

**Integrated Arable Land Optimization through
Portfolio Optimization for Sustainable Agriculture:
A Case Study in Uzbekistan**

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ABSTRACT

In this research, our objective is to enhance the efficiency of agricultural land allocation using portfolio optimization to promote a sustainable agriculture system. We will conduct a case study in Uzbekistan, a country with a semi-arid climate containing limited cultivable land, accounting for less than 10%. Additionally, the region faces constraints in water resources, relying mainly on the Amu Darya and Syr Darya rivers for agricultural water. Given the significant role of agriculture in the country's economy, optimizing land allocation becomes crucial for conserving water, increasing yield, and enhancing productivity. The study focuses on seven crops, namely barley, maize, potato, rice, tomato, watermelon, and wheat, and employs a linear programming model utilizing Markowitz portfolio optimization for land allocation. This financial technique, which integrates parameters such as risk and return, is adapted to optimize land allocation based on crop production values. The approach involves defining initial land based on crop weights, followed by a second model where the initial weight of each crop is optimized based on the outcomes of the initial land model. The optimization in this model is oriented towards maximizing the production value of the crops. In the following, a sensitivity analysis is conducted to the obtained outcomes from the model, by varying the risk parameter (λ) to assess its impact on land allocation and optimization. Finally, a model estimating the water consumption of each crop according to their respective land distribution is presented.

Keywords: Agriculture, Crop Production, Land-use, Portfolio Optimization

ÖZ

Bu arařtırmada amacımız, sürdürülebilir bir tarım sistemini teşvik etmek için portföy optimizasyonunu kullanarak tarımsal arazi tahsisinin verimliliğini artırmaktır. Yarı kurak iklime sahip, ekilebilir arazilerin sınırlı olduđu ve %10'dan az bir alana sahip olan Özbekistan'da bir örnek çalışma yapacağız. Ayrıca bölge, tarımsal su için esas olarak Amu Darya ve Syr Darya nehirlerine bağımlı olduğundan su kaynaklarında kısıtlamalarla karşı karşıyadır. Tarımın ülke ekonomisindeki önemli rolü göz önüne alındığında, arazi tahsisinin optimize edilmesi suyun korunması, verimin artırılması ve üretkenliğin artırılması açısından hayati önem taşıyor. Çalışma, arpa, mısır, patates, pirinç, domates, karpuz ve buğday olmak üzere yedi ürüne odaklanmakta ve arazi tahsisi için Markowitz portföy optimizasyonunu kullanan doğrusal bir programlama modeli kullanmaktadır. Risk ve getiri gibi parametreleri birleřtiren bu finansal teknik, bitkisel üretim değerlerine göre arazi tahsisini optimize edecek şekilde uyarlanıyor. Yaklaşım, ilk arazinin ürün ağırlıklarına göre tanımlanmasını ve ardından her mahsulün başlangıç ağırlığının, ilk arazi modelinin sonuçlarına göre optimize edildiđi ikinci bir modeli içerir. Bu modeldeki optimizasyon, mahsullerin üretim değerini maksimuma çıkarmaya yöneliktir. Aşağıda, arazi tahsisi ve optimizasyon üzerindeki etkisini değerlendirmek için risk parametresi (λ) deđiřtirilerek modelden elde edilen sonuçlara yönelik bir duyarlılık analizi yapılmıřtır. Son olarak, her bir ürünün su tüketimini ilgili arazi dađılımına göre tahmin eden bir model sunulmaktadır.

Anahtar Kelimeler: Tarım, Bitkisel Üretim, Arazi Kullanımı, Portföy Optimizasyon

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TABLE OF CONTENTS

ABSTRACT	iii
ÖZ	iv
ACKNOWLEDGEMENT	v
LIST OF TABLES	ix
LIST OF FIGURES	x
1 INTRODUCTION	1
1.1 Thesis Overview.....	1
1.2 The Aim of the Study	1
1.3 Introduction to Agriculture	2
1.3.1 Land Management	2
1.3.2 Water Management.....	3
1.4 Optimization.....	4
1.4.1 Portfolio Optimization	4
1.5 The Importance of the Study	7
2 LITERATURE REVIEW.....	9
2.1 Portfolio Optimization	9
2.2 Land Management.....	12
2.3 Irrigated Agriculture.....	15
2.4 Gap Research and Contributions of the Study	16
3 UZBEKISTAN.....	18
3.1 General Information	18
3.2 Uzbekistan Divisions	19
3.3 Weather Conditions.....	20

3.4 Crop Production	21
4 MARKOWITZ'S PORTFOLIO SELECTION MODEL	22
4.1 Optimization Problem Formulation	23
4.2 Efficient Frontier	23
4.3 Relation of the Efficient Frontier to the DEA Method	24
4.4 Efficient Portfolios	24
4.5 Illustrative Example for Markowitz	25
4.5.1 Given Data	25
4.5.2 Formulation and Solution	26
5 PROBLEM STATEMENT	29
5.1 Problem Overview and the Study Region	29
5.2 Collection of Data	30
5.2.1 Barley	30
5.2.2 Maize	33
5.2.3 Potato	36
5.2.4 Rice	37
5.2.5 Tomato	40
5.2.6 Watermelon	41
5.2.7 Wheat	43
5.2.8 Arable Land Allocated to Each Crop	46
6 SOLUTION APPROACH	48
6.1 Markowitz Portfolio Application in Agriculture	48
6.2 The Mathematical Formulation for Land	49
6.2.1 Definitions	49
6.2.2 Mathematical Model	49

6.2.3 Numerical Calculation	50
6.2.4 Solution.....	50
6.3 The Mathematical Formulation for Production Value	52
6.3.1 The Mathematical Formulation	53
6.3.2 Numerical Calculations and Solution	55
6.4 Sensitivity Analysis.....	57
6.5 The Mathematical Model for Water Consumption	60
6.5.1 Definitions and Variables	60
6.5.2 Mathematical Formulation	60
7 CONCLUSION	63
REFERENCE.....	65
APPENDICES	72
Appendix A: MATLAB Results for DJI Allocation	73
Appendix B: MATLAB Results for the Initial Land Model.....	74
Appendix C: MATLAB Results for Land Use Based on Production Value.....	75
Appendix D: Price Estimation Results in MATLAB.....	76

LIST OF TABLES

Table 1: Land Categories in Uzbekistan	19
Table 2: The Average, Minimum, and Maximum Land-use	46
Table 3: The Lower and Upper Bounds Land-Use	47
Table 4: Optimal Allocated Land to Each Crop.....	52
Table 5: The Average Yield Acquired From FAO.	53
Table 6: The Average Estimated Prices for Each Crop	55
Table 7: The Average Price, Yield and Optimized Land for Each Crop	55
Table 8: Allocated Area Based on Production Value	57
Table 9: Allocated Area to Each Crop in Different Lambda Values	58
Table 10: The Optimal Values Generated with Different Lambda Values.....	59

LIST OF FIGURES

Figure 1: Administrative Divisions in Uzbekistan.....	20
Figure 2: The Standard Efficient Frontier	24
Figure 3: The DJI Code in MATLAB.....	27
Figure 4: The Efficient Frontier of DJI Stocks Apple & Microsoft.....	28
Figure 5: Changes in Harvested Area and Production of Barley	31
Figure 6: Distribution of Barley Cultivation [41]	32
Figure 7: Yield And 5-year Moving Average of Barley	33
Figure 8: Changes in Harvested Area, and Production of Maize.....	34
Figure 9: Distribution of Maize Cultivation [41].....	35
Figure 10: Yield and 5-year Moving Average Yield of Maize	35
Figure 11: Changes in Harvested Area and Production of Potato	36
Figure 12: Yield and 5-year Moving Average Yield of Potato.....	37
Figure 13: Changes in Harvested Area and Production of Rice	38
Figure 14: Distribution of Rice Cultivation [41].....	39
Figure 15: Yield and 5-year Moving Average Yield of Rice.....	39
Figure 16: Changes in Harvested Area and Production of Tomato	40
Figure 17: Yield and 5-year Moving Average Yield of Tomatoes.....	41
Figure 18: Changes in Harvested Area and Production of Watermelon	42
Figure 19: Yield and 5-year Moving Average Yield of Watermelon	43
Figure 20: Changes in Harvested Area and Production of Wheat	44
Figure 21: Distribution of Wheat Cultivation [41]	45
Figure 22: Yield and 5-year Moving Average of Yield.....	46
Figure 23: Initial Arable Land Code in MATLAB	51

Figure 24: Interpolation of Prices MATLAB Code 54

Figure 25: MATLAB Code of Optimized Arable Land Based on Production Value 56

Chapter 1

INTRODUCTION

1.1 Thesis Overview

This study explores the optimal utilization of arable land in Uzbekistan through applying Portfolio Optimization. The study aims to formulate a mathematical approach to enhance the efficiency of arable land in Uzbekistan, by integrating insights from various disciplines including agricultural studies, mathematical optimization, and mathematical economics. Hence, the research converges these diverse fields through an interdisciplinary perspective allowing for an exploration of agricultural practices, mathematical modelling, and economic principles to derive practical and sustainable solutions for maximizing the potential of available land resources.

1.2 The Aim of the Study

Given Uzbekistan's status as a semi-arid geography with severely restricted arable land and water supplies, the goal of this research is to implement optimization strategies to establish an efficient solution for allocating cultivation land to produce crops. The objective is to meet the demand for agricultural productivity while simultaneously utilizing cultivation land resources efficiently and conserving water resources. The key challenges in this scenario revolve around the significant dependent of agriculture in Uzbekistan on irrigation. We observed that around 80 percent of the Uzbekistan is contained of deserts given its geographical area. Additionally, salinization is an extra issue causing reduction in arable lands causing restricted agricultural land subsequently limited agricultural productivity. Furthermore, water scarcity is

considered a critical issue in Uzbekistan and there is a heavy reliance on the Amu Darya and Syr Darya rivers for irrigation, and the utilization of outdated irrigation systems resulting in water loss.

Another notable issue is the limitation of available data. Due to the lack of thorough information on water management from open access sources impeded our progress.

1.3 Introduction to Agriculture

Agriculture is the practice of utilizing arable land to cultivate crops to produce food. The analysis modern agriculture reveals that it has evolved over time since it has been influenced by technological advancements, environmental concerns, and societal needs. Over the past few years, the increase in population has led to the expansion of urban areas, contributing to a reduction in the amount of arable land [1] Additionally, it is noteworthy to mention that agriculture is the primary consumer of water resources. According to the World Bank report [2], it typically consumes 70% of the world's freshwater on average. In modern agriculture, the main goal is to ensure a consistent food supply, considering two major constraints water and arable land resources while minimizing the negative impact on the environment. Therefore, the researchers have been actively exploring innovative solutions to increase productivity while minimizing its environmental footprint given the resource restrictions.

1.3.1 Land Management

Land management involves efficient approaches which optimizes agriculture resources, and it is crucial for assuring a viable long-term farming. The issue of land-use in agriculture is considered as one of the critical aspects of modern farming and land management. It mainly concentrates on how agricultural land is allocated

according to various policies and the need to balance food production with environmental sustainability.

In the process of allocating land to agriculture purposes, a challenge arises due to the steadily growing of world's population. According to research [3], it is estimated that the population will reach 9 to 11 billion by 2050. Hence, the demand for food is expected to rise in the future and in order to meet the demand, [4] estimated that agriculture land should be increased around 320 to 850 million hectares.

1.3.2 Water Management

Water management in agriculture is another critical issue due to the significant demand for water in crop production and livestock farming. Thus, one of the major concerns is the scarcity of water for agricultural purposes considering the population growth [5] and consequently the demand for water is estimated to rise[4]. Due to the unbalanced distribution of water resources, many regions around the globe specifically in Central Asia, already face water shortages, resulting in over-consuming their water resources [6]. The water stress could have a major impact on agricultural productivity and lead to conflicts over water allocation among different sectors.

Irrigation which is the artificial utilization of water is an essential method for agriculture, especially in arid and semi-arid regions. Irrigation water can be supplied from groundwater using specific equipment or from surface water that the main source would be through rivers or lakes. It is important to choose the right irrigation system since an inefficient irrigation method can lead to water contamination and wastage. Improving irrigation efficiency through modern techniques like drip irrigation or precision watering systems is crucial to conserve water.

1.4 Optimization

In general, optimization is a mathematical discipline focused on finding the best feasible solution given that we have a problem with a set of constraints. Optimization techniques have been extensively applied in various fields to design efficient systems and to improve decision-making processes for managers and resource allocation. Constrained optimization originated with the work of George Dantzig, an American mathematical scientist (1914-2005). Dantzig pioneered the field by introducing the simplex algorithm, which he developed in 1947 and subsequently published his paper in 1949 [7].

Portfolio optimization is one of the techniques which has been widely used in finance and aims to maximize the returns at a given level of risk which will be discussed more in detail in the following section.

1.4.1 Portfolio Optimization

• Portfolio Theory

In the financial market, a portfolio consists of any asset that can be traded such as stocks, bonds, and cash. An investor decides how to allocate the money to maximize the expected return according to their risk tolerance which the whole concept represents an optimization problem [8]. Every optimal portfolio that demonstrates a balance between risk and return generates a series of Pareto optimal combinations, known as the Pareto efficient frontier in the Markowitz portfolio selection model. In other words, this frontier represents the set of portfolios where no other portfolio can offer higher returns without an increase in risk, or conversely, reduce risk without sacrificing potential returns [9].

The concept of Pareto optimality, which is often referred to as Pareto efficiency first was introduced by Vilfredo Pareto an Italian civil engineer and economist who lived from 1848 to 1923. He applied this principle in his research on economic efficiency and income distribution. In general, Pareto optimality characterizes a scenario in which no action or allocation would enhance one situation without diminishing another [10].

•**Brief Biographical of Harry M. Markowitz**

Harry Max Markowitz was an American economist who lived from 1927 to 2023 and introduced the Portfolio Theory, also known as Modern Portfolio Theory (MPT), which offers a mathematical model for allocating money among various assets.

While he was studying a Ph.D. program in Economics at the University of Chicago, his primary consideration for his dissertation centered on the application of mathematical methods to interpret the stock market. He conceived the foundation of the MPT approach after encountering John Burr Williams's Theory of Investment Value [11], [12].

John Burr Williams was an American economist (1900-1989) who graduated from Harvard University. The theory of investment value which came from his Ph.D. thesis proposing that a stock's value should equal the present value of its future dividends, considering the uncertainty of future dividends. He introduced the concept of valuation through Discounted Cash Flow (DCF) with an emphasis on dividend-based valuation [13].

Markowitz interpreted the primary concept of John Burr Williams's book, suggesting that the value of a stock should equal the present value of its forthcoming dividends, as future expected dividends considering the uncertainty of future dividends. In this

scenario, if an investor aims to maximize the anticipated value of a portfolio, they could achieve this by investing solely in a single security. Nevertheless, investors typically opt for diversification as they are mindful of both risk and return, choosing to spread their investments across various assets [12]. He developed the model based on Pareto optimality delineating a set of all portfolios that offer the highest expected return for a given level of risk, given the need for tools to determine the optimal level of diversification, which is known as the efficient frontier, and it serves as an investment tool that plots the level of diversification required to achieve the highest return within an investor's acceptable risk level. Portfolios positioned on the "efficient frontier" of the graph are considered efficient because they offer the maximum return for the given risk tolerance. Portfolios outside this efficient region either carry too much risk for the expected return or provide too little return for the associated risk. This concept inspired Markowitz to write his influential article "Portfolio Selection" [14], ultimately leading to his Nobel Prize in Economics.

Harry Markowitz was awarded the Economic Sciences Prize for his groundbreaking contribution, which was initially presented in "Portfolio Selection" (1952) and it was subsequently elaborated in his book "Portfolio Selection: Efficient Diversification" (1959). Markowitz's theory was designed as a guide for investment managers, addressed the optimal allocation of money among assets with variable expected returns and risks. Over time, this theory has served as the basis for subsequent research in financial economics. According to data obtained from internet searches, Markowitz's original paper 'Portfolio Selection' has been cited in 19,016 articles on Google Scholar. [15]. Overall, Markowitz's contributions have significantly influenced academic research and the financial market.

To sum up, in this theory, we want to find the optimum way of investing a particular amount of money in a set of available assets, and each different way of diversifying this money between those assets is called a portfolio. In order to solve this portfolio selection problem, Markowitz presented the mean-variance model by computing the expected returns and the variance of returns known as the risk of assets, in which return and risk should be considered together[15]. The portfolios that offer the minimum risk for a given level of return are called efficient portfolios and the optimal solution gives the best way of investing our money. Therefore, to achieve a particular amount of return, the investor should choose the portfolio with the smallest variance which reflects risk, and this problem will be formulated as a mean-variance optimization which will be discussed more in detail.

1.5 The Importance of the Study

In general, this study provides a tool guide for decision-makers and agricultural practitioners to choose the way of diversifying resources among various crops. In this scenario, optimizing allocation and utilization of the right amount of water and land is challenging due to the difficulties in matching the balance between the minimum amount of irrigation water and the maximum irrigated cropped land in a semi-arid region like Uzbekistan where rainfall is insufficient to support crop growth and yield, thus the allocation of assets must be scheduled properly. Consequently, optimizing land-use with efficient irrigation systems directly would aim to conserve water resources.

This concept leads to sustain crop yield and long-term farming while eliminating water waste in irrigation which is vital to agriculture sustainability, environmental quality, and the national economy. Efficient and sustainable crop cultivation is essential for

food security, economic stability, and environmental preservation in Uzbekistan. Optimizing the use of available resources will not only enhance agricultural productivity but also contribute to the overall resilience of the nation's agricultural sector.

Chapter 2

LITERATURE REVIEW

The optimization of arable land and water consumption in agriculture represents the critical issue that the agriculture industry has faced with resource restriction given the global population growth which leads to increasing demand for food security. Therefore, as the climate change, resource scarcity, and environmental degradation has become the recent global challenges to deal with, it becomes imperative to explore sustainable practices that enhance agricultural productivity. This literature review aims to delve into the diverse strategies and technological advancements employed to optimize the utilization of arable land in agriculture. The goal is to provide insights that can inform decision-makers, and practitioners in their efforts to make the best decisions regarding issues of food production and resource management.

2.1 Portfolio Optimization

A range of studies have considered the optimization of arable land allocated to crops in agriculture utilizing various assumptions and methods. Within this body of literature, we are assessing papers that have considered the utilization of portfolio optimization method and incorporation of risk aversion parameter in their research. In the financial field point of view, taking huge risks usually leads to losing a considerable amount of profit. In the case of dealing with taking huge risks, [16] conducted a review paper examining methodologies for capturing risk perception and analyzing risk attitude and they outlined approaches to integrate the influence of these factors in risk programming of agriculture systems. They explored two methods such

as Stochastic simulation and farm-risk programming. Stochastic simulation is found to be effective in handling parametric and non-parametric distributions. In farm risk programming methods, Utility efficient programming (UEP), accommodating various function forms, also emerges as an applicable approach. Consequently, the integration of stochastic simulation with UEP enhances model's ability for complex decision-making when practitioners face with catastrophic risks.

By considering the portfolio optimization specifically as the research approach, [17] implied a heuristic approach based on neural network to solve a portfolio selection problem which was transformed into a mixed quadratic and integer programming problem by including cardinality and bounding constraints and mapping its efficient frontier. The obtained results were compared to other heuristic methods and showed that the neural network model demonstrates better solutions, particularly when requiring well-diversified portfolios with low investment risk.

Some researchers considered the application of MPT approach in natural scenario cases in which return and risk criteria both are considered i.e., [18] developed a flexible portfolio optimization model for some crops under water consumption restriction. To evaluate the model, they conducted a case study on a farm, located in Pajaro Valley of California, where three crops have been cultivated. [19] included conventional and organic farming as potential alternatives for cultivating bananas in the portfolio selection approach, the researchers developed land use strategies to maximize the expected return at a given level of accepted risk for a case study conducted in Ecuador. [20] emphasized the necessity of considering risk in the context of ecosystem delivery services in Maoming, China by considering the impact of land management strategies on delivery systems, The researchers defined suitable areas for agricultural production,

dividing them into grid cells. Multiple land management patterns were devised, and MPT was employed to derive optimal portfolios for each scenario. The researchers treated land management patterns as assets, with the average ecosystem supply serving as the expected returns. Ultimately, they found that the agricultural supply service could be enhanced by 21.89%. Their study demonstrated that the MPT approach is valuable for quantifying expected returns and risks, offering decision-makers an efficient solution. [21] address the spatial variability in returns from natural resources, emphasizing its impact on equitable distribution of value to individuals and communities. They utilize financial portfolio theory to this context, aiming to quantify the trade-off between resource returns and social equity, defined as a more uniform distribution of resource value. The study shows that achieving increased spatial equity often requires nonlinear reductions in resource returns by conducting a marine case study of the Channel Islands in California. The research highlights two advantages of portfolio approaches such as they prevent false expectations which results in improving outcomes, and they have the potential to enhance either resource return or social equity while maintaining the other. [22] investigate the impact of uncertainty, through a non-linear portfolio optimization, on the distribution of timber harvests and net revenues with consideration of two risk sources, hazard probabilities of trees and price volatility, over a 30-year period. For the optimization, they compared three different objective functions consisting maximizing net present value (NPV), maximizing the certainty equivalent (CE), and maximizing the value-at-risk (VAR). Their results showed that VAR maximization led to smoother and more consistent harvest volumes and net revenues, while NPV maximization resulted in sudden, unbalanced outputs. Risk aversion had less impact when the initial forest structure resembled a fully regulated forest, but risk aversion proved to be a more effective

hedge against variability than age-class structure, with reduced coefficients of variation for harvests and net revenues.

In conclusion, this review highlights the diverse applications of portfolio optimization in the agricultural sector. The above-mentioned studies reveal the effectiveness of portfolio optimization in addressing their specific challenges and the findings suggest adoption this optimization technique to enhance the efficiency of systems.

2.2 Land Management

As time is moving forward, the agricultural harvesting area is diminishing due to factors such as urbanization expansion and land salinization, prompting researchers to concentrate on optimizing arable land. This issue addressed by [23], assessing the efficient utilization of arable land, particularly considering seven major crops: potato, maize, wheat, sugar beet, rye, barley, and sunflower, occupying approximately 70 percent of Hungary's arable land. They formulated a chance-constrained stochastic programming model to address the issue. Ultimately, their findings suggested that it is feasible to allocate 17.5 percent of the total arable land for alternative purposes. [24] address the agricultural cultivation-scale problem in an arid region, Linze County in Gansu Province, northwest China. They developed a Two-Stage Stochastic Chance-Constrained Fractional Programming model to optimize the economic benefit and irrigated crop area given water availability and demand. The results indicate that the appropriate agricultural cultivation scale in Linze County ranges from 10,961 to 16,855 hectares under various scenarios, providing valuable insights for decision-makers in managing water resources and agricultural land use. [25] takes a more technical approach, proposing an optimization-simulation method for crop pattern determination and irrigation water allocation, focusing on soil environment protection.

Thus, they developed a multi-objective non-linear chance-constrained programming model. The approach manages uncertainties and trade-offs in land and water resources for diverse crops in irrigation districts. The method was applied to the Heihe River basin oasis in northwest China, resulting in optimal land and water resource allocation and reducing soil carbon emissions and enhancing efficiency. [26] focuses on the Hablehroud watershed in Iran where has potential economic growth but it is adversely affected by inappropriate land-use allocation. Therefore, the research aims to identify optimal land-use allocation using multi-criteria decision-making (MCDM) methods given the ecological and economic challenges in the region. They employ weighted goal programming (WGP), lexicographic goal programming (LGP), and compromise programming (CP) to evaluate economic, social, and environmental criteria. The obtained results according to environmental criteria, indicate an overall decrease in all methods while based on economic criteria, they observed better results in all methods. Finally, improvements in watershed management policies based on the optimal land-use patterns, with consideration of economic, and environmental factors is recommended by the authors.

The issue of diminishing arable lands and restricted water resources, creates a scenario where the demand for food surpasses the availability of them. Therefore, to solve the above-mentioned issue, new agricultural techniques and methods have emerged. One such innovative practice is vertical farming, initially introduced as a conceptual solution by Despommier [27]. Regarding to the emerging of this land-use method, [1] argues the merits and drawbacks of vertical farming and reveals its dual impact on both the economy and the environment within a country. Vertical farming emerges as an environmentally sustainable practice, particularly when employing hydroponics.

From environmental point of view, this method eliminates the need for soil, contributing to a reduction in the use of fossil fuels and subsequently lowering carbon emissions. Furthermore, vertical farming aligns with water recycling practices, leading to reduced water consumption, a critical consideration given limited water resources. From an economic perspective, vertical farming offers several advantages such as enhancement in Productivity, and there is a potential decrease in the foundational costs associated with fertilizers and pesticides. The inherent resilience of vertical farms becomes evident in their ability to mitigate losses during disasters such as floods or droughts. Additionally, the transportation costs linked to the transfer of crops could see a notable reduction. Regarding the drawbacks, there are substantial initial investment requirements driven by elevated land prices in urban areas. Moreover, there is a notable energy consumption issue, although this aspect could be improved utilizing renewable energy sources. It's important to note that the applicability of vertical farming is restricted for certain crops like wheat and maize. Moreover, the maintenance of vertical farms necessitates a skilled workforce.

In summary, this literature mentions the critical need for innovative and sustainable agricultural practices in the case of diminishing arable lands and water scarcity. Each approach addresses the land-use issue based on specific regional challenges and emphasizing on the importance of considering environmental, economic factors in devising effective strategies. However, the development of agriculture land-use necessitates ongoing research to ensure a resilient and sustainable future for global food production.

2.3 Irrigated Agriculture

Irrigated agriculture involves conserving water since improper usage can lead to the mitigation of water sources and consequently jeopardizing the long-term viability of agricultural activities. [28] provides a systematic review of innovative strategies to decrease water consumption in agriculture, highlighting the potential of controlled-environment agriculture, hydroponics, and precision farming.

In the Central Asian region given that most parts are semi-arid and arid irrigated agriculture is essential. [29] conducted a case study in the Khorezm region in Uzbekistan in 2006 and found that irrigation systems were mainly designed based on large cotton farms and they are not suitable for small, diversified agriculture practices since it leads to unsustainable excessive water usage. The study reveals that the designed water allowance for the irrigation systems exceeded the actual required water amount by 46 percent and as a result, they suggested revising outdated irrigation systems and water allowance. [6] evaluate the renewable water resources in the Middle Eastern, North African, and Central Asian countries and assess the potential of expanding agriculture area in each country considering irrigated and rainfed farming to identify the most sustainable farming methods. The researchers mentioned Uzbekistan as one of the countries that can expand the cultivation area only by using fossil water resources. They suggest that farmer education on crop, land, and water management is essential to prevent decreasing yield rates and increasing soil salinity. Also, aligning irrigation water use with crop value, such as investing in vegetables due to higher quality and price, is recommended for efficient water use. This paper [30] focuses on water resource sustainability in farming systems, emphasizing the crucial role of cropping patterns in water management. The main approach is a multi-objective fractional goal programming method considering two optimization ratios

‘ $\frac{\text{net return}}{\text{water consumption}}$ ’, and ‘ $\frac{\text{labour employment}}{\text{water consumption}}$ ’, and according to the results they suggested the efficiency of this approach to get sustainable cropping patterns. [31] address the critical need for water management in the Chirchik river basin in Uzbekistan, primarily driven by water deficit and salt damage. They developed a model using Geomorphology Based Hydrological Model and they also conducted a land-use classification. [32] discuss the need for irrigated rice systems that use less water than traditional flooded rice, focusing on irrigated aerobic rice systems where rice grows in no flooded and no saturated soil under supplemental irrigation. The study aims to identify promising varieties and quantify yield potential, water use, field water outflows, and water productivity. The research was conducted at the International Rice Research Institute in the Philippines over six seasons (2001–2003). The study concludes that the concept of aerobic rice shows promise, especially for farmers facing water scarcity and unable to grow flooded lowland rice. [33] addresses Iran's severe water shortage, exacerbated by droughts and excessive water extraction in Qazvin province. The aim is to identify an optimal water-saving cultivation pattern in the field cropping subsector. The following objectives were considered optimal: a 15% increase in gross profit, a 10% reduction in agricultural water consumption.

This review represents various aspects in irrigated agriculture including water conservation strategies, challenges in designing an irrigation system, and the influence of efficient cropping patterns on saving water. The collected findings emphasize on the importance of adopting water-efficient practices in irrigated agriculture globally.

2.4 Gap Research and Contributions of the Study

All the examined research, emphasize the necessity for a comprehensive and adaptable approach to optimize the utilization of arable land and water. The constraints in

agricultural resources present a critical challenge, particularly in arid regions such as Uzbekistan since resource limitations persist, despite the country's good potentials and financial capacity for investing in modern agricultural practices, as well as the adoption of efficient irrigation methods over outdated ones. Therefore, this study aims to provide a framework for selecting land allocations based on the assessed risk, with the goal of achieving efficient production rates while maintaining control over water usage by conducting a case study in Uzbekistan.

Chapter 3

UZBEKISTAN

3.1 General Information

In this study we are conducting a case study in Uzbekistan located in Central Asia with a total area of 448,978 Km² and Tashkent is its capital city. Uzbekistan is a doubly landlocked country which means it is surrounded by five landlocked countries including Kazakhstan, Kyrgyzstan, Tajikistan, Afghanistan, and Turkmenistan. In addition, none of its rivers lead to the sea. Only a small fraction, accounting for less than 10% of its land, is dedicated to crop cultivation, primarily in river valleys. Previously, the main cultivation area was in the Aral Sea basin, where has tragically dried up and ranking as one of the most severe environmental catastrophes in the world. The remaining part of the Uzbekistan is the vast Kyzylkum Desert and mountains [34].

Uzbekistan is known for its predominantly agricultural economy in Central Asia. Cotton is a major crop and has been a significant contributor to Uzbekistan's economy. However, the Uzbekistan's government has recognized the need for allocation of land to other primary crops and has been implementing reforms to shift away from excessive reliance on cotton and instead of that efforts have been made to encourage crop diversification like fruits, vegetables, and cereals.

Uzbekistan has been faced challenges related to water scarcity since they heavily rely on irrigation for agriculture. Amu Darya and Syr Darya basins are the main sources of water that support a variety of crops. The diversion of water from those two rivers for agricultural purposes has contributed to the shrinking of the Aral Sea one of the significant issues in the region. According to [6], Uzbekistan is among the countries which water usage and demand is higher than their overall available water resources and they're currently consuming their fossil water resources.

In Table 1, we provided the data related to the diversification of land-use in Uzbekistan [35] which can be concluded that the ratio of land allocation for cultivating is less than 10%.

Table 1: Land Categories in Uzbekistan

Land area (ha)	Agriculture land (ha)	Water sources area (ha)	Irrigated land (ha)
42,540,000	3,677,000	2,200,000	3,357,000

3.2 Uzbekistan Divisions

Uzbekistan country is composed of 13 regions 1. Andijan Region, 2. Bukhara Region, 3. Jizzakh Region, 4. Qashqadaryo Region, 5. Navoiy Region, 6. Namangan Region, 7. Samarqand Region, 8. Surxondaryo Region, 9. Sirdaryo Region, 10. Tashkent Region, 11. Fergana Region 12. Khorezm Region and 13. Republic of Karakalpakstan. You can see the administrative divisions of Uzbekistan in Figure 1.



Figure 1: Administrative Divisions in Uzbekistan

The area in Uzbekistan contains high mountains, extended across the border zones in the eastern and southern parts of the country where Fergana, Namangan and Andijan valley sit between those mountains on the other hand, the republic of Karakalpakstan is mostly consisted of desert.

3.3 Weather Conditions

Uzbekistan's climate is continental with average temperature exceeding 25°C in summer and dropping below 8°C in winter. it is evident that in Uzbekistan the agriculture would be impractical without the implementation of irrigation systems since the annual rainfall is less than 300mm.

The key factors impacting land use in Uzbekistan are population growth, urbanization, agricultural practices, industrialization, and climate change. [36]

3.4 Crop Production

Agriculture plays a crucial role in the economy of Central Asian countries, in this case contributing to 18.5 percent of Gross Domestic Production (GDP) in Uzbekistan.[37]

In Uzbekistan, crops are classified based on their cultural significance and their food consumption background. In this context, the classifications include cereals, cotton, fruits, grapes, meat, melons (encompassing melon's various types), potatoes, vegetables, and wheat. Notably, based on the statistical data obtained from Uzbekistan, the cereals category comprises maize, barley, rice, and sorghum, while potatoes and wheat are distinguished from their broader group which are vegetables and cereals category [35]. According to [35], the total production volumes from all regions of Uzbekistan for the considered categories in 2014 are as follows.

- Cereals: 8,050,500 tones
- Wheat: 6,956,000 tones
- Cotton: 3,400,200 tones
- Potatoes: 3,011,300 tones
- Vegetables: 2,452,400 tones
- Melons: 9,286,700 tones
- Fruits: 1,696,100 tones
- Grapes: 1,441,200 tones
- Meat: 1,906,300 tones

Chapter 4

MARKOWITZ'S PORTFOLIO SELECTION MODEL

In defining the set of variables and vectors according to [38], let's first consider a portfolio with i securities denoted as S where $i = 1, 2, 3, \dots, n$ hence, the vector S (1) consisting of i assets that each of them is identified by their return value denoted as vector R (2). The weight of each portfolio is expressed as vector U (3) where U_i is denoted as the proportion of money invested on each asset S_i .

$$S = [S_1, S_2, \dots, S_n] \quad (1)$$

$$R = [R_1, R_2, \dots, R_n] \quad (2)$$

$$U = [U_1, U_2, \dots, U_n] \quad (3)$$

In this context, we assume that the total weight equals to the summation of all allocated weights as shown in the following equation.

$$W = W_1 + W_2 \dots + W_n \quad (4)$$

To address the investor's interest in determining the distribution of funds among assets, we focus on normalized portfolios, ensuring that the total amount $W = 1$.

We need to calculate the mean and covariance of returns of a portfolio with i assets:

The mean value $\mu_i = \frac{R_1 + R_2 + \dots + R_n}{n} = E(R_i)$

The covariance $C_{ij} = Cov(R_i, R_j)$ is calculated through the following equation (5):

$$C_{ij} = E(R_i * R_j) - E(R_i)E(R_j) \quad (5)$$

Variance – Covariance matrix = $\begin{bmatrix} c_{11} & c_{12} & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix}$, Where Variance = c_{ii} .

The expected return of the portfolio will be a linear equation which is shown as follows

(6):

$$R_p = \sum_{i=1}^N \mu_i W_i \quad (6)$$

Finally let us denote $\lambda \in [0,1]$ as the risk aversion parameter [15].

In the MPT approach, we can interpret that the objective function penalizes the expected return based on the level of risk.

4.1 Optimization Problem Formulation

$$\text{Max} = \sum_{i=1}^N \sum_{j=1}^N \mu_i W_i - \lambda W_i^T C_{ij} W_i \quad (7)$$

Subject to:

$$\sum_{i=1}^N W_i = 1 \quad (8)$$

$$W_i \geq 0 \quad (9)$$

The variance of a portfolio is a measure of the portfolio's volatility which is obtained through the following equation (10):

$$V_p = W^T * C_{ij} * W \quad (10)$$

Underlying assumptions:

- It is assumed that the variance covariance matrix is a positive semidefinite matrix.
- It is presumed that exist at least two assets, denoted as i and j , with distinct mean returns $\mu_i \neq \mu_j$.

4.2 Efficient Frontier

A diagram depicting the efficient frontier can be plotted based on the set of points (V_i, μ_i) corresponding to all efficient portfolios which is presented in Figure 2.

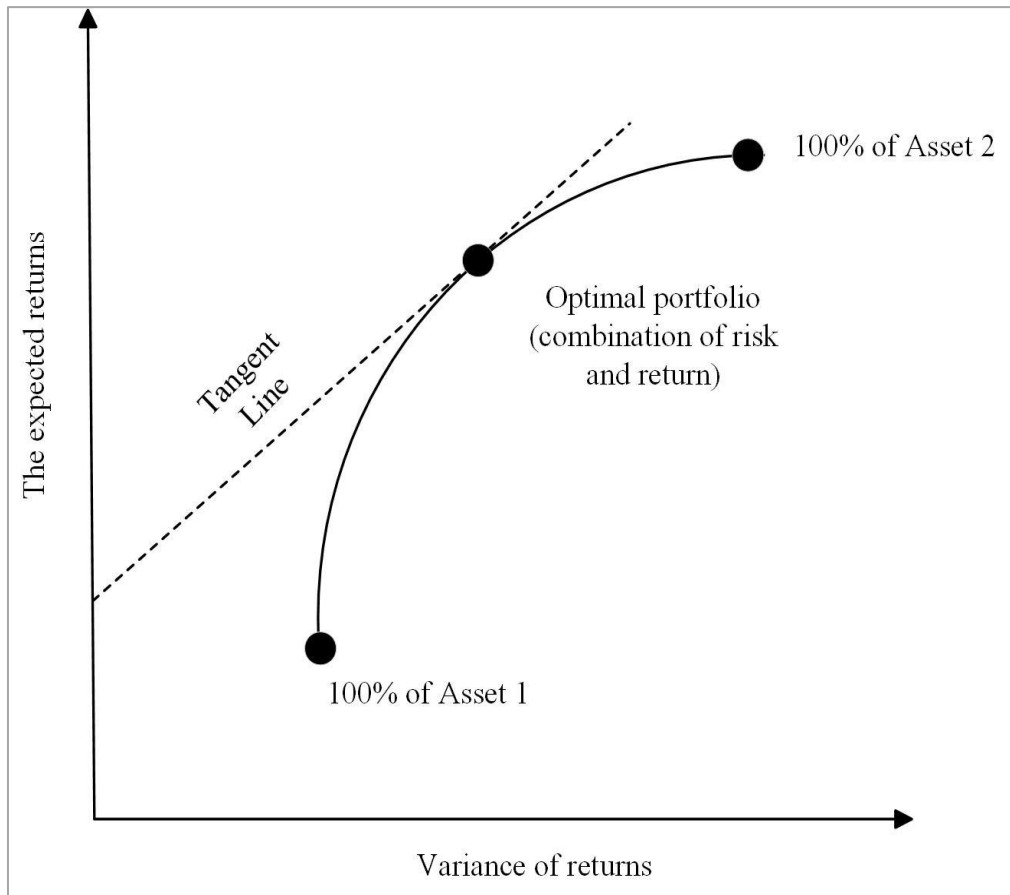


Figure 2: The Standard Efficient Frontier

4.3 Relation of the Efficient Frontier to the DEA Method

Data envelopment analysis, known as DEA, is a non-parametric linear programming method in operations research developed by Charnes, Cooper and Rhodes (1978) used to estimate production efficiency.

In general, Non-parametric methods evaluate feasible input and output combinations among the existing decisions. DEA which is one of the most common non parametric approaches calculates the highest achievable output for a given input set [39].

4.4 Efficient Portfolios

As per the model we outlined, the optimal weight vector in a normalized portfolio is denoted as U_i^* and it is considered efficient if there's no alternative feasible vector U

containing a superior expected return with lower variance value known as risk level [40]. To put it in mathematical terms, it can be demonstrated as follows.

$$\mu_i^* \geq \mu_i \quad (11)$$

$$V_i^* \leq V_i \quad (12)$$

μ_i : Denoted as the expected return of a portfolio.

V_i : The variance of the portfolio.

In summary, Markowitz's optimization model is a mathematical framework that allows investors to systematically construct portfolios that aim to achieve their desired level of risk and return. By considering the covariance between assets and the trade-offs along the efficient frontier, investors can create diversified portfolios that optimize their risk-return profiles, making the most rational and informed investment decisions. This model has been foundational in shaping modern portfolio management practices and remains a cornerstone of investment theory.

4.5 Illustrative Example for Markowitz

In this section, the pricing data for two stocks, Apple and Microsoft, has been collected over a time span of 200 days, from December 14, 2022, to October 2, 2023. The ensuing section presents the acquired data, and the model is constructed utilizing portfolio optimization theory. Subsequently, the formulated model is resolved using Matlab.

4.5.1 Given data

The average closing price for each company is computed as follows.

$$m_A = 166.3635$$

$$m_M = 296.4639$$

m_A : The mean closing price of Apple company.

m_M : The mean closing price of Microsoft company.

In the Dow Jones Industrial (DJI), the closing price of a stock signifies the last traded price before the market closes its daily trading session.

The Variance-Covariance matrix is provided below.

$$\text{Covariance (A, M)} = \begin{bmatrix} 361.76 & 692.94 \\ 692.94 & 1407.04 \end{bmatrix}$$

The expected return for each company can be obtained using the following formula.

$$E(R_i) = \sum_{i=1}^2 m_i * w_i \quad (12)$$

w_i : Denotes the weight invested in each stock.

4.5.2 Formulation and Solution

The Linear Programming (LP) model is presented as follows.

$$\text{Max} = \sum_{i=1}^2 m_i * w_i - \lambda * w_i * \text{Cov} \quad (13)$$

λ : Denotes as the risk aversion parameter.

Subject to,

$$w_1 + w_2 = 1 \quad (14)$$

$$w_i \geq 0 \quad (15)$$

I have resolved the model using MATLAB software. In the following page, I've included the MATLAB code along with a detailed explanation of the process within the code, and the outcome is presented as follows.

```

1 % Given data
2 mean = [166.3635; 296.4639];
3 Cov = [361.76, 692.94;
4        692.94, 1407.04];
5 lambda = 0.1;
6
7 fun = @(w) -sum((mean' * w) - lambda * (w' * Cov * w));
8 % s.t.
9 Aeq = [1, 1];
10 beq = 1;
11 lb = [0; 0];
12 ub = [1; 1];
13 W0 = [0.5; 0.5];
14
15 options = optimoptions('fmincon', 'Display', 'iter');
16 W_optimal = fmincon(fun, W0, [], [], Aeq, beq, lb, ub, [], options);
17
18 disp('Optimal Weights:')
19 disp(W_optimal)
20 portfolio_return = mean' * W_optimal;
21 portfolio_variance = W_optimal' * Cov * W_optimal;
22 disp('Optimal Portfolio Return:')
23 disp(portfolio_return)
24 disp('Optimal Portfolio Variance:')
25 disp(portfolio_variance)

```

Figure 3: The DJI Code in MATLAB

In Figure 3, the process begins by defining the mean vector and the variance-covariance matrix for the two specified companies, with parameter lambda set to 10^{-1} . In line (8), 'fun' is a function representing the optimization objective function of the optimization problem, this operator minimizes the value hence the formula is multiplied by a negative sign. The vector 'W' represents the assigned weights, and lambda serves as a parameter reflecting the trade-off between risk and return. Subsequently, constraints are established: 'Aeq' and 'beq' refers that the sum of weights (W) should equal 1. 'lb' and 'ub' are vectors defining the lower and upper bounds on the weights which is between 0 and 1. 'W₀' represents the initial weights assigned to each company. The optimization is executed using MATLAB's optimization function 'fmincon' and then displays the optimal weights, expected portfolio return, and portfolio variance.

The results are as follows.

- The Optimal Weights:

$$w_1 = 0.1661$$

$$w_2 = 0.8339$$

The optimal weights for the DJI portfolio are distributed as 16.61% for Apple company and 83.39% for Microsoft.

- Optimal Portfolio Return:

274.8559

- Optimal Portfolio Variance:

$1.1804 * 10^3$

In conclusion, this combination of weights results in a portfolio with the highest expected return for the associated level of risk (variance). It represents an efficient allocation of assets within the given constraints.

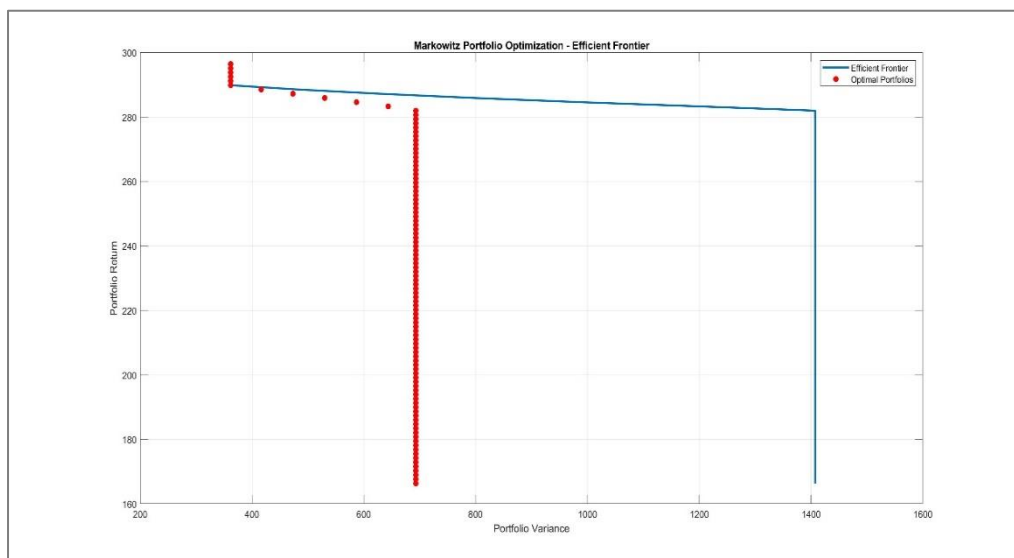


Figure 4: The Efficient Frontier of DJI Stocks Apple & Microsoft

Chapter 5

PROBLEM STATEMENT

5.1 Problem Overview and the Study Region

As we discussed in Chapter 1 and 3, Uzbekistan is characterized by a semi-arid climate and faces a significant challenge in optimizing crop cultivation due to limited arable land and water resources. The reliance on the Amu Darya and Syr Darya rivers for irrigation, along with salinization issues and outdated irrigation systems, leads to a complex problem that requires systematic optimization.

The agricultural sector in Uzbekistan plays a pivotal role in the nation's economy, contributing significantly to its Gross Domestic Production (GDP). However, the scarcity of arable land and water resources has affected the sector's potential for a steady growth. The reliance on traditional irrigation methods and the impact of salinization on soil fertility are critical issues affecting crop yields.

The primary challenge addressed in this study encourages us to develop an optimization framework for crop cultivation considering seven different crops including Barley, Maize, Potato, Rice, Tomato, Wheat, and Watermelon in Uzbekistan, given the limited availability of arable land and water resources. The goal is to maximize agricultural productivity and land use while optimizing water usage.

5.2 Collection of Data

In our study, we focused on seven crops: Barley, Maize, Potato, Rice, Tomato, Wheat, and Watermelon. The data we examined spanned from Food and Agriculture Organization (FAO) 1992 to 2021.

For each specific crop I provided some information and figures to understand the situation better.

5.2.1 Barley

Barley is an important cereal crop in Uzbekistan, and the country has a history of cultivating barley to meet both domestic demand and support its agricultural economy.

Figure 5, Shows the harvested area and production of barley since 1992. In the beginning the production value of barley was 394,700 tonnes which was the highest value in over 30 years, and it decreased gradually until 1995. In the next 5 years we observed a significant decrease to 91,200 t. There was a fluctuation trend around 150000 t from 2000 until 2021 which reached to 958,282 t. Barley cultivation occupies a significant portion of Uzbekistan's arable land. Harvested Area according to Figure 5, peaked in 1994 and reached near 300,000 ha then it decreased up to 50,000ha and the amount of cultivated area fluctuated around 100,000ha until 2021. In overall, there is a decreasing pattern in both harvested area and production value.

It is note-worthy mentioning that Barley is grown alongside other cereal crops such as wheat and contributes to the country's overall grain production supply and livestock feed.

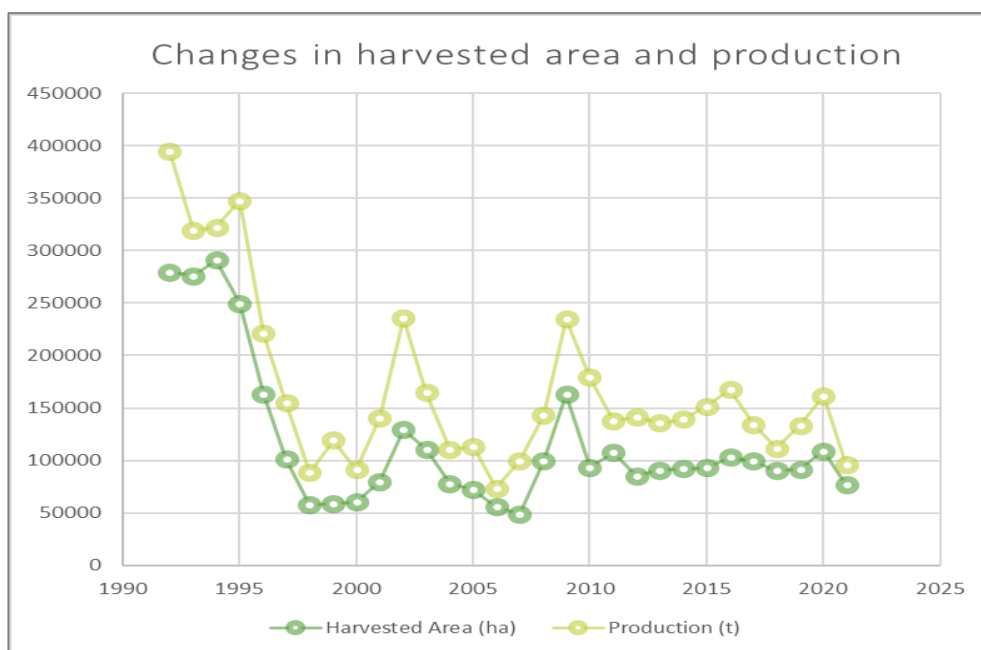


Figure 5: Changes in Harvested Area and Production of Barley

Barley is well-suited to a range of climates, and Uzbekistan's diverse climate allows for the cultivation of barley in different regions. Barley is often grown in both rain-fed and irrigated areas. According to Figure 6, 50 percent of total production comes from Jizzakh also in other regions such as Khashadarya and Samarkand it can be produced.

Uzbekistan's barley production is generally aimed at meeting domestic needs. Since the focus is often on ensuring a stable supply for domestic consumption and livestock feed, the country may import barley based on market conditions. According to data acquired from FAO the imported quantity of barley was 56,564 tonnes with 11,096,000 dollars on average from 2012-2021 while there is not much information regarding the exported quantity of barley, and we assume that the country would mainly import barley to meet the demands.

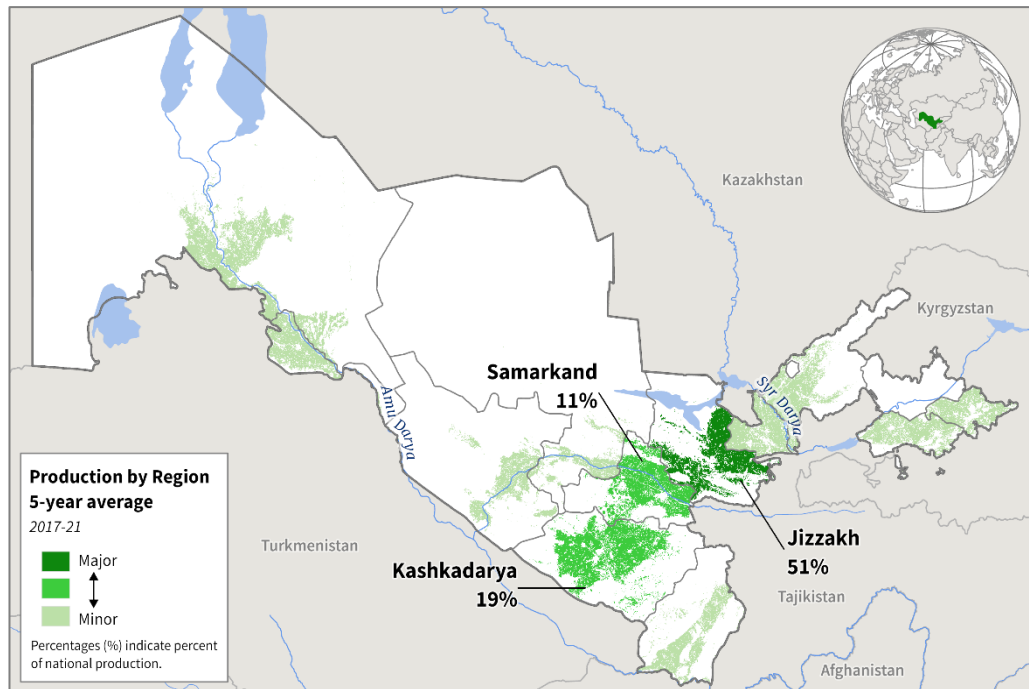


Figure 6: Distribution of Barley Cultivation [41]

Figure 7 shows the yield and 5-year moving average of Barley which helps to identify the underlying trends over 30years time. By analysing the moving average of barley, we realise that there is a general increasing pattern up to 2002 followed by a decreasing trend until 2021. By observing the actual value of yield, we can see that the years 1999, 20007 and 2010 can be considered as peak years which yield was approximately $2 \frac{t}{ha}$. After 2010 there was a significant decreasing in the yield value, and it reached to $1.25 \frac{t}{ha}$ in 2021.

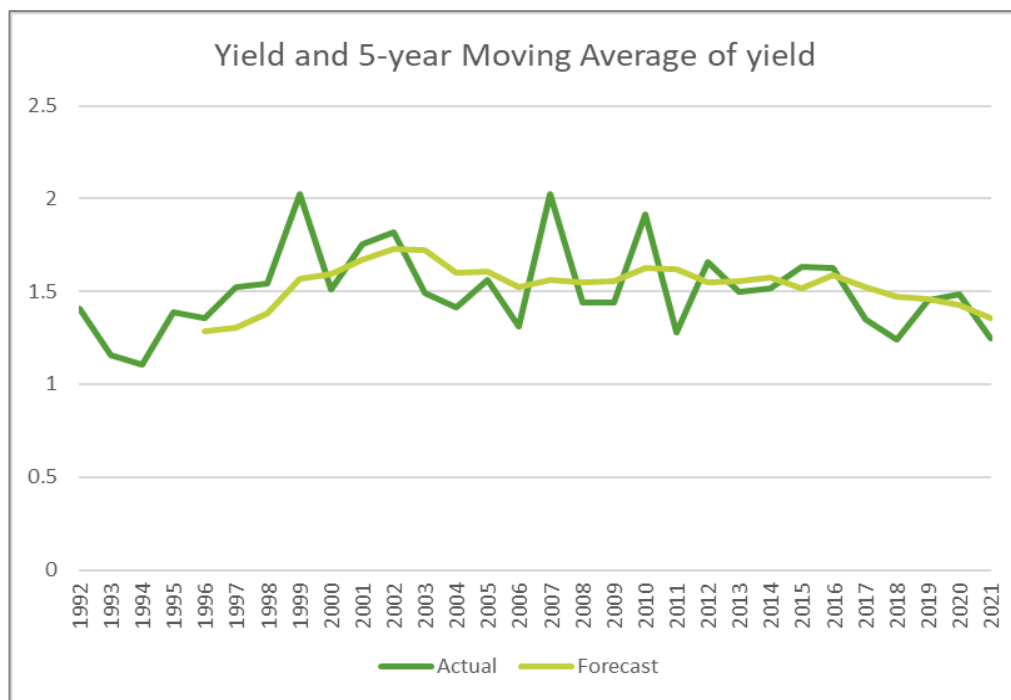


Figure 7: Yield And 5-year Moving Average of Barley

5.2.2 Maize

maize (corn) is an important cereal crop in Uzbekistan, and the country cultivates maize for various purposes, including food, livestock feed, and industrial uses since maize is a raw material to produce corn oil and other industrial products. Maize cultivation occupies a substantial portion of Uzbekistan's arable land. Farmers may grow maize alongside other crops. According to Figure 8, the cultivation area allocated to maize has been around 45,000 ha. In the beginning it was 100,000 ha, but it has decreased over years. In the production pattern of the maize there is a significant upward movement towards the end of the period, and it reached to nearly 600,000 t.

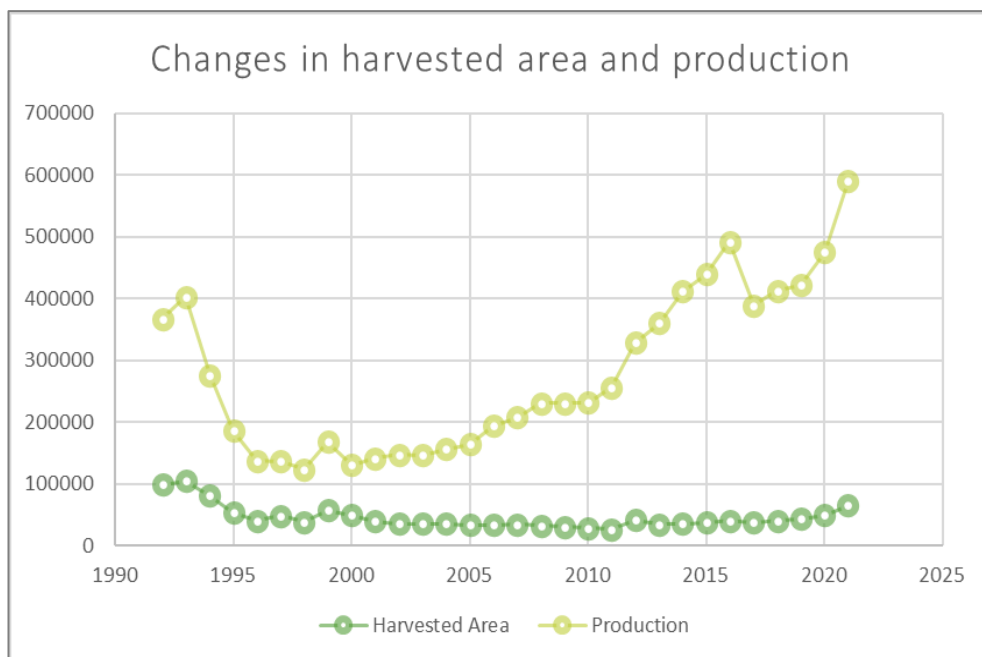


Figure 8: Changes in Harvested Area, and Production of Maize

Maize is well-suited to a variety of climates, and Uzbekistan's diverse climate allows for the cultivation of maize in different regions. It is often grown in both rain-fed and irrigated areas. Maize mainly has been produced in southern regions of Uzbekistan similar to other cereals in regions like Andijan, Ferghana, Tashkent, Namangan and some other southern parts. Figure 9 illustrates the regions contributing to maize production based the quantity has been produced.

While Uzbekistan is generally self-sufficient in maize production, there may be variations in production levels from year to year. By analysing the data gathered from FAO The country may import maize based on market conditions and domestic demand which was 38,935 tonnes with 10,593,700 dollars on average from 2012-2021.

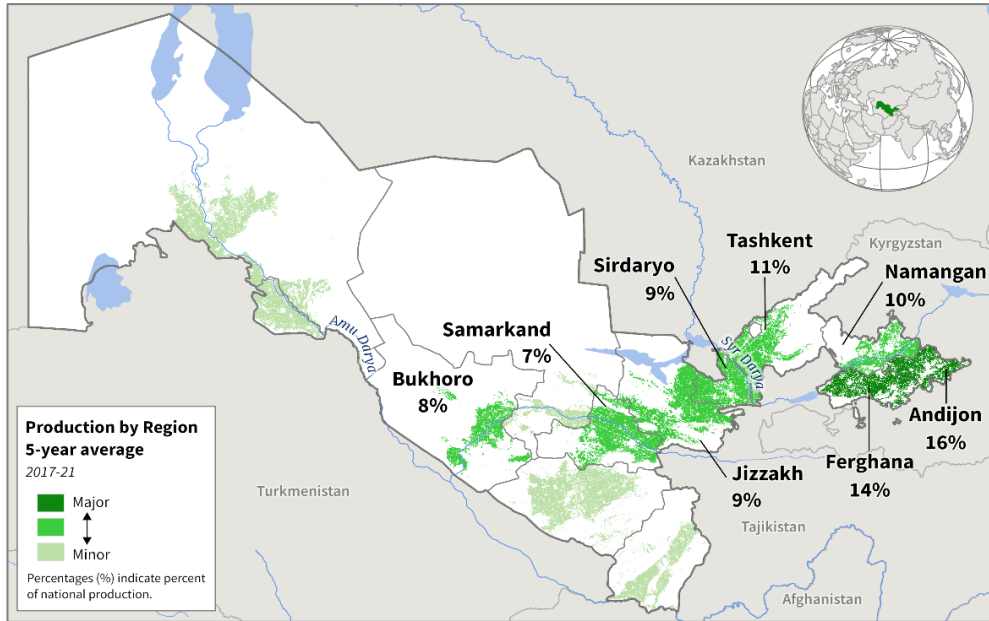


Figure 9: Distribution of Maize Cultivation [41]

Figure 10 shows the yield and 5-year moving average yield of barley. We observe an upward pattern by considering the moving average of barley. By analysing the actual yield values, we see that it exceeded $12 \frac{t}{ha}$ in 2016 followed by a sudden decrease over the next 5 years and it reached to $9.07 \frac{t}{ha}$ in 2021.

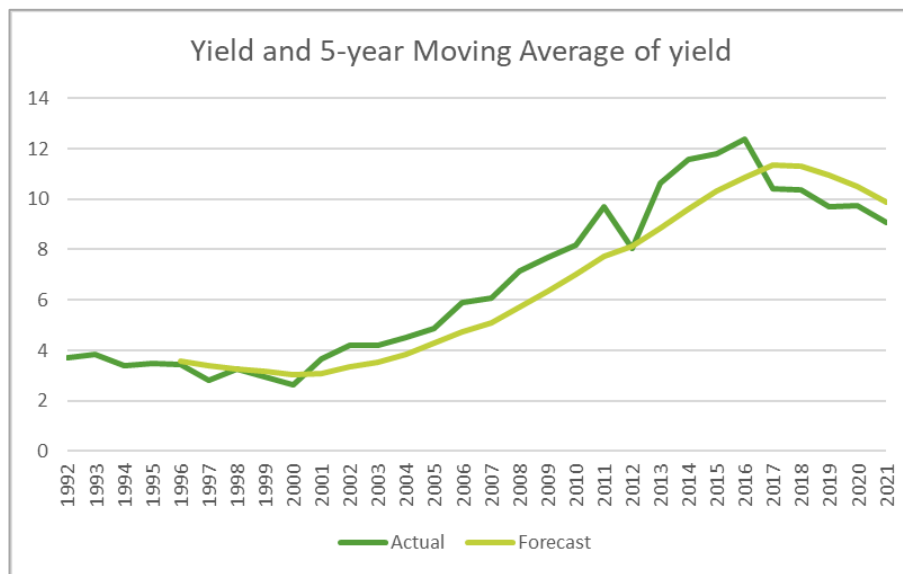


Figure 10: Yield and 5-year Moving Average Yield of Maize

5.2.3 Potato

Uzbekistan has been a significant producer of potatoes in Central Asia, and they are an important staple in the Uzbek diet, and their cultivation plays a crucial role in the country's agriculture. Figure 11, Shows harvested area, and production of Potatoes since 1992. The production of potatoes had an increasing trend during the 30 years interval, starting from less than 500000t in 1992 and it reached to 3285646t in 2021. The agricultural area was pretty stable around 64000ha.

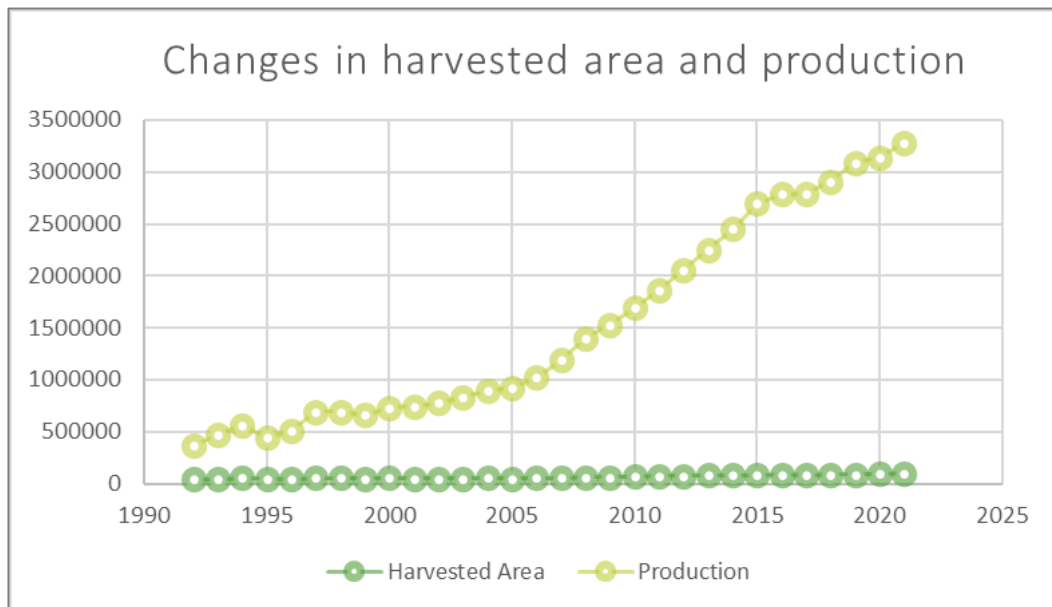


Figure 11: Changes in Harvested Area and Production of Potato

Potatoes are typically grown in various regions of Uzbekistan Production volumes are high in the Central zone region of Tashkent and the Middle Western zone region of Samarqand, followed by the Fergana Valley regions of Andijan, Fergana and Namangan. The production and importing levels contribute to domestic consumption and, to some extent, less quantity was exported. Due to FAO datasets 209,146 tonnes were imported with 27,359,200 dollars on average from 2012-2021 while the exported value was 1,175 tonnes with 270,000dollars income. The country's agricultural sector

has traditionally been important to its economy, and efforts have been made to modernize farming methods.

Figure 12, Shows an increasing pattern in the 5-year moving average of potato in overall and stable in the last three years and by considering the actual values of yield, the peak year would be 2017 with $35.7 \frac{t}{ha}$.

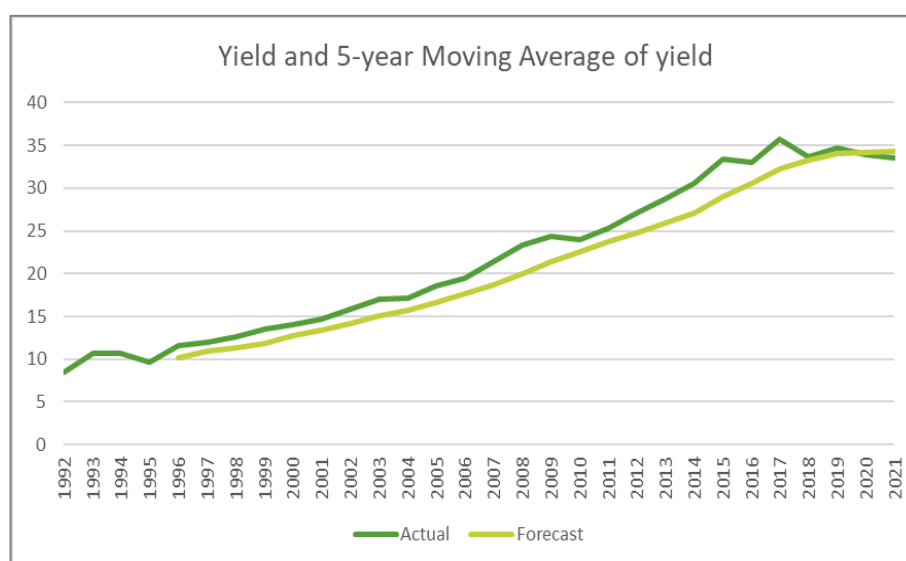


Figure 12: Yield and 5-year Moving Average Yield of Potato

5.2.4 Rice

It's important to note that Uzbekistan's climate and geography pose some challenges to rice cultivation, and the country is not among the top global producers of rice. Uzbekistan has an arid and semi-arid climate, which is not particularly well-suited for rice cultivation. Rice requires a lot of water, and in regions with limited water resources, the cultivation of rice may be challenging. Despite the arid conditions, Uzbekistan has invested significantly in irrigation infrastructure. The government has implemented various irrigation projects to support agriculture, including rice cultivation. Large-scale irrigation systems have been developed to provide water for

crops, including rice. The reason is that rice is an important staple in the Uzbek diet. It is a key component of traditional dishes, and there is domestic demand for locally grown rice. Figure 13, Shows harvested area and production of rice since 1992. In 1992 the production of rice was 583,000 t but there were fluctuations during the next 30 and in 2021 it decreased to 334,226 t. Also, the area allocated to rice has a decreasing pattern at the beginning it was nearly 200,000 ha but it decreased to 48883 ha in 2021.

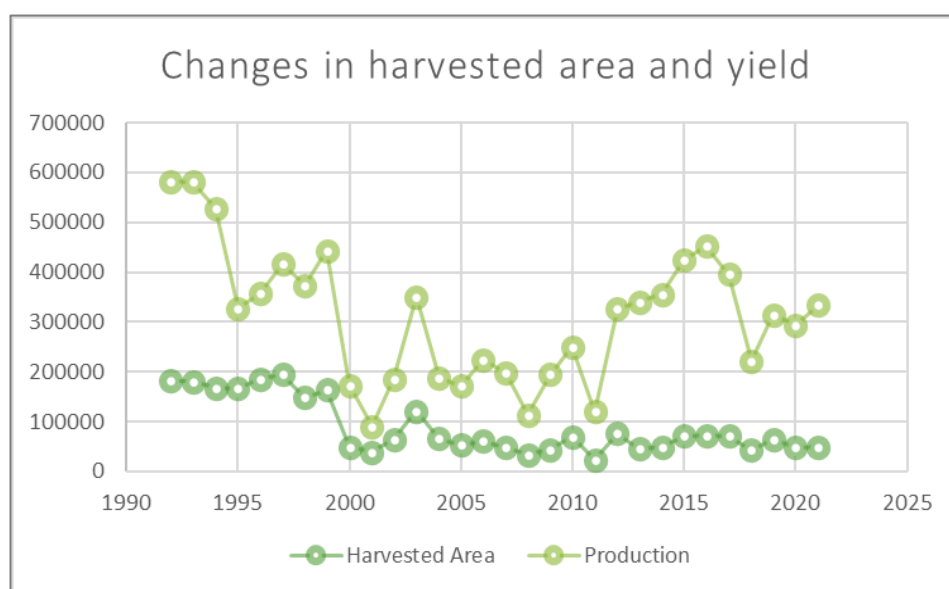


Figure 13: Changes in Harvested Area and Production of Rice

The production levels of rice were to satisfy the domestic consumption though to some extent some were imported based on FAO data the average value imported was 1,024 tonnes with the value of 200,000 dollars from 2012-2021.

According to Figure 14, 46 percent of rice production is provided in Khorezm region. Karakalpakstan, Andijan and Ferghana are among the regions that cultivate rice based on the production amount, respectively.

Shows the yield and 5-year moving average of yield of rice. By following the moving average, we realise that there is an upward moving average pattern followed by a decrease and again an increasing trend in the last three years. According to the actual yield values, the peak year is 2013 with $7.58 \frac{t}{ha}$.

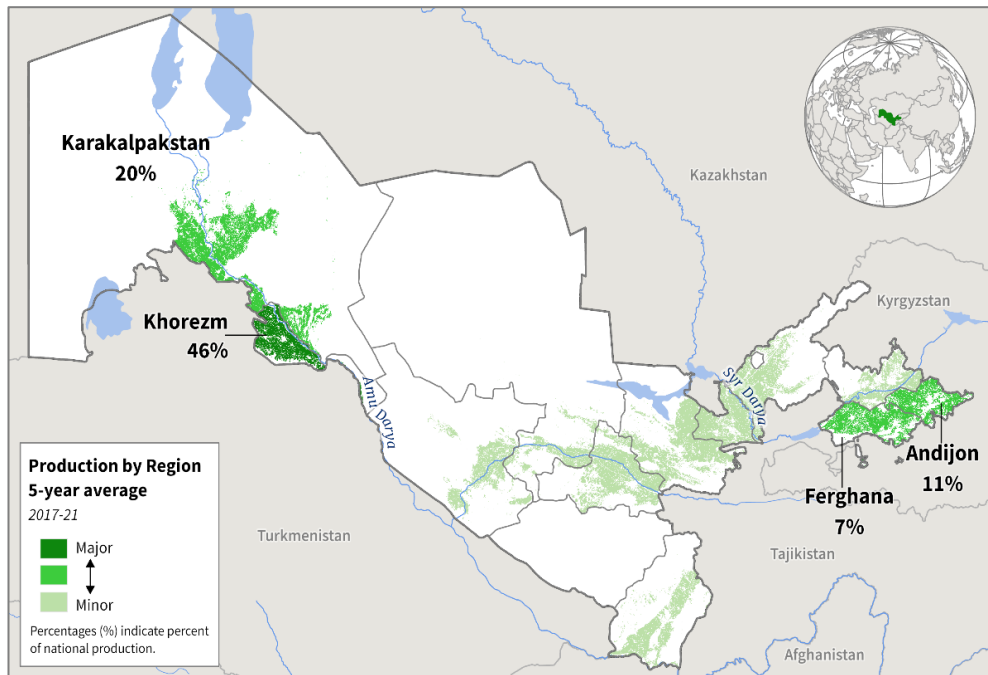


Figure 14: Distribution of Rice Cultivation [41]

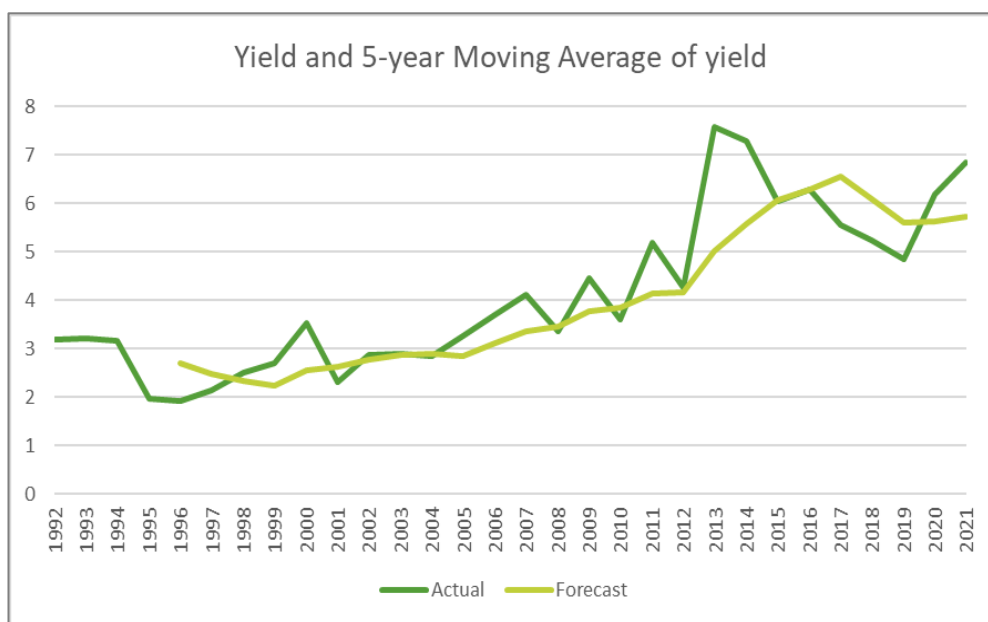


Figure 15: Yield and 5-year Moving Average Yield of Rice

5.2.5 Tomato

Uzbekistan's favourable climate and agricultural practices contribute to the cultivation of tomatoes, among other crops. Uzbekistan's diverse climate, including warm temperatures in many regions, provides favourable conditions for tomato cultivation as they thrive in a warm and sunny environment. The main production volumes of tomatoes are from regions of Tashkent and Samarqand.

Tomatoes are grown both in irrigated and rain-fed areas and greenhouse cultivation as one of the modern agricultural practices, are also employed to enhance yields and extend the growing season. Figure 16 illustrates changes in harvested area and production of tomatoes over 30 years. The production had an increasing trend up to 2016 with the value of 2,796,189 t followed by a decreasing pattern and it reached to 2,206,641 t in 2021. And the harvested area allocated to producing tomatoes was around 50,000 ha.

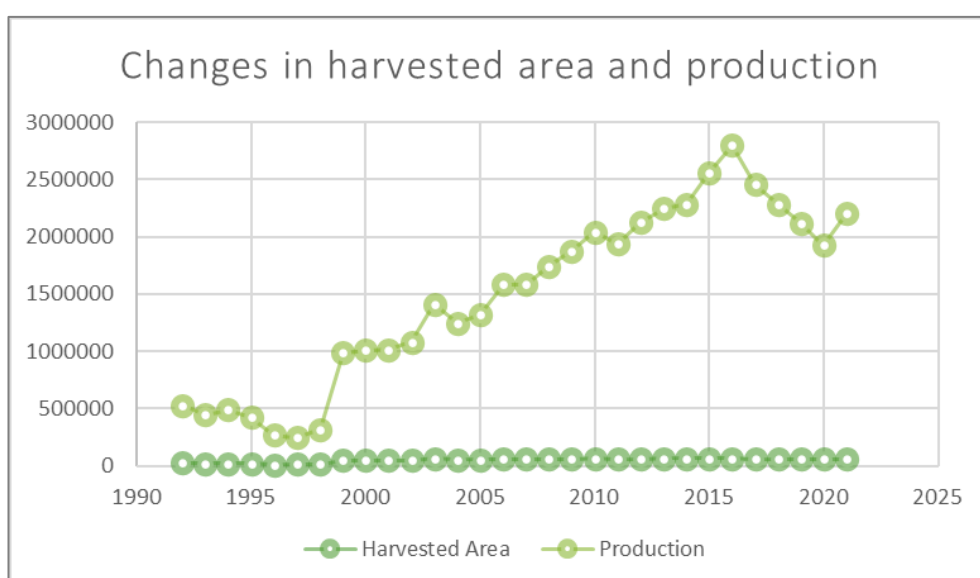


Figure 16: Changes in Harvested Area and Production of Tomato

A significant portion of tomato production is consumed domestically, though some quantity of tomatoes also has been exported to neighbouring countries. According to FAO, 48,765 t were exported with average income of 37,799,800 dollars 2012-2021.

Moreover, Figure 17, shows the yield and 5-year moving average of tomato and by following the moving average pattern we can observe an upward trend in the first almost 25 years followed by a decreasing in the last 5 years. The highest value of yield is related to 2016 with $43.6 \frac{t}{ha}$.

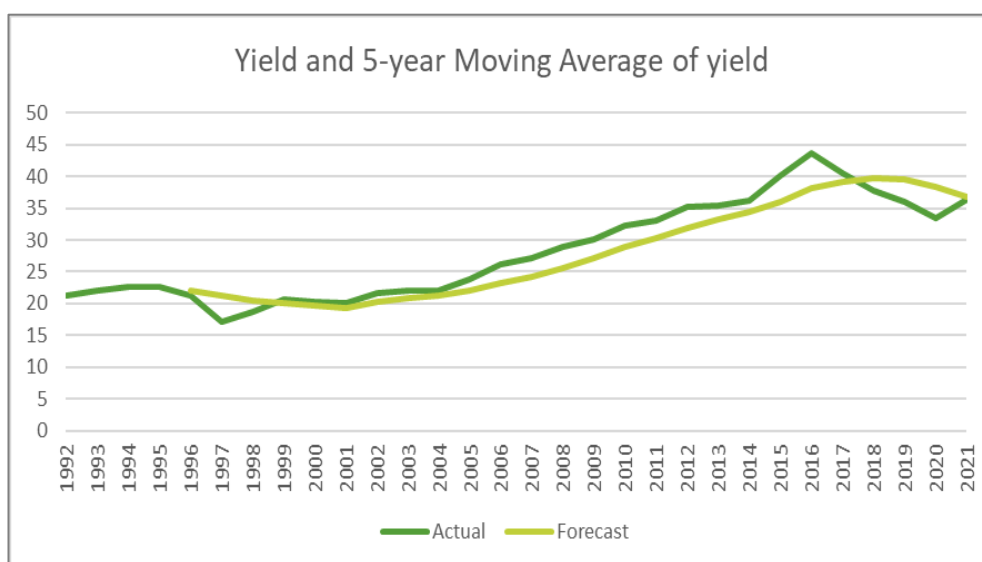


Figure 17: Yield and 5-year Moving Average Yield of Tomatoes.

5.2.6 Watermelon

Uzbekistan is known for its significant production of watermelons as shown in Figure 18, The watermelon production exceeded 2,000,000 tonnes in 2017 and 2018 and overall, it was more than 1,000,000 in the last 10 years. Uzbekistan's watermelon production is not only for domestic consumption but also for export. The country exports watermelons to neighbouring countries and beyond. According to the data acquired from FAO, has been observed a significant increase in watermelon exports

which 49,241 tonnes were exported, and the exported value earned was 11,779,000 dollars in 2020. Thus, watermelon cultivation contributes to the country's economy and provides income for many farmers.

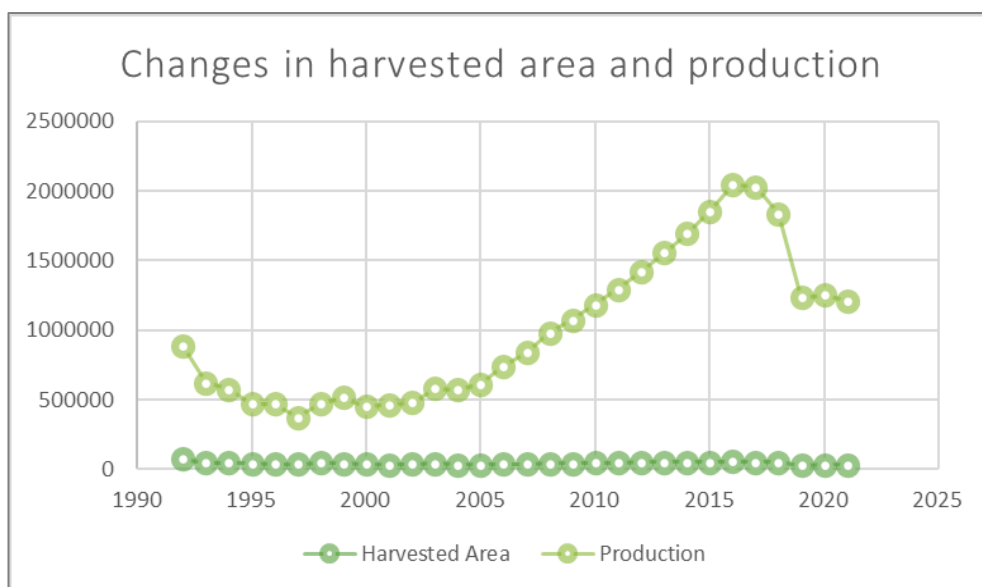


Figure 18: Changes in Harvested Area and Production of Watermelon

Cultivation of watermelon is widespread mainly in the following regions based on the production values: Jizzakh, Surkhundarya, Tashkent, Qashqadaryo and Khorezm, respectively [35] As shown in Figure 18, in total the agricultural area allocated to watermelon was stable around 45000 ha.

The country has favourable climatic conditions for watermelon cultivation, and this crop has become an important part of Uzbekistan's agricultural sector.

Irrigation methods utilized for producing watermelons:

- Furrow Irrigation
- Sprinkler Irrigation
- Drip Irrigation
- Mulching Agro-Technology

[42], used a software to estimate the water consumption of watermelon and according to their results 271 mm is the irrigation water requirement for watermelon.

Figure 19, Shows the yield and 5-year moving average yield of watermelon which by following the moving average pattern we can see an upward trend. Considering the actual yield values, it's obvious that up to 2019 the yield values were increasing and the year 2019 was the peak year with yield of 41.6 t/ha but followed by a sudden decrease and it reached to 34.98 $\frac{t}{ha}$ in 2021.

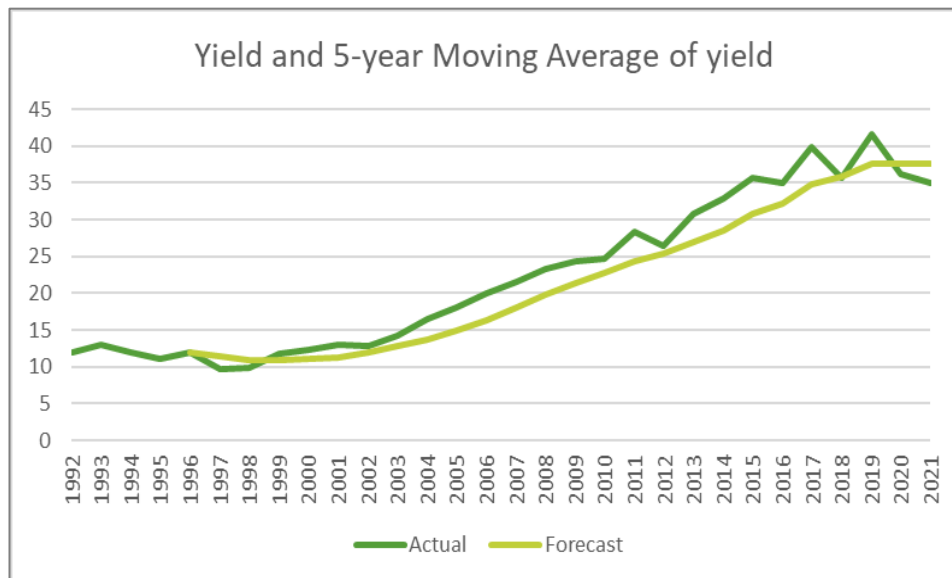


Figure 19: Yield and 5-year Moving Average Yield of Watermelon

5.2.7 Wheat

Wheat is a crucial crop in Uzbekistan as it is a major component of their diet, and the country has a long history of wheat cultivation, based on the data acquired from FAO, which is shown in Figure 20, the production of wheat was 1000,000 tonnes in 1992 and since then we are observing an increasing trend and from 2005 until 2021 the production of wheat exceeded 5,000,000 tonnes annually. Wheat is cultivated on a large scale, and it occupies a significant portion of the arable land in Uzbekistan,

according to Figure 20, On average 1,300,000 ha were allocated to wheat cultivation. Both rain-fed and irrigated agriculture contribute to wheat production.

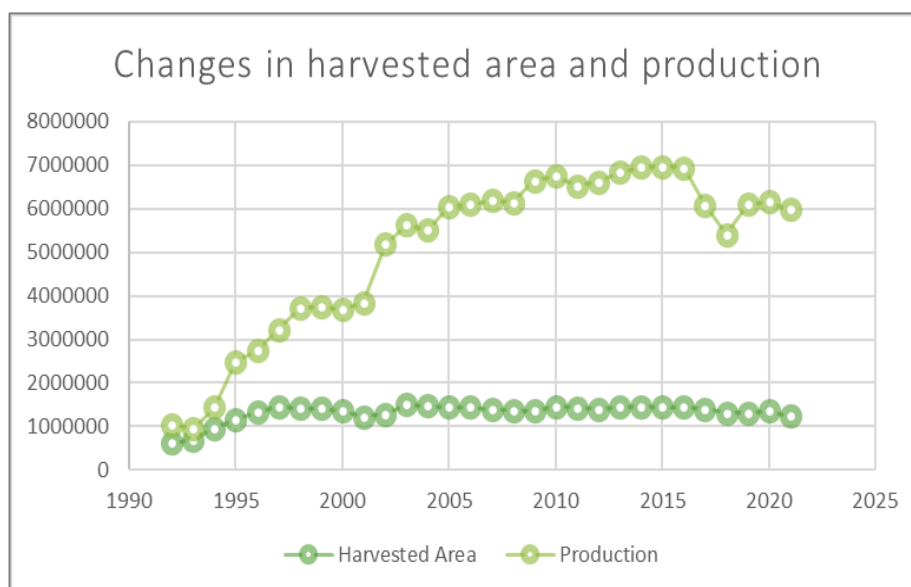


Figure 20: Changes in Harvested Area and Production of Wheat

Uzbekistan is one of the leading wheat-producing countries in Central Asia. Wheat is a staple food and Uzbekistan is both producer and consumer of wheat. The country generally produces enough wheat to meet its domestic demand, which is considered substantial, but there may be variations in production levels from year to year. Additionally, Uzbekistan may export or import wheat based on market conditions. Based on the data available on FAO they imported 1,620,111 tonnes with 286,436,000 dollars and 129,831 tonnes were exported with 26,752,000 dollars from 2012-2021 on average.

Uzbekistan's diverse climate, including arid and semi-arid regions, is suitable for wheat cultivation. The country experiences warm temperatures, and wheat is often grown in irrigated areas.

Figure 21, Illustrates the distribution of wheat production across the country which nearly 80 percent of the total production is in the southern parts of the Uzbekistan including Kashkdarya, Samarkhand, Surkhundarya, Ferghana, Andijon, Bukhoro, Jizakh, Tashkent and Namangan.

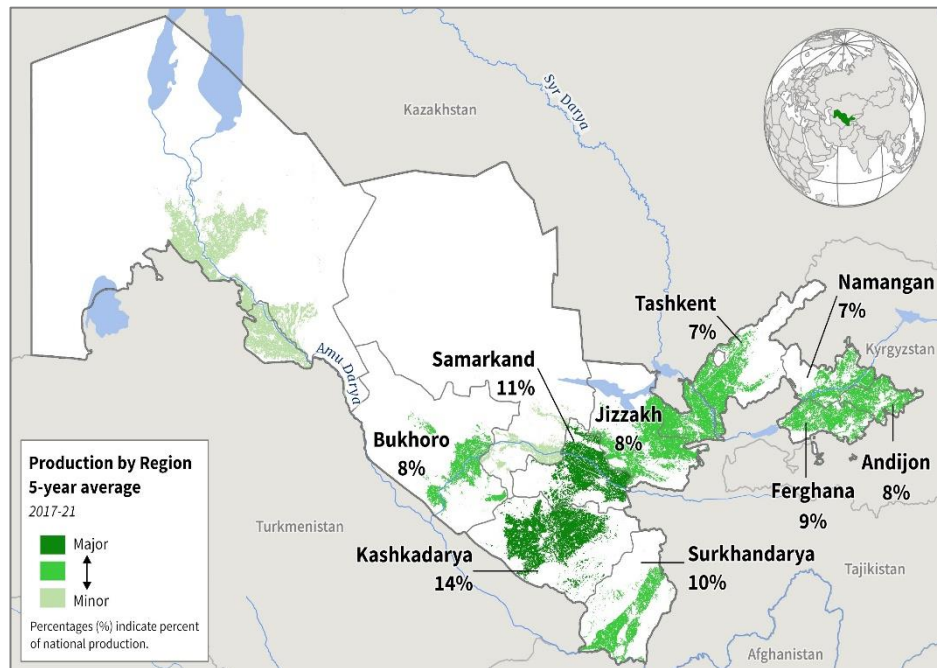


Figure 21: Distribution of Wheat Cultivation [41]

Figure 22, Shows the yield and 5-year moving average yield of wheat. The moving average reveals an increasing trend until 2009, followed by a steady pattern until 2021. Also, by examining the actual value of the yield we can observe the upward movement reaching to near $5 \frac{t}{ha}$ from $1.7 \frac{t}{ha}$ with the peak value of $5 \frac{t}{ha}$ occurred in 2009. It is notable that a sudden decrease has happened in 2016 dropping to $4 \frac{t}{ha}$ during the next two years despite this the overall trend was fluctuating in the last five years.

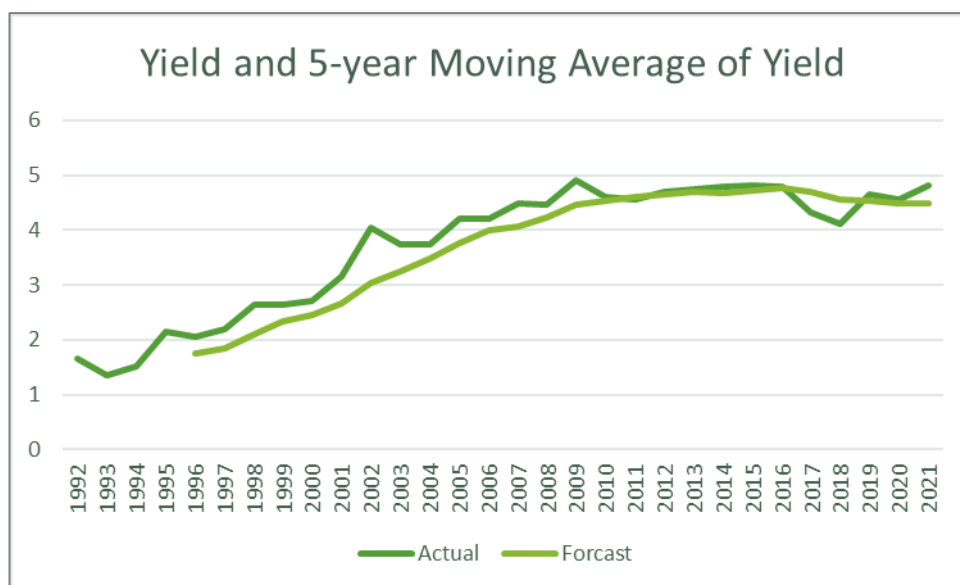


Figure 22: Yield and 5-year Moving Average of Yield

5.2.8 Arable Land Allocated to Each Crop

After a detailed examination of each individual crop, this section involves the computation of the required numerical data. This data related to allocated land for the crops under study is presented in Table 2. The average land allocation from 1992 to 2021 is calculated and the table also includes details such as the minimum, maximum and the total amount of land that has been utilized in this duration.

Table 2: The Average, Minimum, and Maximum Land-use

Crops	The average arable land (1992-2021) (ha)	Minimum arable land (ha)	Maximum arable land (ha)
Barley	116,966.7	49,200	290,900
Mize	45,107.03333	26,471	105,000
Potato	63,806.16667	42,930	98,169
Rice	88,204.13333	23,100	195,200
Tomato	49,295.86667	12,600	64,149
Watermelon	44,199.46667	29,608	73,800
Wheat	1,320,805.4	626,990	1,507,600
Total	1,728,384.767		

Utilizing the data provided in Table 2, we established both lower and upper bounds agricultural land can be allocated to each crop, serving as constraints for the initial land allocation model in the next chapter. Table 3 presents the specified lower and upper bounds for each crop.

Table 3: The Lower and Upper Bounds Land-use

Crops	The lower bounds (ha)	The upper bounds (ha)
Barley	40,000	300,000
Mize	20,000	120,000
Potato	30,000	120,000
Rice	20,000	220,000
Tomato	10,000	80,000
Watermelon	25,000	90,000
Wheat	600,000	1,600,000

Chapter 6

SOLUTION APPROACH

6.1 Markowitz Portfolio Application in Agriculture

In this study, the goal is to maximize the overall arable lands utilized for cultivating the seven crops that are under investigation. Therefore, here the objective is to optimize the arable land, the lifeblood of agricultural production. In the first model, we propose a LP Markowitz portfolio model to optimize the initial land-use of each crop at a given level of risk. Secondly, the initial optimized values acquired from the first model would be applied in the second model which will consider the production value of the crops and the objective function is to maximize the land allocation according to their production value.

This introduction to the application of the Markowitz portfolio model in agriculture explores the possibilities and challenges of adopting a quantitative approach to maximize the productivity of arable lands. At the end, we will delve into if this model can help agricultural stakeholders, from individual small farmers to large-scale agribusinesses, make informed decisions about land allocation, crop selection, and resource allocation. Moreover, we will examine the effect of risk parameter on land allocation by conducting a sensitivity analysis. At the end of this chapter, a LP model regarding the water consumption of each crop according to the diversified land will be provided.

6.2 The Mathematical Formulation for Land

6.2.1 Definitions

Index

i and j defined as the set of crops barley, maize, potato, rice, tomato, watermelon, and wheat ($i = 1, \dots, 7$) and ($j = 1, \dots, 7$).

Variables

L_i = The size of the arable land of crop i in hectares

μ_i = The average arable land has been used in 30 years for crop i in ha

C_{ij} = Covariance of two crops i and j

λ = Parameter of risk aversion of the decision maker

6.2.2 Mathematical Model

$$\text{Max} = \sum_{j=1}^7 \sum_{i=1}^7 \mu_i L_i - \lambda L_i^T C_{ij} L_i \quad (16)$$

Subject to:

$$\sum_{i=1}^7 L_i = 1,625,000 \text{ ha} \quad (17)$$

$$40,000 \leq L_1 \leq 300,000 \quad (18)$$

$$20,000 \leq L_2 \leq 120,000 \quad (19)$$

$$30,000 \leq L_3 \leq 120,000 \quad (20)$$

$$20,000 \leq L_4 \leq 220,000 \quad (21)$$

$$10,000 \leq L_5 \leq 80,000 \quad (22)$$

$$25,000 \leq L_6 \leq 90,000 \quad (23)$$

$$600,000 \leq L_7 \leq 1,600,000 \quad (24)$$

Equation (16) represents the objective function aiming for optimization of arable land allocated to each crop i ($i = 1, \dots, 7$).

Equation (17) is the initial constraint indicating that the sum of assigned weights for each crop should equal 1,625,000 hectares based on the predetermined allocations.

Equations (18)-(24) show the minimum and maximum land required for the cultivation of crop i ($i = 1, \dots, 7$).

6.2.3 Numerical Calculation

In the computations the initial step involves determining the average arable land used for each crop. This calculation is according to data set available in FAO spanning from 1992 to 2021. As previously specified, let μ_i denote as the average utilized land for crop i and L_i represent the size of arable land dedicated to crop i . In the context of portfolio optimization, L_i is treated as the weight assigned to each crop. Hence, the estimation of μ_i and L_i are derived as follows.

$$\mu_i = [116,966.7 \ 45,107 \ 63,806.17 \ 88,204.13 \ 49,295.87 \ 44,199.47 \ 1,320,805.4]$$

$$L_i = [300,000 \ 20,000 \ 30,000 \ 20,000 \ 80,000 \ 25,000 \ 1,150,000]$$

6.2.4 Solution

The mathematical model developed in the 6.2 section is solved utilizing MATLAB software. The code written in the MATLAB environment is depicted in Figure 23.

```

InitialLand.m x markowitz.m x +
1 mu_i = [116966.7 45107 63806.17 88204.13 49295.87 44199.47 1320805.4];
2 Cov_ij = [4656367932.96 960911021.11 -412083136.57 2289373793.83 -742870705.21 252732376.23 -11991924865.65;
3 960911021.11 367452227.96 -80176701.76 666375296.44 -197522115.79 60479242.984 -3660203468.19;
4 -412083136.57 -80176701.76 280271787.04 -512582675.37 175643980.06 6988394.20 1161016995.72;
5 2289373793.83 666375296.44 -512582675.37 3086604789.02 -864432533.74 130256549.35 -5720309434.12;
6 -742870705.21 -197522115.79 175643980.06 -864432533.74 320310753.36 -16171953.04 2076013198.33;
7 252732376.23 60479242.984 6988394.20 130256549.35 -16171953.04 82680813.91 -736253625.61;
8 -11991924865.65 -3660203468.19 1161016995.72 -5720309434.12 2076013198.33 -736253625.61 44608408828.25];
9 L_i = [300000 20000 30000 20000 80000 25000 1150000];
10 lambda = 0.000001;
11 fun = @(L_i) -sum((mu_i * L_i') - lambda * (L_i * Cov_ij * L_i'));
12 % s.t.
13 Aeq = ones(1, 7);
14 beq = 1625000;
15 lb = [40000; 20000; 30000; 20000; 10000; 25000; 600000];
16 ub = [300000; 120000; 120000; 220000; 80000; 90000; 1600000];
17
18 % Solution
19 options = optimoptions('fmincon', 'Display', 'iter');
20 L_i_optimal = fmincon(fun, L_i, [], [], Aeq, beq, lb, ub, [], options);
21 disp('Optimal Portfolio Allocation Sizes (hectares):');
22 disp(['L_1: ' num2str(L_i_optimal(1)) ' hectares']); % Barley
23 disp(['L_2: ' num2str(L_i_optimal(2)) ' hectares']); % Maize
24 disp(['L_3: ' num2str(L_i_optimal(3)) ' hectares']); % Potato
25 disp(['L_4: ' num2str(L_i_optimal(4)) ' hectares']); % Rice
26 disp(['L_5: ' num2str(L_i_optimal(5)) ' hectares']); % Tomato
27 disp(['L_6: ' num2str(L_i_optimal(6)) ' hectares']); % Watermelon
28 disp(['L_7: ' num2str(L_i_optimal(7)) ' hectares']); % Wheat
29

```

Figure 23: Initial Arable Land Code in MATLAB

In Figure 23, The following procedure was executed: Initially, the given data is specified based on the computations. The mean vector for crops, denoted as μ_i ($i = 1, 2, \dots, 7$) and variance-covariance vector of crops i and j is defined as Cov_{ij} ($i = 1, 2, \dots, 7$), ($j = 1, 2, \dots, 7$). The variable L_i represents the size of arable land for crop i which is considered as the weight of each crop in the portfolio optimization. Initially, the risk aversion parameter, λ , is set to be 10^{-6} . The objective function is defined in line 14. Subsequently, constraints are introduced, where 'Aeq' and 'beq' enforce the condition that the sum of weights (arable land size for crop i) should not exceed the total agricultural land available. Vectors 'lb' and 'ub' are vectors establish the lower and upper bounds on the arable land allocated to each crop which each vector is specified for crop i .

The optimization is executed using MATLAB's optimization function 'fmincon'. Consequently, the outcomes related to the optimized allocation of arable land for each crop are as follows.

Table 4: Optimal Allocated Land to Each Crop

$L_i (i = 1, 2, \dots, 7)$	Crop type	Optimal Portfolio Allocation (ha)
L_1	Barley	300,000
L_2	Maize	120,000
L_3	Potato	120,000
L_4	Rice	220,000
L_5	Tomato	80,000
L_6	Watermelon	90,000
L_7	Wheat	695,000

6.3 The Mathematical Formulation for Production Value

In this context, the estimation of land use is based on maximizing production value through a step-by-step calculation. Initially, a land use model was formulated in the preceding section, yielding optimized land allocations for each crop i . Subsequently, a second model is developed to determine the optimized arable land use for each crop given their price and yield value.

The production amount for a particular crop can be acquired using the following formula:

Production Value = Quantity Produced * Market Price

Production Quantity(t) = Area of Land (ha) * Crop Yield ($\frac{t}{ha}$)

To accomplish this goal, it is imperative to estimate the prices associated with each crop to facilitate the computation of production value.

The new set of variables are as follows.

$\mu(y)_i$ = The average yield of crop i in 3 years ($\frac{t}{ha}$).

$\mu(p)_i$ = The average price of crop i in 3 years (UZS).

L_i = The optimized harvesting area.

$\lambda = 0.000001$

6.3.1 The Mathematical Formulation

$$\text{Max } z = \sum_{i=1}^7 \sum_{j=1}^7 \mu(y)_i \mu(p)_i L_i - \lambda L_i^T C_{ij} L_i \quad (25)$$

Subject to,

$$\sum_{i=1}^7 L_i = 1,625,000 \text{ ha} \quad (26)$$

$$40,000 \leq L_1 \leq 300,000 \quad (27)$$

$$20,000 \leq L_2 \leq 120,000 \quad (28)$$

$$30,000 \leq L_3 \leq 120,000 \quad (29)$$

$$20,000 \leq L_4 \leq 220,000 \quad (30)$$

$$10,000 \leq L_5 \leq 80,000 \quad (31)$$

$$25,000 \leq L_6 \leq 90,000 \quad (32)$$

$$600,000 \leq L_7 \leq 1,600,000 \quad (33)$$

In the following table, data related to yield for each crop i is collected from FAO data set.

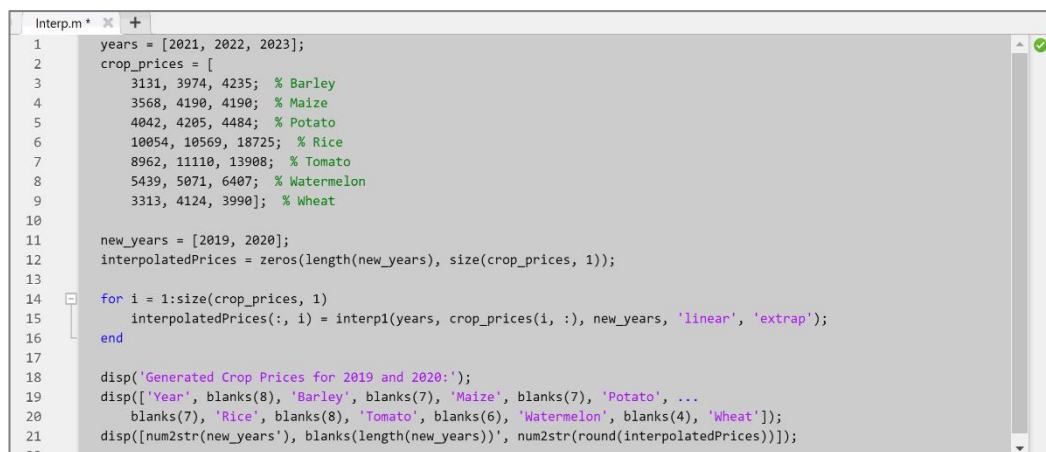
Table 5: The Average Yield Acquired From FAO.

Crops	Average yield 2019 ($\frac{t}{ha}$)	Average yield 2020 ($\frac{t}{ha}$)	Average yield 2021 ($\frac{t}{ha}$)	Tot. Average ($\frac{t}{ha}$)
Barley	1.45	1.48	1.25	1.39
Maize	9.7	9.76	9.07	9.51
Potato	34.64	33.95	33.47	34.02
Rice	4.85	6.19	6.84	5.96
Tomato	36.01	33.4	36.45	35.29
Watermelon	41.63	36.15	34.99	37.59
Wheat	4.65	4.56	4.82	4.68

I gathered price data for each crop from [43] covering the years 2021-2023. However, precise prices for the years 2019 and 2020 were unavailable. Consequently, I employed

an interpolation technique, assuming a linear trend between the available dataset points. Utilizing the interpolation function in MATLAB and referencing the available data from 2021 to 2023, I estimated the prices of crop i for the years 2019 and 2020.

In Figure 24, the MATLAB code is presented for the mentioned procedure. Initially, I defined a matrix containing crop prices for the years 2021, 2022, and 2023. A vector, 'new_years' is created to encompass the years 2019 and 2020 for which estimated prices were sought. The matrix 'interpolatedPrices' was initialized with zeros that each row representing the interpolated prices for all crops in a specific year. Subsequently, a 'for' loop iterates through each row of crop prices, utilizing the 'interp1' function for linear interpolation of each crop's prices between the years 2021, 2022, and 2023. The resulting interpolated prices for the 'new_years' (2019 and 2020) are then stored in the 'interpolatedPrices' matrix.

The image shows a screenshot of a MATLAB script editor window titled 'Interp.m'. The code defines a matrix 'crop_prices' for years 2021, 2022, and 2023 for seven crops: Barley, Maize, Potato, Rice, Tomato, Watermelon, and Wheat. It then defines 'new_years' as [2019, 2020] and initializes an 'interpolatedPrices' matrix. A for loop uses 'interp1' to interpolate prices for each crop. Finally, lines 18-21 use 'disp' to format and display the output, including crop names and rounded prices for the new years.

```
1 years = [2021, 2022, 2023];
2 crop_prices = [
3     3131, 3974, 4235; % Barley
4     3568, 4190, 4190; % Maize
5     4042, 4205, 4484; % Potato
6     10054, 10569, 18725; % Rice
7     8962, 11110, 13908; % Tomato
8     5439, 5071, 6407; % Watermelon
9     3313, 4124, 3990]; % Wheat
10
11 new_years = [2019, 2020];
12 interpolatedPrices = zeros(length(new_years), size(crop_prices, 1));
13
14 for i = 1:size(crop_prices, 1)
15     interpolatedPrices(:, i) = interp1(years, crop_prices(i, :), new_years, 'linear', 'extrap');
16 end
17
18 disp('Generated Crop Prices for 2019 and 2020:');
19 disp(['Year', blanks(8), 'Barley', blanks(7), 'Maize', blanks(7), 'Potato', ...
20     blanks(7), 'Rice', blanks(8), 'Tomato', blanks(6), 'Watermelon', blanks(4), 'Wheat']);
21 disp([num2str(new_years), blanks(length(new_years)), num2str(round(interpolatedPrices))]);
22
```

Figure 24: Interpolation of Prices MATLAB Code

The lines 18-21, Displays a header for the output, generated crop prices for the years 2019 and 2020, including the names of crops and the corresponding prices, which were interpolated using linear interpolation. The 'function' is used to round the

interpolated prices to the nearest integer for display. The results related to the estimation of prices is shown in the table below.

Table 6: The Average Estimated Prices for Each Crop

Crops	Average Annual prices 2019 (UZS)	Average Annual prices 2020 (UZS)	Average Annual prices 2021(UZS)	Tot. Average (UZS)
Barley	1,445	2,288	3,131	2,288
Maize	2,324	2,946	3,568	2,946
Potato	3,716	3,879	4,042	3,879
Rice	9,024	9,539	10,054	9,539
Tomato	4,666	6,814	8,962	6,814
Watermelon	6,175	5,807	5,439	5,807
Wheat	1,691	2,502	3,313	2,502

Table 7: The Average Price, Yield, and Optimized Land for Each Crop

Items	Barley	Maize	Potato	Rice	Tomato	Water-melon	Wheat
Price	2,288	2,946	3,879	9,539	6,814	5,807	2,502
Yield	1.39	9.51	34.02	5.96	35.29	37.59	4.68
Land	300,000	120,000	120,000	220,000	80,000	90,000	625,000

In Table 7, comprehensive results are presented, encompassing the average estimated prices and average yields for the years 2019 to 2021. Additionally, the optimized land allocations obtained from the first model have been incorporated into this table. The total production value which has no unit will be computed as follows.

$$\text{Tot. P} = \sum_{i=1}^7 P_i Y_i L_i = 79,680,189,300 \quad (34)$$

6.3.2 Numerical Calculations and Solution

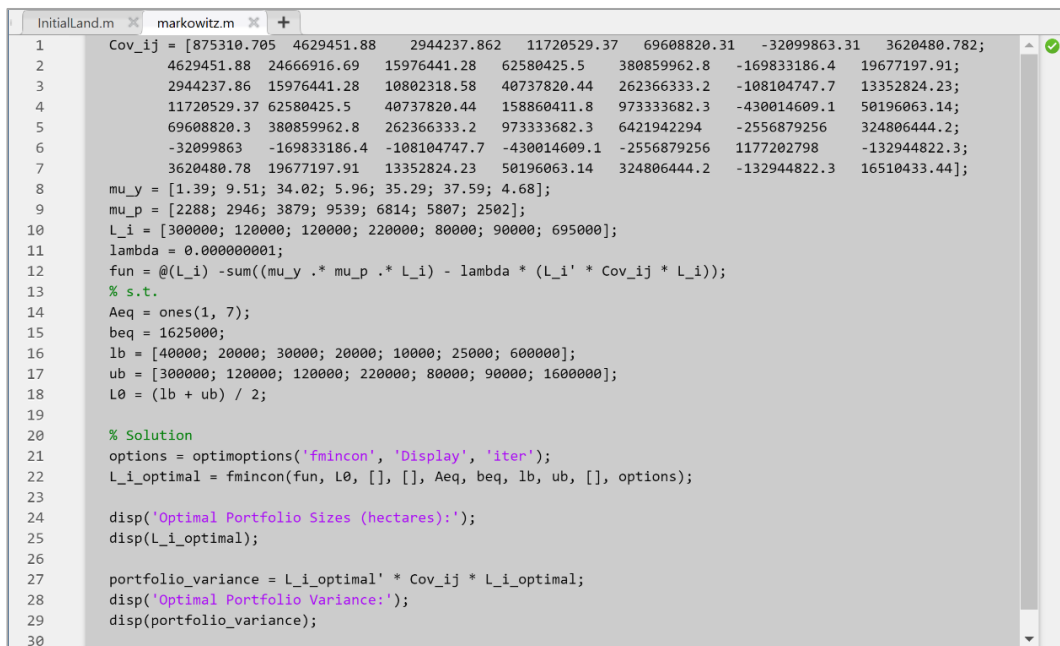
Same as the previous model, we developed the code to solve this one also in MATLAB software. Figure 25, Shows the mathematical code regarding the production value model. As shown in the Figure 25, First we described the given data which are mean of prices and yield from years 2019-2021 plus the optimized land values acquired from

previous model for crop i . I also calculated the variance covariance matrix of sum product of price and yield.

$$mean_p = [2288 \ 2946 \ 3879 \ 9539 \ 6814 \ 5807 \ 2502]$$

$$mean_Y = [1.39 \ 9.51 \ 34.02 \ 5.96 \ 35.29 \ 37.59 \ 4.68]$$

$$Land = [300000 \ 120000 \ 120000 \ 220000 \ 80000 \ 90000 \ 695000]$$



```

1  Cov_ij = [875310.705  4629451.88  2944237.862  11720529.37  69608820.31  -32099863.31  3620480.782;
2  4629451.88  24666916.69  15976441.28  62580425.5  380859962.8  -169833186.4  19677197.91;
3  2944237.86  15976441.28  10802318.58  40737820.44  262366333.2  -108104747.7  13352824.23;
4  11720529.37  62580425.5  40737820.44  158860411.8  973333682.3  -430014609.1  50196063.14;
5  69608820.3  380859962.8  262366333.2  973333682.3  6421942294  -2556879256  324806444.2;
6  -32099863  -169833186.4  -108104747.7  -430014609.1  -2556879256  1177202798  -132944822.3;
7  3620480.78  19677197.91  13352824.23  50196063.14  324806444.2  -132944822.3  16510433.44];
8
9  mu_y = [1.39; 9.51; 34.02; 5.96; 35.29; 37.59; 4.68];
10 mu_p = [2288; 2946; 3879; 9539; 6814; 5807; 2502];
11 L_i = [300000; 120000; 120000; 220000; 80000; 90000; 695000];
12 lambda = 0.000000001;
13 fun = @(L_i) -sum((mu_y .* mu_p .* L_i) - lambda * (L_i' * Cov_ij * L_i));
14 % s.t.
15 Aeq = ones(1, 7);
16 beq = 1625000;
17 lb = [40000; 20000; 30000; 20000; 10000; 25000; 600000];
18 ub = [300000; 120000; 120000; 220000; 80000; 90000; 1600000];
19 L0 = (lb + ub) / 2;
20
21 % Solution
22 options = optimoptions('fmincon', 'Display', 'iter');
23 L_i_optimal = fmincon(fun, L0, [], [], Aeq, beq, lb, ub, [], options);
24 disp('Optimal Portfolio Sizes (hectares):');
25 disp(L_i_optimal);
26
27 portfolio_variance = L_i_optimal' * Cov_ij * L_i_optimal;
28 disp('Optimal Portfolio Variance:');
29 disp(portfolio_variance);
30

```

Figure 25: MATLAB Code of Optimized Arable Land Based on Production Value

Here, λ the risk aversion parameter is considered to be 10^{-6} to begin with. Then, in line 14 the objective function of the model is defined. Subsequently, constraints are established: 'Aeq' and 'beq' refers to the first constraint that the sum of weights (the size of arable land of crop i) should be less than or equal to the total agricultural land available in the country. 'lb' and 'ub' are vectors define the lower and upper bounds on the arable land that can be allocated to crop i which each vector is specified for crop i .

The optimization is executed using MATLAB's optimization function 'fmincon' and then displays the optimal weights, expected portfolio return, and portfolio variance.

Therefore, the results regarding the optimized arable land allocated to crop i according to their production value are as follows.

Table 8: Allocated Area Based on Production Value

$L_i (i = 1, 2, \dots, 7)$	Crop type	Allocated land (ha)
L_1	Barley	300,0000
L_2	Maize	20,000
L_3	Potato	120,000
L_4	Rice	20,000
L_5	Tomato	10,000
L_6	Watermelon	90,000
L_7	Wheat	1,065,000

The total expected portfolio yield for the optimizes allocation approximately would be $5.3 * 10^{10} \frac{t}{ha}$ which is obtained by the summation of the average yield of each crop to the optimized allocated land. The optimal portfolio variance representing the measure of risk is $1.08 * 10^{19}$.

6.4 Sensitivity Analysis

In the following section, we are conducting a sensitivity analysis by considering different lambda values. By decreasing lambda values, we compare the total optimal value, optimal portfolio yield and optimal portfolio variance. This action will help decision-makers to consider various options and make the best decision at a given level of risk.

Table 9: Allocated Area to Each Crop in Different Lambda Values

lambda value	The area allocated to each crop (ha)						
	Barley	Maize	Potatoes	Rice	Tomatoes	Water-melons	Wheat
10^{-6}	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
10^{-7}	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
10^{-8}	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
10^{-9}	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
$9 * 10^{-10}$	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
$8 * 10^{-10}$	300,000	20,000	120,000	20,000	10,000	90,000	1,065,000
$7 * 10^{-10}$	300,000	67,000	120,000	20,000	10,000	90,000	1,018,000
$6 * 10^{-10}$	300,000	120,000	120,000	20,000	10,000	90,000	965,000
$5 * 10^{-10}$	300,000	120,000	120,000	20,000	10,000	90,000	965,000
$4 * 10^{-10}$	300,000	120,000	120,000	20,000	10,000	90,000	965,000
$3 * 10^{-10}$	300,000	120,000	120,000	20,000	10,000	90,000	965,000
$2 * 10^{-10}$	300,000	120,000	120,000	20,000	10,000	90,000	965,000
10^{-10}	300,000	120,000	120,000	112,710	10,000	90,000	872,290

Some insights regarding the results achieved in Table 9 are as follows:

- The land allocation for each crop remains quite consistent across different lambda values.
- The allocations for Barley, Potatoes, Tomatoes, and Watermelons remains without any change for all lambda values ranging from 10^{-6} to 10^{-10} .
- It can be concluded that lambda continues to have a limited impact on the land allocation.
- The consistent land allocation across different lambda values suggests that, in our specific optimization problem case study, lambda might not have a significant impact on the trade-off between maximizing production value and minimizing portfolio risk.

Table 10: The Optimal Values Generated with Different Lambda Values.

lambda value	Optimal value	Production Value	Optimal portfolio variance
10^{-6}	$-1.0784 * 10^{13}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
10^{-7}	$-1.0307 * 10^{12}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
10^{-8}	$-5.5362 * 10^{10}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
10^{-9}	$4.2171 * 10^{10}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
$9 * 10^{-10}$	$4.3255 * 10^{10}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
$8 * 10^{-10}$	$4.4338 * 10^{10}$	$5.3008 * 10^{10}$	$1.0837 * 10^{19}$
$7 * 10^{-10}$	$4.6088 * 10^{10}$	$5.3774 * 10^{10}$	$1.0989 * 10^{19}$
$6 * 10^{-10}$	$4.7936 * 10^{10}$	$5.4639 * 10^{10}$	$1.1171 * 10^{19}$
$5 * 10^{-10}$	$4.9054 * 10^{10}$	$5.4639 * 10^{10}$	$1.1171 * 10^{19}$
$4 * 10^{-10}$	$5.0171 * 10^{10}$	$5.4639 * 10^{10}$	$1.1171 * 10^{19}$
$3 * 10^{-10}$	$5.1288 * 10^{10}$	$5.4639 * 10^{10}$	$1.1171 * 10^{19}$
$2 * 10^{-10}$	$5.2405 * 10^{10}$	$5.4639 * 10^{10}$	$1.1171 * 10^{19}$
10^{-10}	$5.7174 * 10^{10}$	$5.8824 * 10^{10}$	$1.6505 * 10^{20}$

By examining the outcomes achieved in Table 10 obtained across various lambda values, certain observations and insights come to light:

- As lambda decreases from 10^{-6} to 10^{-10} , the optimal value of the objective function increases. This implies that as the impact of variance, which is controlled by lambda reduces, the algorithm tends to prioritize maximizing the production value.
- The production value remains constant at $5.3008 * 10^{10}$ for lambda values ranging from 10^{-6} up to $8 * 10^{-10}$ also it is constant at $5.4639 * 10^{10}$ for lambda values spanning from $6 * 10^{-10}$ until $2 * 10^{-10}$. This insight indicates that changes in lambda are not affecting the production value significantly.
- The optimal portfolio variance remains constant at $1.0837 * 10^{19}$ for lambda values ranging from 10^{-6} up to $8 * 10^{-10}$ also it is constant at $1.1171 * 10^{19}$ for lambda values spanning from $6 * 10^{-10}$ until $2 * 10^{-10}$. This indicates that changes in lambda are not affecting the risk (variance) associated with the portfolio. The optimization is likely driven more by maximizing the production value.

- The negative optimal value of the objective function indicates a maximization problem. As lambda decreases, the objective function becomes more positive, suggesting a trend toward maximizing the production value.
- The case with $\lambda = 10^{-10}$ stands out. It results in the highest optimal value, production value, and portfolio variance. This extreme case indicates that, in this optimization setup, minimizing the impact of the covariance term significantly contributes to maximizing the production value.
- It can be concluded that there is a trade-off between production value and portfolio risk. Higher lambda values prioritize minimizing portfolio risk, while lower lambda values emphasize maximizing the production value with potentially higher risk.

6.5 The Mathematical Model for Water consumption

In the following section, I provide a linear programming model for optimizing the water consumption of each specified crop according to the land allocated to them.

6.5.1 Definitions and Variables

L_i = The amount of land area allocated to crop i

Iw_i = Denoted as irrigation water requirement for each crop

Sw = The amount of surface water resources available

Gw = The amount of ground water resources available

α = The efficiency ratio for irrigation from surface water

β = The efficiency ratio for irrigation from ground water resources

6.5.2 Mathematical Formulation

Mathematical linear programming model in water consumption

$$\text{Min } Z = \sum_{i=1}^7 Iw_i * x_i \quad (35)$$

Subject to,

$$\sum_{i=1}^7 L_i = 1,625,000 \text{ ha} \quad (36)$$

$$\sum_{i=1}^7 Iw_i * x_i \leq \alpha * Sw + \beta * Gw \quad (37)$$

Equation (35) expresses the objective function, representing the cumulative water consumption for cultivating each crop on their allocated land areas.

The initial constraint in equation (36) enforces the limitation that the combined land allocation must not surpass the predetermined total specified in the initial land model.

Equation (37) encapsulates the constraint that the total water consumption should not exceed the aggregate of available surface water and groundwater resources, considering their respective irrigation efficiency ratios.

The lack of comprehensive information on water management from sources like FAO, Aqua stat, and Uzbekistan statistics, we were unable to successfully run the model and obtain results.

While I did have data on the overall waterbody area in Uzbekistan, specified information on the available groundwater and surface water for agricultural use were missing. Additionally, details on the amount of water utilized for irrigation from Amu Darya and Syr Darya for various crops were not provided. Due to the uncertainty of water usage, the exact quantity of irrigated water for individual crops in Uzbekistan, as well as the influence of weather conditions on water consumption, remained unknown. One assumption is that this factor might vary in different years.

It's crucial to identify the irrigation methods employed for each crop and measure the efficiency ratio of using surface water and groundwater resources. Unfortunately, due to these data limitations and time constraints, I couldn't execute the model and generate results in the last attempt.

Chapter 7

CONCLUSION

The main goal of this study is to optimize the agricultural land-use for cultivation of seven crops including barley, maize, potato, rice, tomato, watermelon, and wheat in Uzbekistan. To achieve this goal, we formulize the problem through two LP models utilizing portfolio optimization.

The first model, referred to as the initial land-use model, is centered around maximizing the land allocation problem according to the historical data acquired from FAO dataset. Thus, to apply the portfolio optimization method we assigned an initial weight to each crop based on the FAO data achieved and solved the model via MATLAB software. Then the outcomes obtained from the initial model were incorporated into the next model, which aims to maximize the land-use based on the production value of the crops. To gain this goal, we estimated the market price of each crop for years 2019-2021. The developed model was solved using MATLAB. By comparing the results, we observed that the most significant cultivation area assigned to wheat, maize, and potato respectively.

Subsequently, we conducted a sensitivity analysis by varying lambda values to assess the effect of reducing the risk parameter value on the optimal value and land allocation. Upon decreasing the lambda value down to 10^{-10} and the results revealed a steady land allocation pattern for barley, potato, and watermelon while the amount of maize

and rice increased, and wheat land-use decreased. The optimal value rose steadily with decreasing lambda, and it reached to the highest value at 10^{-10} point. Consequently, we inferred that alteration in lambda had minimal impact on the land allocation, and the main emphasis was on increasing the production value. Finally, we proposed a model to control the water consumption of each crop based on the land allocated, given the sensitivity of Uzbekistan to water resources.

- Research Limitations

The research has faced limitations regarding the lack of reliable data in crucial domain of the land-use data in Uzbekistan which some of them were inaccurate. Additionally, the data related to the water sources also were not sufficient. Inaccurate and unreliable information in these key areas poses a significant challenge, for following the research. The accuracy and effectiveness of the research outcomes are inevitably constrained by the unavailability of robust data in these essential aspects.

- For Future Research Direction

Future research first should focus on enhancing the reliability and accuracy of data in Uzbekistan. This could involve collaboration with reliable sources to acquire updated information. Addressing the insufficiency of data related to water sources, a comprehensive analysis of water resources in Uzbekistan could be a future direction. This might involve studying both surface and groundwater availability, evaluating irrigation efficiency ratios, and considering climate patterns. Also, alternative optimization techniques and algorithms can be explored to compare the efficacy of different methods in achieving sustainable land-use and water conservation goals.

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APPENDICES

Appendix A: MATLAB Results for DJI Allocation

```
Command Window

Iter F-count      f(x)  Feasibility  First-order  Norm of
          step                    optimality
0         3  -1.525467e+02  0.000e+00   7.103e+00
1         6  -1.557915e+02  0.000e+00   6.395e+01  7.036e-01
2         9  -1.558178e+02  2.220e-16   2.932e+01  2.997e-03
3        12  -1.567933e+02  0.000e+00   9.362e+00  1.938e-01
4        15  -1.568162e+02  0.000e+00   1.168e-03  3.479e-02
5        18  -1.568162e+02  1.110e-16   6.748e-05  2.148e-04
6        21  -1.568162e+02  0.000e+00   3.213e-06  5.599e-07

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal Weights:
    0.1661
    0.8339

Optimal Portfolio Return:
    274.8559

Optimal Portfolio Variance:
    1.1804e+03

fx >>
```

Appendix B: MATLAB Results for the Initial Land Model

```
Command Window

Iter F-count      f(x)  Feasibility  First-order  Norm of
          step                optimality    step
  31    256  1.539952e+16  0.000e+00  2.094e+05  1.332e-04
  32    264  1.539952e+16  0.000e+00  1.056e+05  6.694e-05
  33    272  1.539952e+16  0.000e+00  5.314e+04  3.364e-05
  34    280  1.539952e+16  0.000e+00  2.651e+04  1.690e-05

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
Optimal Portfolio Allocation Sizes (hectares):
L_1: 300000 hectares
L_2: 120000 hectares
L_3: 120000 hectares
L_4: 220000 hectares
L_5: 80000 hectares
L_6: 90000 hectares
L_7: 695000 hectares
fx >>
```

Appendix C: MATLAB Results for Land Use Based on Production Value.

```

Command Window
28    232    7.815580e+13    0.000e+00    2.704e+08    1.123e+04
29    240    7.814174e+13    2.328e-10    2.704e+08    5.616e+01
30    248    7.814156e+13    4.657e-10    2.704e+08    2.456e-01

Iter F-count      f(x)  Feasibility  First-order  Norm of
                    step
31    256    7.814109e+13    6.985e-10    2.703e+08    4.886e-01
32    264    7.810046e+13    4.657e-10    2.701e+08    2.334e+03
33    272    7.581649e+13    4.657e-10    2.531e+08    1.384e+05
34    280    7.580568e+13    4.657e-10    2.530e+08    6.920e+02
35    288    7.580563e+13    0.000e+00    3.336e+03    3.459e+00

Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

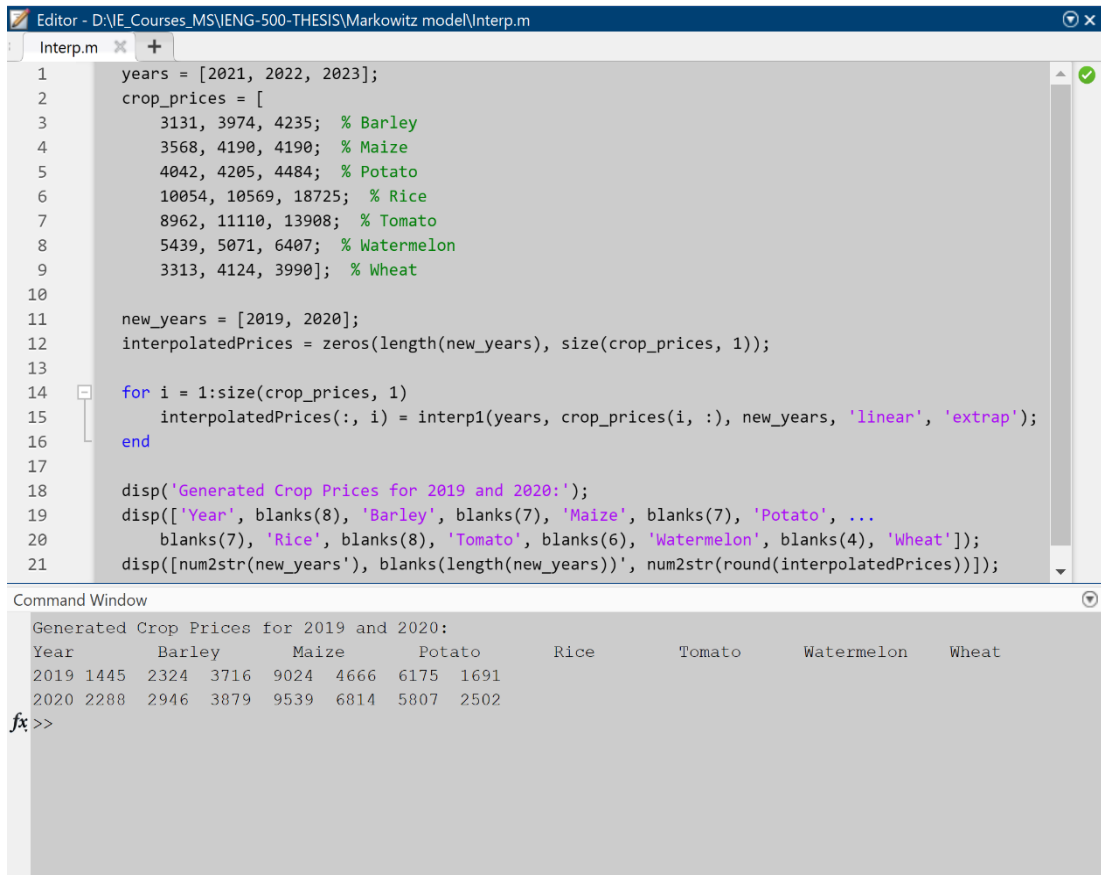
<stopping criteria details>
Optimal Portfolio Sizes (hectares):
1.0e+06 *

0.3000
0.0200
0.1200
0.0200
0.0100
0.0900
1.0650

Optimal Portfolio Variance:
1.0837e+19
fx >>

```

Appendix D: Price Estimation Results in MATLAB



The image shows a MATLAB Editor window with a script named 'Interp.m' and a Command Window below it. The script defines crop prices for 2021, 2022, and 2023, then uses linear interpolation to estimate prices for 2019 and 2020. The Command Window displays the output of the script, showing the generated crop prices for 2019 and 2020 for seven crops: Barley, Maize, Potato, Rice, Tomato, Watermelon, and Wheat.

```
1 years = [2021, 2022, 2023];
2 crop_prices = [
3     3131, 3974, 4235; % Barley
4     3568, 4190, 4190; % Maize
5     4042, 4205, 4484; % Potato
6     10054, 10569, 18725; % Rice
7     8962, 11110, 13908; % Tomato
8     5439, 5071, 6407; % Watermelon
9     3313, 4124, 3990]; % Wheat
10
11 new_years = [2019, 2020];
12 interpolatedPrices = zeros(length(new_years), size(crop_prices, 1));
13
14 for i = 1:size(crop_prices, 1)
15     interpolatedPrices(:, i) = interp1(years, crop_prices(i, :), new_years, 'linear', 'extrap');
16 end
17
18 disp('Generated Crop Prices for 2019 and 2020:');
19 disp(['Year', blanks(8), 'Barley', blanks(7), 'Maize', blanks(7), 'Potato', ...
20     blanks(7), 'Rice', blanks(8), 'Tomato', blanks(6), 'Watermelon', blanks(4), 'Wheat']);
21 disp([num2str(new_years'), blanks(length(new_years))', num2str(round(interpolatedPrices))]);
```

Command Window

```
Generated Crop Prices for 2019 and 2020:
Year      Barley      Maize      Potato      Rice      Tomato      Watermelon      Wheat
2019 1445 2324 3716 9024 4666 6175 1691
2020 2288 2946 3879 9539 6814 5807 2502
fx >>
```