,912		0,103	0,9194
	0,204	0,911		0,129	0,913		0,104	0,9221
	0,205	0,911		0,13	0,9181		0,105	0,9236
	0,206	0,911		0,131	0,9224		0,106	0,925
	0,207	0,911		0,132	0,9224		0,107	0,9299
	0,208	0,9174		0,133	0,9254		0,108	0,9306

Table 20: The case of Mexico is simulated for 5, 8, and 10 procurements. The achieved probabilities of non-shortage in case of different procurement quantities are provided.

Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Prob.	Achieved percentage	Regular procurement in percentage of the total population	Probability
	0,188	0,8902		0,113	0,86		0,088	0,8332
100,00%	0,189	0,8922	100,00%	0,114	0,8646	100,00%	0,089	0,8422
	0,19	0,893		0,115	0,8698		0,09	0,8508
Number of procurements	0,191	0,893	Number of procurements	0,116	0,8725	Number of procurements	0,091	0,8574
	0,192	0,8945		0,117	0,8766		0,092	0,8664
	0,193	0,8947		0,118	0,8811		0,093	0,8715
5	0,194	0,8954	8	0,119	0,8853	10	0,094	0,8754
	0,195	0,8961		0,12	0,8894		0,095	0,88
Initial stock in percentage of total population	0,196	0,8975	Initial stock in percentage of total population	0,121	0,891	Initial stock in percentage of total population	0,096	0,8867
	0,197	0,8979		0,122	0,894		0,097	0,8901
	0,198	0,8984		0,123	0,896		0,098	0,8931
	0,199	0,9009		0,124	0,8982		0,099	0,8996
	0,2	0,9014		0,125	0,9011		0,1	0,9029
0,447	0,201	0,9022	0,3583	0,126	0,9033	0,3226	0,101	0,9061

	0,202	0,9036		0,127	0,9059		0,102	0,9088
	0,203	0,9039		0,128	0,9069		0,103	0,9133
	0,204	0,9041		0,129	0,9096		0,104	0,9167
	0,205	0,9048		0,13	0,9115		0,105	0,9179
	0,206	0,9068		0,131	0,9136		0,106	0,9213
	0,207	0,907		0,132	0,9162		0,107	0,923
	0,208	0,9077		0,133	0,9179		0,108	0,9259

Chapter 6

CONCLUSION

The COVID-19 pandemic is the first one in the history of mankind in that detailed data are available about all countries of the world. It is a good opportunity to describe the behavior of the countries in general and also the behavior of the individual countries in particular. The countries have similar properties and dynamics in spite of the fact that the shapes of the total numbers of COVID-19 cases and vaccinations seem different. The differences are caused by particular events. However, the underlying logic of the pandemic is the same in every country. The shapes can be obtained from the same function. The results of this thesis can be used in the forecast of the course of a future pandemic after the outbreak.

The countries bear the costs of vaccination in case of a pandemic. The thesis intends to reduce public expenses. The main contributions of the research are as follows:

- The increase in the proportion of the vaccinated population is analyzed in the COVID-19 pandemic.
- It is shown in selected 20 countries that the countries have individual behavior.
- However, the cumulative distribution function of the Cauchy distribution is a suitable tool to model the increase of the aforementioned proportion in the case of each country.
- A simple model is elaborated to minimize the IHC if the number of procurements is fixed. The model can be solved for a country within one minute by an

Excel solver. The numerical results are presented for the 20 countries. Three results are provided for each country such that the number of procurements is 3, 4, and 5, respectively.

The stocking procedure of vaccines in an individual country is investigated. The outbreak of a pandemic is assumed. Vaccination can stop the pandemic. Thus, there is a sudden high demand for the vaccine which causes uncertainty in deliveries. The Hungarian inventory model is a proper tool to determine the optimal IP of an individual country. The results of the model are analyzed on the data of the COVID-19 pandemic. As both the spreading and vaccination of COVID-19 have special non-linear properties, a non-linear version of the Hungarian inventory model is applied. The outcomes of the model were checked by simulation of three countries as follows: Denmark, Hungary, and Mexico. The results of the simulation show that an even higher probability of non-shortage can be achieved than what the model promises.

As a regression analysis; the available time series of COVID-19 data is investigated in 17 countries. The numerical results related to Argentina are reported in a detailed way. Only the final results obtained for the other 16 countries are reported in tables. They can be interpreted along the lines of Argentina's example. The 17 countries are different in culture, size, population, and geographical location. Even the values of the parameters as the function of time have different shapes. The most important finding of the thesis is that in all countries, the course of the epidemic can be described by the same function, namely *arcus tangent* despite these differences. This applies to the number of cases, the number of vaccines used, and the number of people vaccinated and fully vaccinated. The phenomenon is due to the fact that the process is saturated since the number of cases is limited by the size of the population. This limitation of

the process also exists in the event that a person may get the disease again, as was the case with COVID-19. The *arcus tangent* function is one of the sigmoid functions. The sigmoid functions are usable for modeling saturation. Very high correlations were achieved between the original time series and the regression functions. The facts that (i) the smallest correlation value is greater than 0.94 and (ii) the huge majority of the correlations are greater than 0.99, show the very strong relation between this saturation process and the *arcus tangent* function.

6.1 Recommendations and Future Scope

The IP of vaccines will be an important issue in case of future pandemics. The present results show that the demand in time can be approximated by *arcus tangent* or other sigmoid functions. It also means that the demand has no uniform distribution in time. A more efficient IP can be elaborated by taking into account this fact. Different policies of countries can be harmonized as well. Every country taking part in cooperation can save money in this way.

The detailed numerical values provided by this research make possible the reconstruction of the results.

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APPENDICES

Appendix A: Informations About Waves of Countries

Table 1: Information of First Wave

Countries	Interval of Days	Parameters				Correlation	Total QE
		a	b	c	z		
Italy	1-369	0.0399393	299.0297128	0.510847062	1.036384751	0.995090087	0.314065117
Germany	1-395	0.028108879	324.6638369	0.485279663	1.177754882	0.997809392	0.157732
Ukraine	1-347	0.026038696	271.4386819	0.449290995	1.236210102	0.999960308	0.002978776
India	1-356	0.025086808	232.9758128	0.421444879	1.160569938	0.998784309	0.091676405
Iran	1-370	0.011714206	297.9347768	0.425948227	1.524782604	0.997234252	0.182215896
Nigeria	1-215	0.027188821	131.751204	0.387298001	1.303718306	0.999324942	0.03979559
Turkey	1-330	0.120881817	274.7748097	0.559762764	0.959780205	0.993950664	0.3717285
Hungary	1-309	0.051294454	265.3707866	0.466845365	1.179495678	0.99881561	0.062800938
UK	1-314	0.027863373	289.6571752	0.492463676	1.503597127	0.988091455	0.467896744
USA	1-249	0.014888568	190.9523485	0.374468017	1.654689764	0.998284537	0.089278938
Mexico	1-302	0.007742663	228.2368523	0.30836431	1.954177299	0.996043705	0.219296828
Portugal	1-301	0.035444216	266.0414721	0.504568631	1.24424146	0.996413439	0.141449023
Malta	1-294	0.027262632	249.0253818	0.460342233	1.236568932	0.999225426	0.03296247
Canada	1-411	0.018483946	338.7084742	0.469082834	1.311742444	0.99732956	0.197371658
Ireland	1-175	0.086176169	47.30085239	0.400022658	1.081012668	0.999353325	0.029070568
Denmark	1-137	0.03284315	44.58856872	0.313902968	1.343429575	0.999193823	0.028232543
Finland	1-178	0.054138201	77.04807994	0.391030516	1.187787172	0.998824123	0.069828854
Czechia	1-273	0.077300091	241.1172012	0.484000585	1.065175569	0.999560958	0.01546086
Cyprus	1-322	0.040377808	290.2975923	0.475936354	1.272060755	0.99755272	0.099708034
Peru	1-86	0.058784348	74.3693839	0.406478727	1.56773365	0.997528811	0.034622007
France	1-116	0.125747154	79.30944227	0.458415282	1.124574501	0.994139994	0.215153981

Table 2: Information of Second Wave

Countries	Interval of Days	Parameters				Correlation	Total QE
		a	b	c	z		
Italy	370-522	0.029873	44.49353	0.299221	1.451517	0.999599	0.013889
Germany	396-541	0.041564614	47.83210718	0.37048044	1.279579534	0.998853881	0.04083725
Ukraine	348-460	0.03622556	49.38534706	0.337583185	1.419135387	0.999910153	0.002480906
India	357-517	0.059931249	105.6195647	0.441030298	1.167521944	0.999679735	0.014962566
Iran	371-537	0.033926856	59.94854182	0.364877545	1.18549784	0.999508111	0.009561859
Nigeria	216-469	0.04395854	111.3476632	0.442192885	1.122484155	0.999769115	0.018963113
Turkey	331-478	0.055592082	68.39035034	0.430135028	1.151207188	0.99988619	0.004878734
Hungary	310-537	0.058385932	74.10545379	0.452369804	1.110115403	0.999478191	0.035119717
UK	315-486	0.046432933	27.32290207	0.275252149	1.35914852	0.999887822	0.003389804
US	250-460	0.02162948	87.69734646	0.336743551	1.348099693	0.999713423	0.014621468
Mexico	303-466	0.018885206	6.961406667	0.019680986	2.356860644	0.999033076	0.026295768
Portugal	302-458	0.082694083	24.1900757	0.367062437	1.152378756	0.99918912	0.020139521
Malta	295-487	0.038094723	57.54778337	0.422073126	1.160352199	0.995629404	0.183073304
Canada	412-544	0.0424709	41.85957629	0.342385972	1.334181988	0.999568998	0.013932814
Ireland	176-265	0.061061743	58.42785266	0.418138727	1.273639337	0.999760597	0.005101915
Denmark	138-342	0.03284315	157.3838469	0.435534191	1.356460111	0.999215894	0.036807609
Finland	179-487	0.010817424	214.5862235	0.354456981	1.676291404	0.998164879	0.131268666
Czechia	274-425	0.013479411	73.1940429	0.243293916	2.055934373	0.998137826	0.061796071
Cyprus	323-452	0.034357403	80.73200302	0.397417171	1.405542794	0.998673495	0.042976569
Peru	87-306	0.019110211	81.98664031	0.33947806	1.359265504	0.999593334	0.019294996
France	117-336	0.049358329	165.8581077	0.454687564	1.153458771	0.9995682	0.022251007

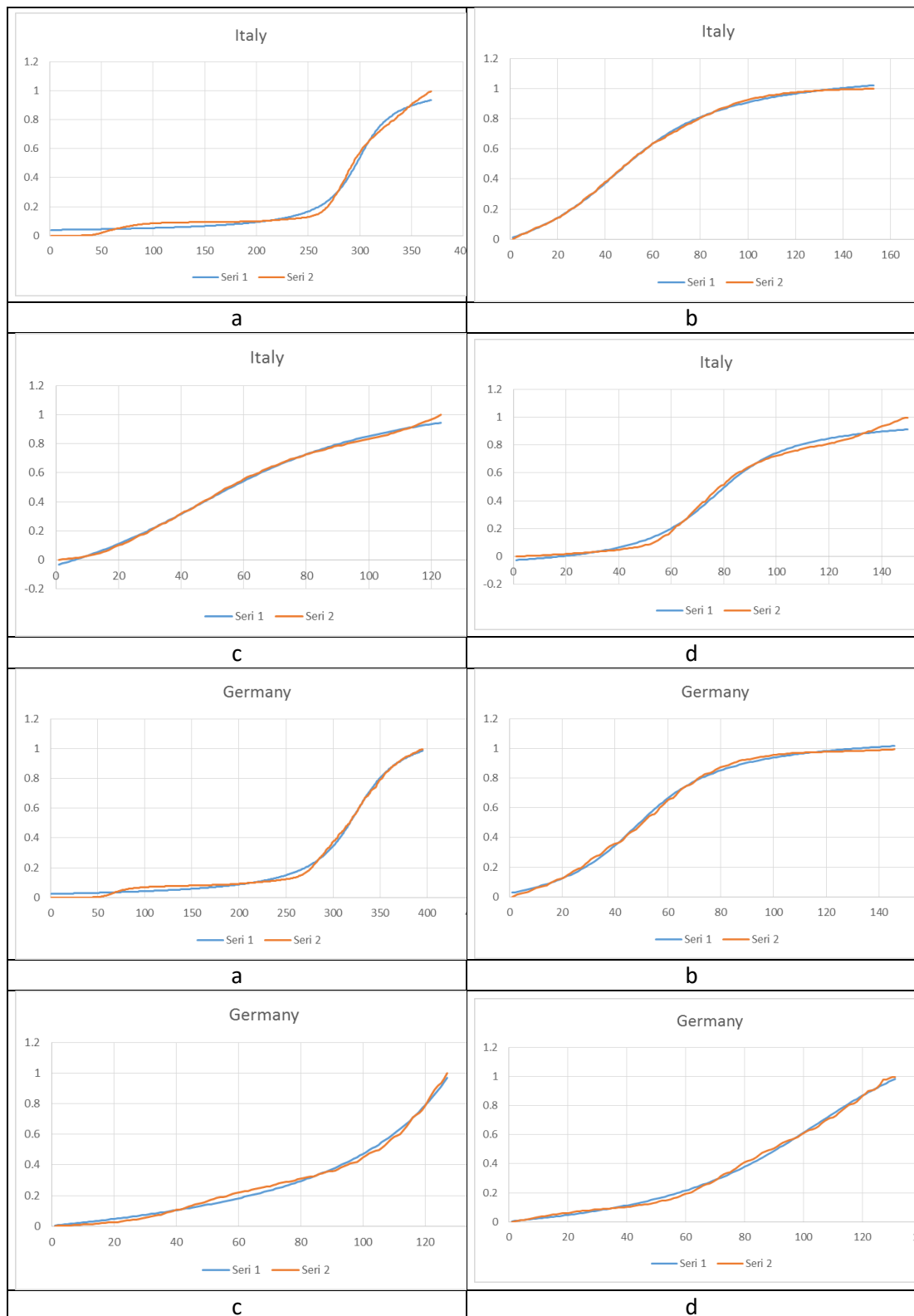
Table 3: Information of Third Wave

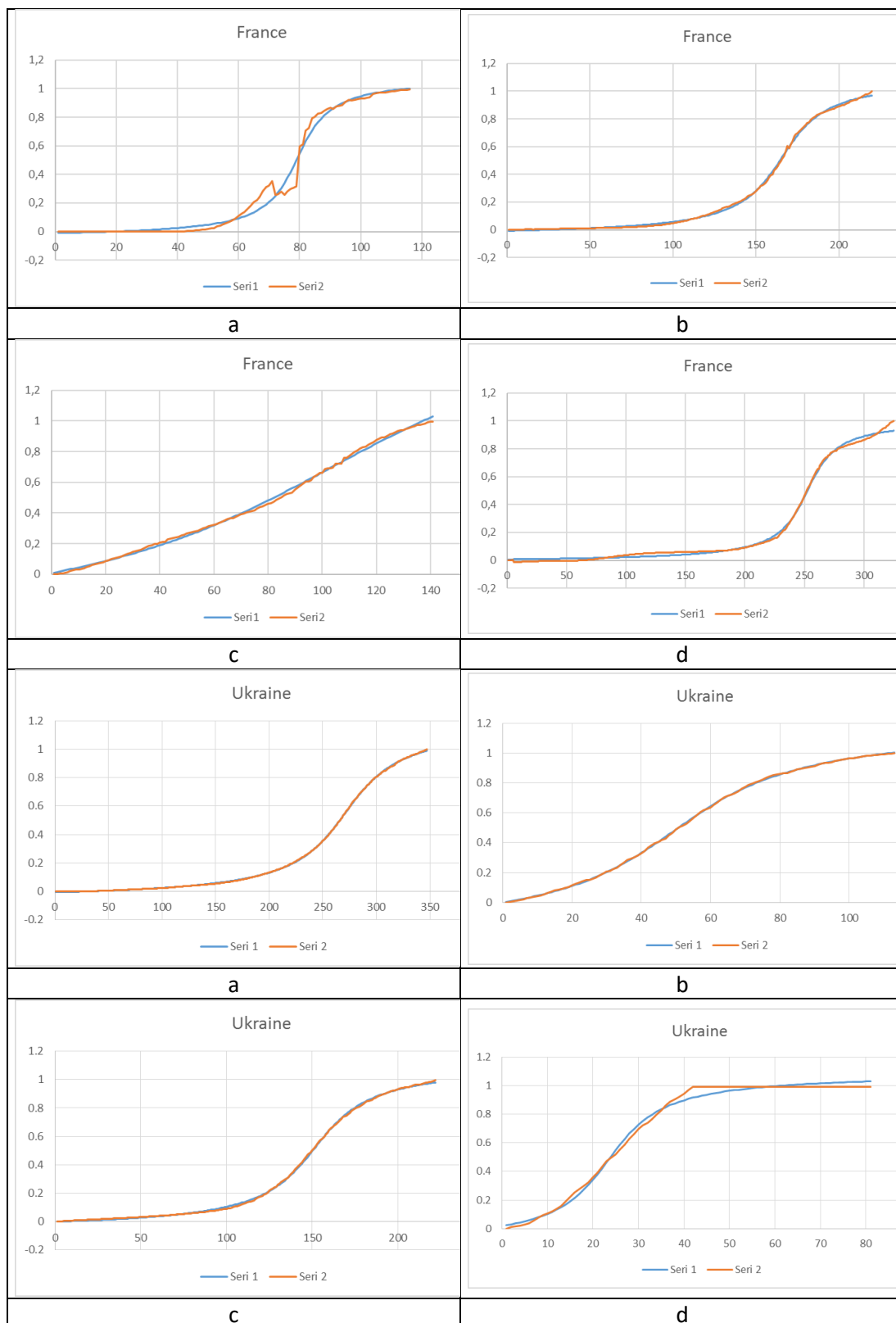
Countries	Interval of Days	Parameters				Correlation	Total QE
		a	b	c	z		
Italy	523-645	0.021168987	46.09819674	0.224588679	1.720525679	0.999030707	0.023809436
Germany	542-668	0.114300713	169.40817	0.483952196	19.99342045	0.99619012	0.061028481
Ukraine	461-682	0.040787822	151.7225558	0.448507616	1.163370024	0.999814801	0.010781896
India	518-693	0.017044397	39.301143	0.196454457	1.762356646	0.999768399	0.007491846
Iran	487-691	0.02610258	57.43102366	0.302431701	1.390459439	0.999820392	0.008537852
Nigeria	470-626	0.035663376	77.06045839	0.361858262	1.283274141	0.999178752	0.035360215
Turkey	479-652	0.011552284	103.8351658	0.26729139	2.058769629	0.999802825	0.007060285
Hungary	538-672	0.06469078	94.74120075	0.436098846	1.183730875	0.999475774	0.018070678
UK	487-678	0.005546235	171.6681058	0.231503899	3.700891087	0.999330127	0.023681428
US	461-693	0.022115738	142.6741346	0.40941592	1.203736582	0.997274299	0.133911309
Mexico	467-656	0.028558175	69.71123414	0.339537282	1.309754886	0.999810936	0.009013093
Portugal	459-600	0.027490151	54.81470305	0.296614118	1.435501399	0.999576302	0.013634678
Malta	488-610	0.032550027	1.607298964	0	2.307597944	0.998778421	0.021452467
Canada	545-682	0.020863129	68.19137014	0.283240884	1.580015382	0.998844728	0.032993085
Ireland	266-355	0.120396448	51.54519845	0.444698741	1.034862304	0.998819457	0.028726482
Denmark	343-493	0.014344333	88.5530853	0.294135588	1.951827553	0.998431119	0.049336528
Finland	488-640	0.019811297	93.38652314	0.320976535	1.579060348	0.997449036	0.081640729
Czechia	426-680	0.069402681	212.8133222	0.499165892	1.067579521	0.99902748	0.043049569
Cyprus	453-610	0.041554934	50.49039968	0.319314104	1.265746124	0.998105664	0.074695372
Peru	307-633	0.016082191	79.30098656	0.291703371	1.377465364	0.999756547	0.015936656
France	337-477	0,012741054	103,1257891	0,296447648	2,343306459	0,998957138	0,028731521

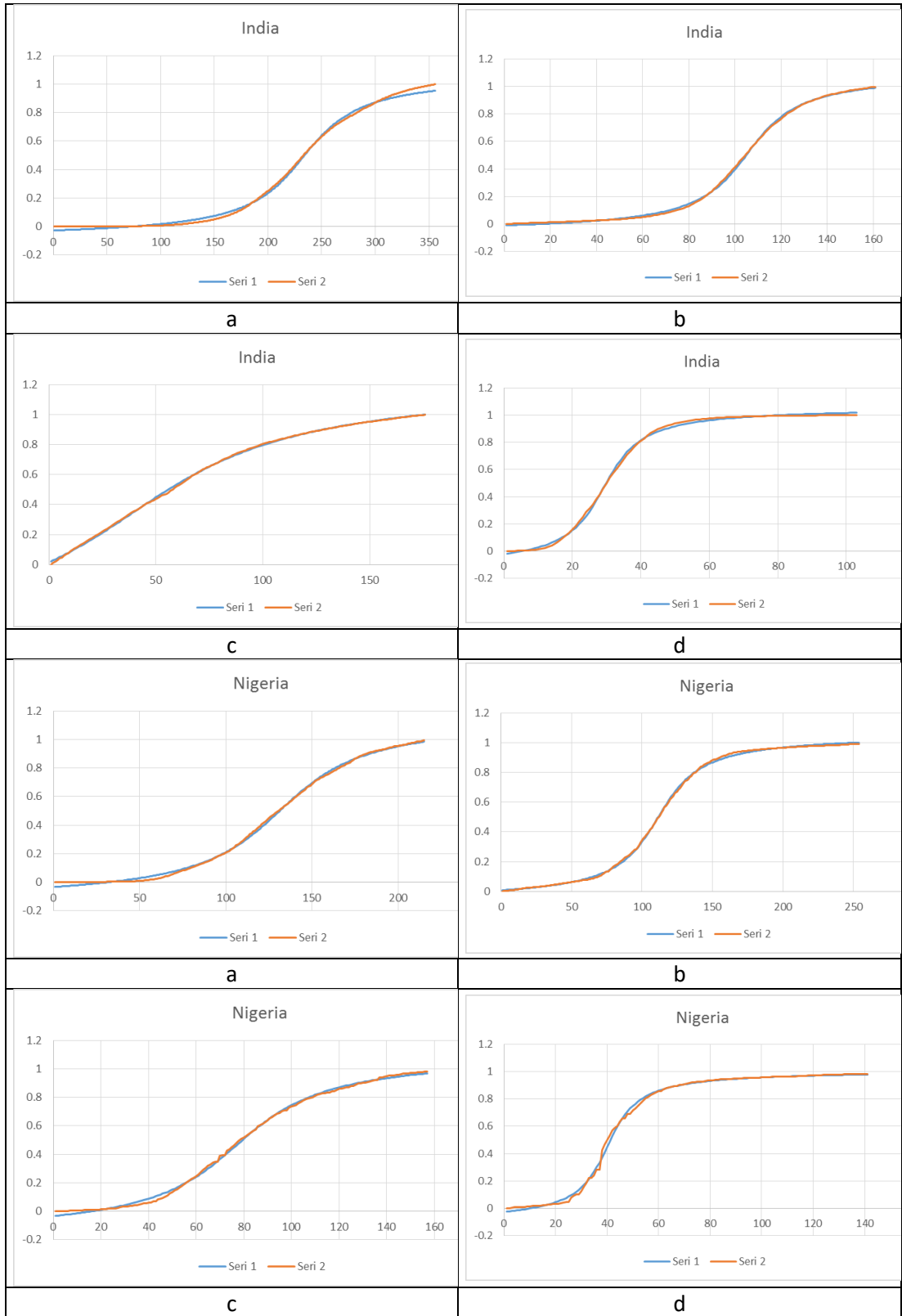
Table 4: Information of Fourth Wave

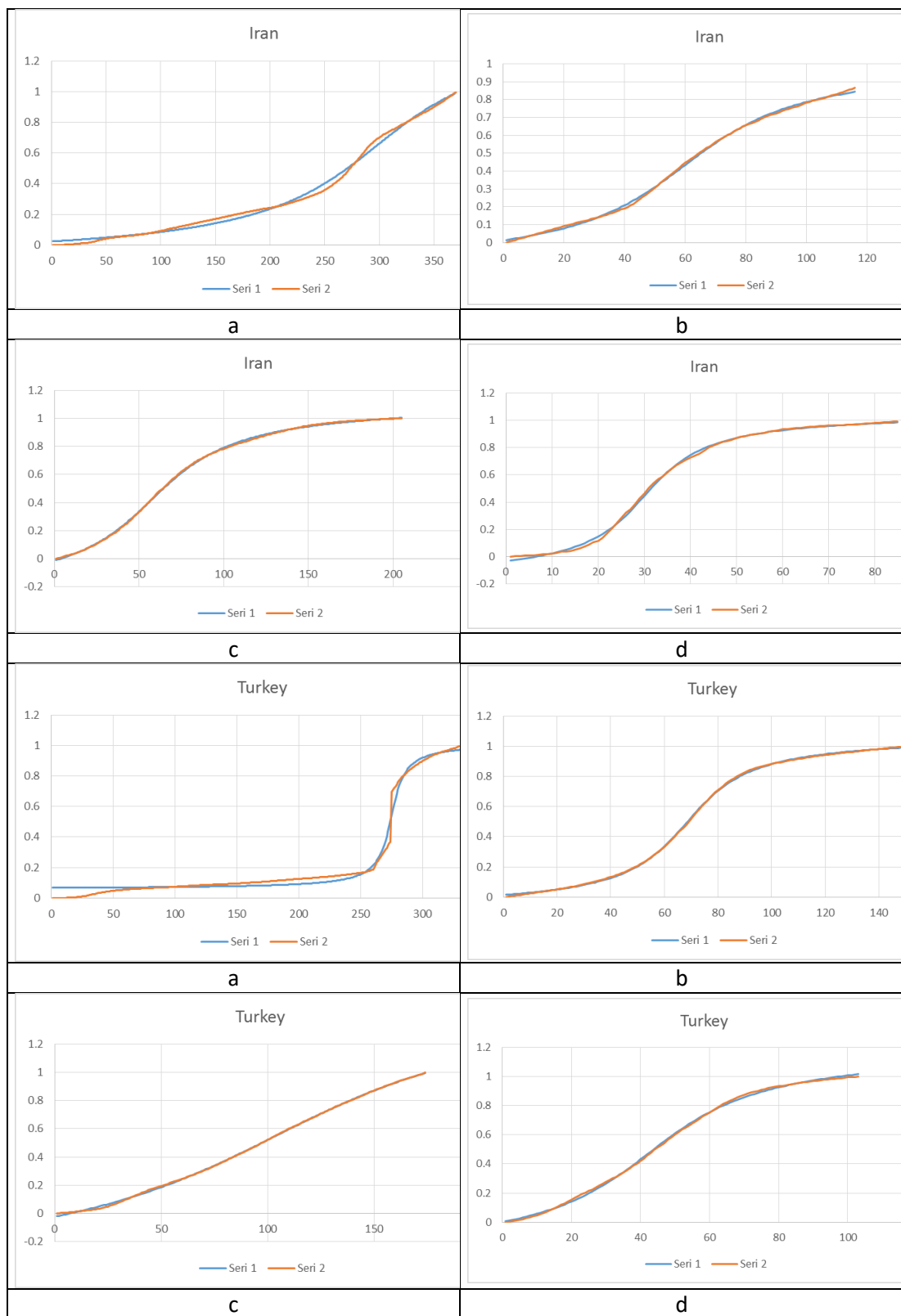
Countries	Interval of Days	Parameters				Correlation	Total QE
		a	b	c	z		
Italy	646-795	0.046714159	76.99439548	0.38899205	1.141534993	0.996837049	0.122925661
Germany	669-799	0.02373825	103.5037937	0.378444516	1.747148525	0.998376984	0.039121297
Ukraine	683-763	0.11151042	23.44943609	0.398365792	1.212285164	0.996700388	0.069829605
India	694-796	0.121997871	29.15244197	0.394988521	1.183864643	0.999306003	0.020914866
Iran	692-776	0.096963103	30.12763742	0.368990791	1.215410212	0.999413287	0.014300897
Nigeria	627-767	0.112150955	40.13111856	0.407287278	1.112218267	0.999056709	0.038409064
Turkey	653-755	0.039850097	42.5137091	0.330757877	1.439359367	0.999529173	0.011976379
Hungary	673-762	0.065480297	25.14023224	0.316946469	1.315259691	0.999522385	0.009534201
UK	679-795	0.0177598	23.40439752	0.103936921	2.161867538	0.997049688	0.060201691
US	694-804	0.076053894	29.64794583	0.359597252	1.191169262	0.999692395	0.007725044
Mexico	657-767	0.05904582	42.37428789	0.348979932	1.290757069	0.999144446	0.02745425
Portugal	601-764	0.062568214	94.56626216	0.43331084	1.109908009	0.999412417	0.028953077
Malta	611-759	0.056802032	59.70728155	0.38100573	0.995362011	0.99256244	0.235201257
Canada	683-803	0.048727416	34.24460176	0.293310603	1.324646161	0.998390844	0.04454475
Ireland	356-766	0.02429498	322.8887702	0.477984297	1.143602661	0.997906291	0.16673662
Denmark	494-768	0.048640436	213.1215652	0.467240952	1.164227067	0.999797411	0.013617556
Finland	641-797	0.017928283	110.3474489	0.329264688	1.733907563	0.996398129	0.11393948
Czechia	681-765	0.051007636	21.13724254	0.235230233	1.50620192	0.998435249	0.027239456
Cyprus	611-758	0.012378142	118.9330662	0.288557127	2.370387225	0.994338692	0.152746486
Peru	634-760	0.119519683	55.35901979	0.439664727	1.109198725	0.99954908	0.02070994
France	478-802	0,053706004	252,3368092	0,483032224	1,02904864	0,998574121	0,098941619

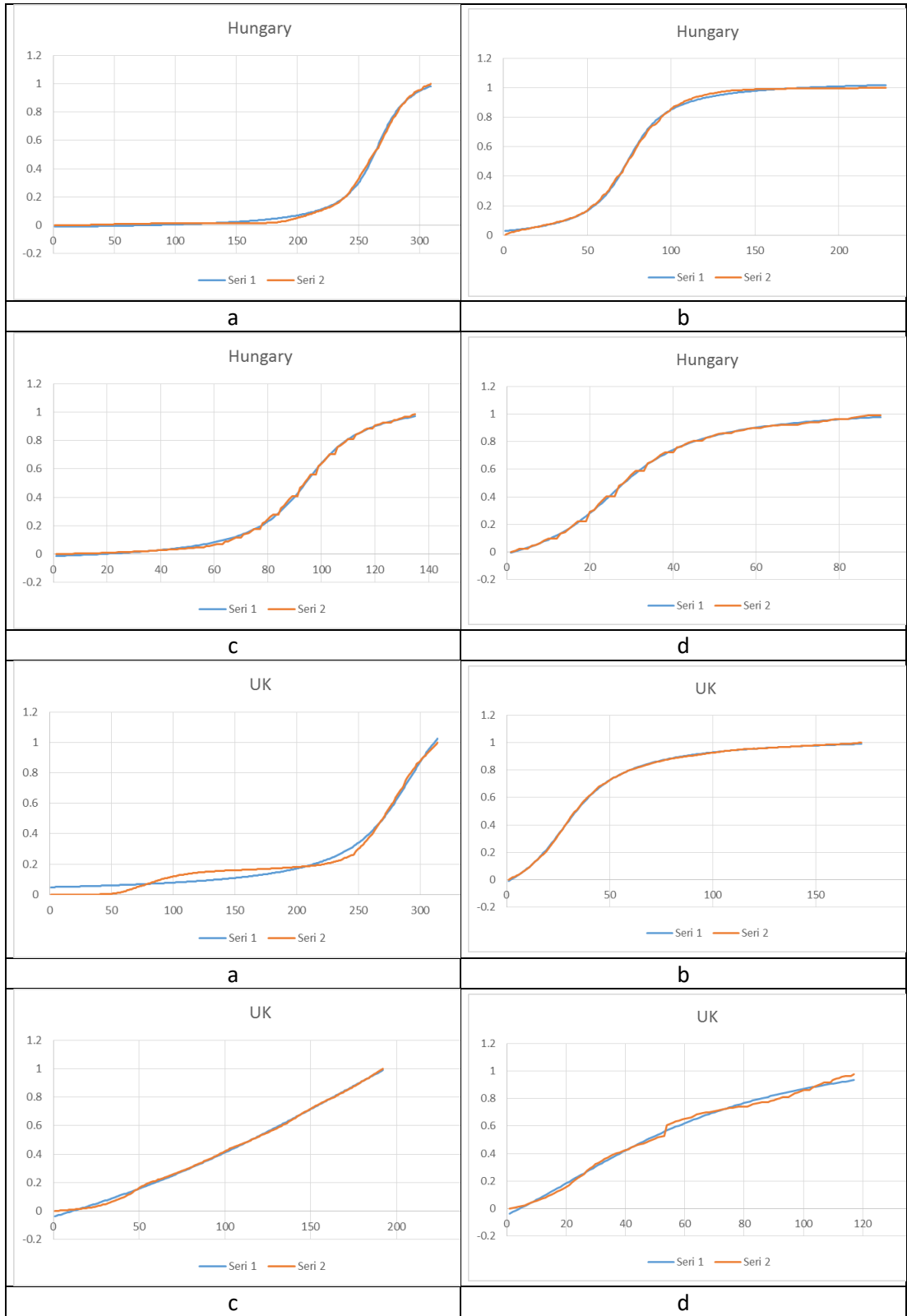
Appendix B: Curve Fitting of Countries for Waves

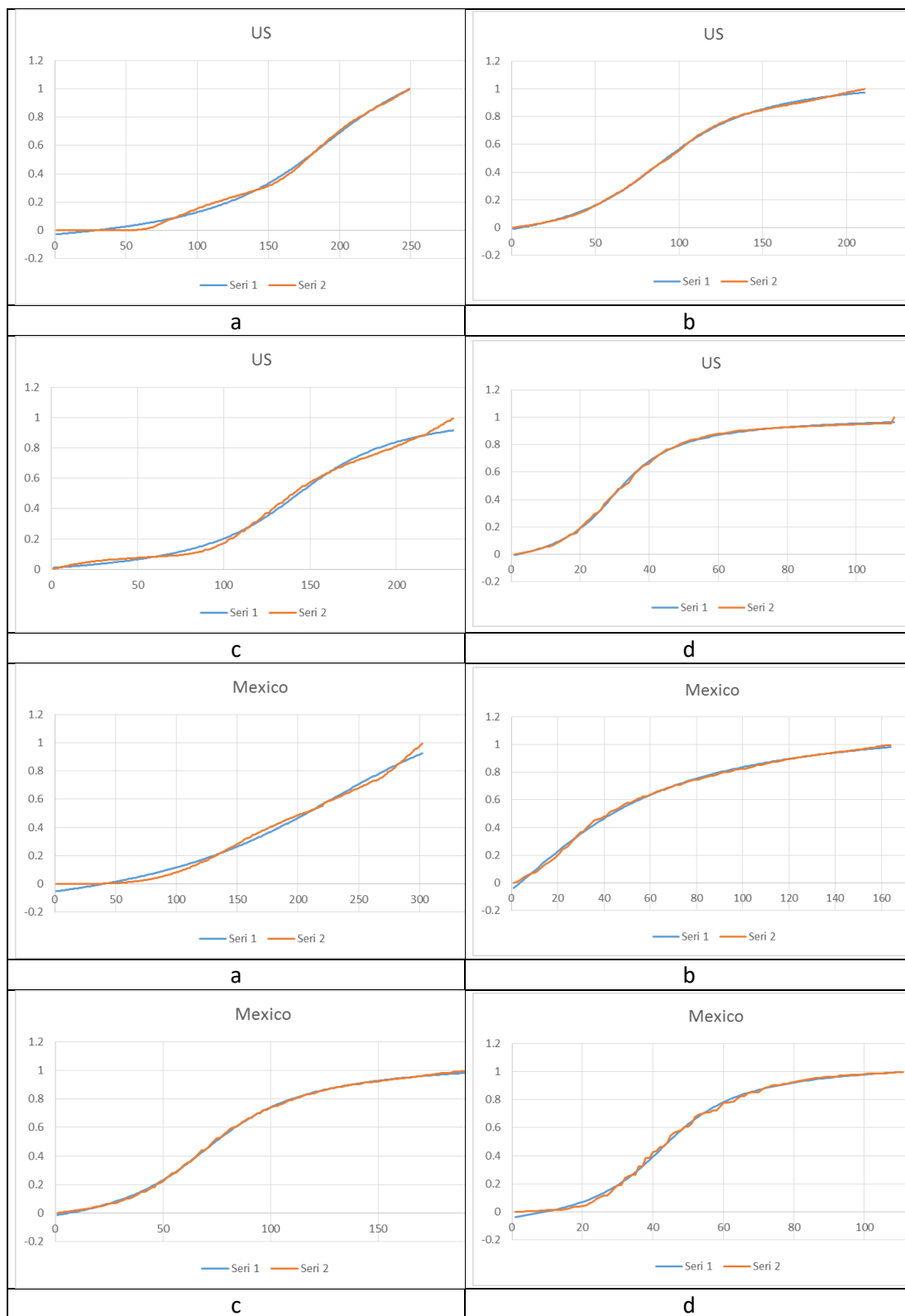


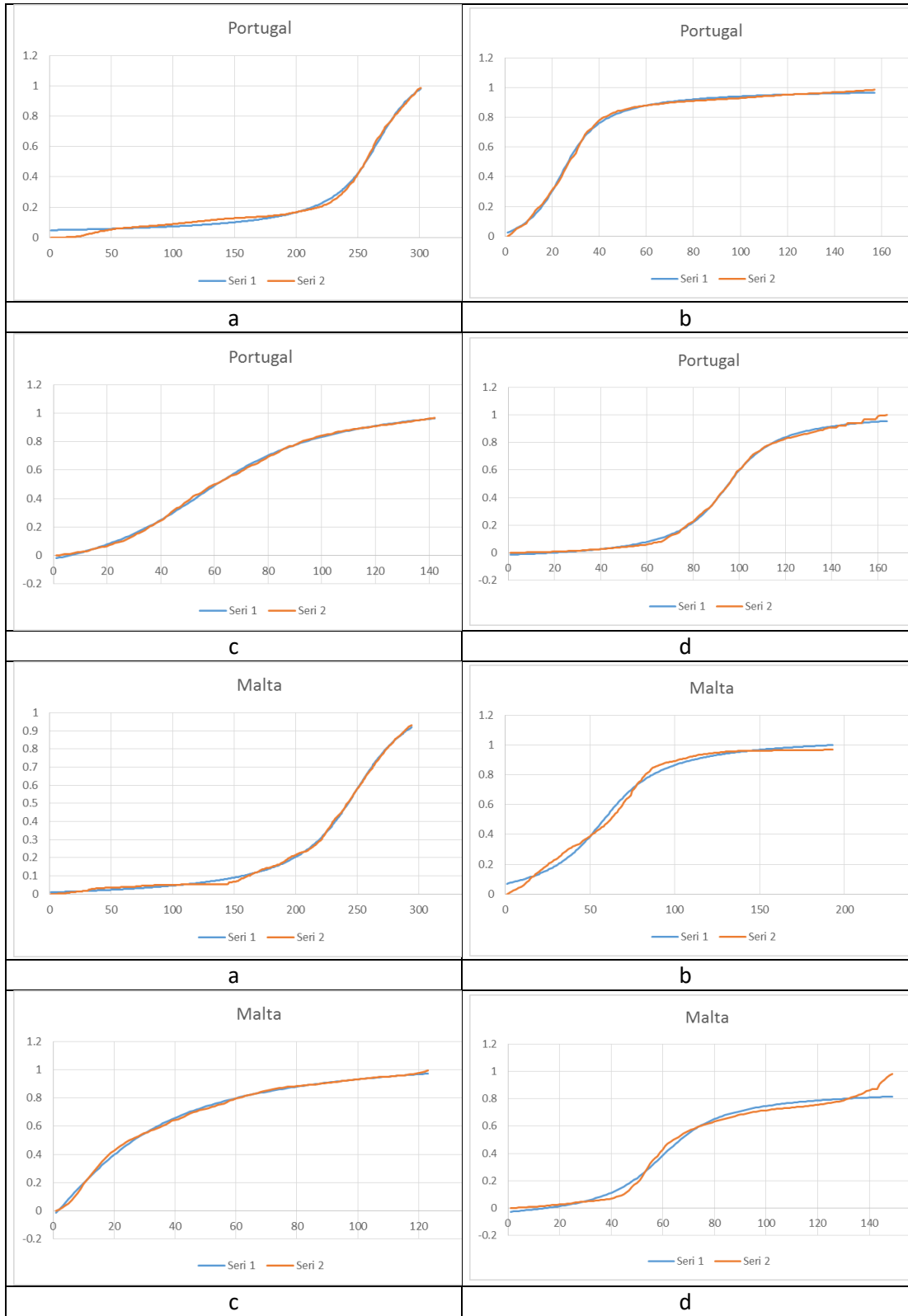


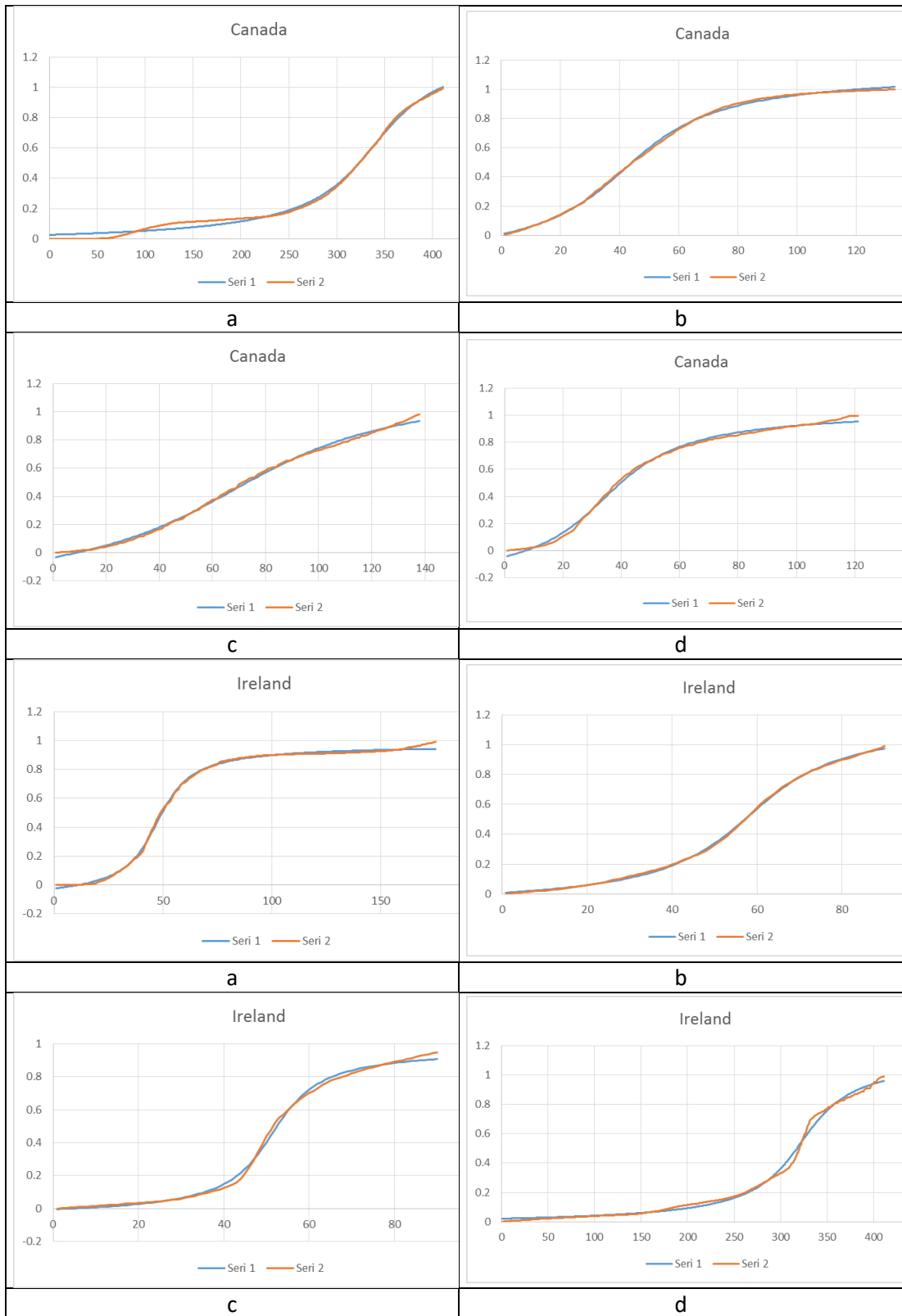


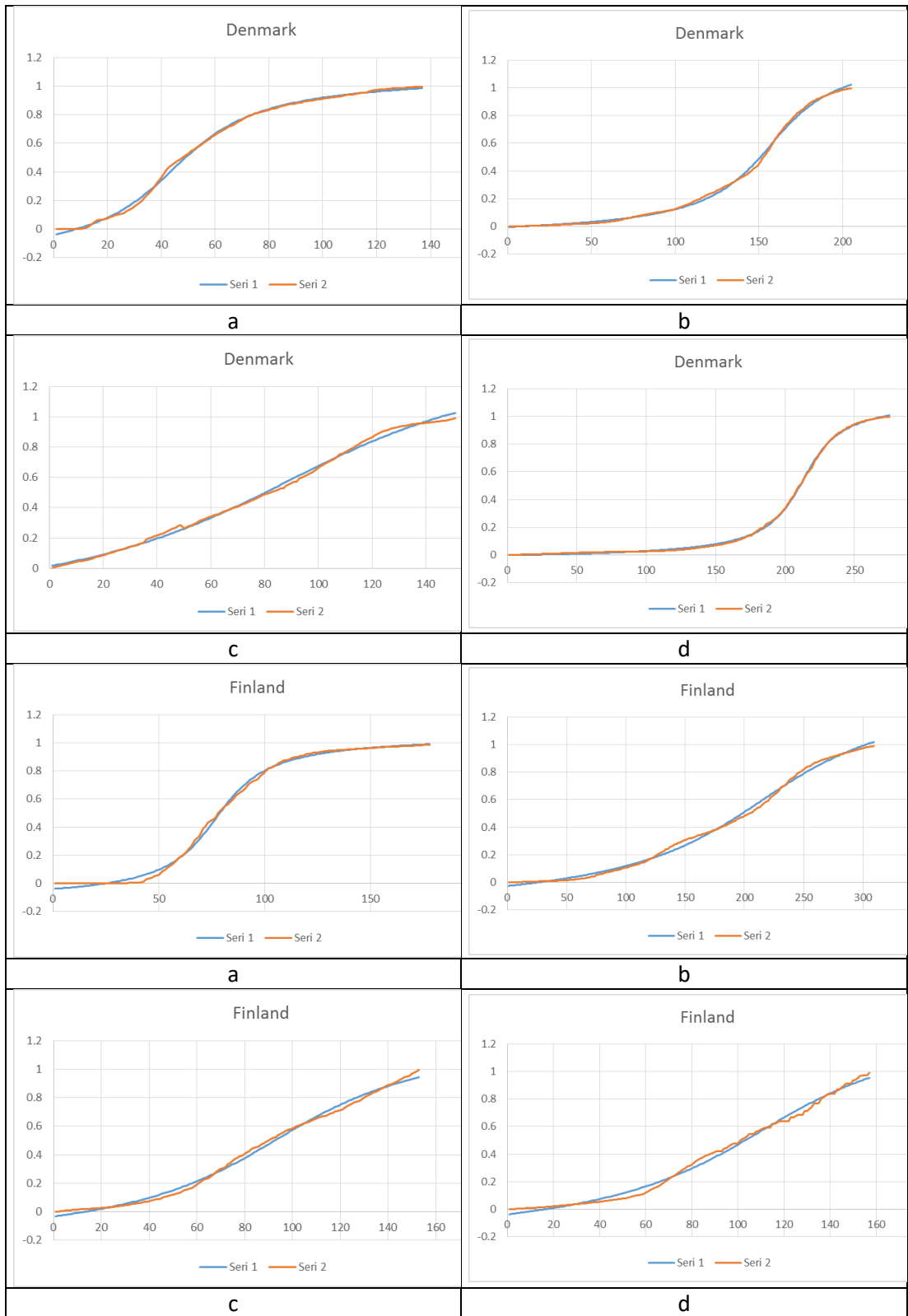


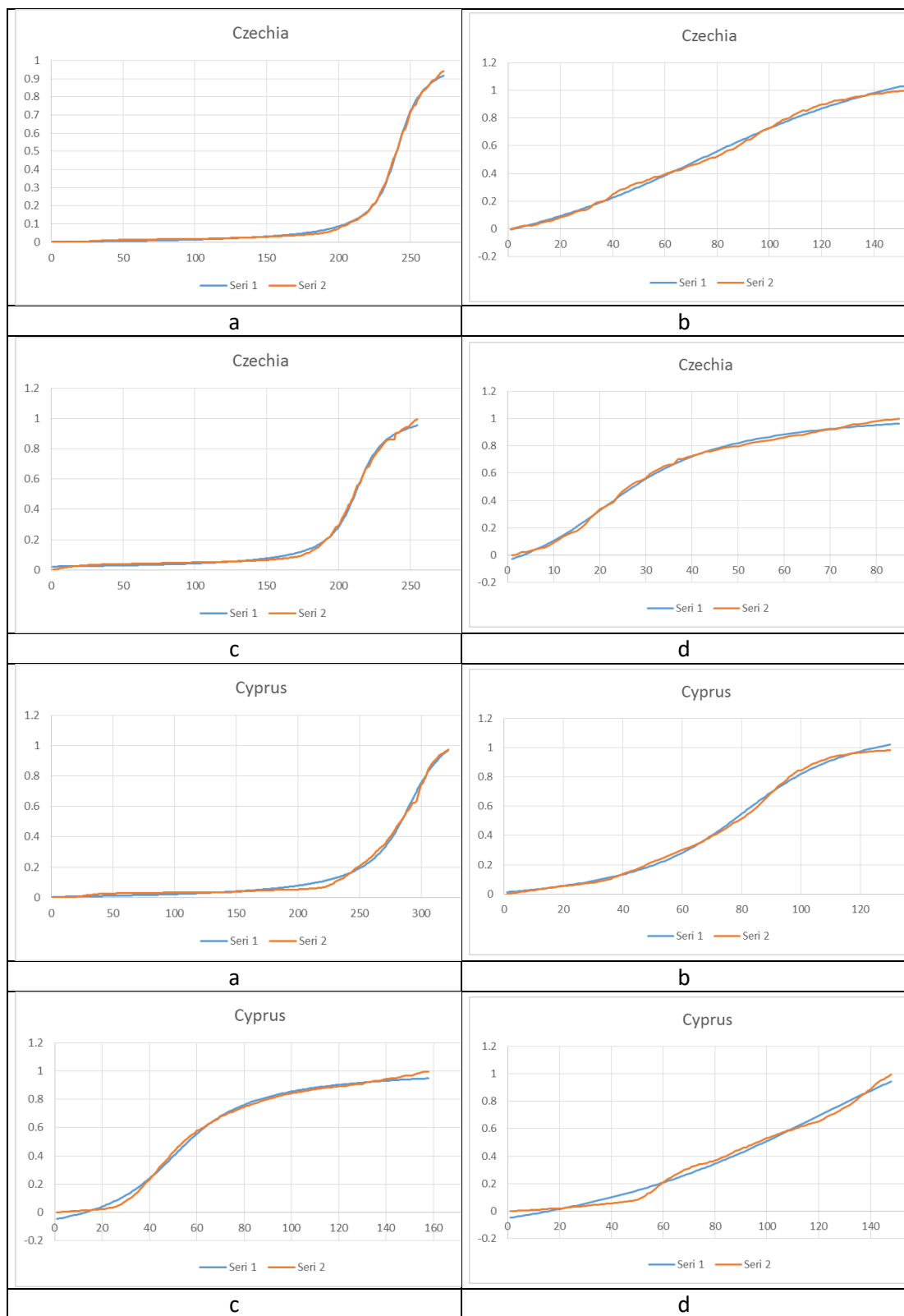












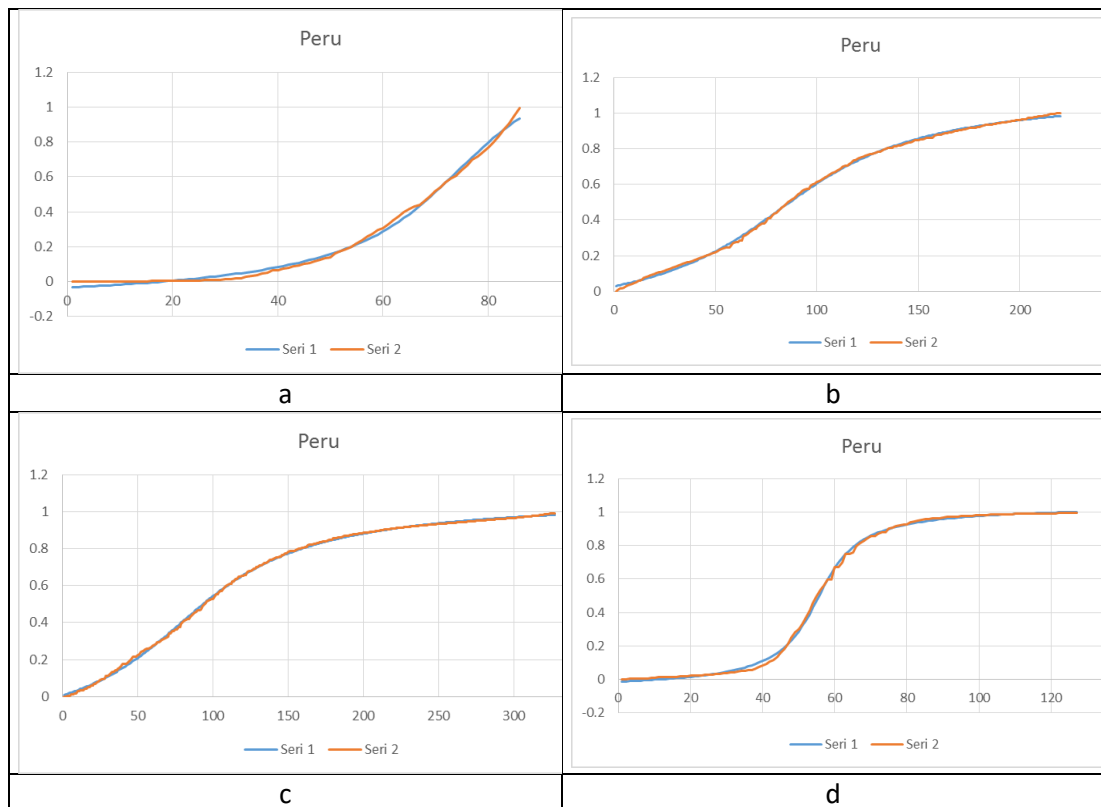


Figure 1: (a) First wave, (b) Second wave, (c) Third wave, (d) Fourth wave