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The dual adjustment approach with an application to the consumption function

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ABSTRACT

For theoretical and statistical reasons, it is important to decompose some series into dual components in order to understand their permanent and temporary movements as well as their dual co-movements. This study, therefore, aims to introduce the dual adjustment approach for the nonstationary macroeconomic variables. In line with this aim, the concept of common Hodrick-Prescott (HP) trend and a simple test for the existence of such relationship (Common HP trending) are also provided. The dual adjustment approach provides an alternative to the cointegration analysis for some cases, e.g., consumption function, by relaxing the implicit assumption of the singular adjustment in cointegration analysis. Our empirical results indicate that while personal consumption expenditure and disposable income are not cointegrated in the US over the period 1929–2017, these variables have a common HP trend. Additionally, it is shown that there is some evidence of dual adjustment in the behavior of US aggregate consumption.

1. Introduction

It is important to decompose some series into dual – permanent and transitory – components to understand their permanent and temporary movements as well as their dual co-movements. For instance, a *strong* version of Permanent Income Hypothesis requires that while permanent consumption is determined by permanent income, transitory consumption and transitory income are unrelated (Friedman, 1957). However, it is also plausible that permanent and transitory components of consumption may have separate co-movements and hence dual adjustment. For example, in a frequently used alternative model of Permanent Income Hypothesis, consumption not only responds to the changes in permanent income (as in a strong version of the hypothesis), but also to the changes in transitory income (see Flavin, 1993: 654–655 for more detail).

As we shall explain later, a standard cointegration relation *implicitly* imposes a common restriction such that the transitory components have *same particular* relationship with the permanent (trend) components and hence cointegration necessitates a *singular* adjustment. Without any doubt the idea of cointegration has greatly enhanced our understanding of the behavior of nonstationary variables and has opened the door to a large number of applications in macroeconomics and other areas. However, it may not be a feasible choice for modeling the dual co-movements of certain variables (e.g., personal consumption and income) due to its intrinsic singular nature. For instance, contrary to the expectations of some prominent researchers, Baltagi (2008) found no cointegration between consumption (C) and personal disposable income (Y) by using US annual data from 1950 to 1993.[1] To put it differently, this result is quite puzzling since the earlier results (e.g., Engle and Granger, 1987; Stock, 1988)[2] provided support for cointegrating relationship among C and Y in US.[3] However, this "puzzle" could be solved easily by considering a dual structure of the co-moving variables within a suitable analytical framework.

This study, therefore, aims to introduce the dual adjustment approach for the nonstationary macroeconomic variables. In line with this aim, the concept of common Hodrick-Prescott (HP) trend

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[1] Baltagi (2008: 368) pointed to the omitted variable problem as a possible cause of this result. Also see Enders (2015) for other possibilities for lack of cointegration.

[2] However, it should be noted here that there are some differences between these studies in terms of model specification, type of data and the sample period.

[3] However, the evidence of cointegration between C and Y is rather poor in Stock's (1988: 404) analysis (Engle-Granger tests reject the null hypothesis of no cointegration between total consumption and disposable income at the 10% significance level).

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– i.e., the series which are stationary around the common HP (Co-HP) trend – and a simple test for the existence of such relationship are also provided.[4] In doing so, this approach provides an alternative to the concept of cointegration for some cases, e.g., consumption function, by relaxing the implicit assumption of the singular adjustment in cointegration analysis.

There are also other critical reasons and motivations for utilizing the dual adjustment approach and Co-HP trending. First of all, the debate on the adequate representation of the nonstationary variables is not settled yet and an alternative perspective will be useful for modeling nonstationary variables with an explicit account of some critical criticisms of the prevailing dominant approach (unit roots and cointegration). For instance, despite intensive efforts the trouble for discriminating between (non-linear) deterministic and stochastic trends still prevails (see, e.g., Bierens, 2000 and Haldrup et al., 2013). Moreover, Bierens (2000) provided important criticisms on the dominant approach for motivating deterministic nonlinear co-trending, most of which are also applicable to Co-HP trending. Some of those criticisms are the implausibility of the assumption of unchanging structure of the economy and the limitations of the standard cointegration analysis (see, Bierens, 2000: 323, for more detail). Additionally, Berenguer-Rico and Gonzalo (2014) recently noted that the concept of integration and cointegration are not appropriate for non-linear worlds and they suggested new concepts for handling non-linearity.

Another related crucial issue to the previously mentioned criticisms is the characterization of the trend. As succinctly expressed by the distinguished econometrician P.C.B. Phillips in a frequently cited statement, the main challenge is that "no one really understands trends, even though most of us see trends when we look at economic data" (Phillips, 2003: C35).

Motivated by all these issues, we regard Co-HP trending and dual adjustment approach as an alternative to the dominant approach for modeling nonstationary variables. As emphasized by Granger (2004:18) "[i]t is rare for one model to be superior for all purposes … Different users will have different tastes, beliefs, and needs, and will prefer certain types of models"; therefore, like consumers, different users usually prefer to make a choice among alternatives. In line with this view, White and Granger (2011) have carefully shown that there are numerous distinct ways for defining and modeling trends in time series data. They also emphasized their concern for *the economics (as well as non-economics,* e.g., *politics) of trend* as follows: "we recommend that apparent trends, … cycles not be considered in isolation, that is, just in terms of the time series variable itself. Whenever possible, we recommend attempting to relate apparent trends to underlying phenomena, whether economic, demographic, political, legal, technological or physical" (White and Granger, 2011:33). They also encouraged *indirect approaches*, for instance, "one might define trends as 'that which trend filters remove'" (White and Granger, 2011:14), by pointing to the use of Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1980, 1997). This point was also made by Nelson and Plosser (1982: 154, footnote 16) more than three decades ago, while considering the application of signal extraction methods to unobserved components models.

Considering all these, this study utilizes the *indirect approach* by defining trends as that which HP trend filters remove. Despite its criticisms[5] HP trends have several ideal properties and they may work well in various situations (see, e.g., Mills, 2003). Additionally, the HP transformation is purely statistical, linear and "same" (judgment-free) for all series (Kydland and Prescott, 1990: 8). Moreover, De Jong and Sakarya (2016) obtained new results which provide "a justification for the use of the HP filter, as it implies that inference based on the cyclical component can be asymptotically correct" (De Jong and Sakarya, 2016: 314). However, special care should be taken for choosing the appropriate value for the smoothing parameter (see, e.g., Phillips and Jin, 2015).

Motivated by the above issues, this paper develops a dual adjustment approach that enables us to consider separate dual co-movements – for permanent and transitory components of observed variables – and hence the possibility of *dual adjustment*. By utilizing *the dominant common factors* framework (Granger et al., 2006), the concept of a common HP trend (or Co-HP trending) will be developed as an alternative perspective for modeling a pair of nonstationary variables. Put differently, the concept of Co-HP trending will allow us to link "apparent trends" to the "underlying phenomena", which is in line with the above mentioned recommendation of White and Granger (2011). This is important since it is difficult to characterize the trends of time series variables in isolation, particularly in univariate approaches like unit root testing as emphasized by Hansen (1995).[6] Therefore, Co-HP trending also has another advantage that unit root testing is not required.

In sum, this study proposes a simple approach for estimation and testing of dual adjustment with an application to the consumption function using the US data over the 1929–2017 period. Our empirical results indicate that while personal consumption and disposable income are not cointegrated in the US over the sample period, these variables have a common HP trend and this trend acts as an attractor in a way that consumption is stationary around the common permanent (Co-HP trend) component. Additionally, it is shown that the transitory components of consumption and income are significantly related and there is some evidence of dual adjustment in the behavior of US aggregate consumption. Thus, our results are in line with those studies (e.g., Campbell and Mankiw, 1989; Flavin, 1993) arguing that both Permanent Income Hypothesis and traditional consumption function play some roles in explaining the aggregate consumption behavior.

The rest of the study is organized as follows. Section 2 introduces the concept of dual adjustment, common HP Trend and its comparison to cointegration. Section 3 presents the empirical steps and applies it to the personal consumption expenditure and disposable income relation in the US. Finally, Section 4 concludes the paper.

2. Dominant common factors, common HP trend and the dual adjustment approach

Since the 1990s, there has been an enormous effort to enhance the early coverage of the idea of cointegration. The most notable

[4] It should be mentioned at the outset that the usage of the term "common HP trend" is conceptually different from the utilization of the idea of *common trend* that is used in the literature of multivariate detrending. For instance, Kozicki (1999) developed a methodology to detrend a group of time series under common trend restrictions – with particular focus on the business cycle research – without requiring the testing of the validity of those restrictions. In contrast, parallel to the standard Engle-Granger cointegration analysis, the existence of a non-spurious relationship (Common HP trending) needs to be tested in the dual adjustment approach.

[5] See, for example, Mills (2003) and the references cited therein for the criticisms of HP filter (Also see Phillips and Jin, 2015; Pedersen, 2001). As opposed to the criticisms of several researchers, in a recent paper by De Jong and Sakarya (2016), it is shown that HP filter is capable of removing a unit root from a time series.

[6] Hansen (1995) emphasized that unit root tests may have low power since they do not take into account the information provided by the related variable (covariate). Hence he offered a new test by including covariate to the standard unit root (ADF) tests. In doing so, he used the cointegration concepts for deriving unit root tests (see, Maddala and Kim, 1998: 231–233 for more detail).

one, in line with the aim of this study, among many developments is co-featuring (common features)[7] or co-movements (e.g., co-trending, co-cycling, co-breaking) among variables (see Centoni and Cubadda, 2011 and Mills, 2003, for a comprehensive review).[8] For instance, Chapman and Ogaki (1993) and Bierens (2000) developed the concepts of linear and nonlinear deterministic co-trending, respectively. However, these ideas have received scant attention in the related literature mainly because there is widespread discontent with the deterministic trends since the 1980s (see, for example, Enders, 2015: 182).

Even though the idea of Co-HP trending could be considered as a special case within the spirit of common features, it has important differences. For example, while a common trend and common cycle framework of Vahid and Engle (1993) is developed within the framework of unit roots and cointegration, the definitions of trend [as I(1) variables] and "cyclical" or "transitory" [first differences of I(1) variables][9] components are highly restrictive.[10] Additionally, as we shall explain later, for Co-HP trending and dual adjustment there is no need to impose restrictions on dual components.

Nevertheless, the dominant common factors framework of Granger et al. (2006) will be utilized to develop the concept of a common HP trend (or Co-HP trending) as an alternative perspective for modeling a pair of nonstationary variables. Thus, in order to understand the main motivation behind the concept of a common HP trend, it is, initially, useful to define the concept of the dominant property (DP). According to Granger et al. (2006: 44), the DP could be defined "as being in a component of process. If a series has several properties, it will be the DP that, in general will determine the relationship of the variable with others, and how it fits into models and equations".[11] They considered properties satisfying the following axioms:

- If X_t has the DP, then kX_t will have the DP for any non-zero value of k ($k \neq 0$). Similarly, if X_t does not have the DP, then kX_t will not have the DP.
- If both X_t and Y_t do not have the DP, then $S_t = X_t + Y_t$ will not have the DP.
- If X_t has the DP and Y_t does not have the DP, then $S_t = X_t + Y_t$ will have the DP.

It is also *generally* true that if both X_t and Y_t have the DP, then $S_t = X_t + kY_t$ will have the DP. However, there is a special case for the DPs in which they appear as *common factors* as follows

$$Y_t = kP_t + V_{1t} \tag{1}$$

$$X_t = P_t + V_{2t} \tag{2}$$

where P_t has the DP, V_{1t} and V_{2t} do not have the DP and, as before, k is a non-zero constant.

Considering the above axioms, in this *common* DP framework both X_t and Y_t have the DP, but $Z_t = Y_t - kX_t\ (= V_{1t} - kV_{2t})$ does *not* have the DP. In other words, *a particular* linear combination of two variables with common DPs may not have the DP. This result can be used to justify the ideas of several important topics including cointegration and co-trending (Granger et al., 2006). For instance, if DP is I(1) [and V_{1t} and V_{2t} are I(0)] then *X* and *Y* are said to be cointegrating. Similarly, if DP is a trend then X and Y are said to be co-trending.[12]

Considering the aim of this study, it is instructive to provide more detail on the cointegration (the benchmark case) within the *common* DP framework. For instance, if we consider Granger's (1993: 312) common factor representation, P_t is the permanent (or long-memory) component and V_{1t} and V_{2t} are the transitory (or short-memory) components. Granger (1993) stated that *a particular* linear combination of two variables (X_t and Y_t), $Y_t - kX_t\ (Z_t = 0)$ is an attractor and could be viewed as a (long-run) equilibrium between two stochastic variables; i.e., the process of X_t and Y_t will tend towards a point on the attractor. Clive W. J. Granger has noted in many of his writings that cointegration will only occur if two series have the "common factor" representation as stated above (see, for example, Granger and Lee, 1989; Gonzalo and Granger, 1995). However, one important point, which is neglected in the literature, is that the above common factor restriction on the permanent (dominant) component requires that the transitory component should also have *same particular* relationship. That is, since $Z_t = Y_t - kX_t = V_{1t} - kV_{2t}$, permanent and transitory components are assumed to be related (or "adjusted") with the same parameter (k), over the short and long time period. Thus, the standard cointegration analysis imposes the implicit assumption of the *singular adjustment* on two different components. We can explain this point in more detail with an example from the consumption function.

Let's consider a simplified version of Permanent Income Hypothesis (PIH) as follows (see Stock, 1988: 402),

$$C_t = kY_t^P + \nu_{Ct} \tag{3}$$

$$Y_t = Y_t^P + \nu_{Yt} \tag{4}$$

where *C* is consumption and *Y* is disposable income, Y^P is permanent disposable income and ν_C and ν_Y are transitory consumption and income, respectively.

Note that, in this simple set-up, current consumption comprises of permanent consumption and transitory consumption ($C = C^P + \nu_C$)[13] and permanent consumption is assumed to be proportional to the permanent disposable income ($C^P = kY^P$). In other words, *C* and *Y* share a common (stochastic) trend. Stock (1988: 401) noted that "since the PIH implies that consumption and disposable income will share a *common* stochastic trend, these variables will be cointegrated as defined by Engle and Granger (1987)." Thus, after

7 See Engle and Kozicki (1993).

8 Another crucial development is the introduction of the asymmetric adjustment to the dominant approach, e.g., threshold cointegration models and non-linear unit root tests (see, for instance, Enders, 2015 for a review). Additionally, there are also other important developments like the concepts of fractional integration and cointegration which have extended the scope of the standard unit root and cointegration analysis. Nevertheless, all these developments are beyond the scope of this paper.

9 In footnote #1, Vahid and Engle (1993: 341) admit that their use of the term "cycle" is inaccurate.

10 Vahid and Engle (1993), by following Stock and Watson (1988), utilize a multivariate version of Beveridge and Nelson [BN] (1981) decomposition. However, many researchers are discontented with the BN decomposition since it presumes a specific ARIMA model for observed series and it is not unique (see Enders, 2015, for more detail). That is, "decomposition separately applied to each variable will not yield the same trend for each" (Enders, 2015: 253). More importantly, as we shall explain later, Beveridge and Nelson decomposition requires that permanent and transitory components of variables should be perfectly negatively correlated (Enders, 2015) and hence it is not suitable for the empirical analysis of Permanent Income Hypothesis.

11 The idea of the DP is closely related to the concept of common feature developed by Engle and Kozicki (1993).

12 As noted before, Chapman and Ogaki (1993) and Bierens (2000) developed the concepts of linear and nonlinear deterministic co-trending, respectively. It should be also noted that Stock and Watson's (1988) analysis has shown that cointegrated variables share common stochastic trends and these trends are cancelled out.

13 Similarly, total disposable income (Y) comprises of permanent income (Y^P) and transitory income (ν_Y); that is, $Y \equiv Y^P + \nu_Y$ (see Friedman, 1957: 26).

verifying that both C and Y contain unit roots, he tested for presence of cointegration relationship between C and Y; i.e., whether $Z = C\text{-}kY$ is stationary.

Consequently, in the standard cointegration analysis C is assumed to be proportional to Y, where k is treated as the constant of proportionality (that is $C = kY$ is an attractor). However, since $C^P = kY^P$, this is only possible if and only if $\upsilon_C = k\upsilon_Y$. That is, both permanent and transitory components are related with a same constant of proportionality. In other words, this implies that marginal propensity (in this case, also average propensity) to consume out of permanent and transitory components are identical! This is in conflict with Milton Friedman's idea of PIH. In his seminal study, Friedman (1957:26) assumed that the correlation between the transitory components is zero (which implies that k should be zero for transitory component) and he explicitly stated that significant positive correlation between transitory components "would greatly weaken the [Permanent Income] hypothesis and reduce its potential usefulness" (Friedman, 1957:29). Of course, as admitted by Friedman, the zero correlation is a strong assumption and it requires empirical justification.[14]

Thus, it is highly plausible to have a relationship like $\upsilon_C = h\upsilon_Y$ where h is the constant of proportionality for transitory components $k \neq h \geq 0$ (possibly $k \geq h$). Nevertheless, this possibility is either missed or omitted by many researchers who solely rely on the standard cointegration analysis like Engle-Granger approach, due to its *singular nature* ($k = h$). Unfortunately, as noted above, the possibility of the dual adjustment [$C^P = kY^P$ and $\upsilon_C = h\upsilon_Y$ where $k \neq h$] is not compatible with the standard cointegration analysis, e.g., as conducted by Stock (1988), in which the attractor is $C = kY$ (that is, C is assumed to be proportional to Y, with $k = h$ is treated as the *common* constant of proportionality). Thus, in the case of dual adjustment $C = kY$ could not be a valid attractor and this can potentially explain the puzzling results − no cointegration between consumption and personal disposable income − noted in the introduction. In other words, the standard cointegration analysis implies a *singular* adjustment ($k = h$) and hence it is a special case of dual adjustment for I(1) variables.

To sum up, considering the above simple version of PIH, consumption should be proportional to permanent income over the long-run and hence $C = kY^P$ is the valid attractor rather than $C = kY$. That is, since the transitory component of consumption is undoubtedly stationary it is clear from Equation (3) that PIH requires $C\text{-}kY^P$ should be stationary (see Enders, 2015: 345 for a similar exposition). Additionally, it is also possible to test the validity of dual adjustment hypothesis of consumption behavior [$C^P = kY^P$ and $\upsilon_C = h\upsilon_Y$ where $k \neq h$] with the dual adjustment approach as explained below (see Section 3).

Fortunately, one can use the idea of common DP to develop new concepts for co-movements of variables. In what follows we shall attempt to do that while developing the concept of Co-HP trending. However, before doing that, in line with the aim of this study we need to provide an appropriate conceptual framework for the dual components (permanent vs. transitory).

Following the popular tradition, one can decompose the series (e.g., Y and X) into dual components as follows:

$$Y_t \equiv Y_t^P + Y_t^T \tag{5}$$

$$X_t \equiv X_t^P + X_t^T \tag{6}$$

where the variables with superscript P and T denote permanent (trend) and transitory (cycle) components, respectively, and permanent and transitory components of each series are assumed to be independent or uncorrelated. Note that such decomposition is in line with the vision of Milton Friedman's decomposition (see Friedman, 1957: 26) and widely acknowledged in macroeconomics. Additionally, in line with the seminal study by Nelson and Plosser (1982), while permanent (trend) components are considered as nonstationary, transitory ones are treated as stationary in the above framework.

As noted before, considering the suggestion of White and Granger (2011:14) we will follow the indirect approach and utilize the Hodrick-Prescott (HP) filter to define (and measure) the trend or permanent component of the variables. Despite some criticisms HP filter is a popular tool for decomposition of time series variables in macroeconomic analyses (see Mills, 2003 and the references cited therein) and, as mentioned in the introduction, HP trends have several ideal properties (e.g., inferences from the cyclical components are asymptotically correct, see, De Jong and Sakarya, 2016) and it may work well in various situations.[15] One important advantage of HP is that it utilizes the same method to purge the trend from the data and makes no presumption regarding the model for observed series. Another important, but neglected, characteristic of HP filtered data is that they meet "weak rationality" criterion, i.e., ex-post proxy for rational expectations (see Ash et al., 2002 and Grant and Thomas, 1999).[16] Additionally, it is important to note that Milton Friedman underlined the importance of zero correlation between permanent and transitory components of consumption and income (see Friedman, 1957: 26) thus HP is also suitable from this point of view.[17] Considering all these issues, HP is the preferred method of decomposition in this study but this does not necessarily mean that it is the best one. Moreover, it is important to reemphasize that special care should be taken for choosing the appropriate value for the smoothing parameter (see, e.g., Phillips and Jin, 2015).

Thus, we can now use the idea of common DP to develop the concept of Co-HP trending as follows,

$$Y_t = \beta_0 + \beta_1 X_t^P + Y_t^T \tag{7}$$

$$X_t \equiv X_t^P + X_t^T \tag{8}$$

where X_t^P is the HP trend of X [DP], Y_t^T and X_t^T are transitory I(0) components and they do not have the DP and β_0 and β_1 are nonzero constants.

[14] According to the *modern* theory of consumption (the life cycle and permanent income hypothesis/LC-PIH) "marginal propensity to consume out of permanent income is large and that the marginal propensity to consume out of transitory income is very small …. The LC-PIH is a very attractive theory, but it does not give a complete explanation of consumption behavior. Empirical evidence shows that the traditional [Keynesian] consumption function appears to also play a role." (Dornbusch et al., 2012: 341). See, for instance, Campbell and Mankiw (1989) and Flavin (1993) for such evidence.

[15] See, for instance, Mills (2003) for more detail on the flexibility and the important properties of HP.

[16] This is a valuable property since the adaptive formation of expectations, which was also used (and pioneered) by Milton Friedman, particularly, for the determination of permanent income, was largely discredited with the introduction of the idea of rational expectations to macroeconomics. Nevertheless, weak rationality is more plausible since "holding agents to a criterion of quickly learning from their mistakes is too stringent when agents have incomplete information concerning the nature of those mistakes" (Grant and Thomas, 1999: 332).

[17] As noted before, Beveridge and Nelson (1981) decomposition, which is quite popular in cointegration analysis (see Stock and Watson, 1988), assume a specific model (i.e., particular ARIMA set-up) for observed series and requires that permanent and transitory components of variables are perfectly negatively correlated (see Enders, 2015, for more detail). Thus, it is not suitable for the empirical analysis of PIH.

Considering the earlier axioms of *common* DP framework, X and Y have the DP which is a HP trend, but a particular linear combination Y-$(\beta_0 + \beta_1 X^p)$ does *not* have the DP [Recall that $Y^T = Y - (\beta_0 + \beta_1 X^p) \sim I(0)$]. In this case, X and Y are said to be Co-HP trending. In other words, X and Y have a common HP trend ($Y_t^P = \beta_0 + \beta_1 X_t^p$) and $Y = \beta_0 + \beta_1 X^p$ is an attractor. It should be also noted here that, whenever possible, the trends (permanent components) should be related to the underlying phenomena within the framework of Co-HP trending. Of course, this requires empirical verification (see Section 3).

Finally, it is important to emphasize that in our suggested framework there is no restriction on the relationship between the transitory components. For instance, they can have a different relationship $Y_t^T = \beta_2 X_t^T$, where $\beta_2 \neq \beta_1$ and hence dual adjustment. However, transitory components may also be unrelated ($\beta_2 = 0$). In the next section, we will present the empirical procedures for the dual adjustment approach in detail with an application.

3. Empirical issues and application

In this section, initially, the empirical steps are provided for testing and estimation of Co-HP trend within the dual adjustment framework, and then the use of the suggested tests and estimation procedures are illustrated with an application to the personal consumption expenditure and disposable income for the US data over the 1929–2017 period.

3.1. Empirical steps

The empirical steps of the basic dual adjustment approach can be motivated by comparing it to the standard Engle-Granger (EG) cointegration approach, which is considered to be the benchmark case.

Let's consider the following simple bivariate relationship,

$$Y_t = \beta_0 + \beta_1 X_t + u_t \quad (9)$$

where Y and X are nonstationary variables (e.g., I(1))[18] and u is the disturbance term that is serially correlated. The disturbance term can be specified as a first-order autoregressive serial correlation (AR(1)) as follows, $u_t = \rho u_{t-1} + \varepsilon_t$, where ρ represents the correlation between u_t and u_{t-1} and ε is a white noise disturbance term.

In Engle-Granger cointegration test, the following regression equation is used as a test equation,[19]

$$\Delta u_t = \gamma u_{t-1} + \varepsilon_t \quad (10)$$

where $\gamma = \rho - 1$.

The residuals from Equation (9) are used to estimate the above test equation. If the null hypothesis of no cointegration ($\gamma = 0$ or $\rho = 1$) is rejected then this means that these residuals are stationary ($|\rho| < 1$) with mean zero and hence Equation (9) is an attractor.[20]

As noted before, the *preliminary step* in the dual adjustment approach is to decompose permanent and transitory components of variables under consideration. As explained before, HP is the preferred method for such a decomposition. However, the crucial step in applying the HP method is the choice of the smoothing parameter (λ). Even though Hodrick and Prescott (1997) suggested certain values for different frequencies of data (e.g., λ = 100 for annual data) other researchers offered alternative values, for instance, in a frequently cited study by Ravn and Uhlig [RU] (2002), the suggested value of λ for annual data is 6.25. According to Mills (2003: 95), the optimal value of the smoothing parameter should be selected between 5 and 10 for annual data.[21] Therefore, in this study we will consider the two most frequently used values, as suggested by HP (λ_{HP})& RU (λ_{RU}), as well as the lower (λ_{OL}) and upper (λ_{OU}) limits suggested by Mills (2003). Additionally, Monte Carlo studies suggest that the critical values of Engle-Granger tests could also be used in the case of Co-HP Trend analysis under certain conditions.[22]

In line with the standard Engle-Granger cointegration approach (benchmark case), *Co-HP Trend analysis* involves two steps:

In the first step, OLS is applied to the following relationship,

$$Y_t = \beta_0 + \beta_1 X_t^P + u_t \quad (11)$$

where Y and X are observed nonstationary variables, and X_t^P is the HP trend of X as defined above [23] and u is the disturbance term which is serially correlated ($u_t = \rho u_{t-1} + \varepsilon_t$).

In the second step, as in Engle-Granger cointegration test, the following regression equation is used as a test equation,[24]

$$\Delta u_t = \gamma u_{t-1} + \varepsilon_t \quad (12)$$

where $\gamma = \rho - 1$.

The residuals from Equation (11) are used to estimate the above test equation. If the null hypothesis of *no common HP trend* ($\gamma = 0$ or $\rho = 1$) is rejected then this means that these residuals are stationary ($|\rho| < 1$) with mean zero. It should be noted that now Equation (11) is an attractor rather than Equation (9), as in EG approach. That is, the common permanent or trend component (co-HP trend) acts as an attractor in a way that the Y_t sequence is stationary around the trend component.

The dual adjustment approach contains an additional step to analyze the short run relationship (via transitory [T] or "gap" components).[25] Therefore, the following equation is estimated with OLS,

$$Y_t^T = \beta_2 X_t^T + v_t \quad (13)$$

where Y_t^T and X_t^T are HP filtered transitory [I(0)] or "gap" components, and v is the disturbance term.[26]

[18] It should be noted here that the preliminary step of cointegration analysis is to confirm that the variables under consideration have the same degree of integratedness. In this simple bivariate case, cointegration analysis requires the empirical verification – with unit root tests – that both X and Y contain unit roots, i.e., they are I(1).

[19] Of course, in practice, errors are approximated to be white noise by augmenting Equation (10) with additional lagged terms (Δu).

[20] In such a case, the Granger Representation Theorem assures the existence of error correction representation. In cointegration analysis, short-run relationships are analyzed within the context of error correction model.

[21] See, for instance, Pedersen (2001) and Du Toit (2008) and the references cited therein for more detail on optimizing (and estimating) the smoothing parameter.

[22] Monte Carlo studies (not reported) – which are conducted based on plausible values of smoothing parameter for annual data (ranging from 2 to 400) and closely following Mills (2003) – indicate that for I(1) or near I(1) variables (random walk, random walk with a drift and AR(1) with ρ = 0.95) EG tests provide quite good approximations for critical values and at lower values of lambda critical values are lower in absolute values. Of course, there is a need for a detailed analysis for estimation of precise critical values; however, this is beyond the scope of this paper. Nevertheless, the results from the Monte Carlo studies can be provided upon request from the author.

[23] Considering the nature of the HP filter, X_t^P can be treated as an exogenous variable.

[24] Again, in practice, errors are approximated to be white noise with additional lagged terms (Δu). The optimal lag length could be selected with information criteria like Akaike Information Criteria (AIC) or Schwarz Information Criteria (SIC).

[25] It should be noted that such a gap representation is used in several applications of macroeconomics (see, for instance, Ismihan, 2016a, for the popular gap version of Okun's law).

[26] A dynamic version of Equation (13), e.g., Autoregressive Distributed Lag (ARDL) model, could also be estimated in order to analyze the role of dynamic adjustment. Moreover, Vector Autoregressive (VAR) model may possibly yield more flexibility for modeling.

Finally, it should be noted that there is no restriction on the relationship between transitory components; i.e., $\beta_1 \neq \beta_2$ (*dual* adjustment). In contrast, in standard cointegration analysis based on I(1) variables, the "singular" adjustment restriction ($\beta_2 = \beta_1$) is implicitly imposed. Nevertheless, in our framework the presence of the dual adjustment could be tested by using a simple *t*-test; that is, if the null hypothesis of singular adjustment is rejected then this means that there is an evidence of *dual adjustment*.[27]

3.2. Application

We illustrate the dual adjustment approach with an application to the consumption function for the US. As noted before, while the earlier results (e.g., Engle and Granger, 1987; Stock, 1988) provided support for a cointegrating relationship among consumption and personal disposable income in US, Baltagi (2008) found no cointegration between these variables (C and Y) when using US annual data from 1950 to 1993, and hence this result seems to be quite puzzling. In this section we use the extended annual data (1929–2017) on the personal consumption expenditure and disposable income for the US and we will attempt to shed some light on the "puzzle" by using the above concepts.[28]

At this point, before illustrating the empirical exercise, it is important to explain the theoretical possibilities. The Permanent Income Hypothesis maintains that permanent consumption is determined by permanent income:[29]

$$C_t^P = \beta_0 + \beta_1 Y_t^P \tag{14}$$

where Y^P and C^P are permanent disposable income and consumption, respectively.

Considering the fact that total current disposable income and consumption (Y and C, respectively) comprises of the respective permanent and transitory components ($Y \equiv Y^P + Y^T$ and $C \equiv C^P + C^T$),[30] we can re-state PIH as follows:

$$C_t = \beta_0 + \beta_1 Y_t^P + C_t^T \tag{15}$$

where all variables are as defined before.

Considering the above version of PIH, consumption is primarily determined by permanent income over the long-run. Since the transitory consumption (C^T) is undoubtedly stationary, it is clear from Equation (15) that PIH requires $C_t - (\beta_0 + \beta_1 Y_t^P)$ should be stationary and this implies the existence of Co-HP Trend between C and Y in the dual adjustment approach.[31] However, we call this the weak version of PIH (WPIH), considering the possibility that the transitory components of consumption and income could also be related as follows:

$$C_t^T = \beta_2 Y_t^T \tag{16}$$

Therefore, it is more intuitive to consider a general framework, *dual consumption function*, in which dual components (permanent and transitory) are allowed to have separate dual co-movements and hence the *dual adjustment*. Thus, we can consider several hypotheses by using the above general set-up as follows.

- Strong version of PIH (SPIH) $[0 < \beta_1 < 1,\ \beta_2 = 0]$.

In addition to the stationarity of $C_t - (\beta_0 + \beta_1 Y_t^P)$, if the null hypothesis of $\beta_2 = 0$ is not rejected, then it can be concluded that SPIH is valid. That is, consumption is smoothed via Equation (15) over the long-run but transitory components are unrelated. As mentioned before, SPIH is the version favored by Friedman (1957).

- Dual Adjustment Hypothesis (DAH) $[0 < \beta_1 \neq \beta_2 < 1]$.

Nevertheless, in case WPIH is valid, if the transitory components are significantly related (i.e., the null hypothesis of $\beta_2 = 0$ is rejected) then we can conclude that dual components (permanent and transitory) have separate dual co-movements. However, this is confirmed after eliminating the possibility of common (singular) adjustment. Finally, it should be noted here that the spirit of DAH is *broadly* in line with that of the model provided by Flavin (1993: 654–655).

- Singular Adjustment Hypothesis (SAH) $[0 < \beta_2 = \beta_1 < 1]$.

In this case, transitory and permanent components are related with a same slope parameter ($\beta_2 = \beta_1$ and hence $C_t^T = \beta_1 Y_t^T$), which implies (from equation (15)) that $C_t = \beta_0 + \beta_1 Y_t$. In other words, while marginal propensity to consume out of permanent income and transitory income are different in DAH, they are identical in SAH. Thus, this claim or SAH ($\beta_2 = \beta_1$) could be tested by using a simple *t*-test as suggested in Section 3.1. It should be also noted that, SAH could be called as the *Dynamic Keynesian Consumption Function* in which WPIH is also embedded. The empirical verification of SAH also requires the stationarity of $C_t - (\beta_0 + \beta_1 Y_t)$; that is, it *necessitates* the existence of cointegration between C and Y if they both contain unit roots.[32]

Now, we can follow the steps suggested in Section 3.1 in order to empirically discriminate between the above hypotheses. Nevertheless, it is important to mention at the outset that *C* and *Y* contain unit roots (i.e., they are I(1))[33] and when we apply the standard

[27] The null hypothesis of singular adjustment can specified as $\beta_2 = \beta_2^*$, where β_2^* is the hypothesized value of β_2 under the null hypothesis and it can possibly be obtained from the estimated value of the parameter β_1 in the long-run relationship in Equation (11) [Recall that our claim under the null hypothesis is singular adjustment, i.e. $\beta_2 = \beta_1$]. We can also perform a similar confirmatory *t*-test by using the null hypothesis of $\beta_1 = \beta_1^*$, where β_1^* can be obtained from the estimated value of the parameter β_2 in the gap relationship [Equation (13)]. However, in order to perform this test we need to utilize the relevant (& valid) parameter estimates and standard errors for the long-run relationship from the three-step Engle-Yoo procedure. Such tests are developed in a follow-up study (Ismihan and Küçüker, 2017). See Harris and Sollis (2003) for more detail on the three-step Engle-[Granger]-Yoo procedure.

[28] C and Y are real personal consumption expenditures and real disposable personal income, respectively (in billions of chained 2009 dollars). The data are from FRED II (St. Louis Fed) and the main source is US Bureau of Economic Analysis (BEA). Nevertheless, the data can be obtained upon request from the author.

[29] It should be noted that in a simplified version of PIH β_0 is treated as zero. However, this is not necessary (see Friedman, 1957).

[30] As succinctly stated by Mankiw (2013: 484): "Permanent income is the part of income that people expect to persist into the future. Transitory income is the part of income that people do not expect to persist. Put differently, permanent income is average income, and transitory income is the random deviation from that average". In our framework, in line with these definitions, permanent and transitory income are obtained by the HP filter in which $E(Y) = Y^P$ and $Y^T = Y - Y^P$. As noted before, the HP filter has a useful property in that it satisfies the "weak rationality" criterion.

[31] That is, the common permanent or the trend component (Co-HP trend) acts as an attractor in a way that the C_t sequence is stationary around the common permanent (trend) component.

[32] Nevertheless, it should be noted here that this is a necessary but not a sufficient condition for the validity of SAH. Variables (in our example consumption and income) may also display singular adjustment even though they are not cointegrated but if they share a common, e.g. HP, trend. Also note that in such a case we may say that the lack of cointegration can be possible due to the misspecified common trend. However, the lack of cointegration cannot guarantee the dual adjustment behavior; therefore, we should use the suggested simple *t*-test for handling the empirical verification of the presence of dual adjustment.

[33] ADF tests with and without deterministic trends indicate that C and Y both contain unit roots at the 5% significance level. These results are in line with the results of Baltagi (2008).

Engle-Granger cointegration (CI) test we found no evidence of CI at the 5% significance level and hence our findings are in line with those of Baltagi (2008).[34] Thus, these results indicate that C and Y have a "spurious" relationship and we cannot go further with the standard CI analysis.[35,36] However, this result may imply that the Singular Adjustment Hypothesis is not supported by the US data.[37] Fortunately, the dual adjustment approach will allow us to empirically check the existence of other possibilities; namely, WPIH, SPIH, and DAH. Table 1 provides the empirical results.

Initially, we test for the existence of Co-HP Trend and hence the validity of WPIH. The results of Co-HP trend analysis are provided for the four different smoothing parameters; namely, λ_{HP}, λ_{RU}, λ_{OL} and λ_{OU}. As expected, the estimated values of parameters β_0 and β_1 are quite similar to each other as well as to those from EG analysis. Additionally, the null hypothesis of *no common HP trend* is rejected in all cases, i.e., with the four different smoothing parameters. This, in turn, means that $C_t - (\beta_0 + \beta_1 Y_t^P)$ is stationary and, hence, WPIH is empirically valid. That is, the common HP trend acts as an attractor in a way that C_t is stationary around the common permanent component (Co-HP trend).[38]

When we consider the transitory components, the null hypothesis of $\beta_2 = 0$ is rejected in all four cases of lambda (λ).[39] Considering all these results, SPIH is not empirically supported but DAH seems to be valid.

an alternative to the concept of cointegration in some cases by relaxing the implicit assumption of the singular adjustment in cointegration analysis. Additionally, this perspective seriously considers the recommendation of White and Granger (2011) by developing the concept of Co-HP trending such that apparent trends are related to the underlying phenomena.

This study also proposes a simple approach for testing and estimation of dual adjustment – parallel to the standard Engle-Granger cointegration approach – with an application to the consumption function using the US data over the 1929–2017 period. Our results indicated that personal consumption and disposable income are not cointegrated, i.e., they have a "spurious" relation, and we could not go further with the standard cointegration analysis. However, it is shown that these variables have a common HP trend (or permanent component) and this trend acts as an attractor in a way that consumption is stationary around the common permanent component. Additionally, it is shown that the transitory components of consumption and income are significantly related and are broadly in line with the dual adjustment hypothesis. Thus, our results are consistent with those studies (e.g., Campbell and Mankiw, 1989; Flavin, 1993) arguing that both PIH and traditional consumption function have *some* roles to play in explaining the aggregate consumption behavior in USA.

Table 1
EG and Co-HP trend tests.

	Engle-Granger (EG) CI Analysis	Co-HP Trend Analysis			
		λ_{HP}	λ_{RU}	λ_{OL}	λ_{OU}
β_0 (s.e.)*	−110.5391 (18.8108)	−111.9967 (24.1998)	−111.4243 (19.5683)	−111.3698 (19.4087)	−111.5531 (19.9816)
β_1 (s.e.)	0.9176 (0.0031)	0.9179 (0.0040)	0.9178 (0.0032)	0.9178 (0.0032)	0.9178 (0.0033)
EG/Co-HP**	−3.1455	−4.4151	−4.0031	−3.9480	−4.1102
β_2 {Adj. s.e}***	–	*0.8617* {0.1139}	*0.6127* {0.1377}	*0.5937* {0.1371}	*0.6554* {0.1376}
Singular Adj. Test	–	[0.6231]****	[0.0293]	[0.0203]	[0.0597]
Lambda	–	100	6.25	5	10

*s.e. = standard errors.
**EG and Co-HP tests are based on the procedures set-out in Section 3.1. Critical values of EG tests (MacKinnon, 1991), based on 89 observations, are −4.02, −3.41 and −3.09 at the 1%, 5% and 10% significance level, respectively. Optimal lag length is determined by SIC (max lag = 11).
***Newey-West standard errors (s.e.).
****P-values for the simple *t*-test for the null hypothesis of singular adjustment (see Section 3.1 for more detail).

4. Conclusion

This paper develops a dual adjustment approach which enables us to consider separate dual co-movements – for permanent and transitory components of the observed variables – and hence the possibility of *dual adjustment*. In doing so, this approach provides

We hope that by introducing the dual adjustment approach and the concept of Co-HP trend we can initiate some discussion of related matters. Nevertheless, there are several challenging empirical issues. In this study, we followed White and Granger (2011:14) and utilized *the indirect approach* by defining trends as that which HP trend filters remove. In other words, HP is the preferred method of decomposition in this study for several reasons noted earlier but this does not necessarily mean that it is the best one. Therefore, the dual adjustment approach can be possibly extended by utilizing other popular filters like Baxter-King (BK) filter (Baxter and King, 1999) [see Ismihan and Küçüker, 2017]. However, the quest for a better decomposition method is still a worthwhile endeavor.

Additionally, as noted before, Engle-Granger tests and their critical values could be approximately used for the Co-HP Trend analysis under certain cases. However, there is a need for detailed Monte Carlo analyses for the estimation of precise critical values as well as to consider more cases than the ones covered in the paper. These are left for future research.

Moreover, this study considered a symmetric specification, which possesses limitations in many applications. For example, with the seminal work of Neftci (1984) a large body of literature has shown that many macroeconomic variables display asymmetric dynamic adjustment over the phases of business cycles (see, e.g.,

[34] We also found no evidence of CI when we changed the sample recursively starting with period of 1929–2008 to 1929–2017.

[35] It should be noted here that Vahid and Engle (1993) attempted to test for the presence of common cycles between cointegrated variables by considering first differences of those 1(1) variables within the restrictive context of Beveridge and Nelson decomposition. Thus, if the variables are not cointegrated then we cannot go further with Vahid and Engle's analysis to check for the existence of common cycles.

[36] It should be noted that logarithmic transformation is usually preferred in cointegration analysis, i.e., Ln(C) & Ln(Y) rather than C & Y. However, in some cases as in our case, theoretical considerations could imply a certain functional form.

[37] The results from the simple t-tests for the presence of singular adjustment are also broadly in line with this conclusion.

[38] We confirmed the existence of Co-HP trending with the four different smoothing parameters when we changed the sample recursively starting with period of 1929–2008 to 1929–2017.

[39] Empirical results based on static model ($C_t^T = \beta_2 Y_t^T$) suffered from autocorrelation and heteroscedasticity. Thus, Newey-West standard errors are reported in Table 1.

Morley, 2009 for a review). Consequently, Threshold Autoregressive (TAR) models have received an extensive interest in the domain of cointegration (see, e.g., Balke and Fomby, 1997 and Enders and Siklos, 2001). Therefore, the dual adjustment models (approach) can be possibly extended by utilizing several models including the simple TAR, Momentum (M)-TAR, Band-TAR, RD-TAR (Ismihan, 2016b) and similar models. Of course, these are left for future research.

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