

**Study on the Size-dependent Thermomechanical
Stability Behavior of Functionally Graded Multilayer
Shallow Nano-Arches**

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ABSTRACT

In the present exploration, the effect of surface stress type of size dependency on the nonlinear in-plane stability characteristics of functionally graded (FG) laminated composite arches subjected to uniform radial pressure together with a temperature rise is examined. To accomplish this motivation, the nonlocal strain gradient (NSG) continuum elasticity is implemented to capture the nonlocal stress tensor and the strain gradient size dependencies and the Gurtin-Murdoch continuum elasticity is applied to capture the surface elasticity with the surface residual stress within a higher-order shear flexible curved beam theory in the presence of geometric type of nonlinearity. The effective properties of the employed nanocomposites are approximated via the Halpin-Tsai homogenization scheme corresponding to different lamination patterns and are considered in the principle of virtual work to establish the surface elastic-based nonlinear differential equations of the stability problem. The NSG-based and surface elastic-based radial load-deflection and radial load-axial load nonlinear equilibrium paths are traced corresponding to several parametrical case studies. As a result, both effects of the nonlocal stress tensor and strain gradient size dependency on the value of lower and upper limit radial pressures are more significant than the lower and upper limit resultant axial forces. The temperature rise causes to increase the radial loads associated with the upper limit and the first bifurcation points, while it leads to reduce the values of radial load related to the lower limit and the second bifurcation points.

Keywords: Surface elasticity; Curved beams; Nanostructures; Nonlinear stability; Nanocomposites; Temperature rise.

ÖZ

Şu anda yapılan araştırmada, yüzey gerilimi tipinin boyuta bağlı etkisi, homojen olmayan fonksiyonel gradyant (FG) lamine kompozit kemerlerin doğrusal olmayan düzlem içi kararlılık özellikleri üzerinde, homojen bir radial basınç ve sıcaklık artışına maruz kalmaları durumunda incelenmektedir. Bunu gerçekleştirmek için, yerel olmayan gerilme gradyanı (NSG) sürekli elastikiyeti ve Gurtin-Murdoch sürekli elastikiyeti kullanılarak, yüzeydeki artık gerilim ve yüzey elastikiyetini yansıtmak için, geometrik olmayan tipte bir yüksek mertebeden eğilimli eğri kiriş teorisi içinde yüzey elastikiyeti yakalanır. Kullanılan nanokompozitlerin etkili özellikleri, farklı laminasyon desenlerine karşılık gelen Halpin-Tsai homojenleştirme yöntemi ile yaklaşık olarak hesaplanır ve sanal iş ilkesi içinde yüzey elastik tabanlı doğrusal olmayan denge denklemleri oluşturmak için kullanılır. Yüzey elastik tabanlı NSG temelli ve radial yük-eğilim ve radial yük-eksenel yük doğrusal olmayan denge yolları, birkaç parametrik durum çalışmasına karşılık olarak takip edilir. Sonuç olarak, yerel olmayan gerilme tensörünün ve gerilme gradyanı boyutunun alt ve üst limit radial basınç değerleri üzerindeki etkisi, alt ve üst limit sonucu aksiyel kuvvetlerden daha önemlidir. Sıcaklık artışı, üst limit ve birinci bifurkasyon (çatallanma) noktaları ile ilişkili radial yükleri arttırırken, alt limit ve ikinci bifurkasyon (çatallanma) noktalarına bağlı radial yük değerlerini azaltmaktadır.

Anahtar Kelimeler: Yüzey esnekliği; Eğimli kirişler; Nanoyapılar; Doğrusal olmayan kararlılık; Nanokompozitler; Sıcaklık artışı.

DEDICATION

To My Family

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I would like to express my deepest gratitude to my supervisor, mentor, role model Assoc. Prof. Dr. Babak Safaei for his continuous advice and encouragement throughout my PhD Studies. Also, I would like to express my very sincere sense to my co-supervisor Prof. Dr. Saeid Sahmani and also, Prof. Dr. Qasim Zeeshan for their guidance, and motivation. I would like to express my appreciation to Prof. Dr. Tien-Chien Jen, Prof. Dr. Subrata Kumar Panda, and Assist. Prof. Dr. Ali Shefik for being my committee members as well as providing me their invaluable support and guidance. Moreover, I would like to thank my best friend Meryem Hürmek for being next to me on this journey through all ups and downs. I also would like to offer my thanks to my supportive colleagues and academic staff at the Mechanical Engineering Department.

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LIST OF ABBREVIATIONS

AC	Alternating Current
CNT	Carbon Nanotube
CUF	Carrera Unified Formulation
DC	Direct Current
FE	Finite Element
FEM	Finite Element Method
FG	Functionally Graded
FG-GPLRC	Functionally Graded Graphene Platelet Reinforced Composite
FGM	Functionally Graded Materials
GDQ	Generalised Differential Quadrature
GNF	Graphene Nanofiller
GNPL	Graphene Nano-Platelets
GPL	Graphene Platelets
NCS	Nonlocal Couple Stress
NFC	Natural Fibre Composite
NSG	Nonlocal Strain Gradient
SBM	Solution Ball Milling

Chapter 1

INTRODUCTION

Small-scaled functionally graded multilayer shallow arches are innovative structures that have gained significant attention in recent years due to their superior mechanical properties and potential applications in various fields of engineering [1,2]. The structure consists of multiple layers of materials with varying properties, strategically arranged to achieve optimal performance under different loading conditions [3–5]. The potential applications of small-scaled functionally graded multilayer shallow arch include aerospace, civil engineering, and mechanical engineering, where they can be used in various structural components, such as bridges, roofs, and aircraft wings [6,7]. Apart from all these applications, Najar et al. [8] proposed a pressure sensor application of the arch microbeams by running it by electrically. Also, Khalili et al. [9] proposed another application of shallow arch for the surface heating. The development of small-scaled functionally graded multilayer shallow arch has been driven by the need for innovative structural solutions that can provide high strength and stability while reducing the overall weight and volume of the structure [10].

Advanced materials have been widely utilized in various recently proposed systems and structures having novel capabilities. For instance, Pavase et al. [11] reported an overview about advances of conjugated polymer nanocomposites using for chemical sensors in food spoilage detection. Sahmani et al. [12–14] manufactured new bio-nanocomposite bony beam-type implants having specific mechanical and biological

characters. Yuan et al. [15] discussed about highly dynamic characteristics of nanocomposite hydrogels using for biomedical applications. Gupta et al. [16] explored the effect of metal oxide nanoparticles on thermophysical properties employing for thermal energy storage applications. Sahmani et al. [17,18] predicted analytically and experimentally the influence of magnetic nanoparticles on the mechanical as well as biological properties of hydroxyapatite nanocomposite scaffolds coated by ibuprofen drug. Tu et al. [19] proposed a nanocomposite material containing lead titanate nanowires dispersing within a polyamide-imide matrix using for high-temperature energy harvesters. Kulkarni and Kumari [20] utilized the solvent casting technique to fabricate highly flexible nanocomposites for piezoelectric nanogenerator applications. Published data regarding functionally graded shallow arches have been gathered from the Scopus database in Fig. 1, which shows a drastic increase in papers on this topic. In the Fig. 1 (a) above, published documents between 2013 and 2023 (January) about shallow arches and functionally graded multilayer shallow arches were obtained from Scopus database has been shown. This graph illustrates that the research interest about shallow arches are continued almost at the same level year by year. The figure clearly shows that the shallow arches and functionally graded materials (FGM) started attract the attention of the researchers lately because it can be seen that there are 21 published papers in January 2023 which is the one quarter of the previous years. This graph shows us the researchers lost their interest about shallow arches between 2014-2016. Then, the published papers are reached a top point in 2020 with 116 papers. Fig. 1 (b) shows that the statistics from the database of Scopus for papers published by country. Fig. 1 (c) reveals the number of published works by type (publication distribution among conferences, journals, and book chapters); 1402 are articles, 330 paper

published as conference paper, 36 of these published as book chapter and 29 as review type.

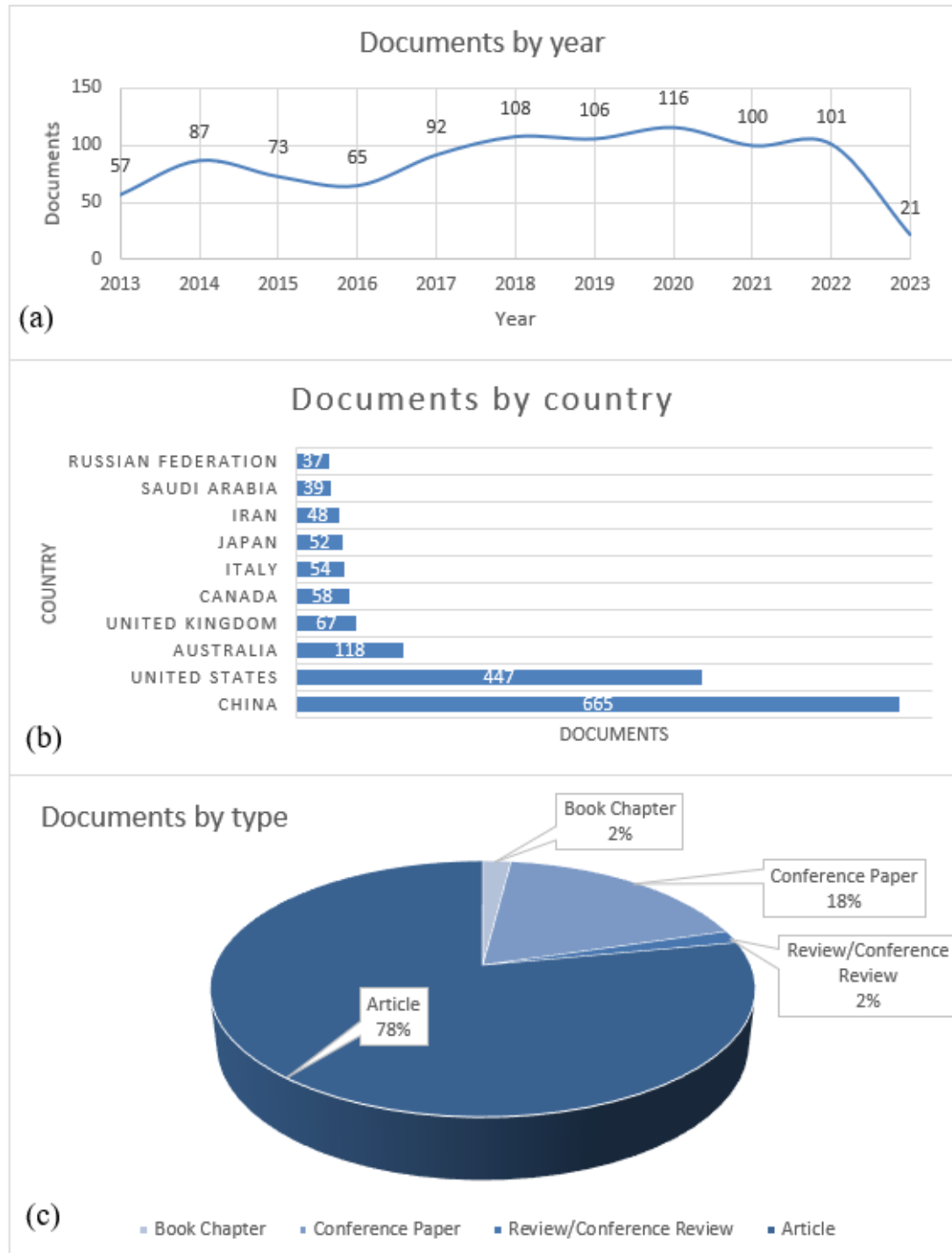


Figure 1: (a) Documents by year, (b) documents by country, and (c) documents by type.

Structures at nanoscale have quite different mechanical behavior compared with that in macroscopic scale, which has been comprehensively described [21,22]. When structural atoms are near or on free surfaces, their equilibrium requirements differ from those inside material. This difference leading additional stress adhere to the surface of the structure and affect the structural behavior [23–25] Surface stress effects in nanostructures are significant due to their high surface area to volume ratio. Gurtin et al. [26,27] applied continuum elasticity theory to introduce a generic theoretical framework to consider surface stress effects on nanostructure mechanical behavior. On the basis of those study, in simulations, the free surface layer is considered as a perfect bonded mathematical membrane with thickness of zero possessing various material characteristics from underlying bulk, and the surface stress is characterized via two parts including the surface residual stress and the surface Lamé constants.

1.1 Functionally Graded Materials

Equally, FG composite materials are greatly used in several applications because of their astonishing characteristics and their ability to provide composite materials with high surface area to volume ratios [28]. Also, they are applied in the mechanics of small-scaled structures (including micro and nano levels) for engineering, bioengineering, and aerospace applications [29]. Tornabene et al. [30] utilized the generalized differential quadrature-based time stepping technique for transient dynamics of arches. Yi et al. [31] explored the effect of surface stress on the large-amplitude free vibrations of porous FG composite nanoshells including the vibrational mode interactions. Allahkarami et al. [32] anticipated the dynamic stability of bi-directional FG porous cylindrical shells embedded by an elastic foundation. Li et al. [33] analyzed the thermal nonlinear behavior of porous nanocomposite cylindrical shells incorporating an initial dental imperfection. In these contexts, very recently, Fan

et al. [34] presented the surface stress effects on the critical buckling loads of FG composite skew nanoplates subjected to shear loading. Shahmohammadi et al. [35] studied the geometrically nonlinear responses of double-curved FG shallow shells reinforced with carbon nanotube composite. Yuan et al. [36] examined the nonlinear dynamic stability characteristics of FG composite truncated conical shells at microscale based upon the nonlocal strain gradient (NSG) elasticity theory. Xie et al. [37] explored the free vibrational and dynamic responses of FG circular micro-arches based on quasi-3D theory. Moghaddasi and Kiani [38] explored free and forced vibrations of laminated composite arches under the effect of moving load. Xie et al. [39] represented the surface elastic-based nonlinear secondary resonance of hard periodical excited FG porous silicon nanobeams. Li et al. [40] studied the nonlinear response of FG curved beams with discontinuous stiffness. Rajasekaran et al. [41] examined the mechanical characteristics of bi-directional FG sandwich tapered arches under thermomechanical environments. Li et al. [42] investigated the nonlinear free vibration response of FG nanostructures by considering the surface stress effect. In the other work, Fan et al. [43] proposed the postbuckling characteristics of porous composite nanoplates made of a FG inhomogeneous material in a thermal environment. Yang et al. [44] predicted the postbuckling behavior of hydrostatically pressurized FG composite microshells on the basis of the NSG continuum theory.

FGMs are a class of advanced engineered materials that exhibit spatial variation in their microstructure and properties, such as composition, mechanical strength, thermal conductivity, and electrical conductivity [45,46]. Unlike conventional homogeneous materials, which have uniform properties throughout their volume, FGMs are designed to have tailored properties that vary gradually or abruptly in a controlled manner

[47,48]. The concept of FGMs is a way to improve the performance of ceramic materials in high-temperature applications [49,50]. It recognized that by varying the composition of a material along a specific direction, it was possible to create a gradient in properties such as thermal expansion, strength, and fracture toughness [51,52]. According to Rodrigues et al. [53], FGMs have a substantial advantage over traditional composites due to their ability to retain the original material characteristics on both sides of the material. The design and fabrication of FGMs typically involve selecting appropriate constituent materials and developing a processing method that allows for the controlled variation of their properties. One common approach is to use powder metallurgy or casting techniques to deposit layers of different materials on top of each other, with each layer having a slightly different composition. The composition and thickness of each layer can be precisely controlled to create a desired property gradient. Another approach is to use additive manufacturing techniques such as 3D printing to create complex FGM structures with precise control over the composition and microstructure. In this case, the FGM can be designed and fabricated to meet specific performance requirements. Fig. 2 illustrates the improvement of material towards functionally graded materials.



Figure 2: Improvement of material towards FGMs

1.2 Structure of Arches

Among different structural element classes, curved beams are widely applied in various engineering domains. Therefore, at the macroscopic level, Pi et al. [54–57]

performed a systematic analysis on the mechanical properties of homogeneous and isotropic arches. Liu et al. [58] investigated the lateral-torsional buckling of FG porous arches having different cross sections subjected to arbitrary radial concentrated loads. Li et al. [59,60] studied composite arch thermal instability. Yang et al. [3,61–68] presented investigations on FG composite arches with considering its properties of free vibration, dynamic buckling and nonlinear static buckling under different external loads. In addition, at the microscopic level, nano-/micro-arches have great practical potential in smart structures. Tajaddodianfar et al. [69,70] investigated the chaotic vibrations of a bistable resonator containing a doubly clamped shallow arch under simultaneous static DC and harmonic AC distributed electrostatic actuations. Vahidi-Moghaddam et al. [71] established a new nonlinear dynamic model to explore arch micro resonator behaviors under electrostatic actuation and casimir force. Babaei et al. [72] investigated the nonlinear snap-through instability behavior of FG composite shallow micro-arches containing integrated surface piezoelectric layers by employing the modified couple stress theory. Ouakad et al. [73] applied meshless Galerkin decomposition technique to study snap-through buckling behavior of electrostatic shallow arch micro-actuator. Marandi and Karimipour [74] explored the free oscillations of FG sandwich nanobeams having flexible core on the basis of the modified couple stress continuum mechanics. On the other hand, surface stress effect on nonlinear in-plane buckling of fixed shallow FG composite curved nanobeams under thermomechanical load has not been studied previously, despite its practical importance [75].

Comprehending the mechanical characteristics of these arch-based structures is of great importance, as they are used in several real-world applications such as

spaceships, aircrafts, bridges, etc., where they are exposed to numerous fluctuating loads. Consequently, extensive reports are available in open literature concerning the mechanics and responses of arch-type and beam-type structures under the effects of static and dynamic loadings, including thermomechanical load, and in several environments [76]. Babaei et al. [77] studied the nonlinear thermal flexural response of FG composite curved tubes. Trinh and Kim [78] introduced a three variable refined shear flexible formulations for oscillations and bending characteristics of FG porous of doubly curved shells. Lu et al. [79] predicted the influences of uneven support movements and rotations on the nonlinear instability of fixed arches representing modified slenderness switching. Zhang et al. [80] predicted the nonlinear in-plane stability of shallow sandwich arches incorporating the first order-type of shear flexibility subjected to a uniform radial load. Brischetto et al. [81] introduced two-dimensional generalized differential quadrature models for free vibrations of FG inhomogeneous plates and cylinders.

Since the beginning of modern world, beams and plates have found large application area as constructive components in current engineering businesses for example mechanical, aerospace, civil, marine and airplane [82]. In addition, circular curved beams are significant structural elements in these applications. That is why their mechanical properties are very important for engineering design and manufacture [83]. Arches, in general, outperform straight beams because of their twisted centerline, which raises the entire rigidity of the construction. By dividing pressures into mostly compressive stresses and keeping tensile stresses within reasonable bounds, they are able to cover enormous regions [84]. The majority of arches are built to function linearly while carrying service loads. They are, nevertheless, prone to significant

deformations due to their slenderness, particularly when external stresses rise over the service point. To create a structural model Polit et al. [85] has been used different length-to-radius and span-to-thickness ratio configurations to create the geometries which are illustrated in Fig. 3 (a) length-to-radius and (b) span-to-thickness.

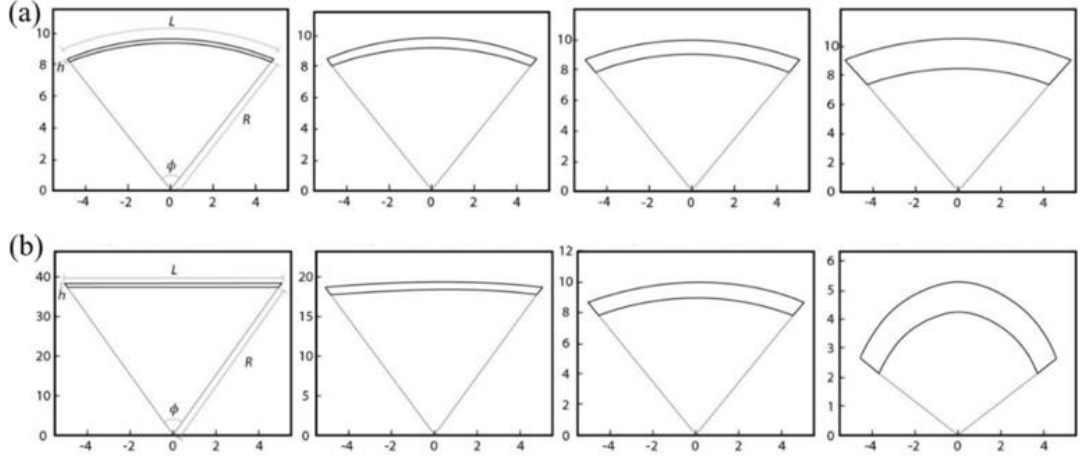


Figure 3: Geometry of fixed (a) length-to-radius proportion from thin to thick and (b) span-to-thickness proportion from shallow to deep.

The structure might suffer disastrous effects from a stability decrement, commonly acknowledged as snap-through buckling [86]. The reaction of the arch must be effectively described by a geometrically nonlinear analysis since linear analysis cannot foresee this kind of instability. When it came to working on the arches and presenting solutions for the instances of distributed and concentrated forces, respectively, Timoshenko and Biezno were among the pioneers in 1935 and 1938, respectively. On the basis of the idea of buckling, Marguerre analyzed a few instances in 1938. Different kinds of arches and lateral loads were taken into consideration by Fung and Kaplan. Numerous studies have been done on the subject since then, taking into account variations in the stiffness of the arch, the stiffness of the support, and other cases outside of those of simple arches. Generally, the exterior layer is made of ceramic and the interior layer is made of metal for the construction of arch models as shown in Fig.

4 (a). Li et al. [87] proposed an functionally graded material arch which is inverted by a new type of distribution of metal and ceramic. The team dispersed the ceramic on the lower part and the metal on the upper part as shown in Fig. 4 (b). Traditional model of functionally graded material arch versus functionally graded material inverted arch is illustrated below [87].

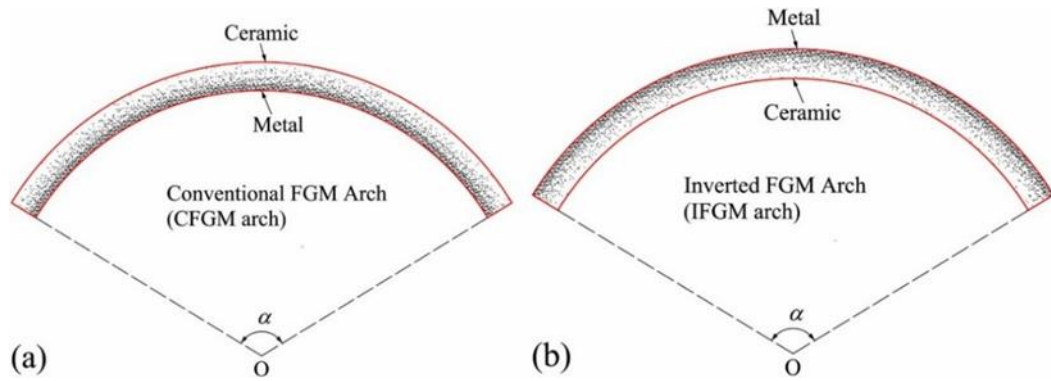


Figure 4: (a) Traditional model of functionally graded material arch versus (b) functionally graded material inverted arch.

One of the most effective shapes used frequently in contemporary engineering designs is the arch and it is used to support the general skeletal framework, aviation, space vehicles, and military defense engineers use the design of arch as the main constructional component owing to the precise and strict standards on the total structure's weight and strength [88]. Fig. 5 below indicates the main skeleton parts that have been used by design engineers for the designing of fighter jets (a), spacecrafts (b) and automobile leaf springs (c) [88,89].

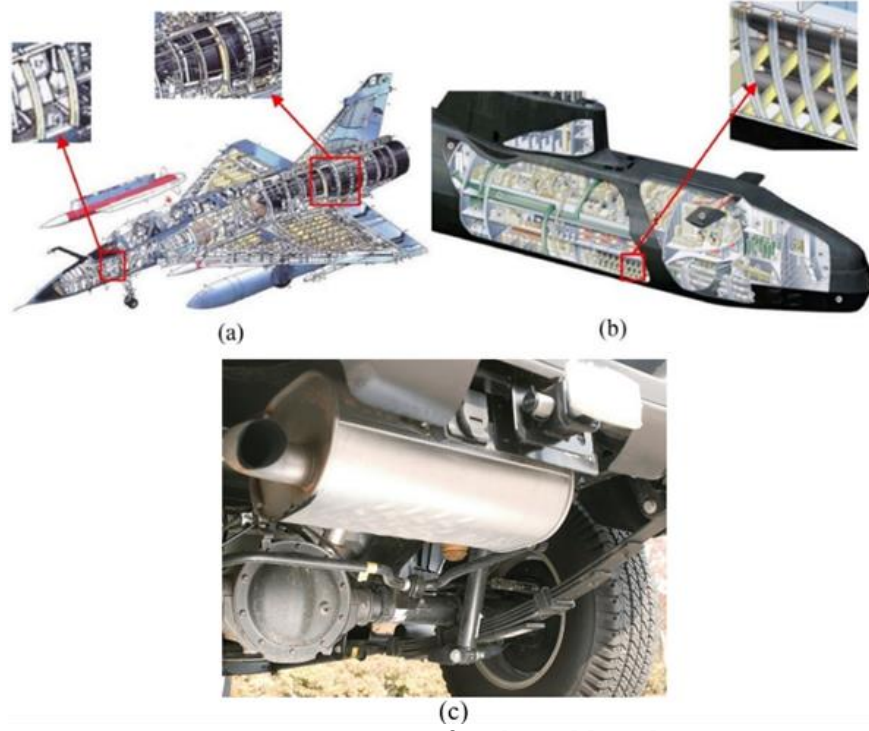


Figure 5: Various types of real-world applications.

When the structural elements are made up from FGMs, micro size and structural effects have a vital role [90]. Şimşek et al. [91] stated that it is possible to significantly decrease the thermal stresses and stress accumulation at contact with open surfaces by grading the compositional transition across a material interface. According to Rahmani et al., [92] surface effects are expected to be substantial and can significantly alter macroscopic characteristics for structures with micron sizes due to the growing surface-to-bulk ratio. Shahverdi et al. [93] stated that the theory of nonlocal strain gradient elasticity is useful for modelling micro and nano structures. Because FGMs work better mechanically, thermally, and electrically than conventional structural materials, they are used for micro- and nano-structures due to the recent rapid advancement of nanotechnology [94,95]. Tang et al. [96] stated that nano/micro-devices differ significantly from macro-devices in that they are both smaller and have different mechanical characteristics. Based on molecule dynamics and continuum mechanics, scholars have shown a strong interest in nanomaterials [97,98]. It is well

known that arched buildings can be unstable, and they are a typical illustration of the behavior known as snap-through buckling [99]. Sarhosis et al. [100] stated that large-scale arched structures are particularly prone to instability, especially those whose loading capacity is inextricably tied to the deformations endured.

1.3 Analysis of Shallow Arches

Nonlocal strain gradient (NSG) theory is an advanced framework for modeling the mechanical behavior of materials that takes into account both strain gradients and nonlocal effects which is used for small-scaled structures. This theory combines the traditional concepts of continuum mechanics with the emerging fields of microscale and nanoscale mechanics, where the material properties are influenced by the internal microstructure of the material. Phung-Van et al. [101] researched a isogeometric model to observe the length scale and size-dependency effects of nanostructures for FG composite nanoplates which are reinforced by carbon nanotube regarding the NSG theory. Phi et al. [102] analyzed the nonlinear dynamic features of functionally graded composite micro-beams strengthened by using carbon nanotubes. Nonlocal effects, on the other hand, arise due to the fact that the response of a material at a particular point can be influenced by the surrounding material. Nonlocal strain gradient theory takes both of these effects into account by incorporating integral equations that account for both the nonlocal and strain gradient effects [103]. The nonlocal strain gradient theory has been used to model a wide range of material behaviors, including deformation and fracture in metals, ceramics, and polymers. It has been found to be particularly useful in modeling materials at the micro- and nanoscale, where the material properties can vary significantly over small distances [104]. Additionally, this theory can provide more accurate predictions of material behavior compared to traditional continuum mechanics, making it an important tool for engineers and scientists working in the field

of materials science [105]. In recent years, there has been increasing interest in the development of new materials with improved mechanical properties. Nonlocal strain gradient theory has the potential to play a critical role in the design and optimization of such materials [106]. Also, Ma et al. [107] used nonlocal stress–strain gradient elasticity theory for nonlinear analysis of isogeometric functionally graded microplates with alternating thickness. Yang et al. [108] carried out research by applying impulsive loading to functionally graded arches for nonlinear forced vibration and dynamic buckling. Investigating nanocomposite materials by using NSG theory is one of the most popular approaches to this area [109,110].

The design and analysis of the structure require the use of advanced computational tools and material science principles to optimize the structure's performance under different loading conditions. [111]. The functionally graded nature of small-scaled functionally graded multilayer shallow arch enables the creation of arches that are highly resistant to bending, buckling, and deformation, while maintaining a low weight and a shallow depth [67]. Yang et al. [66] employed graphene platelet reinforced functionally graded arch structure to analyze the nonlinear buckling behavior which undergoes a central point load and reached a conclusion as the load of buckling reduces, the power law index of the rotational constraints increases. The Fig. 6 below illustrates the proposed arch model mesh with the deformation through the thickness and length.

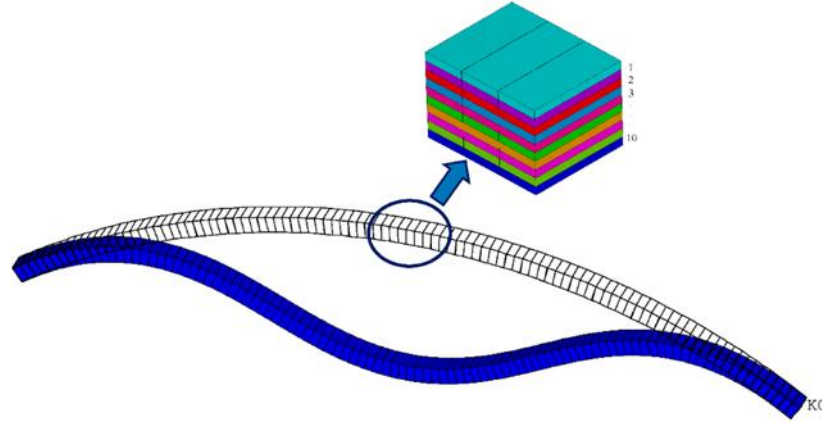


Figure 6: Proposed arch model mesh with the deformation.

As surface stress having significant influences on nanostructures, many studies are reported on the mechanical properties of nanostructures considering the effect of surface stress. Fan et al. [43] investigated thermal post buckling characteristics of functionally graded (FG) porous composite nanoplates considering surface stress effects. Sahmani et al. [112] established surface elastic-based conical shell model to explore large-amplitude composite nanoshell oscillations with in-plane heterogeneity taking into account surface stress effects. Assadi et al. [113] presented an investigation on single-crystalline rectangular nanoplate size-dependent vibrations with cubic anisotropy considering nonlocal elasticity and combined surface stress effects. Li et al. [42], Sahmani et al. [114] and Sarafraz et al. [115] captured modal interactions using surface stress elasticity for nonlinear vibrations of FG composite nanoshells including modal interactions. Tong et al. [116] developed a general higher-order shear deformable plate model for stability analysis of non-isotropic nanoplates taking into account the surface stress effects. Yue et al. [117] discussed surface-induced and surface stress effects piezoelectric nanobeam behaviors. Shaat et al. [118,119] studied the bending behavior of nano-sized plates using the modified couple-stress theory considering microstructural and surface energy effects. McFarland et al. [120] presented the surface stress effects on the resonance behavior of microcantilevers.

Ansari et al. [121,122] and Sahmani et al. [123–127] comprehensively studied the effects of surface stress on structural buckling and free and forced vibrations of nanostructures. Sahmani et al. [128] proposed a surface elastic-based shell formulation for axial buckling analysis of silicon nano shells validated via molecular dynamics simulations. Yang and Li [129] examined the magnetoelectroelastic bending and free oscillations of a circular Kirchhoff circular plates including the surface stress effects. Sarafraz et al. [130] explored the both subharmonic and super harmonic nonlinear resonance of nanobeams in the presence of the surface stress effects. Sahmani et al. [131] investigated the surface stress effect on the radial post buckling behavior of nanosized shells surrounded by an elastic medium. Fan et al. [34] analyzed shear buckling response of FG inhomogeneous skew nanoplates incorporating the effects of surface stress. Yuan et al. [132] studied the surface elastic-based torsional oscillations of inhomogeneous nanowires having multiple defects. Sahmani et al. [133] and Wang et al. [134] predicted the nonlinear bending and nonlinear free oscillations of FG composite nanoplates having a variable thickness. Wang et al. [135] established a boundary layer-based shell model for nonlinear post buckling analysis of axially compressed silicon nano shells. Zhao et al. [136], Rao et al. [137], Li et al. [138], Alshenawy et al. [139,140] developed size-dependent shell models for nonlinear stability characteristics of FG piezoelectric shells at small scale. Yang et al. [141] constructed a unified size-dependent Timoshenko beam model incorporating the surface stress effects.

Nonetheless, analyses of involving buckling and post-buckling mechanics and stability of small-scaled arch structures can only be possible and precise when considering the effect of nonlinearity [142]. Some studies are available concerning the nonlinear, in-

plane stability and instability analyses of FG composite-based structures under the effects of either thermal, mechanical and thermomechanical loadings. Fazzolari et al. [143] constructed a one-dimensional hierarchical Ritz for free vibrations of circular and elliptical shell-type and beam-type structures. Ali et al. [144] extracted the natural frequencies of geometrically imperfect FG composite arches under temperature changes. Fattahi et al. [145] presented a NSG-based beam formulation for nonlinear secondary resonant dynamics of periodically excited FG porous microbeams. Li et al. [146] calculated analytically the nonlinear stability behavior of heated FG heterogeneous arches. Sahmani et al. [114] formulated a surface elastic-based shell model to predict the modal interaction in the nonlinear primary resonance of nanoshells. Huang et al. [147] explored the nonlinear buckling of FG composite laminated shallow arches having elastic rotational constraints. Song and Li [148] explored the nonlinear stability of fixed-fixed FG arches exposed to both thermal and mechanical loadings using Kirchhoff's assumption and shooting method. The geometrical large amplitude nonlinear response analysis of shallow thick circular arches under the effects of thermomechanical loadings resting on a three-parameter elastic foundations was reported by Babaei et al. [149] utilizing third-order shear deformation theory and incorporating two-step perturbation technique. Yuan et al. [150] developed a nonlocal elastic-based plate model for shear stability analysis of FG inhomogeneous skew nanoplates. By taking into consideration the effects of thermal surrounding, Babaei et al. [151] evaluated the geometrical nonlinear bending behaviors of FG shallow curved tubes under the influence of both thermal and mechanical loadings and uniform lateral pressure based on von Kármán assumption, higher order shear deformation theory and two-step perturbation technique. Rao et al. [137] used numerical method to investigated the electro-thermomechanical nonlinear

stability characteristics of axially compressed FG panels at nanoscale by considering combinations of axial mechanical load. Also, Zhao et al. [136] by considering axial-lateral and numerical method, studied nonlinear stability behavior of nanocomposite micro shells containing randomly dispersed nanofillers.

Wan et al. [152] employed Timoshenko beam theory to investigate geometrically nonlinear responses of FG curved beams composed of variable curvatures. Liu et al. [153] suggested an analytical method to examine the stability and nonlinear responses of FG porous arches reinforced graphene platelets (GPLs) subjected to a radial uniformly distributed loading. Taking into account geometric nonlinearity, the structural responses of arches were derived based on Euler-Bernoulli beam theory and virtual work method. Sahmani and Safaei [112] studied the nonlinear free vibrations of in-plane FG inhomogeneous conical nano shells in the presence of surface stress effect. Based on Reddy's beam model and taking into account von Kármán strain–displacement relations, Babaei et al. [154] analyzed the nonlinear behaviors of FG shallow arches resting on a three-parameter nonlinear elastic foundation under the effect of thermomechanical involving a uniform lateral pressure and uniform temperature change, utilizing two-step perturbation technique to obtain the nonlinear responses of FG shallow arches with both edges clamped. By taking into consideration both analytical and numerical methods, Yang et al. [68] explored the asymmetric static and dynamic thermal instability buckling characteristics of pinned-fixed FG reinforced graphene nanoplatelet composite arches subjected to thermal and arbitrary-radial point loadings utilizing energy method and modified Halpin–Tsai micromechanical model. Sun et al. [155] employed a meshfree mathematical model for the nonlinear strain

gradient-based lateral buckling and post buckling of random reinforced nanocomposite micro shells.

Moreover, a few research works on in-plane buckling stability and instability behaviors of FG-based structures are available in open literature. In this context, Yang et al. [156] studied nonlinear in-plane buckling and post buckling instability responses of FG multilayered composite reinforced graphene nanofillers subjected to central point loading using the principle of virtual work. Yue et al. [157] established a NSG-based quasi-3D beam model for the nonlinear dynamic stability analysis of agglomerated nanocomposite beams at microscale. Nikrad et al. [158] investigated the nonlinear in-plane thermoelastic behaviors of FG porous curved beams by employing first-order shear deformation theory and nonlinear green strains. Then they derive the governing equations of motion and modified rule-of-mixture to take into account material properties of porous FG beams, Rayleigh–Ritz and Newton–Raphson methods to obtain the nonlinear in-plane responses of porous FG beams. Zuo et al. [159] represented the nonlinear large-amplitude oscillations of FG porous microplates including different strain gradient tensors. Alshenawy et al. [160] developed a three-dimensional strain gradient-based meshfree model for nonlinear thermo-electromechanical stability analysis of FG piezoelectric microshells. An analytical methodology is proposed by Tung and Duc [161] to investigate the nonlinear behavior of FG curved shallow panels under the effects of thermomechanical loadings resting on an elastic foundation.

1.4 Graphene Nanofillers

Graphene possesses many fascinating physical and mechanical properties, such as exceptionally high young's modulus, high strength to weight ratio, and excellent thermal conductivity [162]. Owing to their superior mechanical properties, e.g., exceptionally high young's modulus, high strength, large specific surface area, and good thermal conductivity, graphene and its derivatives such as graphene GPLs are excellent reinforcing nanofillers for composite materials with an excellent combination of high stiffness, light weight and multi-functionality [163–165]. Graphene is plenty in nature and less expensive when synthesized in larger scale [166]. In addition, spreading GPL nanofillers with a reduced ingredient into polymer matrix with a suitable non-uniform dispersion model can reach a preferable support effect than the traditional uniform model [167,168]. Graphene nano-platelets (GNPLs) used in many engineering applications due to their lightweight material properties to improve the mechanical properties [169]. Using GPLs as nanofillers to improve bonding with matrix accepted to be an effective strategy to reach a better structural performance which enable us to achieve more efficient load transfer and further improvement in strength of the reinforced material [170,171]. According to Song et al. [172], theoretically and experimentally experiments proved that adding a very little amount of graphene into the polymer matrix can improve its mechanical, electrical and thermal properties on a large scale.

Recently, the graphene nanofiller (GNF) is widely used to reinforce the functionally graded materials. According to Sanes et al. [173], the production of nanocomposites by using nanofillers can add new practical features to its matrix as well as keeps the original positive characteristics of the materials. Even fillers first added to the

composites to reduce the cost, Bhattacharya et al. [174] stated that fillers became an important part of fabrication process due its improvements on the mechanical properties of the material. Nguyen et al. [175] summarized the main advantages of graphene nanofillers as being cost-effective, lightweight, good thermal and electrical conductivity, and high mechanical strength. Also, Wang et al. [176] stated that nanofiller reinforced nano composites have been used in many engineering industries such as electrical, aerospace, mechanical, and medical to design structural elements. Nanofiller reinforced nanocomposites shows excellent performances over conventional materials due to the advantages summarized. Table 1 has been prepared according to the various sources such as; [177–181] to compare the properties of graphene nanofillers with a traditional material (PAN based carbon fiber).

Table 1: Properties of graphene nanofillers and PAN based carbon fiber [177–181].

Property	Carbon Fiber	Graphene Nanofiller
Physical Structure	Filaments (Fibers)	Platelet ~ 1nm
Chemical Structure	Hexagonal	Hexagonal
Elastic Modulus (TPa)	100–500 (GPa)	01 (in-plane)
Tensile Strength (GPa)	2.5–7.0	130 ± 10
Thermal Conductivity (W/(mK))	21–125	(4.84 ± 0.44) x 10 ³ to (5.30 ± 0.48) x 10 ³
Electrical Conductivity (S/m)	—	7200
Density (g/cm³)	1.75–1.93	1.8–2.2

1.5 Fabrication of Graphene Reinforced Arch Structures

Functionally graded multilayer shallow curved beams have gained a lot of interest in last decade due to their inimitable mechanical features and potential usage area in diverse fields of engineering. The fabrication of these structures is a complex process that involves multiple steps and techniques. Functionally graded materials have piqued the curiosity of a wide range of industries during the last few decades. [182,183]. A

special kind of composite which has material qualities that differ in one or more directions from one material to another [184,185]. Pascon et al. [186] stated that the concept of FGM has been proposed for the Japanese aerospace program to develop a plate with high mechanical and thermal strengths in 1984. To enhance the qualities of the original material, the FGM arch is built in stages. In most cases, the components that go into a FGM are chosen such that they complement one another. Larkin et al. [187] stated that FGMs typically consist of a metal and a ceramic material. Jha et al. [188] stated that the ceramic portion is thermally and corrosion resistant, while the metallic section is fracture robust and weldable.

Li et al. [189] stated that FGMs can be characterized and considered as continuous materials versions in more than one dimension which results in alterations in their material properties. Functionally graded piezoelectric elements are a unique form of FGM that can be used in energy-harvesting applications [190]. In addition, Larkin et al. [191] indicated that this contrasts with conventional composites, which have an even dispersion of the material. Naebe et al. [192] mentioned that a FGM is a sort of developed engineering material composed for a specific aim or performance of use which is used in harsh conditions. Gupta et al. [193] introduced an analysis on functionally graded materials and proposed the material properties alteration between traditional composite and FGM. In addition, Bohidar et al. [194] published an article on functionally graded materials and indicated the possible property variation between conventional composites and functionally graded materials. In contrast to typical composites, a single scattered phase that is not evenly distributed throughout the matrix can achieve a single FGM, whereas a double FGM requires more than one phase. Regardless of the change distribution density among the employed phases and

the matrix, the continuous gradient is always obtained which is illustrated in Fig. 7 below.

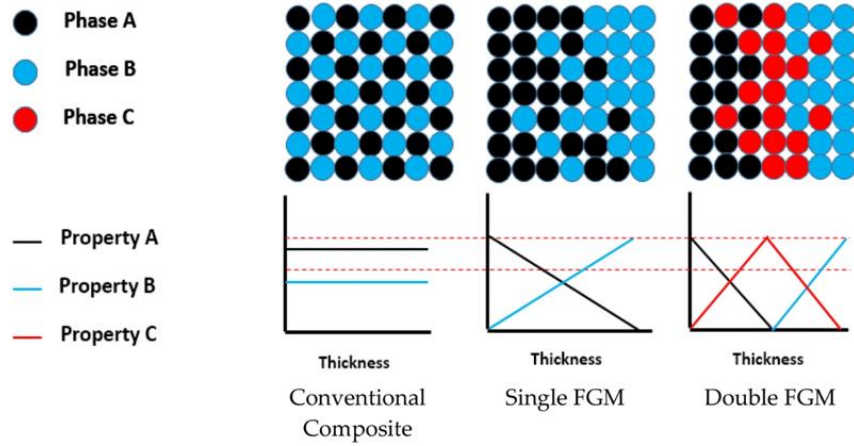


Figure 7: Characteristic alterations in traditional composite and FGMs.

In addition, functionally graded composite material, according to Su and Heshmati [195,196], is one of the most advanced and recently created materials. Its heterogeneous properties enable it to serve a range of technical objectives, including efficient stress reduction in a number of industrial operations. What is more, Thang et al. [197] stated that FGMs are one of the most robust options for laminated structures due to their natural superiority. Wang et al. [198] mentioned the advantages of preserving constructive robustness, avoiding growing stress, and prohibiting interface distribution. Gao et al. [199] proposed a mathematical model and an answer for the nano scale functionally graded composite core and actuator piezoelectric layer at a stable electric and magnetic field. Also, FGMs have been classified as multi-phase materials which have metallic and ceramic phases [200–203]. The different types of nanofiller reinforced functionally graded material structures are possible to be formed along a particular gradient in size, shape, fraction of volume and guidance of the nanofillers scattered in a specific matrix has proposed by Miyamoto et al. [204] back in 1999 and revised by Zhao et al. [205] which is indicated in Fig. 8 below.

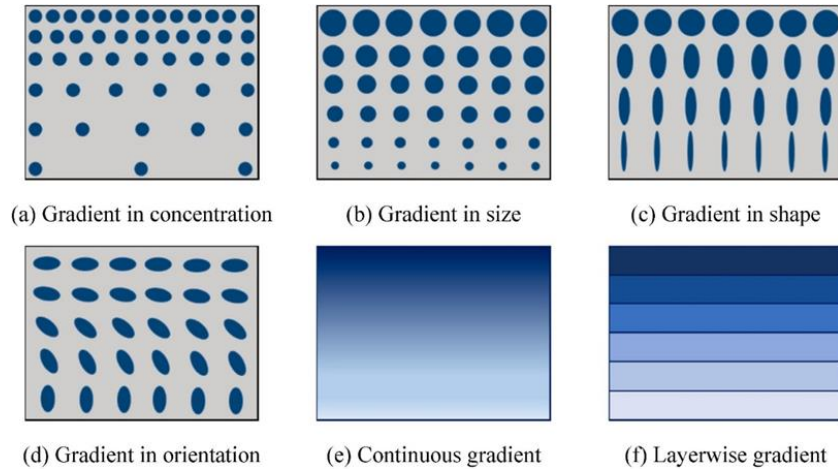


Figure 8: Different types of FGM structures.

Yang et al. [206] introduced a two-step fabrication process for the manufacturing of graphene platelet reinforced functionally graded composite. This process begins with arrangements of graphene platelet nanofillers and mixing the epoxy matrix. After this, graphene platelet and epoxy are mixed together along a proper deploying and stirring process. To create the first layer with a pre-designed GPL weight percentage, the well-mixed nanocomposite liquid is poured into a mold. To get the second single layer with a different GPL weight fraction, the aforementioned procedure is repeated. Before being entirely hardened, these two layers are linked together by hot pressing. The multilayer FG-GPLRC can be created by using this approach to acquire and subsequently bond multiple single layers together [207]. Fig. 9 below illustrates the schematic of this two-step fabrication process.

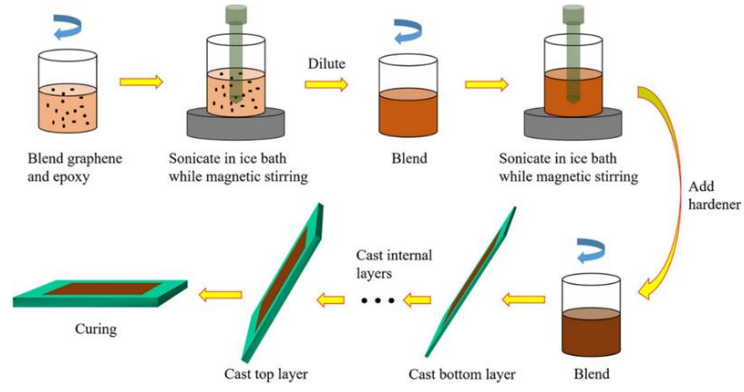


Figure 9: Two-step fabrication process.

In addition, Baig et al. [208] summarized the nanofiller dispersion techniques as solid-state processing, colloidal processing, and colloidal & solid-state hybrid processing. The powder metallurgy process has been adapted by Munir et al. [209] for the solid-state processing and illustrated the schematic process in Fig. 10 (a), whereas, the solution ball milling (SBM) process has been applied for the uniform dispersion of CNT by Chen et al. [210] for the colloidal & solid-state hybrid processing and indicated in Fig. 10 (b).

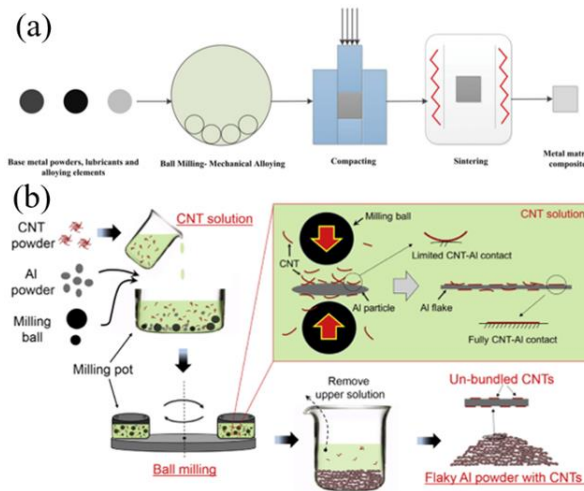
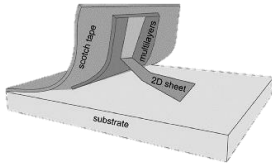
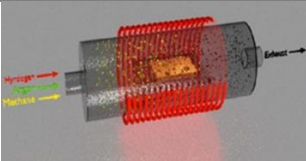
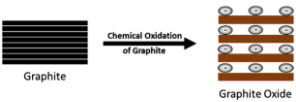
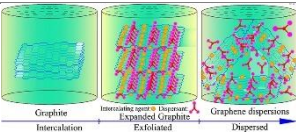
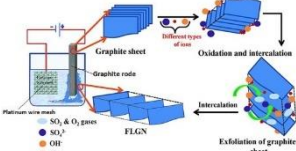


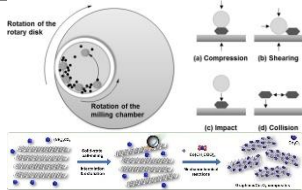
Figure 10: (a) The powder metallurgy process for the solid-state processing and (b) the solution ball milling (SBM) process for the equal distribution of CNT.

Zhang et al. [211] used 3D printing method to create curved surface by continuous natural fiber reinforced composite. Sahraei et al. [212] stated that it is critical to spread

equally the nanofillers in the powder mixing procedure to achieve ideal dispersion at the final composite material. The choice of the most appropriate processing method is one of the most important step to take before the production part of nanocomposite due to many factors that affects the fabrication [213]. According to Lu et al. [214] the reinforcement made by graphene can display a major reinforcement effect on the mechanical characteristics of nanocomposites due to achieving a greater specific surface area along with preferable interfacial interactions with the matrix. Rafiee et al. [215] approved above statement experimentally by achieving outstanding results. In addition, Li et al. [216] conducted a comparison study and CNT and graphene and results approved that the Young's modulus and tensile strength of graphene reinforced composites were higher. Several researches about preparation methods have been compared in Table 2 below for the benefits, drawbacks and schematics of graphene synthesis techniques [217–219].

Table 2: Benefits, drawbacks and schematics of graphene synthesis techniques.

Methods	Process	Benefits	Drawbacks	Ref.
Mechanical Exfoliation		High-quality Simplest process	Small-scale production	[220,221]
Chemical Vapor Deposition		Supereminence Major graphene usage	Complicated process Major energy demand	[222]
Chemical Oxidation (Hummer's method)		Fast reaction Fewer defect	High contamination and degradation	[223]
Liquid-Phase Exfoliation		Serial and increasing manufacture	Poor solubility Eco-friendly	[224]
Electrochemical Exfoliation		One stage production Environmentally friendly	Expensive	[225,226]

Mechanochemical Reaction		Serial manufacture Supereminence	High energy consumption	[227,228]
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Additive manufacturing has been used for manufacturing micro/nano size components. Parthasarathy et al. [229] introduced a method as an example of additive manufacturing called electron beam melting process of porous FG models for the application of biomedical engineering. Fig. 11 (a) illustrates the schematic for electron beam melting process of proposed design. Yan et al. [230] employed laser metal deposition method as additive manufacturing to create functionally graded metallic materials. A schematic illustration of fabricating customized material by laser metal deposition method shown below in Fig. 11 (b). The question marks in the figure states that using premixed powder blends, the composition ratios before and after deposition can be different during the laser metal deposition process.

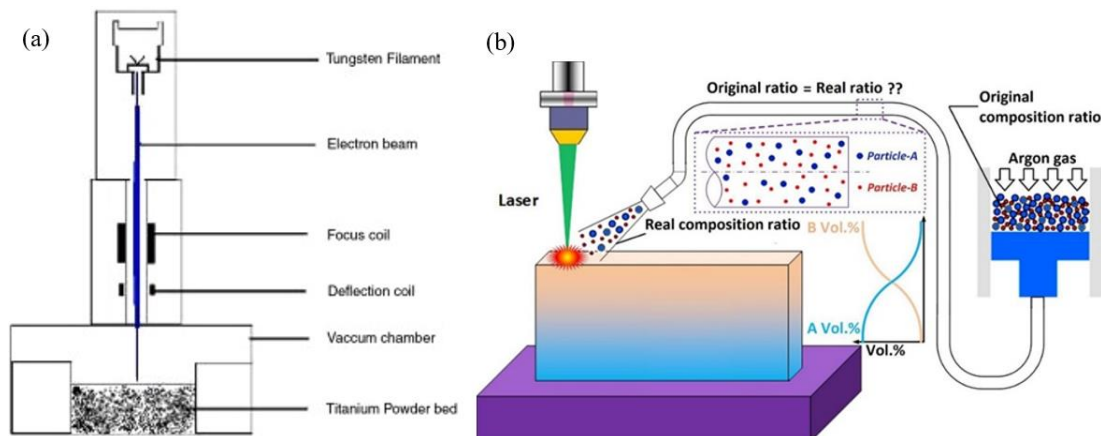


Figure 11: (a) Schematic of an electron beam melting machine and (b) fabricating customized material by laser metal deposition.

1.6 Research Objective and Novelty

Taking into consideration the brief literature survey provided above, one can conclude that the in-plane stability behaviors of FG composite arches at small-scale level subjected to thermomechanical loadings are hardly considered. Further, to the best of our knowledge, NSG theory is hardly employed in nonlinear stability analyses of micro-arches exposed to thermomechanical loadings. In the current investigation, the NSG-based nonlinear in-plane stability behavior of FG multilayer composite micro/nano-arches under thermomechanical loading conditions is analyzed.

On the basis of the main objective of the current investigation, for the first time, the Gurtin-Murdoch surface elasticity theory is employed within the framework of the higher-order shear flexible curved beam model to analysis the nonlinear stability characteristics of thermomechanical loaded FG laminated curved nanobeams. In this regard, the surface elastic-based radial load-deflection and radial load-axial load nonlinear equilibrium curves are traced together with indication of the upper and lower limit points as well as the first and second bifurcation points corresponding to different curved beam thicknesses resulting in various surface stress significant.

1.7 Research Organization

The following describes the structure of this thesis. Firstly, Chapter 1 explained the main theme of the thesis as introduction and which described the literature survey methodology, along with a brief summary of FGMs, structure of arches, analysis of shallow arches, reinforcing materials as graphene nanofillers, fabrication of the proposed arch model and novelty of the research. Chapter 2 will cover the literature review which includes the main equations of proposed model, reinforced micro and nano arch models. In addition, mechanical analysis also discussed in this chapter as

nonlinear bending, buckling and post-buckling, linear and nonlinear vibration, and the dynamic instability. In Chapter 3, methodology of the proposed shallow nano-arch model will be explained under two cases as NSG-based and Surface-elastic based FG multilayer nanocomposite shallow arch along with their numerical solving processes. Results and discussion of the proposed study for case 1 and case 2 will be reviewed along with the validation of the current experiments. Last but not least, Chapter 6 will identify the key challenges associated with the testing of nano-scaled functionally graded multilayer shallow arch model.

Chapter 2

LITERATURE REVIEW

2.1 Basic Equations of Functionally Graded Multilayer Shallow Arches

The nonlocal strain gradient (NSG) theory is generally used to define the total stress tensor. Lim et al. [231] defined it for the NSG theory as below in Eq. (1);

$$t_{ij} = \sigma_{ij} + \nabla \sigma_{ijm}^{(1)} \quad (1)$$

in which σ_{ij} explains the classical stress tensor and $\nabla \sigma_{ijm}^{(1)}$ defines the high-order stress tensor. The symbol for the gradient summarized as; $\nabla = \partial/\partial x + \partial/\partial y$. Stress tensors are given in Eqs. (2) and (3).

$$\sigma_{ij} = \int_V \alpha_0(x', x, e_0 a) C_{ijkl} \varepsilon'_{kl} dV' \quad (2)$$

$$\sigma_{ijm}^{(1)} = l^2 \int_V \alpha_1(x', x, e_0 a) C_{ijkl} \varepsilon'_{kl,m} dV' \quad (3)$$

where ε'_{kl} is defines the strain, $\varepsilon'_{kl,m}$ defines the gradient of strain and C_{ijkl} defines the matrix for the stiffness. In addition, l length of the material. Functions of kernel under the effect of gradient of strain summarized as; $\alpha_0(x', x, e_0 a)$ and $\alpha_1(x', x, e_0 a)$. Higher order stress equal to the function of attenuation that is because $e_0 a = e_1 a = ea$.

The equations above are not simply answered by the current methods of mathematics. This is because Ering et al. [232] introduced a method to solve mentioned equations by transferring them into differential equations. Eqs. (2) and (3) can be solved by using a solution for the Eringen's method which is indicated in Eqs. (4) and (5);

$$[1 - (ea)^2 \nabla^2] \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \quad (4)$$

$$[1 - (ea)^2 \nabla^2] \sigma_{ijm}^{(1)} = l^2 C_{ijkl} \varepsilon_{kl,m} \quad (5)$$

Eqs. (4) and (5) subrogate into Eq. (1) and it is derived as Eq. (6) under the influence of NSG theory which is illustrated below;

$$[1 - (ea)^2 \nabla^2] \tau_{ij} = (1 - l^2 \nabla^2) C_{ijkl} \varepsilon_{kl} \quad (6)$$

The Eqs. (6) can be deducted to NSG theory according to Eringen et al. [233] and Aifantis et al. [234] by using $e_0 a = 0$ and $l = 0$.

Graphene platelet is used by Gao et al. [235] as reinforcement material to create curved beam structure. L given as the length of circumference, h is the thickness, R accepted as curve radius and b given as the width of the proposed model which is illustrated in Fig. 18 below. The proposed model is made of six substrate layers and exposed to a uniform load. The proportion of weight diverse layer to layer with an equal deployment of graphene platelets.

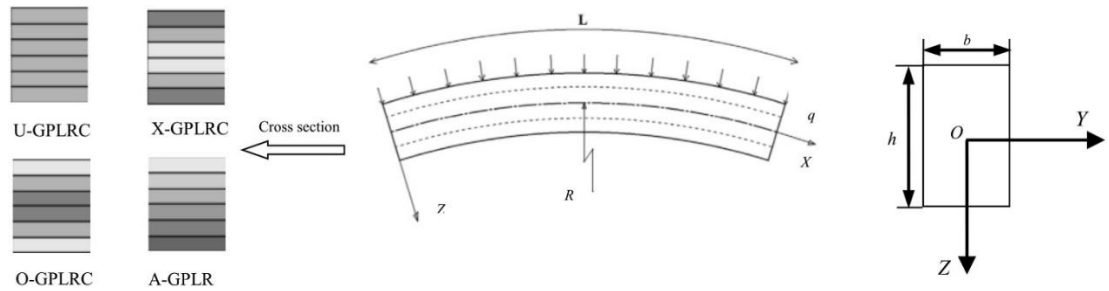


Figure 12: The diagram of proposed model curved beam with different deployment of graphene platelets.

In Fig. 23 above, four ways of graphene platelet deployments are illustrated with uniform and non-uniform distributions as U-GPLRC, A-GPLRC, X-GPLRC and O-GPLRC. The darker color indicates that the graphene platelet distributed more on that

layer. For the k -th layer of graphene platelet volume proportion, the deployment patterns which are illustrated in Eqs. (7), (8), (9), and (10) are as follows;

$$U - GPLRC: V_{GPL}(k) = V_{GPL}^* \quad (7)$$

$$X - GPLRC: V_{GPL}(k) = 2V_{GPL}^* |2k - N_L - 1| / N_L \quad (8)$$

$$O - GPLRC: V_{GPL}(k) = 2V_{GPL}^* \left(1 - \frac{|2k - N_L - 1|}{N_L}\right) \quad (9)$$

$$A - GPLRC: V_{GPL}(k) = V_{GPL}^* (2k - 1) / N_L \quad (10)$$

in which N_L is the layer number in total. In addition, V_{GPL}^* is given as the volume proportion totally and calculated as shown in Eq. (11);

$$V_{GPL}^* = \frac{W_{GPL}}{W_{GPL} + (\rho_{GPL} / \rho_m)(1 - W_{GPL})} \quad (11)$$

where W_{GPL} is the weight proportion of the graphene platelet, ρ_{GPL} is the mass density of graphene platelet and ρ_m is the mass density of the polymer matrix.

Affdl and Kardos et al. [236] stated that the Young's modulus of the specific layer composites can be calculated by using Eq. (12) with a random dispersion of the fillers based on Halpin-Tsai homogenization scheme.

$$E(k) = \frac{3}{8} E_m \frac{1 + \xi_L \eta_L V_{GPL}}{1 - \eta_L V_{GPL}} + \frac{5}{8} E_m \frac{1 + \xi_T \eta_T V_{GPL}}{1 - \eta_T V_{GPL}} \quad (12)$$

where E_m indicates the young's modulus of the polymer matrix and the rest is given as below in Eq. (13);

$$\eta_L = \frac{\frac{E_{GPL} - 1}{E_m}}{\frac{E_{GPL} - 1}{E_m} + \lambda_L}, \eta_T = \frac{\frac{E_{GPL} - 1}{E_m}}{\frac{E_{GPL} - 1}{E_m} + \lambda_T}, \lambda_L = \frac{2L_{GPL}}{h_{GPL}}, \lambda_T = \frac{2b_{GPL}}{h_{GPL}} \quad (13)$$

in which h_{GPL} is the thickness, b_{GPL} is the width, E_{GPL} is the young's modulus, and L_{GPL} is the length of the graphene platelet nanofiller.

Furthermore, Hejazi et al. [237] stated that the mixture rule can be employed to calculate the proportion of Poisson on a specific k-th layer which is indicated in Eq. (14) below.

$$v(k) = v_m(1 - V_{GPL}^{(k)}) + v_{GPL}V_{GPL}^{(k)} \quad (14)$$

where v_{GPL} indicates the Poisson's ratio of graphene platelet and v_m shows the Poisson's ratio of the polymer matrix.

Eslami et al. [238] used The Voigt rule and stated that the proper homogenization method should be employed to explain the thermal and mechanical characteristics of an arch. Kargani et al. [239] summarized these characteristics based on the mentioned rule. E as young's modulus, ρ as mass density, ν as Poisson's ratio, K as thermal conductivity, C_v as specific heat, and α as thermal expansion coefficient should be taken into account as the linear functions of the volume fractions V_c for ceramic and V_m for metal. Kiani et al. [240] expressed the P as a function of the direction of thickness and it is illustrated in Eq. (15) below.

$$P(z, T) = P_m(T) + V_c(z)P_{cm}(T), \quad P_{cm}(T) = P_c(T) - P_m(T) \quad (15)$$

where m indicates the metal property and c ceramic property [241]. Javani et al. [242] employed the model of Touloukian to summarize the material characteristics which are dependent to temperature which is indicated in Eq. (16);

$$P(T) = P_0(P_{-1}T^{-1} + 1 + P_1T + P_2T^2 + P_3T^3) \quad (16)$$

In this case, T is measured in Kelvin and P_i 's are constants and they are specific for each component [243]. Saidi et al. [244] represented V_c as the volume proportion of ceramic and V_m as the volume proportion of metal. These are indicated below in Eq. (17) as a power law dispersion of the components across the thickness of the proposed arch;

$$V_c = \left(\frac{1}{2} + \frac{z}{h}\right)^\zeta, V_m = 1 - V_c \quad (17)$$

According to Naderi et al. [245], ζ is the power law index which cannot be negative and defines the profile distribution feature. Last but not least, at the inner layer of the arch is given as ($z = +h/2$) which is mostly ceramic and at the outer surface of the arch is given as ($z = -h/2$) which is mostly metal.

2.2 Functionally Graded Micro Arches

The structural and geometrical linkage components for extending and bending are included in FGM shallow arches [246,247]. Micro structures attracted too much interest from various fields of research during the last two decades [248]. Prior to falling in the plane of its curve, the FGM shallow arch encounters significant deformation brought on by thermal loads which influences the design's critical buckling temperature [249,250]. Pi et al. [251] carried out a virtual work technique analyze the nonlinear thermoelastic buckling under temperature gradient which concluded that the axial expansion and the curvature changes arise due to temperature gradient. The mechanical, physical, and chemical features of the substance are continuously modified by FGMs over time in the intended spatial orientation, such as; shear modulus, density, Poisson's ratio, Young's modulus, etc. [252–258]. It is possible to construct FGMs include a continuous gradient in both the mechanical and physical properties that allows for specialized features to obtain the required structural representation of the composite [259,260]. Advanced engineering materials recognized as FGMs are created for a particular applications or purpose. To do this, detailed graded compositions, microstructures, and characteristics are provided [261–263]. The mixing ratio and characteristics of FGMs, which comprise two or more unique material constituents, change continuously along one or more directions [264–267]. The performance of functionally graded micro arches relies on various elements,

for example; the arch geometry, material properties, fabrication procedure, and the operating conditions [268]. Therefore, their design and optimization require a multidisciplinary approach that combines mechanics, materials science, microfabrication, and system integration [269]. Khalili et al. [270] applied rapid surface thermal shock to the functionally graded shallow arches to examine the thermally induced vibrations by using Ritz method. Ouakad et al. [73] employed meshless Galerkin decomposition technique on electrostatic shallow micro arch to analyze the snap-through buckling behavior and found that the first and fifth modes of applied method affected the micro-actuator structural performances with a considerably amount. Anirudh et al. [271] carried out an experiment on porous curved beams by employing functionally graded graphene platelets reinforcement material under the shear flexible finite element approach to examine the nonlinear bending attitude. At the end of the research, it is observed that the degree of nonlinear bending characteristics of the beam affected by the graphene platelet geometry which is illustrated in Fig. 13 below in order to contrast the nonlinear maximum deflections with the applied uniform loads under the fixed-hinged condition.

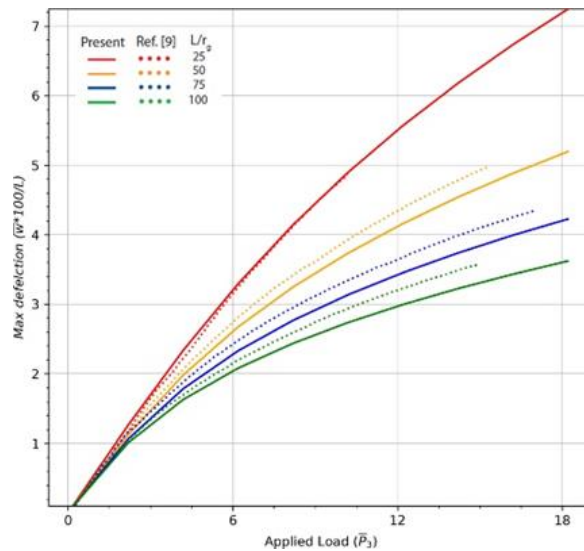


Figure 13: Nonlinear maximum deflections for isotropic beams under the condition of fixed hinges are compared to the applied uniform loads.

Yang et al. [272] employed the nonlocal couple stress (NCS) mechanics of continuum theory into the third-order shear flexible arch in order to analyze the nonlinear thermomechanical in-plane stability attitude of the proposed model. Fig. 14 shows the bifurcation lateral loads for various nonlocal and couple stress parameters and the values of the lateral load at the first two bifurcation points decreases as the couple stress tensor increases under the effect of the nonlocal stress tensor.

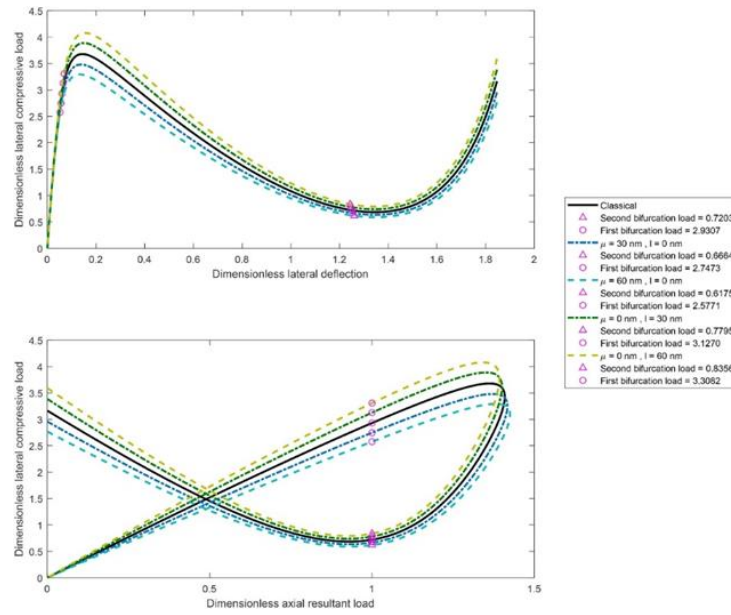


Figure 14: NCS-based and classical dimensionless nonlinear stability paths.

In the Table 3, different geometry and shapes of FG micro arches are elaborated.

Table 3: Comparison of functionally graded multilayer shallow micro arches.

Schematic	Effective Material Properties	Model Principle	Ref.
	$E(z) = E_c V_c + E_m V_m$ $v(z) = v_c V_c + v_m V_m$ $\rho(z) = \rho_c V_c + \rho_m V_m$	Quasi-3D model.	[37]

	$E(z) = \frac{9K_e\mu_e}{3K_e + \mu_e}$ $v(z) = \frac{3K_e - 2\mu_e}{6K_e + 2\mu_e}$ $\rho(z) = \rho_m V_m + \rho_c V_c$	Mori–Tanaka homogenization technique. [273]
	$E_{eff} = \frac{1}{5} A_f E_f V_f + E_m (1 - V_f)$ $v_c = V_f v_f + (1 - V_f) V_m$ $\rho_c = V_f \rho_f + (1 - V_f) \rho_m$	Hamilton's principle. [274]
	$E = E_i V + E_m (1 - V)$ $v = v_i V + v_m (1 - V)$ $\rho = \rho_i V + \rho_m (1 - V)$	Hamilton's principle. [275]
	$E(z) = E_b + (E_t - E_b) \left(\frac{1}{2} + \frac{z}{h} \right)^k$ $v(z) = v_b + (v_t - v_b) \left(\frac{1}{2} + \frac{z}{h} \right)^k$ $\rho(z) = \rho_b + (\rho_t - \rho_b) \left(\frac{1}{2} + \frac{z}{h} \right)^k$	Timoshenko theory. [276]

2.3 Functionally Graded Nano Arches

Nano-scaled functionally graded multilayer shallow curved beams are a topic of major interest in the area of nanotechnology and structural engineering. The design and analysis of such structures have been the subject of many research studies over the past few years. Wang et al. [277] prepared a model by using curved beams reinforced with graphene oxides and analyzed its dynamic responses when subjected to moving loads. Resonance and thermal snap-buckling behavior of FG curved nanobeams have been conducted by She et al. [278]. Ebrahimi et al. [279] used a nonlocal strain gradient approach to analyze a curved beam model for buckling. Zenkour et al. [280] carried out an experiment on sandwich curved nanobeam by using piezomagnetic face-sheets for the size-dependent analysis. According to Arefi et al. [281], nonlocal parameter

increases with radial and transverse deflections after the reduction in stiffness of the curved nanobeam. Sayyad et al. [282] conducted a virtual work on functionally graded curved beams to analyze the static behavior of the beam. The x-y plane of functionally graded curved beam under bending and the variations in elasticity modulus throughout beam thickness are indicated in Fig. 15 and Fig. 16 below.

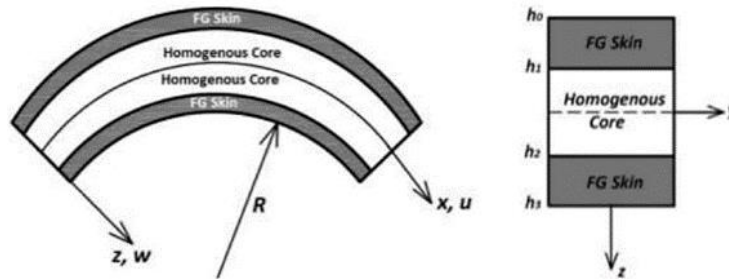


Figure 15: x-y plane of functionally graded curved beam under bending.

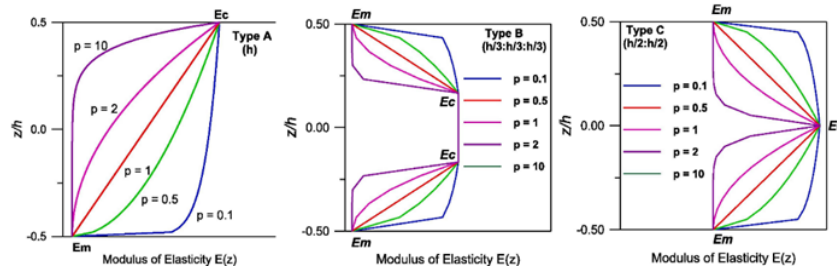


Figure 16: Variations in elasticity modulus throughout beam thickness.

Nanostructured materials have unique mechanical and physical properties that can enhance the performance of these structures [283,284]. For example, the addition of nanoparticles can improve the stiffness, strength, and ductility of the graded layers, resulting in higher load-bearing capacity and improved resistance to deformation [285–287]. Another important aspect of the design of nano-scaled functionally graded multilayer shallow curved beams is the use of different geometries and configurations. Ahmadi et al. [288] employed Hamilton's principle develop the governing equations of the 2D functionally graded curved nanobeam using the theory of first-order shear

deformation to examine the size dependent dynamic behavior of the proposed model by using a meshless method. It can be seen that, opening angle θ_0 and natural frequencies have a considerable effect on the proposed nanobeam. The Fig. 17 below illustrates the mode shapes of two smallest natural frequency of curved clamped-clamped edge nanobeam for $\theta_0 = 50$, $\theta_0 = 60$, $L = 10$, and $L/h = 20$. Also Fig. 18 indicates the clamped-clamped edge curved nanobeam's natural frequency after the applying nonlocal parameter μ effect ($L = 10$, $L/h = 20$, $n_z = 0$).

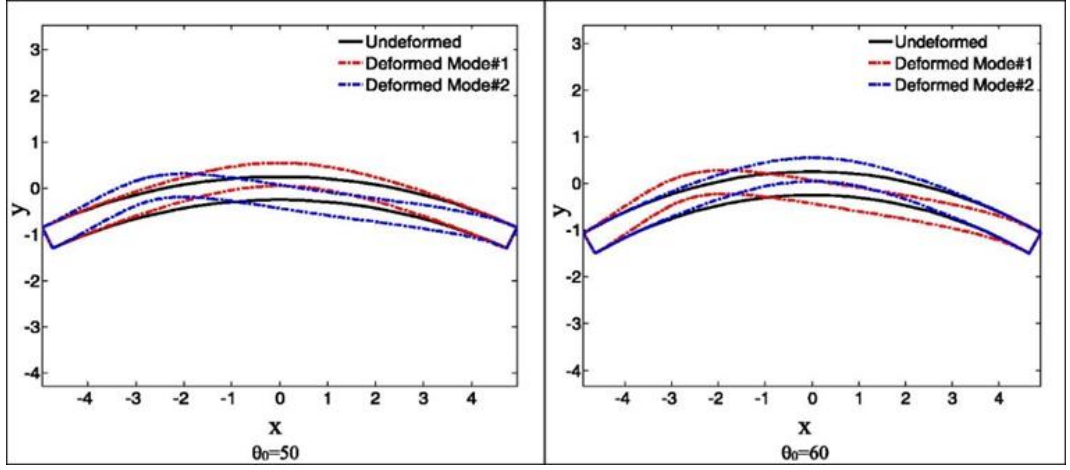


Figure 17: Mode shapes of two smallest natural frequency of curved clamped-clamped edge nanobeam.

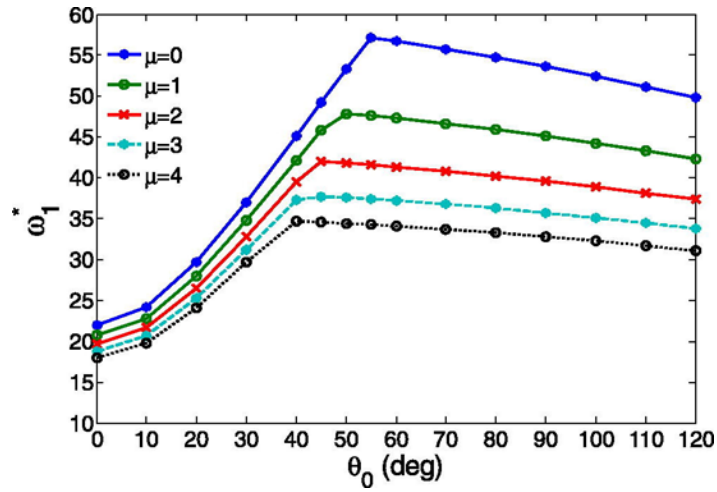
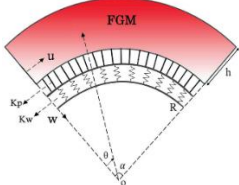
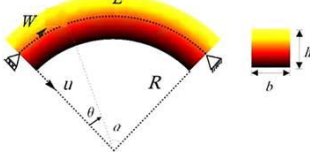
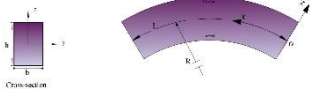
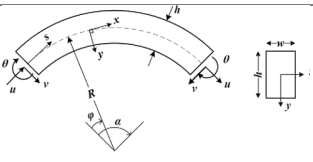
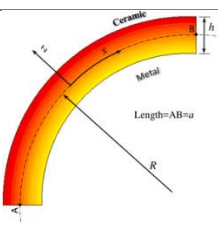


Figure 18: Clamped-clamped edge curved nanobeam's natural frequency after applying the nonlocal parameter effect.

In the Table 4, different geometry and shapes of FG nano arches are elaborated.

Table 4: Comparison of functionally graded multilayer shallow nano arches.

Schematic	Effective Material Properties	Model Principle	Ref.
	$E(z) = E_1[1 - (1 - \frac{E_2}{E_1})V_\beta]$ $V_\beta = (\frac{z}{h} + \frac{1}{2})^\beta$ $E(z) = E_1(E_2/E_1)^{V_\beta}$	Hamilton's principle.	[289]
	$E(z) = (E_c - E_m)(\frac{z}{h} + \frac{1}{2})^p + E_m$ $\alpha(z) = (\alpha_c - \alpha_m)(\frac{z}{h} + \frac{1}{2})^p + \alpha_m$ $\rho(z) = (\rho_c - \rho_m)(\frac{z}{h} + \frac{1}{2})^p + \rho_m$	Hamilton's principle.	[290]
	$E(z) = (E_c - E_m)V_c(z) + E_m$ $G(z) = (G_c - G_m)V_c(z) + G_m$ $V_c(z) = (\frac{z}{h} + \frac{1}{2})^p$	Hamilton's principle.	[291]
	$E(y) = E_{out} + (E_{in} - E_{out})(\frac{1}{2} + \frac{y}{h})^n$ $G(y) = G_{out} + (G_{in} - G_{out})(\frac{1}{2} + \frac{y}{h})^n$	Timoshenko theory.	[292]
	$P_e(z, T) = P_1(T)V_1(z) + P_2(T)V_2(z)$ $V_1(z) = (0.5 + \frac{z}{h})^p$ $V_2(z) = 1 - V_1(z)$	Hamilton's principle.	[293]

2.4 Graphene Reinforced Shallow Arches

With continued advances in the fabrication and characterization of graphene-based composites, it is likely that these materials will find increasing use in a wide range of applications, from structural engineering to biomedical devices. In addition, Huang et al. [294] conducted a crack analysis of multilayer FG composite strips which graphene nanoplatelets-used as reinforcement material and acknowledged that an increase in weight fraction of graphene platelet leads to a decrease in stress intensity factor in

pattern X, whereas, an increase in stress intensity factor in pattern A and pattern O. The distribution patterns are given below in Fig. 19.

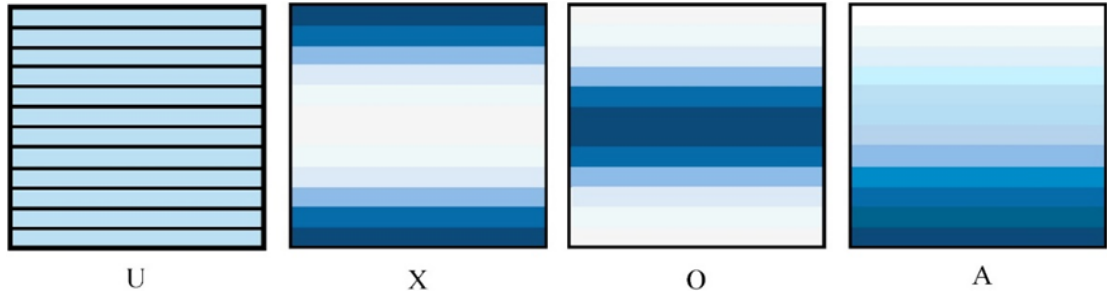


Figure 19: Four GPL distribution patterns.

Sobhy et al. [295] employed a new higher-order theory to analyze magneto-hydrothermal bending graphene and aluminum reinforced functionally graded shallow arches. Baranifard et al. [296] examined the in-plane dynamic behaviors of graphene reinforced curved beams in thermal environments and realized that the radial deflection amplitudes of the structure decrease for all distribution patterns with a minor amount of GPLs increasement into matrix. In addition, Shen et al. [297] conducted an experiment on post buckling attitude of curved composite shells with the addition of graphene in a thermal area which are exposed to the lateral pressure and concluded the experiment when realized that the post buckling equilibrium paths are no longer the bifurcation type. Talebizadehsardari et al. [298] used graphene reinforced polymer nanocomposite curved beams to analyze the elastic wave characteristics and thickness stretching effect of the structure which is the GNPs distribution patterns (a) and its weight fraction variation versus the number of layers (b) illustrated below in Fig. 20.

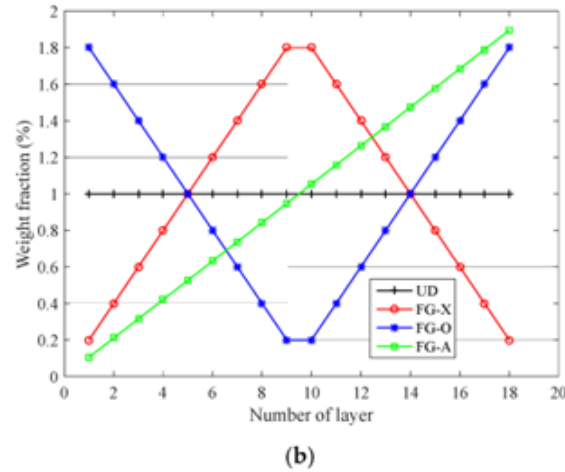
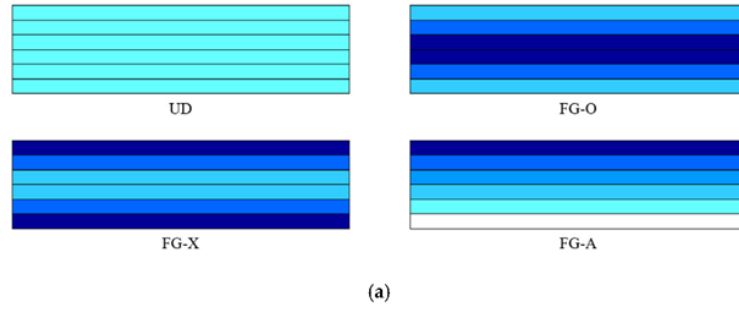


Figure 20: (a) The GNPs distribution patterns and (b) weight fraction variation versus the number of layers.

Numerous studies have shown that graphene and its derivatives have a considerable reinforcing influence on the mechanical characteristics of polymer composites [299]. As an example of various application, Mahani et al. [300] carried out an experiment on graphene platelets reinforced multilayer conical shell for the thermal buckling analysis. In another example, Sobhy et al. [301] has been used graphene reinforced functionally graded multilayer shallow shells to analyze their magneto-electro-thermal bending behaviors. Graphene, on the other hand, is a two-dimensional material that is known for its exceptional strength, stiffness, and toughness [302]. Sobhy et al. [303] employed FG multilayer curved nanobeams with the addition of graphene platelets and analyzed the dynamic behavior under stretching effect on thickness with a magnetic field around it. It can be seen that from Fig. 21 (a) ω^* given as fundamental frequency and (b) Γ given as a buckling load of proposed structure below, the increase in

thickness provides a decrease in fundamental frequency and an increase in buckling load.

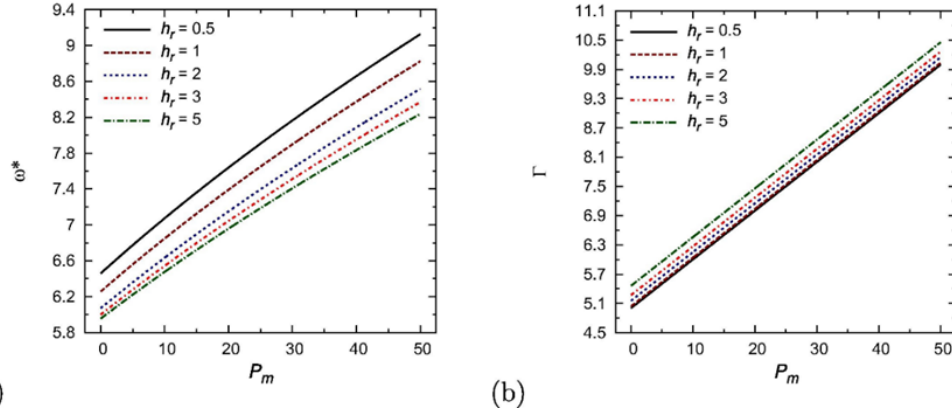


Figure 21: Impact of a magnetic field (a) ω^* given as fundamental frequency and (b) Γ given as a buckling load of proposed structure.

The use of graphene-reinforced shallow arches has significant implications for the design of new materials and structures, with the potential to create even better performance than currently available. Li et al. [304] employed FG porous curved beams with the addition of graphene platelet to analyze the dynamic attitude of the proposed structure. According to the result of the experiment which is illustrated in Fig. 22 below as the change in buckling load with (a) length to width proportion, and (b) thickness of graphene platelet, an upgrade in the weight fraction provides a significant decrease in buckling load.

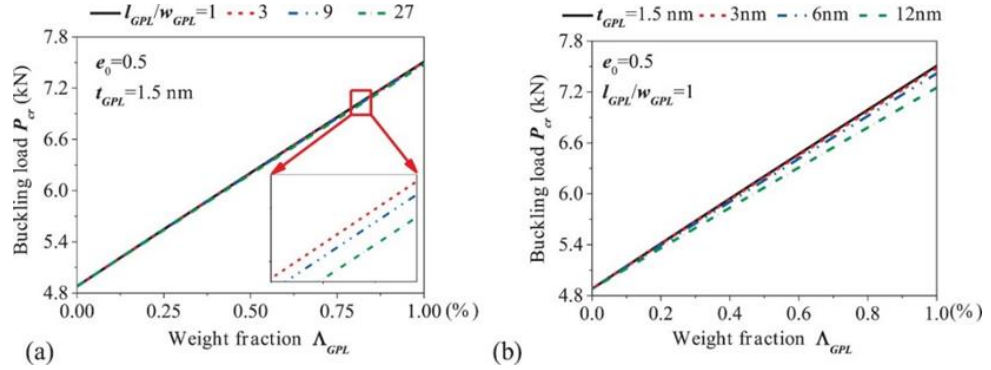


Figure 22: The change in buckling load with (a) length to width proportion, and (b) thickness of graphene platelet.

In the Table 5, different geometry and shapes of FG graphene reinforced shallow arches are elaborated.

Table 5: Various experiments of graphene reinforced shallow arches.

Schematic	Distribution Pattern Formulas	Model	Ref.
	$U - GPLRC: V_{GPL}^{(k)} = V_{GPL}^*$ $X - GPLRC: V_{GPL}^{(k)} = \frac{2V_{GPL}^* 2k - N_L - 1 }{N_L}$ $O - GPLRC: V_{GPL}^{(k)} = 2V_{GPL}^* \left(1 - \frac{ 2k - N_L - 1 }{N_L} \right)$ $V_{GPL}^* = \frac{W_{GPL}}{W_{GPL} + \frac{\rho_{GPL}}{\rho_m}(1 - W_{GPL})}$	Halpin-Tsai micromechanics model	[62]
	$U: V_{GPL}^n = V_{GPL}^*$ $X: V_{GPL}^n = \frac{2V_{GPL}^* 2k - N_L - 1 }{N_L}$ $O: V_{GPL}^n = 2V_{GPL}^* \left(1 - \frac{ 2k - N_L - 1 }{N_L} \right)$ $A: V_{GPL}^n = \frac{V_{GPL}^*(2k - 1)}{N_L}$ $V_{GPL}^* = \frac{W_{GPL}}{W_{GPL} + \frac{\rho_{GPL}}{\rho_m}(1 - W_{GPL})}$	Halpin-Tsai micromechanics model	[147]
	$U - GPLRC: V_{GPL}^k = V_{GPL}^*$ $X - GPLRC: V_{GPL}^k = \frac{2V_{GPL}^* 2k - N_L - 1 }{N_L}$ $O - GPLRC: V_{GPL}^k = 2V_{GPL}^* \left(1 - \frac{ 2k - N_L - 1 }{N_L} \right)$ $V_{GPL}^* = \frac{W_{GPL}}{W_{GPL} + \frac{\rho_{GPL}}{\rho_m}(1 - W_{GPL})}$	Halpin-Tsai micromechanics model	[156]
(a) Geometry and coordinates of the FG-GPLRCs curved beams	$UD: g_{GPL} \left(\frac{N+1}{2} - \left k - \frac{N+1}{2} \right \right) / (2+N)$ $FG - O: 4g_{GPL} \left(\frac{1}{2} + \left k - \frac{N+1}{2} \right \right) / (2+N)$ $FG - X: 4g_{GPL} \left(\frac{1}{2} + \left k - \frac{N+1}{2} \right \right) / (2+N)$ $FG - A: 2kg_{GPL} / (2+N)$	Halpin-Tsai model	[305]

	$ \begin{aligned} &U - GPLRC: V_{GPL}^k = V_{GPL}^* \\ &X - GPLRC: V_{GPL}^k = \frac{2V_{GPL}^* 2k - N_L - 1 }{N_L} \\ &O - GPLRC: V_{GPL}^k = 2V_{GPL}^* \left(1 - \frac{ 2k - N_L - 1 }{N_L} \right) \\ &V_{GPL}^* = \frac{W_{GPL}}{W_{GPL} + \left(\frac{\rho_{GPL}}{\rho_m} \right) (1 - W_{GPL})} \end{aligned} $	Halpin-Tsai micromechanics model [306]
	$ \begin{aligned} &UD: g_{GNP}^* \\ &FG - O: 2g_{GNP}^*(N_L + 1 - 2k - (N_L + 1))/(2 + N_L) \\ &FG - X: 2g_{GNP}^*(1 + 2k - (N_L + 1))/(2 + N_L) \end{aligned} $	Halpin-Tsai model [307]

2.5 Carbon Nanotube Reinforced Shallow Arches

Carbon nanotube reinforced shallow arches are a relatively new class of composite materials that have been attracting significant attention in the field of structural engineering. These composites are characterized by their unique combination of high strength, stiffness, and flexibility, which makes them an ideal candidate for reinforcing shallow arches. Babaei et al. [308] conducted research on carbon nanotube reinforced functionally graded shallow arches to examine the dynamics and instability behavior on nonlinear elastic foundation and realized that an upgrade in the slenderness ratio provides a higher natural frequency with an increase in the density of snap-through instability of the composite arch which is indicated below in Fig. 23. Carbon nanotubes (CNTs) are a type of nanomaterial made up of cylindrical carbon molecules, which are arranged in a hexagonal lattice. CNTs have unique mechanical, electrical, and thermal properties, making them an attractive material for a wide range of applications. When embedded in a polymer matrix, CNTs can significantly improve the mechanical properties of the resulting composite material, including its strength, stiffness, and toughness. Daikh et al. [309] carried out an experiment on carbon nanotube reinforced composite curved beams to analyze its buckling behavior in thermal area and realized

that as temperature goes up, a decrease in the buckling load takes place which is indicated in Fig. 24 as dimensionless buckling load versus temperature graph.

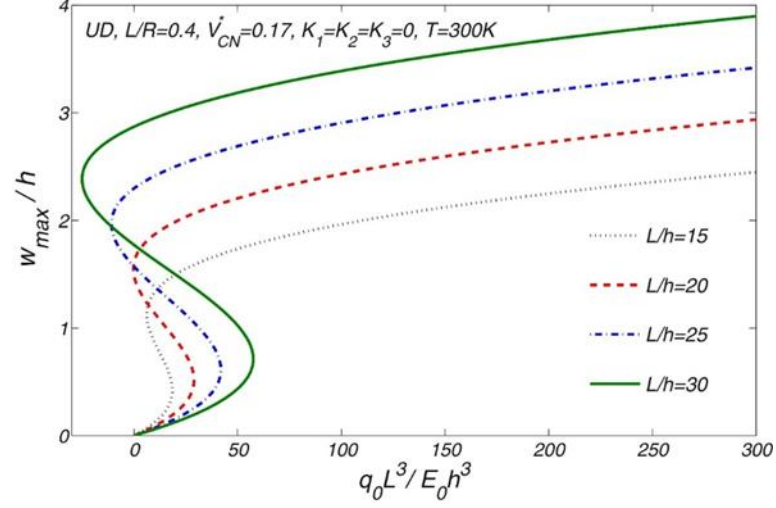


Figure 23: Impact of slenderness ratio on the snap-through buckling of proposed design.

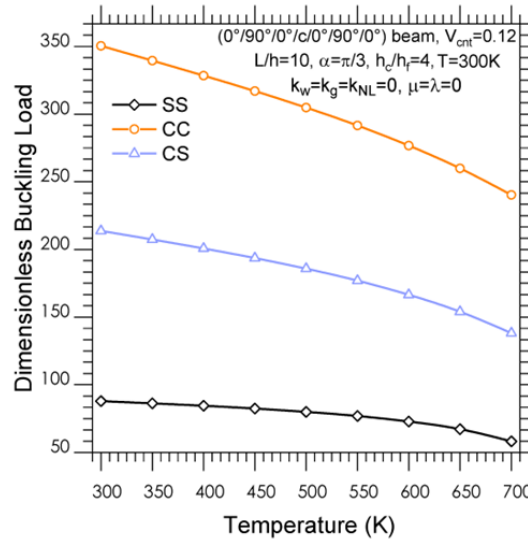


Figure 24: Dimensionless buckling load versus temperature.

The unique properties of CNTs make them ideal candidates for reinforcing curved beams. In carbon nanotube reinforced shallow arches, a layer of CNT-reinforced polymer is applied to the underside of the arch, providing additional support and enhancing the overall structural integrity of the arch. The CNTs align themselves along

the direction of the applied load, resulting in a material that is stronger in the direction of the load. This property is particularly useful in shallow arches, where the load is primarily compressive, and the arch's curvature creates tension on the underside of the arch. [310]. Researchers have explored different approaches for incorporating CNTs into curved beams, including infusion, coating, and direct growth [311]. These methods can create a strong bond between the CNTs and the beam matrix, ensuring efficient load transfer and stress distribution [312]. Viet et al. [313] employed carbon nanotubes as reinforcement material to examine the impact of this reinforcements on the transfer of load due to the number of layers which is illustrated in Fig. 25 below.

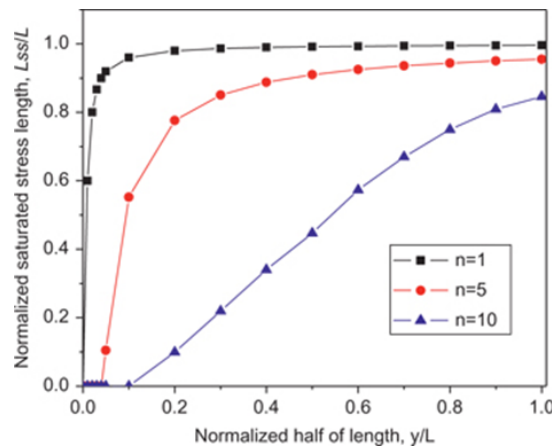


Figure 25: Impact of CNTs on the transfer of load due to the number of layers.

Several studies have investigated the mechanical behavior of CNT reinforced curved beams using numerical simulations and experimental testing [314–316]. The results have shown that CNTs can significantly improve the bending and torsional stiffness of curved beams, as well as their ultimate strength and fatigue resistance [317,318]. Karami et al. [319] stated that an upgrade takes place in the buckling load due to a higher volume fraction of carbon nanotubes which is indicated in Fig. 26 below as (a) UD distribution and (b) FG distribution.

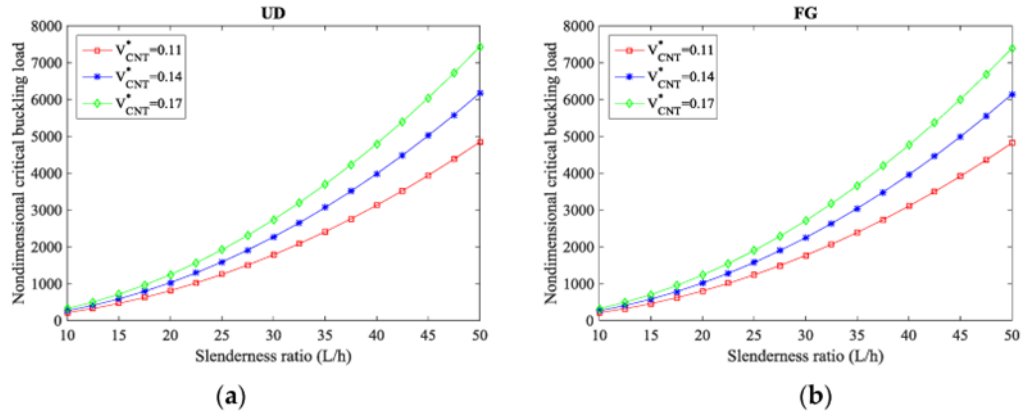
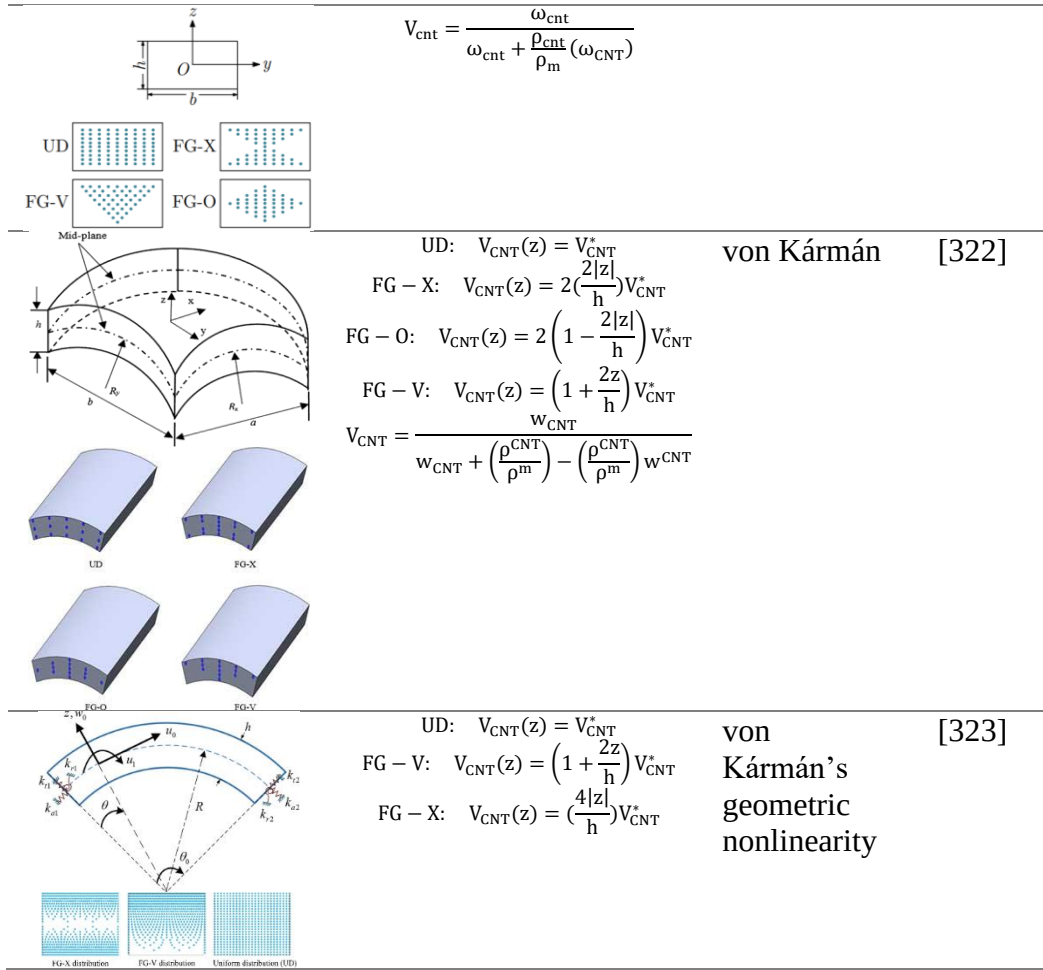


Figure 26: Improvement on buckling load due to various volume fractions (a) UD distribution and (b) FG distribution.

The use of carbon nanotube reinforced shallow arches has the potential to revolutionize the design and construction of shallow arches, improving their strength and durability while reducing their weight and material cost. As such, carbon nanotube reinforced shallow arches represent an exciting area of research and development with significant potential for practical applications in the construction industry. In the Table 6 below, different geometry and shapes of FG carbon nano tube reinforced shallow arches are elaborated.

Table 6: Various experiments of carbon nano tube reinforced shallow arches.

Schematic	Distribution Pattern Formulas	Theory	Ref.
	UD: $V_{CNT} = V_{CNT}^*$ FG - X: $V_{CNT} = 4\left(\frac{ Z }{H_b}\right)V_{CNT}^*$ FG - O: $V_{CNT} = \left(2 - \frac{4 Z }{H_b}\right)V_{CNT}^*$ FG - V: $V_{CNT} = \left(1 - \frac{2Z}{H_b}\right)V_{CNT}^*$ FG - A: $V_{CNT} = \left(1 + \frac{2Z}{H_b}\right)V_{CNT}^*$ $V_{CNT}^* = \frac{\omega_{CNT}}{\omega_{CNT} + \lambda(1 - \omega_{CNT})}$	Reddy shear deformation theory	[320]
	UD: $V_{cnt}(z) = V_{cnt}$ FG - X: $V_{cnt}(z) = \frac{4 z }{h}V_{cnt}$ FG - V: $V_{cnt}(z) = \left(1 + \frac{2z}{h}\right)V_{cnt}$ FG - O: $V_{cnt}(z) = \left(2 - \frac{4 z }{h}\right)V_{cnt}$	Donnell's shallow shell theory	[321]



2.6 Natural Fiber Composite Reinforced Shallow Arches

Nano fiber composite reinforced functionally graded multilayer shallow curved beams are a topic of great interest in the field of nanotechnology and structural engineering [324–326]. These structures are made up of a composite material reinforced with nanofibers and can provide superior mechanical properties compared to traditional homogeneous materials [327–329]. Natural fiber composite reinforced shallow arches are a class of composite materials that utilize natural fibers as reinforcement to enhance the mechanical properties of the composite. These materials have gained increasing attention in the field of structural engineering due to their sustainability, low cost, and potential for producing lightweight and high-performance composites. Natural fiber

composite reinforced shallow arches can be used to reinforce shallow arches in construction applications, providing improved strength, stiffness, and durability [330]. Steinbrecher et al. [331] carried out an experiment on Cosserat fiber beams to analyze the consistent coupling of positions and rotations. In addition, Qui et al. [332] introduced a type of curved beam with fiber Bragg grating for health monitoring and managed to achieve a good performance with stable and reliable device.

Natural fibers such as jute, flax, sisal, and hemp are widely used in the production of NFCs due to their abundance, renewability, and biodegradability. When used as reinforcement in a polymer matrix, NFCs can significantly improve the mechanical properties of the resulting composite material. Garmendia et al. [333] employed advanced composite materials to reinforce historical masonry arches. A physical experiment carried out on the proposed model and presented in Fig. 27 after the failure. Also, load versus displacement graph of mentioned experiment is shown in Fig. 28.

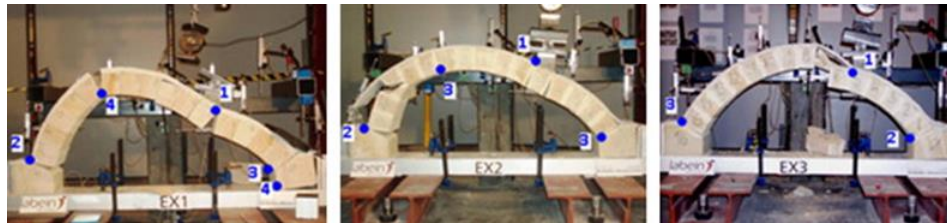


Figure 27: Experiment with damaged points.

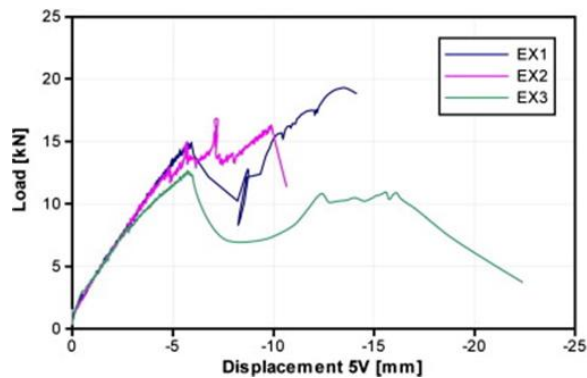


Figure 28: Load versus displacement graph.

Natural fibers offer several advantages over traditional reinforcement materials such as glass fibers and carbon fibers, including lower cost, higher specific strength, and environmental sustainability. In natural fiber composite reinforced shallow arches, a layer of natural fiber-reinforced polymer is applied to the underside of the arch, providing additional support and enhancing the overall structural integrity of the arch. The natural fibers align themselves along the direction of the applied load, resulting in a material that is stronger in the direction of the load. This property is particularly useful in shallow arches, where the load is primarily compressive, and the arch's curvature creates tension on the underside of the arch. On the other hand, Xie et al. [334] employed ultrahigh performance fiber to reinforced a concrete curved beam to conduct an experimental and analytical analysis. It was seen that the increase in curvature lead a variation in failure mechanism of the proposed design. Fig. 29 below indicates the (a) alteration of bend-to-twist proportion and (b) the change in curvature diagrams.

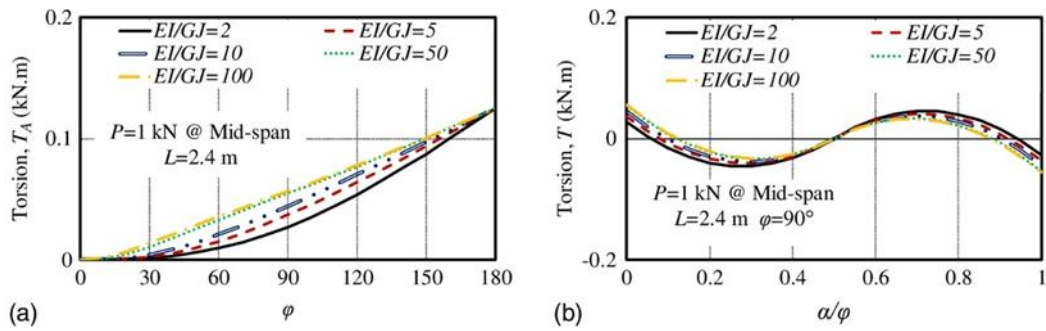


Figure 29: (a) Alteration of bend-to-twist proportion and (b) the change in curvature diagrams.

In another approach, Alecci et al. [335] employed masonry arches and used lime-based mortar composite to reinforce the structure. In addition, Castori et al. [336] employed fiber composites to reinforce masonry arches. Ali et al. [337] introduced a novel composite arch bridge and examined the structural performance of the proposed

design. The finite element vibration analysis of proposed structure is given below in Fig. 30.

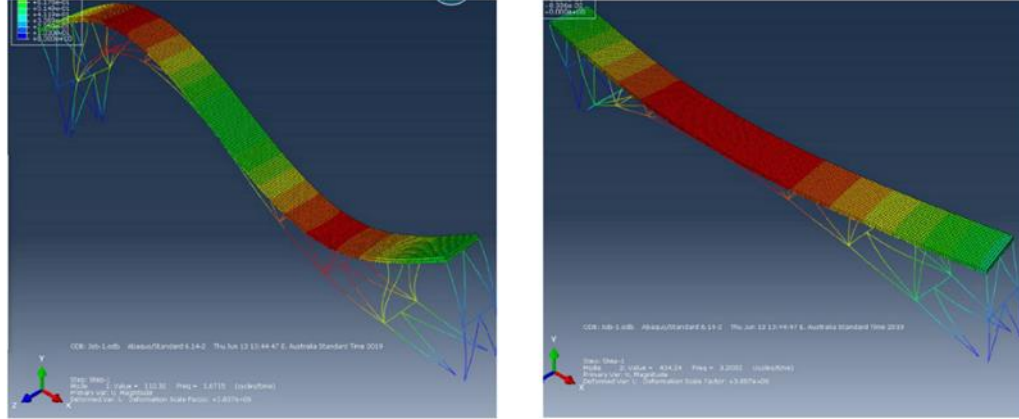


Figure 30: Shapes of the first and second modes of vibration for FE analysis.

Gioffré et al. [338] employed a composite material which is made from a hemp fiber for reinforcing the model of arch to analyze it experimentally and reached a conclusion that hemp-fiber bio-composite is a good reinforcing material for old masonry buildings. Kunj et al. [339] reinforced curved composite beams by using hybrid fabric polymer matrix to analyze its vibrational behavior. Helaili et al. [340] employed braided natural fibers to reinforce curved composite plates to analyze the load bearing behavior. Moreover, Baba et al. [341] introduced curved sandwich beams to analyze the free vibration behavior and realized that an upgrade in the angle of curvature leads to an improvement on the natural frequency.

2.7 Modelling and Simulation

2.7.1 Nonlinear Bending

Bending analysis is a good fire starter for the mechanical analysis of composite structures. Kiss et al. [342] has used arbitrary concentrated radial forces to analyze the nonlinear stability of shallow arches. Nonlinear bending analysis involves the use of mathematical models to simulate the behavior of shallow arches under load. These

models take into account the geometric and material nonlinearities that occur when the arch is subjected to large deformations. Wu et al. [343] conducted an experiment by using the von Kármán nonlinear theory on FG shallow arches which are strengthened by graphene to analyze the bending features with auxetic effects. A good amount of development achieved in the bending characteristics as the graphene volume proportion improved in the top and bottom substrates under the room temperature state. This is shown in Fig. 31 below as a comparison with an existing work and the proposed model; (a) multilayer composite beam with different dispersion patterns of graphene as; UD, FG-X, and FG- Λ ; (b) boundary condition of two arches.

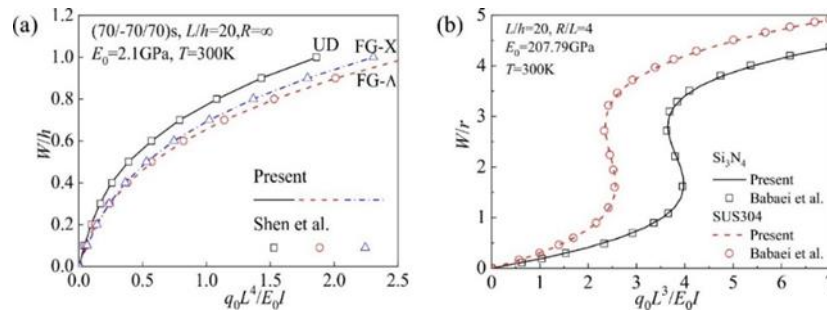


Figure 31: Comparison with an existing work and the proposed model; (a) multilayer composite beam with different dispersion patterns of graphene as; UD, FG-X, and FG- Λ ; (b) boundary condition of two arches.

Another important factor in nonlinear bending analysis of shallow arches is the geometric nonlinearity. This refers to the fact that the arch undergoes large deformations, which can cause changes in its shape and geometry. Nonlinear bending analysis must take into account the changes in the arch's shape and geometry, as well as the resulting changes in the distribution of stresses and strains. Static bending analysis conducted by Talebizadehsardari et al. [344] for CNT reinforced functionally graded polymer composite curved beams and concluded that static deflections were decreased due to the raise in CNT volume fraction. Anirudh et al. [345] has used three different variations of dispersion for porosity and GPLs to analyze the bending in the

beam and resulted with an effect on the beam stiffness due to the weight distribution of GPLs. Uniform normal load applied on the curved beam by Luu et al. [346] to investigate the dynamic behavior of the arch-shaped beam. Keibolahi et al. [347] employed pin-ended deep arches by subjecting one substrate to a thermal shock and keeping the second substrate at the reference temperature to analyze the nonlinear bending behavior. The effect of thermal shock on the vibration of proposed design is illustrated below in Fig. 32 as (a) 310K and (b) 400K.

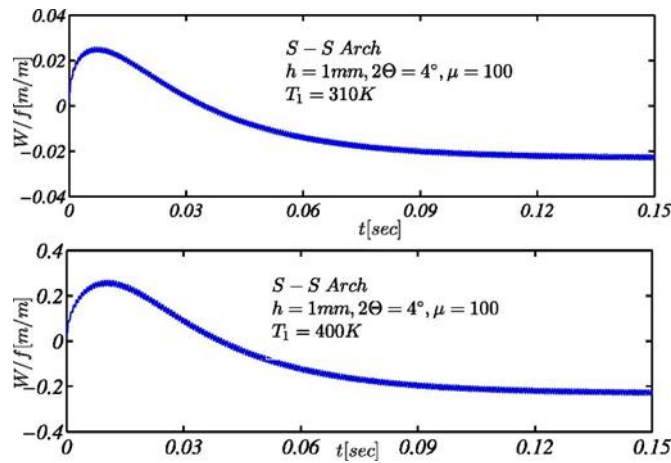


Figure 32: Effect of thermal shock on the vibration of proposed design as (a) 310K and (b) 400K.

Several studies have been conducted on the nonlinear bending behavior of shallow arch structures and proved that as the degree of curvature increases, the arch becomes stiffer and more resistant to deformation. In addition, increasing the thickness of the arch can improve its load-carrying capacity and reduce the risk of failure. Bending characteristics are changed immediately as the curvature increased which causes a greater opposition against the transverse loading. This behavior is examined by Khaniki et al. [348] when conducting an experiment on thick visco-hyper-elastic arches in axial, transverse and rotation motions. It is indicated in Fig. 33 as bending

deformation of proposed model arch with different curvature variations as; (a) axially deformed, (b) transversely deformed and (c) rotationally deformed.

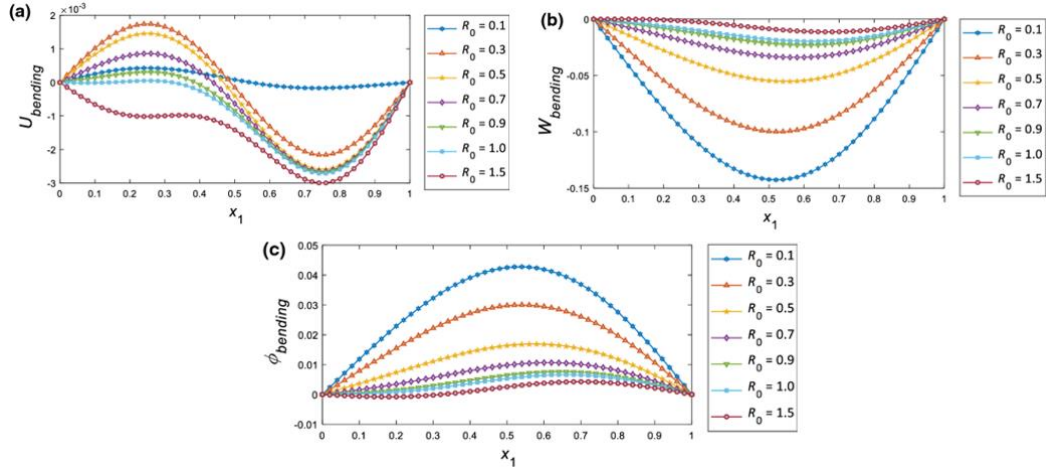


Figure 33: Bending deformation of proposed model arch with different curvature variations as; (a) axially deformed, (b) transversely deformed and (c) rotationally deformed.

2.7.2 Buckling and Post-buckling

By taking into account the stress and twisting characteristics of arches, the nonlinear dynamic buckling attitudes are found to be the primary determinants of the curve-carrying valence. Various studies investigated the dynamic durability and buckling attitude of isotropic curve configuration [54,349–354]. Zhang et al. [355] conducted theoretical, computational, and experimental investigations of the nonlinear in-plane buckling response of orthotropic composite curves with fixed support subjected to radial point loading, and provided analytical answers for the critical buckling load and nonlinear equilibrium approach, which were validated by the finite element model. Asgari et al. [356] used linear temperature gradient to conduct an experiment on functionally graded shallow arches for the analysis of the nonlinear thermal buckling. Eroglu et al. [357] used a variational iteration approach to examine the buckling of snap-through and bifurcation attitudes of FG arches with pin-ended, while Xu et al.

[358] published a CUF-based finite element model to explore the nonlinear performance of isotropic curves reinforced by orthotropic fiberglass layer subjected to -point radial loads in their plane, and demonstrated good alignment between the numerical forecasts and laser-based experiment outcomes.

Timoshenko and Gere et al. [359] introduced the buckling behavior of shallow arches in 1961 and Simitres et al. [360] in 1976. The theory is applied to estimate the buckling load of elasticity on shallow arches. According to Pi et al. [361] it is crucial to be able to correctly anticipate an arch's in-plane elastic buckling stress in order to account for it in the design and avoid in-plane collapse. Pi et al. [362] stated that when the applied force increases on the shallow arch, a nonlinear in-plane structural behavior can be observed on it. Also, Fig. 34 indicates the two buckling modes (a) asymmetric bifurcation and (b) symmetric snap-through modes when the applied load achieves a particular value. According to Pi et al. [363] the deformation impacts should be taken into account before the buckling because they become non-linear.

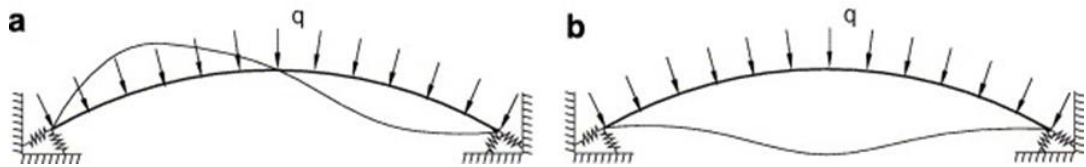


Figure 34: (a) Asymmetric bifurcation and (b) symmetric snap-through modes.

Despite having exceptional load-bearing capacity, arches are susceptible to bowing from outside applied forces, which can lead to dangerous structural instability [364,365]. The non-linear behavior which is snap-through buckling can be seen under even a little amount of curvature [366]. Pinto et al. [367] stated that external applied lateral load or alteration in the thermal environment can cause the shallow arch to undergo snap-through buckling behavior. Virgin et al. [368] stated that the change

through two stable position can be called as the symmetric snap-through buckling. Rao et al. [369] stated that the asymmetric bifurcation buckling causes arch to have one of the asymmetric deformations. Chen et al. [370] stated that these can affect the thickness, axial force, the change in temperature, rise in curvature, surface and size effects. The arch elements can be damaged under different loading types [371]. A structure's capacity to withstand loads is known as structural stability [372]. Kiss et al. [373] stated that the loss of this ability leads to instability, which is a condition in which the structure is no longer in balance with a shift in its geometry. Also, Kiss et al. [374] mentioned that buckling affects both the configuration's slenderness ratio and stiffness. In addition, Li et al. [375] summarized the deformation by stating the compressive stress of the arch wall develops until it reaches to a critical level or beyond which causes arch to buckle and undergo symmetrical snap-through or asymmetrical bifurcation deflection as a result of applied external loads which is shown in Fig. 35 with (a) asymmetrical, and (b) symmetrical deformation. Chen et al. [376] proposed that the arch shape is symmetric (a) after the snap-through, whereas, the arch can be deformed asymmetrically (b) after the asymmetric bifurcation which are shown in Fig. 36.

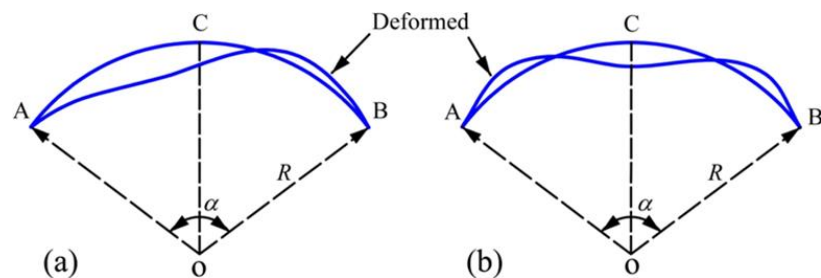


Figure 35: (a) Asymmetric, and (b) symmetric deformation.

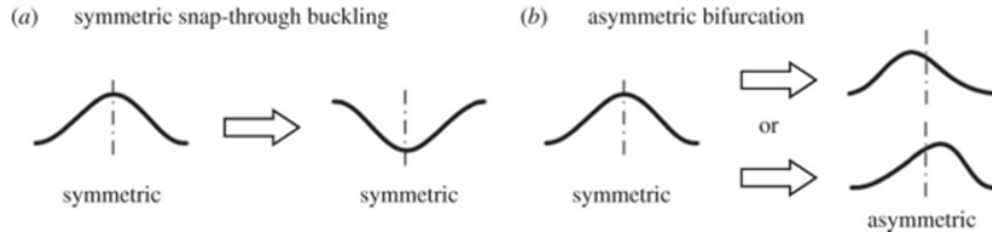


Figure 36: (a) Symmetrically deformed snap-through buckling and (b) asymmetrically deformed bifurcation buckling.

On the other hand, Wu et al. [377] conducted a research on FG graphene nanocomposite plates by spreading more graphene platelets next to the surface layers to investigate the thermal and post buckling behavior. Bateni et al. [378] carried out an analysis on FGM shallow arches to see the stability attitude of the model arches under central concentrated load including pre-buckling deformations. Adam et al. [379] employed shallow arches which are flexibly bonded to investigate the buckling and instability of in-plane arches. Furthermore, Atacan et al. [380] carried out an experiment on curved beams under the influence of lateral midpoint loads to examine the snap-through buckling behavior with the finite deflections. Tsiatas et al. [381] employed memory alloy shallow arches to analyze the buckling attitudes before and after the applied load. It can be seen that the slenderness ratio decreases when the arch radius increases. The proposed stress-strain curve for the shape memory alloy is illustrated in Fig. 37 below and it is obvious that the curve divided into three parts with being the linear part, the phase transformation part and again a linear part with stiffer material under different modulus of elasticity.

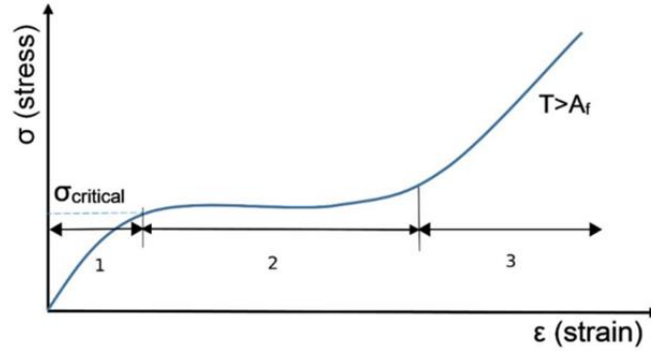


Figure 37: Stress-strain curve of shape memory alloy.

Wu et al. [382] used composite beams with carbon nanotube as reinforcing material to analyze the thermal and post-buckling behavior by using Timoshenko beam theory. After that, Wu et al. [383] carried out an experiment on the buckling attitude after a thermally induced application takes place on the same beams. Then, Wu et al. [384] used multilayer nanocomposite beams with graphene and analyzed the dynamic instability under the thermal environment. Zhang et al. [385] used the fixed shallow arches by using a load thermally and radially to examine the nonlinear in-plane buckling behavior. The increase in temperature affected displacement forces as well as bifurcation buckling behaviors are which are depicted in Fig. 38 below.

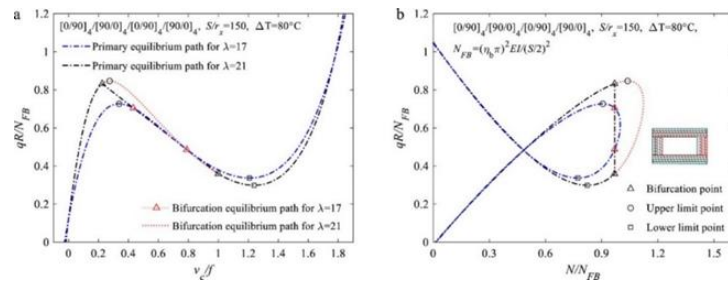


Figure 38: Bifurcation buckling attitudes of fixed-end multilayer composite arches after applying thermal and radial load.

Last but not least, composite arch structure analyzed by Yang et al. [65] by considering the size and weight fraction impacts of the graphene platelet reinforcements on proposed model structure to examine the behavior of buckling analytically. The

increase in weight fraction of graphene platelet showed an increase in buckling load. This influence of proposed model illustrated below in Fig. 39 for limit point buckling.

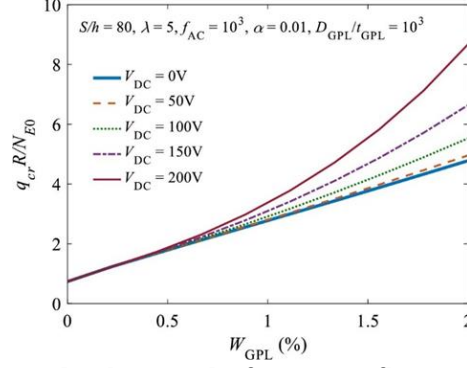


Figure 39: Graphene platelet weight fraction influence on buckling loads.

2.7.3 Linear Vibration

Functionally graded multilayer shallow arches have become a popular choice for structural elements in various fields of engineering due to their excellent mechanical properties. One of the key aspects in designing and analyzing these arches is to understand their vibration behavior. Amir et al. [386] conducted an experiment on graphene platelets reinforced curved microbeams by including the change due to thickness to analyze vibration. Jafari et al. [387] employed composite arches with the reinforcements of graphene platelet to analyze the free and forced dynamic behavior for a quasi-3D beam model under moving load. The results of the experiment indicated that FG-X distribution provided a major frequency under low deflections, whereas, FG-O distribution achieved less frequency with an increased deflection. However, the increase in weight proportion of graphene platelets produced low lateral deflection in higher frequencies. Mukahal et al. [388] carried out an experiment on FG graphene platelets sandwich curved beam to analyze the free vibration of the structure on viscoelastic foundation. Four distribution patterns of GPL have shown below in Fig. 40.

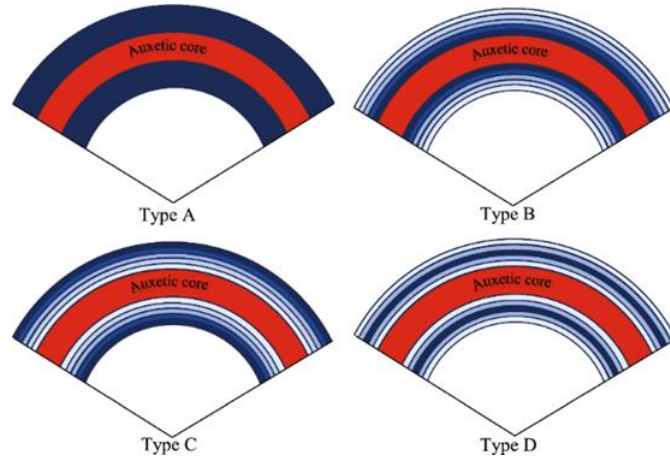


Figure 40: Four types of GPLs distribution sandwich-curved beams.

The inherent frequencies and mode forms of functionally graded arch structures are ascertained by linear vibration analysis [389]. The vibration analysis is typically done by applying the finite element method (FEM) [390]. In this method, smaller parts of the structure can be analyzed, and the motion equations for each element are reproduced under the assumed displacement functions [391]. These equations are then assembled into a global matrix equation that describes the motion of the entire structure [392]. To take into account the variation in material properties across the arch, the characteristics of materials vary simultaneously across each element [393]. This is done by using a power law function to explain the change in material characteristics through its thickness [394]. Furthermore, Malekzadeh et al. [395] employed functionally graded arches to analyze the in-plane free vibrations due to the temperature-dependent properties. Fig. 41 below indicates the effects of clamped circular arches along with the different temperatures on material characteristics which are subjected to uniform increase in temperature.

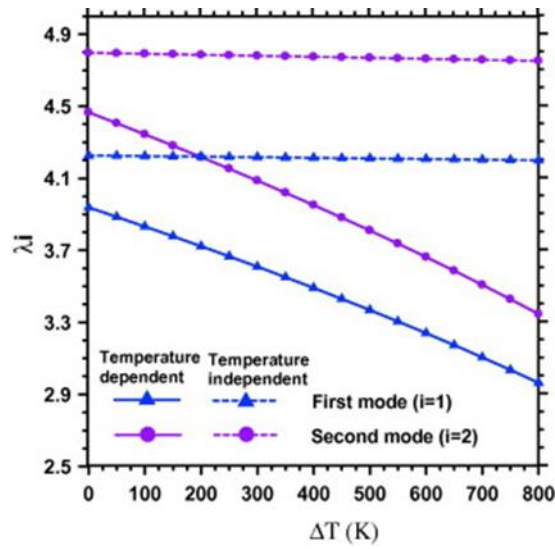


Figure 41: Effects of clamped circular arches along with the different temperatures on material characteristics.

The natural frequency is the frequency at which the structure will vibrate if excited by a small disturbance, and the mode shape is the shape that the structure will take on at that frequency [396]. In addition, Noori et al. [397] introduced a parabolic arch design with unstable cross section to analyze the in-plane free and forced dynamic behaviors of the structure. Fig. 42 below show that the arch frequency gets closer to the load frequency when a sine wave load is applied to the structure.

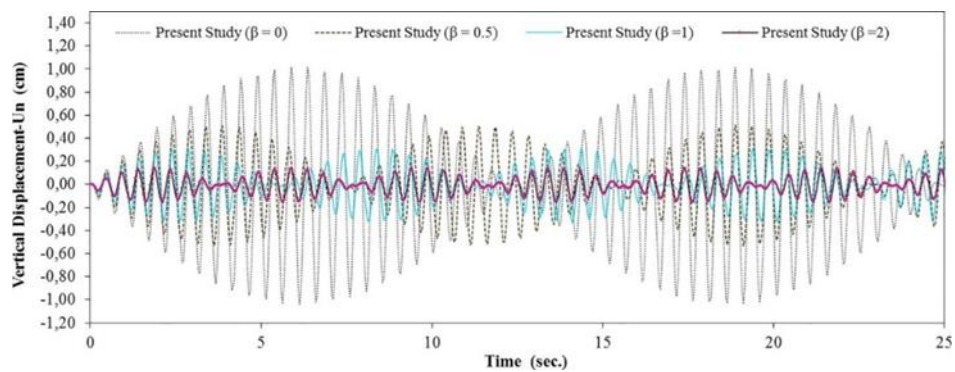


Figure 42: Vertical displacement versus time graph of the arch on a sine load.

2.7.4 Nonlinear Vibration

The nonlinear vibration attitude of functionally graded multilayer shallow arches has focused on developing mathematical models to describe their dynamic response [398]. These models take into account the characteristics of material on every layer and the geometric properties of the arch, such as its thickness, curvature, and span [399]. The models also counts the impacts of external loads and the resulting stress and strain distributions within the arch [400]. Outassafte et al. [401] employed a shallow arch to investigate the non-linear free and forced dynamic behaviors on a geometric scale. Fig. 43 below illustrates (a) the diagram of proposed shallow arch with the assistance of strings at either sides and, (b) the deployment of curvature due to the deflection.

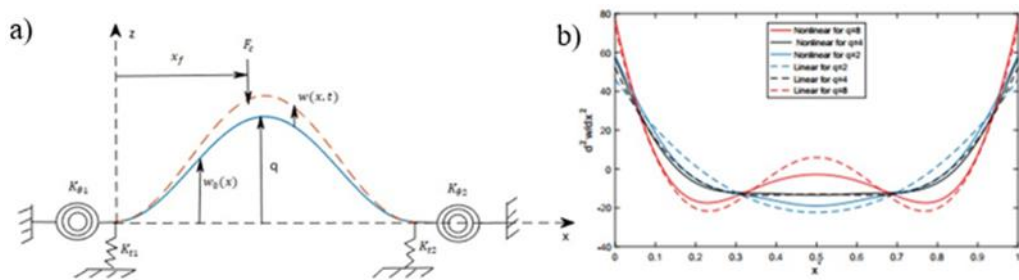


Figure 43: (a) The diagram of proposed shallow arch with the assistance of strings at either sides and, (b) the deployment of curvature due to the deflection.

Recent research has explored the use of various numerical techniques, such as finite element analysis and perturbation ways, to solve the differential equations of nonlinearity that describe the vibration behavior of functionally graded multilayer shallow arches [402,403]. These techniques have been found to provide accurate predictions of the arch's dynamic response under various loading conditions. Lakrad et al. [404] carried out an experiment on a shallow arch structure to estimate the nonlinear vibrations behavior when applying a resonant harmonic excitation with low frequency. Keibolahi et al. [405] applied rapid heating on shallow arches and analyzed

their nonlinear behaviors. Ghayes et al. [406] employed a conical shallow arch to analyze the forced nonlinear mechanic behavior. Moreover, Huynh et al. [407] used an isogeometric approach on FG arches with altered curvatures to estimate the bending, buckling and free vibration attitudes. As a result, Fig. 44 illustrates the change of the non-dimensional frequency with slope index for the proposed parabolic structure with different slenderness ratio (a) $s/h = 10$ and, (b) $s/h = 4$ and realized that the vibration mode shape transition is badly affected by the deployment of the material.

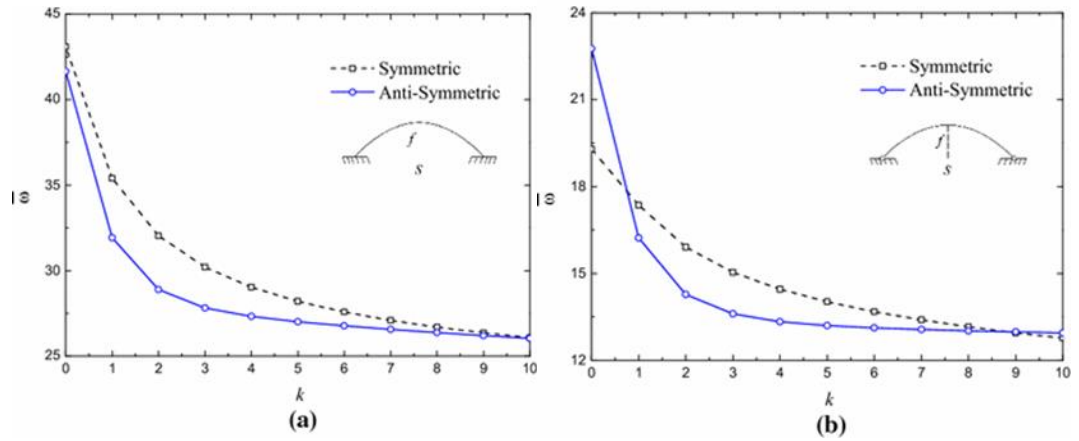


Figure 44: The change of the non-dimensional frequency with slope index for the proposed parabolic structure with different slenderness ratio (a) $s/h = 10$ and, (b) $s/h = 4$.

Last but not least, Kurtaran et al. [408] employed functionally graded deep curved beams to analyze the large displacement static and transient behaviors with generalized differential quadrature method. In Fig. 45 below, a central load with step pulse is applied to the proposed structure for the nonlinear transient analysis and realized that peak displacements increase as k value increases.

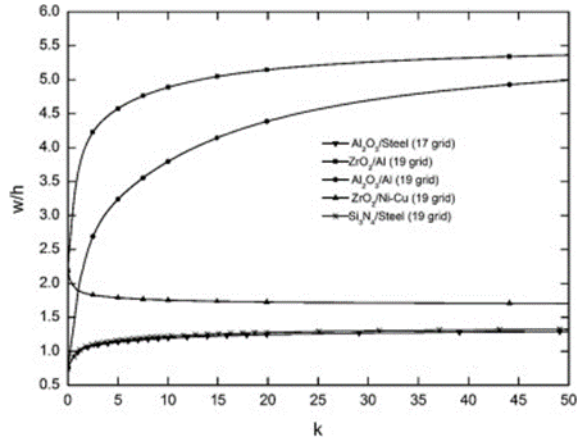


Figure 45: Maximum displacement versus material parameter for the proposed arch.

2.7.5 Dynamic Instability

Dynamic instability of shallow arches occurs when the structure is subjected to external loads that cause it to deflect from its original position. If the deflection is large enough, the arch can become unstable and start to oscillate, which can cause a defect on the structure [409]. The critical load that causes dynamic instability rely on different determinants, for example the material properties of the arch, its geometry, and the type of loading applied. Zhong et al. [410] used circular arch model with an effect of base excitation to conduct research on instability behavior. Zhong et al. [411] carried out an investigation on a shallow circular arch to analyze its in-plane dynamic instability behavior under a uniform load. Virtual work proved that the part of the instability depends on the periodical answers. It can be seen that the period $2T$ is upgraded on the instability part in contrast with the period T for both antisymmetric and symmetric instabilities. The part of symmetric instability for proposed arches with varied rise-span proportions are illustrated in Fig. 46 as (a) period T and (b) period $2T$.

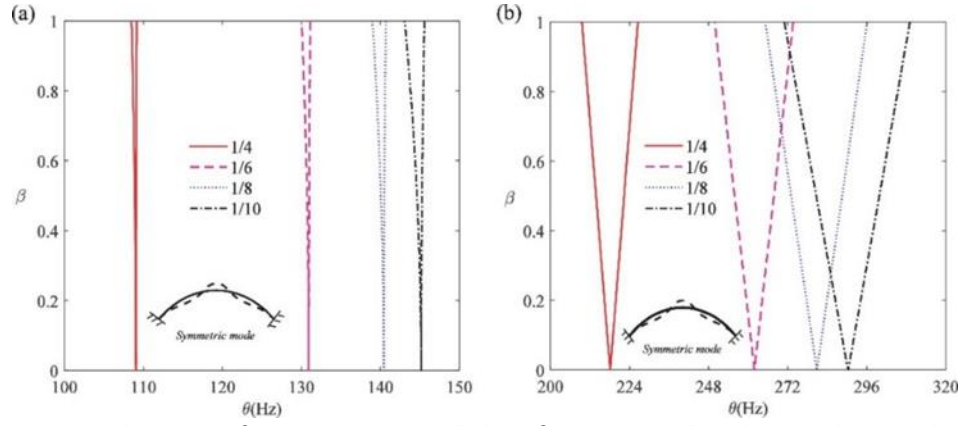


Figure 46: The part of symmetric instability for proposed arches with varied rise-span proportions are illustrated in Fig. 76 as (a) period T and (b) period $2T$.

Lu et al. [412] derived answers theoretically for the fixed arches under point load to examine the nonlinear instability before and after of applying the point load. The bifurcation instability and post-instability behaviors of the proposed solution is indicated in Fig. 47 below as (a) and (b) respectively.

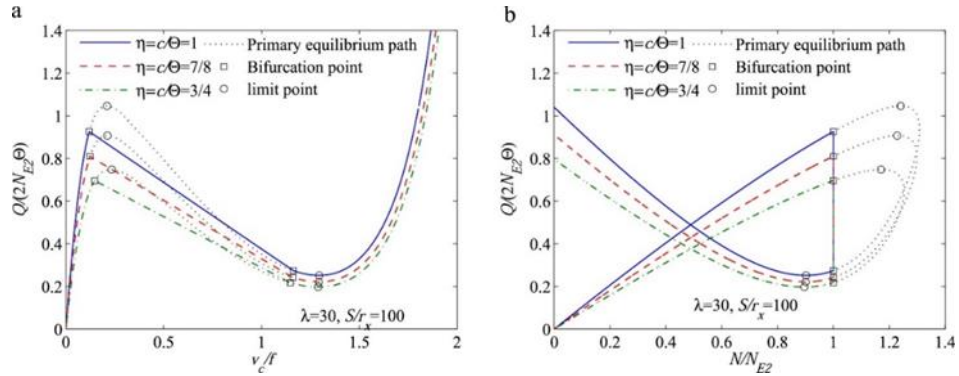


Figure 47: The (a) bifurcation instability and (s) post-instability behaviors of the proposed solution.

Yang et al. [61] employed models of machine learning to analyze the dynamic instability behavior of composite arches. To do this, power-law distribution technique has been used to spread the reinforcement material with atom vacancy defects. Graphene defect decreased the structural performance of the proposed arch model and that is because the distribution of graphene sheets has played an important role on the

performance of the structure. The impact of the defect of graphene on the principal dynamic unstable region of the proposed arch model is illustrated in Fig. 48 below and it is obvious that large defect increases the principal dynamic unstable region of the arch.

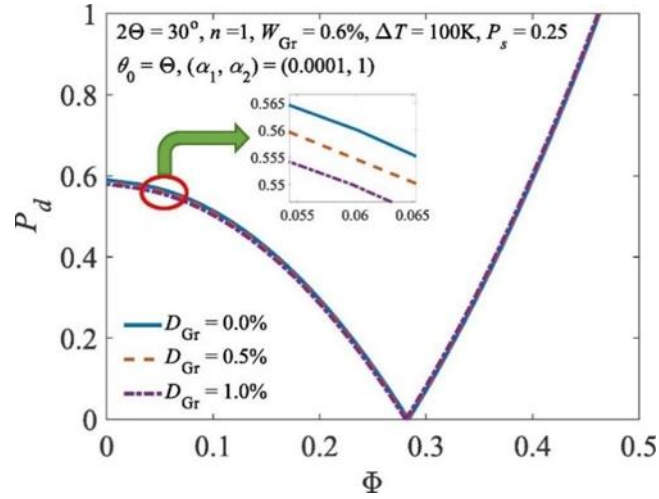


Figure 48: Graphene defect influence on the principal dynamic unstable region of the proposed arch.

Chapter 3

METHODOLOGY

In this research the NSG theory in nonlinear stability analyses of nano-arches exposed to thermomechanical loadings are investigated. The NSG-based nonlinear in-plane stability behavior of FG multilayer composite nano-arches under thermomechanical loading conditions is analyzed. In addition, the Gurtin-Murdoch surface elasticity theory is employed for the first time, within the framework of the higher-order shear flexible curved beam model to analysis the nonlinear stability characteristics of thermomechanical loaded FG laminated curved nanobeams. The investigation will be examined in two cases as Case 1: NSG-based FG multilayer nanocomposite shallow arch model which will allow us to capture the nonlocal stress tensor with strain gradient size dependencies and Case 2: Surface elastic-based FG laminated curved beam model which will allow us to capture the surface elasticity with the surface residual stress.

3.1 Case 1: NSG-based FG Multilayer Nanocomposite Shallow Arch Model

According to Figure 49, a micro/nano-arch made of perfectly bonded nanocomposite layers reinforced with graphene nanofillers under uniformly distributed radial compression in conjunction with thermal environment is taken into consideration. Based upon the richness of nanofillers for each layer, four types of FG multilayer distribution are employed namely as U-GNRC, A-GNRC, X-GNRC, and O-GNRC,

as illustrated in Figure 49. For instance, in O-GLRC pattern, the richness of graphene nanofillers in inner layers are more than that of outer ones.

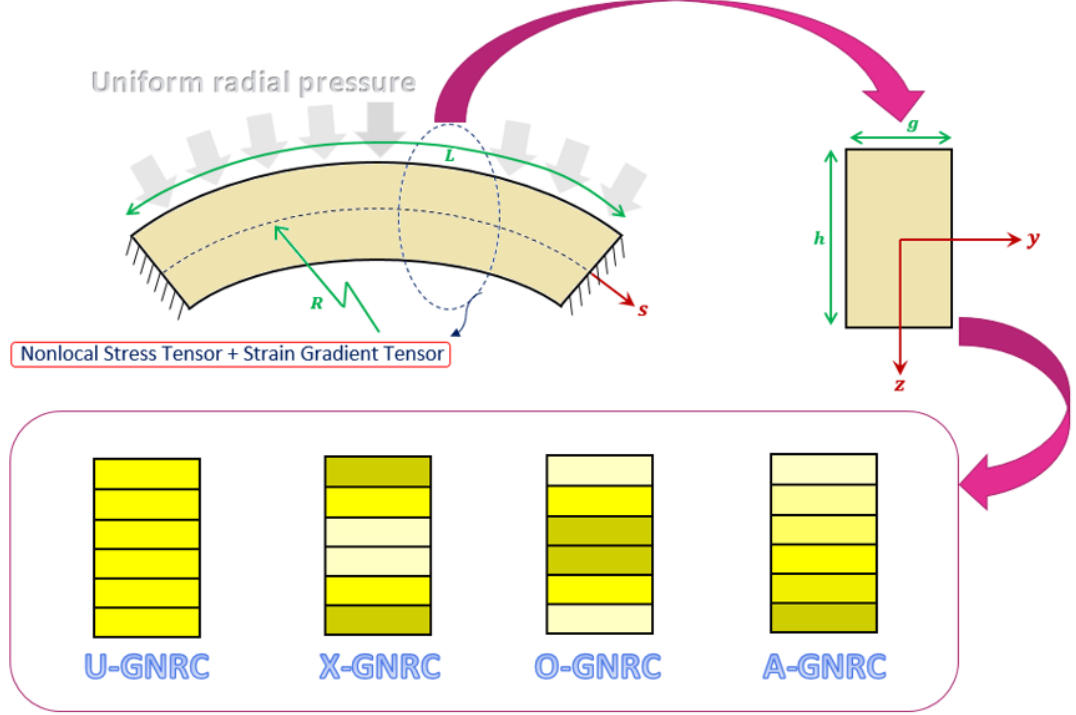


Figure 49: Schematic demonstration of NSG-based model of clamped FG multilayer micro/nano-arches with different FG patterns.

On the basis of the Halpin-Tsai homogenization scheme [413], one can extract the effective Young's modulus of k^{th} layer relevant to FG multilayer nanocomposites as shown below in Eq. (1);

$$E^{(k)} = \left[\frac{3}{8} \left(\frac{1 + \vartheta_L \eta_L V_{GN}^{(k)}}{1 - \eta_L V_{GN}^{(k)}} \right) + \frac{5}{8} \left(\frac{1 + \vartheta_T \eta_T V_{GN}^{(k)}}{1 - \eta_T V_{GN}^{(k)}} \right) \right] E_m \quad (1)$$

in which V_{GN} denotes the graphene nanofiller volume fraction, E_m represents the matrix Young's modulus, ϑ_L and ϑ_T are the geometrical parameters associated with graphene nanofillers as given in Eq. (2);

$$\vartheta_L = \frac{2a}{h_{GN}} \quad , \quad \vartheta_T = \frac{2b}{h_{GN}} \quad (2)$$

where a and b stand for the length and width of graphene nanofillers, and h_{GN} is their thickness. Also, efficiencies are illustrated in Eq. (3) below;

$$\eta_L = \frac{\frac{E_{GN}}{E_m} - 1}{\frac{E_{GN}}{E_m} + \vartheta_L} \quad , \quad \eta_T = \frac{\frac{E_{GN}}{E_m} - 1}{\frac{E_{GN}}{E_m} + \vartheta_T} \quad (3)$$

in which E_{GN} refers to the Young's modulus of graphene nanofillers.

For each considered FG multilayer pattern, the value of V_{GN} associated with k^{th} layer can be derived in the following forms which are illustrated in Eqs. (4a), (4b), (4c), and (4d);

$$U - GNRC: \quad V_{GN}^{(k)} = V_{GN}^* \quad (4a)$$

$$A - GNRC: \quad V_{GN}^{(k)} = \frac{2V_{GN}^*|2k-K-1|}{K} \quad (4b)$$

$$X - GNRC: \quad V_{GN}^{(k)} = 2V_{GN}^* \left(1 - \frac{|2k-K-1|}{K}\right) \quad (4c)$$

$$O - GNRC: \quad V_{GN}^{(k)} = \frac{V_{GN}^*(2k-1)}{K} \quad (4d)$$

where K represents the total layers number, and

$$V_{GN}^* = \frac{W_{GN}}{W_{GN} + \frac{\rho_{GN}}{\rho_m}(1-W_{GN})} \quad (5)$$

in which W_{GN} represents the weight fraction of graphene nanofillers, ρ_{GN} and ρ_m stand for the mass densities of nanofillers and matrix, respectively.

The displacement field for a micro/nano-arch incorporating higher-order shear deformation can be expressed as shown in Eqs. (6a) and (6b);

$$U_1(s, z) = \left(1 - \frac{z}{R}\right)u(s) + z\psi(s) - \frac{4z^3}{3h^2}\left(\psi(s) + \frac{dw(s)}{ds}\right) \quad (6a)$$

$$U_3(s, z) = w(s) \quad (6b)$$

in which u and w refer to the displacement of an arbitrary point located on the middle surface of the micro/nano-arch along with s -axis and z -axis, respectively, and ψ denotes the rotation of this surface about y -axis.

Now, by taking the von Karman type of geometrical nonlinearity, the mechanical components of the traditional strain tensor are written as illustrated in Eqs. (7a) and (7b) below;

$$\varepsilon_{ss} = \frac{du}{ds} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 - \frac{w}{R} + z \frac{d\psi}{ds} - \frac{4z^3}{3h^2} \left(\frac{d\psi}{ds} + \frac{d^2w}{ds^2} \right) \quad (7a)$$

$$\gamma_{sz} = \left(1 - \frac{4z^2}{h^2} \right) \left(\psi + \frac{dw}{ds} \right) \quad (7b)$$

Based upon the NSG-type continuum elasticity proposed by Lim et al. [231], the total strain tensor for a micro/nano-arch can be expressed in the following form indicated in Eq. (8);

$$\Phi_{ij} = \sigma_{ij} - \nabla \sigma_{ijm}^* \quad (8)$$

where σ_{ij} and σ_{ijm}^* represent, respectively, the traditional and high-order stress tensors, and ∇ denotes the symbol of gradient.

Through satisfying the Eringen's method [414], the equivalent differential equations associated with the constitutive equations between the strain tensor and traditional stress as well as the high-order stress tensors can be derived as shown below in Eqs. (9a) and (9b);

$$(1 - \mu^2 \nabla^2) \sigma_{ij} = C_{ijpq} \varepsilon_{pq} \quad (9a)$$

$$(1 - \mu^2 \nabla^2) \sigma_{ijm}^* = l^2 C_{ijpq} \varepsilon_{pq,m} \quad (9b)$$

in which C_{ijpq} is the stiffness matrix of FG multilayer micro/nano-arch. In addition, μ and l in order stand for the nonlocal and strain gradient characteristic parameters. By

inserting equation (9) in equation (8), the NSG-based constitutive equation is achieved as illustrated in Eq. (10);

$$(1 - \mu^2 \nabla^2) \Phi_{ij} = (1 - l^2 \nabla^2) C_{ijpq} \varepsilon_{pq} \quad (10)$$

So, for the considered thermomechanical loaded micro/nano-arches, one will have which is shown in Eq. (11);

$$\begin{pmatrix} \Phi_{ss} - \mu^2 \frac{d^2 \Phi_{ss}}{ds^2} \\ \Phi_{sz} - \mu^2 \frac{d^2 \Phi_{sz}}{ds^2} \end{pmatrix}^{(k)} = \begin{bmatrix} \frac{E^{(k)}}{1-(\nu^{(k)})^2} & 0 \\ 0 & \frac{E^{(k)}}{2(1+\nu^{(k)})} \end{bmatrix} \begin{pmatrix} \varepsilon_{ss} - l^2 \frac{d^2 \varepsilon_{ss}}{ds^2} \\ \gamma_{sz} - l^2 \frac{d^2 \gamma_{sz}}{ds^2} \end{pmatrix}^{(k)} - \begin{pmatrix} \alpha^{(k)} \Delta T \\ 0 \end{pmatrix} \quad (11)$$

where ΔT stand for the temperature change. Moreover, $\nu^{(k)}$ and $\alpha^{(k)}$ represent the Poisson's ratio and thermal expansion coefficient associated with the k^{th} layer, which can be extracted on the basis of the rule of mixture as follows indicated in Eqs. (12a) and (12b);

$$\alpha^{(k)} = \alpha_{GN} V_{GN}^{(k)} + \alpha_m (1 - V_{GN}^{(k)}) \quad (12a)$$

$$\nu^{(k)} = \nu_{GN} V_{GN}^{(k)} + \nu_m (1 - V_{GN}^{(k)}) \quad (12b)$$

Consequently, the variation of strain energy stored in a uniformly radial compressed micro/nano-arch can be expressed as illustrated in Eq. (13);

$$\begin{aligned} \delta \Pi_s &= \int_{\Omega} \{ \sigma_{ss} \delta \varepsilon_{ss} + \sigma_{ss}^* \nabla \delta \varepsilon_{ss} + \tau_{sz} \delta \gamma_{sz} + \tau_{sz}^* \nabla \delta \gamma_{sz} \} d\Omega = \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} \{ \Phi_{ss} \delta \varepsilon_{ss} + \\ &\Phi_{sz} \delta \gamma_{sz} \} dz dL + \int_{-\frac{h}{2}}^{\frac{h}{2}} \{ \sigma_{ss} \delta \varepsilon_{ss} + \sigma_{ss}^* \delta \varepsilon_{ss} \} dz \Big|_0^L = \int_0^L \left\{ N_{ss} \delta \left[\frac{du}{ds} - \frac{w}{R} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 \right] + \right. \\ &\left. M_{ss} \delta \left[\frac{d\psi}{ds} \right] - P_{ss} \delta \left[\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} \right] + Q_{sz} \delta \left[\psi + \frac{dw}{ds} \right] \right\} ds \end{aligned} \quad (13)$$

in which indicated in Eqs. (14a), (14b), (14c), and (14d) below;

$$N_{ss} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Phi_{ss} dz \rightarrow N_{ss} - \mu^2 \frac{d^2 N_{ss}}{ds^2} = A_{11} \left[\frac{du}{ds} - l^2 \frac{d^3 u}{ds^3} - \frac{w}{R} + \frac{l}{R} \frac{d^2 w}{ds^2} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 - \right. \\ \left. l^2 \frac{dw}{ds} \frac{d^3 w}{ds^3} - l^2 \left(\frac{d^2 w}{ds^2} \right)^2 \right] + B_{11} \left[\frac{d\psi}{ds} - l^2 \frac{d^3 \psi}{ds^3} \right] - C_{11} \left[\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} - l^2 \frac{d^3 \psi}{ds^3} - l^2 \frac{d^4 w}{ds^4} \right] - \\ A_{11}^T \Delta T \quad (14a)$$

$$M_{ss} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Phi_{ss} z dz \rightarrow M_{ss} - \mu^2 \frac{d^2 M_{ss}}{ds^2} = B_{11} \left[\frac{du}{ds} - l^2 \frac{d^3 u}{ds^3} - \frac{w}{R} + \frac{l}{R} \frac{d^2 w}{ds^2} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 - \right. \\ \left. l^2 \frac{dw}{ds} \frac{d^3 w}{ds^3} - l^2 \left(\frac{d^2 w}{ds^2} \right)^2 \right] + D_{11} \left[\frac{d\psi}{ds} - l^2 \frac{d^3 \psi}{ds^3} \right] - F_{11} \left[\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} - l^2 \frac{d^3 \psi}{ds^3} - l^2 \frac{d^4 w}{ds^4} \right] - \\ D_{11}^T \Delta T \quad (14b)$$

$$P_{ss} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Phi_{ss} \frac{4z^3}{3h^2} dz \rightarrow P_{ss} - \mu^2 \frac{d^2 P_{ss}}{ds^2} = C_{11} \left[\frac{du}{ds} - l^2 \frac{d^3 u}{ds^3} - \frac{w}{R} + \frac{l}{R} \frac{d^2 w}{ds^2} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 - \right. \\ \left. l^2 \frac{dw}{ds} \frac{d^3 w}{ds^3} - l^2 \left(\frac{d^2 w}{ds^2} \right)^2 \right] + F_{11} \left[\frac{d\psi}{ds} - l^2 \frac{d^3 \psi}{ds^3} \right] - H_{11} \left[\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} - l^2 \frac{d^3 \psi}{ds^3} - l^2 \frac{d^4 w}{ds^4} \right] - \\ H_{11}^T \Delta T \quad (14c)$$

$$Q_{sz} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \Phi_{sz} \left(1 - \frac{4z^2}{h^2} \right) dz \rightarrow Q_{sz} - \mu^2 \frac{d^2 Q_{sz}}{ds^2} = (A_{44} - D_{44}) \left[\psi + \frac{dw}{ds} - l^2 \frac{d^2 \psi}{ds^2} - \right. \\ \left. l^2 \frac{d^3 w}{ds^3} \right] \quad (14d)$$

where for a shallow micro/nano-arch having the width of g , one will have as shown below in Eq. (15);

$$A_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} dz \quad , \quad B_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} z dz \\ C_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} \frac{4z^3}{3h^2} dz \quad , \quad D_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} z^2 dz \\ F_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} \frac{4z^4}{3h^2} dz \quad , \quad H_{11} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{1 - (\nu^{(k)})^2} \right\} \frac{16z^6}{9h^4} dz \\ A_{44} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{2(1 + \nu^{(k)})} \right\} dz \quad , \quad D_{44} = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{E^{(k)}}{2(1 + \nu^{(k)})} \right\} \frac{4z^2}{h^2} dz$$

$$A_{11}^T = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{\alpha^{(k)} E^{(k)}}{1-\nu^{(k)}} \right\} dz, D_{11}^T = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{\alpha^{(k)} E^{(k)}}{1-\nu^{(k)}} \right\} z dz, H_{11}^T = g \sum_{k=1}^K \int_{-\frac{h}{2}}^{\frac{h}{2}} \left\{ \frac{\alpha^{(k)} E^{(k)}}{1-\nu^{(k)}} \right\} \frac{4z^3}{3h^2} dz \quad (15)$$

Additionally, the work carried out by the external uniform lateral pressure takes the following variational form illustrated in Eq. (16);

$$\delta \Pi_w = - \int_0^L q \delta w ds \quad (16)$$

On the basis of the virtual work's principle, one will have Eq. (17) indicated below as;

$$\delta \Pi_s - \delta \Pi_w = 0 \quad (17)$$

Accordingly, by putting the technique of integration by parts to use, it yields the following equations which are illustrated in Eqs. (18a), (18b), and (18c);

$$\delta u: \frac{dN_{ss}}{ds} = 0 \quad (18a)$$

$$\delta w: \frac{dQ_{sz}}{ds} + \frac{d^2 P_{ss}}{ds^2} + \frac{N_{ss}}{R} + N_{ss} \frac{d^2 w}{ds^2} + q = 0 \quad (18b)$$

$$\delta \psi: \frac{dM_{ss}}{ds} - \frac{dP_{ss}}{ds} - Q_{sz} = 0 \quad (18c)$$

The axial resultant force of N_{ss} is constant along s -direction. So, through substitution of equation (14a) in equation (18a) and then integration over the considered domain, one will have the form indicated in Eq. (19);

$$N_{ss} = \frac{1}{L} \int_0^L \left\{ A_{11} \left[\frac{du}{ds} - l^2 \frac{d^3 u}{ds^3} - \frac{w}{R} + \frac{l}{R} \frac{d^2 w}{ds^2} + \frac{1}{2} \left(\frac{dw}{ds} \right)^2 - 3l^2 \left(\frac{d^3 w}{ds^3} \right)^2 - 4l^2 \frac{d^2 w}{ds^2} \frac{d^4 w}{ds^4} - l^2 \frac{dw}{ds} \frac{d^5 w}{ds^5} \right] + B_{11} \left[\frac{d\psi}{ds} - l^2 \frac{d^3 \psi}{ds^3} \right] - C_{11} \left[\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} - l^2 \frac{d^3 \psi}{ds^3} - l^2 \frac{d^4 w}{ds^4} \right] - A_{11}^T \Delta T \right\} ds \quad (19)$$

Moreover, by inserting equation (14) in equations (18b) and (18c), the NSG-based nonlinear differential equations associated with the thermomechanical nonlinear stability of micro/nano-arches can be derived as shown in Eqs. (20a) and (20b);

$$\begin{aligned}
& (A_{44} - D_{44}) \left[\frac{d\psi}{ds} + \frac{d^2w}{ds^2} - l^2 \frac{d^3\psi}{ds^3} - l^2 \frac{d^4w}{ds^4} \right] + C_{11} \left[\frac{d^3u}{ds^3} - l^2 \frac{d^5u}{ds^5} - \frac{1}{R} \frac{d^2w}{ds^2} + \frac{l}{R} \frac{d^4w}{ds^4} + \right. \\
& \left. \frac{dw}{ds} \frac{d^3w}{ds^3} + \left(\frac{d^2w}{ds^2} \right)^2 - l^2 \left(\frac{d^3w}{ds^3} \right)^2 - 2l^2 \frac{d^2w}{ds^2} \frac{d^4w}{ds^4} - l^2 \frac{dw}{ds} \frac{d^5w}{ds^5} \right] + F_{11} \left[\frac{d^3\psi}{ds^3} - l^2 \frac{d^5\psi}{ds^5} \right] - \\
& H_{11} \left[\frac{d^3\psi}{ds^3} + \frac{d^4w}{ds^4} - l^2 \frac{d^5\psi}{ds^5} - l^2 \frac{d^6w}{ds^6} \right] + N_{ss} \left(\frac{1}{R} + \frac{d^2w}{ds^2} - \mu^2 \frac{d^4w}{ds^4} \right) + q - \mu^2 \frac{d^2q}{ds^2} = 0 \quad (20a) \\
& (B_{11} - C_{11}) \left[\frac{d^2u}{ds^2} - l^2 \frac{d^4u}{ds^4} - \frac{1}{R} \frac{dw}{ds} + \frac{l}{R} \frac{d^3w}{ds^3} + \frac{dw}{ds} \frac{d^2w}{ds^2} - l^2 \frac{dw}{ds} \frac{d^4w}{ds^4} - 3l^2 \frac{d^2w}{ds^2} \frac{d^3w}{ds^3} \right] + \\
& (D_{11} - F_{11}) \left[\frac{d^2\psi}{ds^2} - l^2 \frac{d^4\psi}{ds^4} \right] - (F_{11} - H_{11}) \left[\frac{d^2\psi}{ds^2} + \frac{d^3w}{ds^3} - l^2 \frac{d^4\psi}{ds^4} - l^2 \frac{d^5w}{ds^5} \right] + \\
& (D_{44} - A_{44}) \left[\psi + \frac{dw}{ds} - l^2 \frac{d^2\psi}{ds^2} - l^2 \frac{d^3w}{ds^3} \right] = 0 \quad (20b)
\end{aligned}$$

Also, the necessary conditions which should be satisfied at both ends of micro/nano-arches can be expressed for clamped boundary conditions as: $u = w = \frac{1}{R} \frac{dw}{ds} = \psi = 0$.

3.1.1 Numerical Solving Process for Case 1

In order to employ the GDQ-based solution strategy, the following dimensionless parameters which are illustrated in Eq. (21) are taken into consideration:

$$U = \frac{u}{L}, \quad W = \frac{w}{L}, \quad X = \frac{x}{L}, \quad \Psi = \psi \quad (21)$$

Furthermore, in order to apply the associated discretization scheme, the β^{th} derivative of one-dimensional function $\mathcal{F}(x)$ can be achieved via a linear sum of its values in the following form as shown in Eq. (22);

$$\left. \frac{d^\beta \mathcal{F}(x)}{dx^\beta} \right|_{x=x_i} = \sum_{j=1}^n \mathcal{T}_{ij}^{(\beta)} \mathcal{F}(x_j) \quad (22)$$

where n stands for the number of grid points utilized in the discretization process.

Now, a column vector of $\mathbf{g}$ can be taken into account including the nodal values of the one-dimensional function $\mathcal{F}(x)$ as follows in Eq. (23);

$$\mathcal{G} = [\mathcal{G}_j] = [\mathcal{F}(x_j)] = \begin{bmatrix} \mathcal{F}(x_1) \\ \mathcal{F}(x_2) \\ \vdots \\ \mathcal{F}(x_n) \end{bmatrix} \quad (23)$$

Afterwards, a differential matrix operator on the basis of the defined column vector can be introduced as indicated below in Eq. (24);

$$\frac{d^\beta}{dx^\beta}(\mathcal{G}) = \mathcal{D}_x^{(\beta)} \mathcal{G} = [\mathcal{D}_x^{(\beta)}]_{i,j} \{\mathcal{G}_j\} \quad (24)$$

in which yields to Eq. (25);

$$\mathcal{D}_x^{(\beta)} = [\mathcal{D}_x^{(\beta)}]_{i,j} = \mathcal{T}_{ij}^{(\beta)} \quad i, j = 1, 2, \dots, n \quad (25)$$

as the associated weighting coefficients are calculated in the following scheme which is indicated in Eq. (26);

$$\mathcal{T}_{ij}^{(\beta)} = \begin{cases} \mathbf{I}_x & \beta = 0 \\ \frac{\mathcal{P}(x_i)}{(x_i - x_j)\mathcal{P}(x_j)} & \beta = 1, \quad i \neq j, \quad i, j = 1, 2, \dots, n \\ \beta \left(\mathcal{T}_{ij}^{(1)} \mathcal{T}_{ii}^{(\beta-1)} - \frac{\mathcal{T}_{ij}^{(\beta-1)}}{x_i - x_j} \right) & \beta = 2, 3, \dots, n-1, \quad i \neq j, \quad i, j = 1, 2, \dots, n \\ -\sum_{j=1, j \neq i}^n \mathcal{T}_{ij}^{(\beta)} & \beta = 1, 2, \dots, n-1, \quad i = j, \quad i, j = 1, 2, \dots, n \end{cases} \quad (26)$$

where $\mathbf{I}_x$ denotes an identity matrix with dimensions of $n \times n$, and it is indicated in Eq. (27) as;

$$\mathcal{P}(x_i) = \prod_{j=1, j \neq i}^n \{x_i - x_j\} \quad (27)$$

Now, by considering the introduced dimensionless parameters within the domain of $0 \leq X \leq 1$, the meshing procedure based upon the Chebyshev-Guass-Lobatto gridding scheme can be performed as illustrated in Eq. (28);

$$X_i = \frac{1}{2} \left[1 - \cos \left(\frac{(i-1)\pi}{n-1} \right) \right] \quad i = 1, 2, \dots, n \quad (28)$$

As a consequence, the following column vectors can be obtained as shown in Eq. (29);

$$\mathbf{U} = \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{Bmatrix}, \quad \mathbf{W} = \begin{Bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{Bmatrix}, \quad \mathbf{\Psi} = \begin{Bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{Bmatrix} \quad (29)$$

By inserting equations (21), (22), and (29) in equations (19) and (20), the discretized form of the governing nonlinear associated with the NSG-based thermomechanical in-plane stability of micro/nano-arches can be deduced. Consequently, the related set of nonlinear equations can be expressed for the considered domain as illustrated in Eq. (30);

$$\mathfrak{A}: \mathbb{O}^{3n+1} \rightarrow \mathbb{O}^{3n}, \mathfrak{A}(N_{ss}^b, \mathbf{X}) = 0, \mathbf{X}^T = [\mathbf{U}^T, \mathbf{W}^T, \mathbf{\Psi}^T] \quad (30)$$

in which N_{ss}^b refers to the critical bifurcation load.

Now, with the aid of the pseudo arc-length continuation technique, this obtained parametrized problem can be solved after putting the residuals of the clamped boundary condition equations to the residual vector associated with the introduced domain of $\mathfrak{A}(N_{ss}^b, \mathbf{X})$.

In a similar way, the clamped boundary conditions at two ends of micro/nano-arches can be discretized. It should be pointed out that the necessary additional requirements associated with the NSG-based boundary conditions [415] are taken into account in a simplified form for the grid points located at the boundary lines.

3.2 Case 2: Surface Elastic-based FG Laminated Curved Beam Model

As shown in Figure 50, a laminated composite curved nanobeam reinforced with graphene nanofillers subjected to a uniformly distributed radial compressive load together with a temperature rise is considered. In accordance with the richness of nanofillers relevant to each laminate, four kinds of FG lamination pattern are taken

into account namely as U-GNRC, A-GNRC, X-GNRC, and O-GNRC, as can be seen in Figure 50. Accordingly, the richness of graphene nanofillers is the same for all laminates in U-GNRC pattern. On the other hand, the richness of graphene nanofillers decreases from outer laminates to inner ones in X-GNRC pattern, from inner laminates to outer ones in O-GNRC pattern, and from bottom laminates to top ones in A-GNRC pattern.

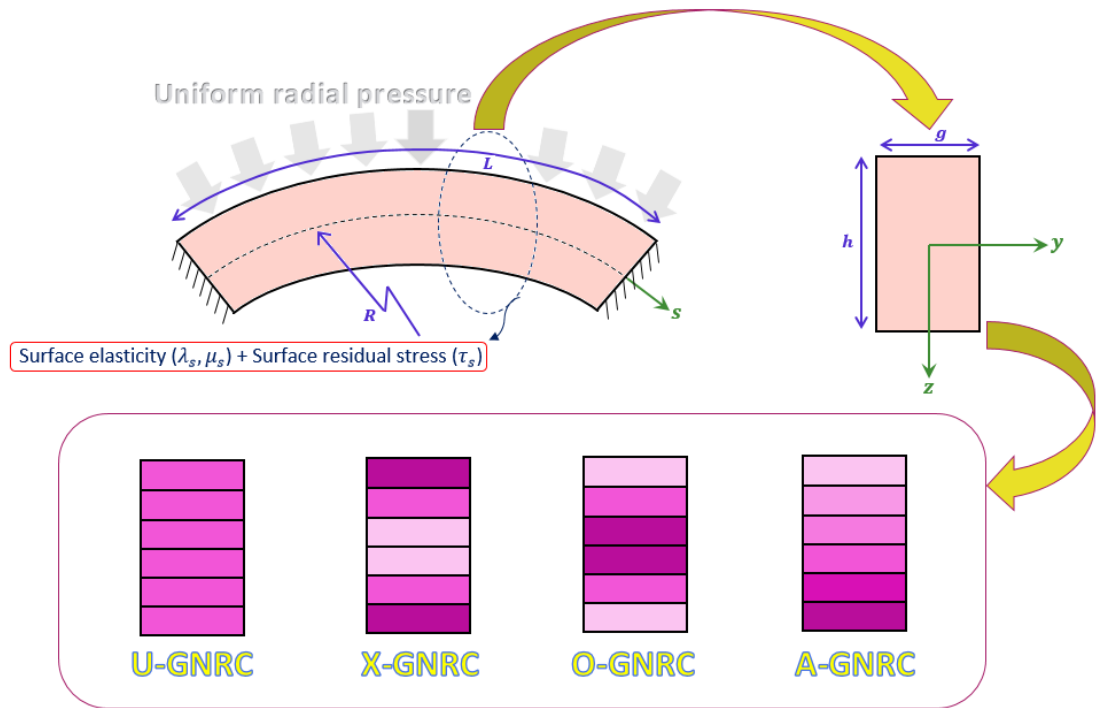


Figure 50: Schematic demonstration of surface elastic-based model of a clamped FG laminated curved nanobeam with different FG lamination schemes.

The equations between (1) to (7) and (12) to (19) are same as Case 1. This is because, the following part only replaced for equation (8) to equation (11) and equation (20) to employ the Gurtin-Murdoch continuum elasticity theory as indicated between equation (31) to equation (36).

Due to an additional energy associated with the cohesive forces between the atoms located on the free surface of a structure, it leads to creation of a surface stress which

becomes more prominent in mechanical responses of nanostructures, in which the ratio of surface to volume is as much as possible. In this regard, the Gurtin-Murdoch continuum elasticity was introduced, based on which the components of the surface stress tensor can be derived as [26,27] illustrated in Eqs. (31a) and (31b);

$$\sigma_{ij}^{S\pm} = 2(\mu_s - \tau_s)\varepsilon_{ij}^{S\pm} + (\tau_s + \lambda_s)\varepsilon_{kk}^{S\pm} + \tau_s\delta_{ij} + \tau_s \frac{dU_i^{S\pm}}{dx_j} \quad (31a)$$

$$\sigma_{iz}^{S\pm} = \tau_s \frac{dU_z^{S\pm}}{dx_i} \quad (31b)$$

where δ_{ij} represents the Kronecker delta, and x_i stands for the coordinate components (s or z).

In this regard, the components of the surface stress tensor for a FG laminated nanocomposite curved nanobeam take the following form illustrated in Eqs. (32a) and (32b);

$$\sigma_{ss}^{S\pm} = (\lambda_s + 2\mu_s)\varepsilon_{ss}^{S\pm} + \tau_s \left(\frac{du}{ds} - \varepsilon_{ss}^{S\pm} \right) \quad (32a)$$

$$\sigma_{sz}^{S\pm} = \tau_s \frac{dw}{ds} \quad (32b)$$

In the traditional curved beam model, the stress component of σ_{zz} is ignored, because its value is negligible. Indeed, in order to satisfy the necessary equilibrium conditions on the free surfaces of a nanostructure, it is essential to take this stress component into account. To this end, a linear variation of σ_{zz} through the thickness of the curved nanobeam is supposed as [416] indicated in Eq. (33);

$$\sigma_{zz} = \frac{1}{2} \left(\frac{d\sigma_{sz}^{S+}}{ds} - \frac{d\sigma_{sz}^{S-}}{ds} \right) + \frac{z}{h} \left(\frac{d\sigma_{sz}^{S+}}{ds} + \frac{d\sigma_{sz}^{S-}}{ds} \right) \quad (33)$$

By inserting equation (32) in equation (33), one will have the equation which is indicated in Eq. (34);

$$\sigma_{zz} = \frac{2\tau_s z}{h} \frac{d^2 w}{ds^2} \quad (34)$$

Consequently, the components of the traditional stress tensor can be achieved in the following forms which is illustrated in Eq. (35) below;

$$\begin{Bmatrix} \sigma_{ss} \\ \tau_{sz} \end{Bmatrix}^{(k)} = \begin{bmatrix} \frac{E^{(k)}}{1-(\nu^{(k)})^2} & 0 \\ 0 & \frac{E^{(k)}}{2(1+\nu^{(k)})} \end{bmatrix} \begin{Bmatrix} \varepsilon_{ss} \\ \gamma_{sz} \end{Bmatrix}^{(k)} - \begin{Bmatrix} \alpha^{(k)} \Delta T - \frac{2\tau_s z \nu^{(k)}}{(1-\nu^{(k)})h} \frac{d^2 w}{ds^2} \\ 0 \end{Bmatrix} \quad (35)$$

where ΔT stand for the temperature change.

Consequently, the surface elastic-based nonlinear differential equations relevant to the in-plane nonlinear stability behavior of FG laminated composite curved nanobeams can be read as shown in Eqs. (36a), (36b), and (36c) below;

$$[A_{11}^* + 2(g+h)(\lambda_s + 2\mu_s)] \left(\frac{d^2 u}{ds^2} - \frac{1}{R} \frac{dw}{ds} + \frac{dw}{ds} \frac{d^2 w}{ds^2} \right) + B_{11}^* \frac{d^2 \psi}{ds^2} - C_{11}^* \left(\frac{d^2 \psi}{ds^2} + \frac{d^3 w}{ds^3} \right) - 2\tau_s(g+h) \frac{dw}{ds} \frac{d^2 w}{ds^2} = 0 \quad (36a)$$

$$\begin{aligned} & C_{11}^* \left[\frac{d^3 u}{ds^3} - \frac{1}{R} \frac{d^2 w}{ds^2} + \frac{dw}{ds} \frac{d^3 w}{ds^3} + \left(\frac{d^2 w}{ds^2} \right)^2 \right] + \left[F_{11}^* + \left(\frac{gh^2}{3} + \frac{2h^3}{15} \right) (\lambda_s + 2\mu_s) \right] \frac{d^3 \psi}{ds^3} - \\ & \left[H_{11}^* + \left(\frac{8gh^2}{27} + \frac{8h^3}{63} \right) (\lambda_s + 2\mu_s) \right] \left(\frac{d^3 \psi}{ds^3} + \frac{d^4 w}{ds^4} \right) + (A_{44}^* - D_{44}^*) \left(\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} \right) + \\ & [A_{11}^* + 2(g+h)(\lambda_s + 2\mu_s)] \left[\frac{1}{R} \frac{du}{ds} + \frac{du}{ds} \frac{d^2 w}{ds^2} - \frac{w}{R^2} - \frac{w}{R} \frac{d^2 w}{ds^2} + \frac{1}{2R} \left(\frac{dw}{ds} \right)^2 + \right. \\ & \left. \frac{1}{2} \frac{d^2 w}{ds^2} \left(\frac{dw}{ds} \right)^2 \right] + B_{11}^* \left(\frac{1}{R} + \frac{d^2 w}{ds^2} \right) \frac{d\psi}{ds} - C_{11}^* \left(\frac{1}{R} + \frac{d^2 w}{ds^2} \right) \left(\frac{d\psi}{ds} + \frac{d^2 w}{ds^2} \right) + 2(g+h) \\ & h\tau_s \left[\frac{1}{R} \left(1 - \frac{1}{2} \left(\frac{dw}{ds} \right)^2 \right) + 2 \frac{d^2 w}{ds^2} - \frac{1}{2} \frac{d^2 w}{ds^2} \left(\frac{dw}{ds} \right)^2 \right] - A_{11}^T \Delta T \left(\frac{1}{R} + \frac{d^2 w}{ds^2} \right) + q = 0 \quad (36b) \end{aligned}$$

$$\begin{aligned} & (B_{11}^* - C_{11}^*) \left(\frac{d^2 u}{ds^2} - \frac{1}{R} \frac{dw}{ds} + \frac{dw}{ds} \frac{d^2 w}{ds^2} \right) + \left[D_{11}^* - F_{11}^* + \left(\frac{gh^2}{6} + \frac{h^3}{30} \right) (\lambda_s + 2\mu_s) \right] \frac{d^2 \psi}{ds^2} - \\ & \left[F_{11}^* - H_{11}^* + \left(\frac{gh^2}{27} + \frac{2h^3}{315} \right) (\lambda_s + 2\mu_s) \right] \left(\frac{d^2 \psi}{ds^2} + \frac{d^3 w}{ds^3} \right) - (A_{44}^* - D_{44}^*) \left(\psi + \frac{dw}{ds} \right) - \\ & 2(g+h)\tau_s \frac{dw}{ds} = 0 \quad (36c) \end{aligned}$$

Moreover, the requirements associated with the clamped boundary conditions at both ends of the considered curved nanobeams can be expressed as: $u = w = \frac{1}{R} \frac{dw}{ds} = \psi = 0$.

3.2.1 Numerical Solving Process for Case 2

The generalized differential quadrature (GDQ) numerical solution strategy is an effective technique to solve nonlinear differential equations relevant to the associated boundary conditions via an efficient discretization pattern as employed by Shu [417]. For utilizing the GDQ-based solution strategy, the following dimensionless parameters are taken into account and illustrated in Eq. (37);

$$\begin{aligned}
 U &= \frac{u}{L} \quad , \quad W = \frac{w}{h} \quad , \quad X = \frac{s}{L} \quad , \quad \Psi = \psi \\
 \beta_1 &= \frac{h}{R} \quad , \quad \beta_2 = \frac{h}{L} \quad , \quad \beta_3 = \frac{L}{R} \quad , \quad q^* = \frac{qL}{A_{00}} \\
 \{a_{11}^*, a_{44}^*, d_{44}^*, b_{11}^*, c_{11}^*, d_{11}^*, f_{11}^*, h_{11}^*\} &= \left\{ \frac{A_{11}^*}{A_{00}}, \frac{A_{44}^*}{A_{00}}, \frac{D_{44}^*}{A_{00}}, \frac{B_{11}^*}{A_{00}h}, \frac{C_{11}^*}{A_{00}h}, \frac{D_{11}^*}{A_{00}h^2}, \frac{F_{11}^*}{A_{00}h^2}, \frac{H_{11}^*}{A_{00}h^2} \right\} \\
 a_{11}^{**} &= \frac{2(g+h)(\lambda_s + 2\mu_s)}{A_{00}} \quad , \quad d_{11}^{**} = \frac{\left(\frac{gh^2}{2} + \frac{h^3}{6}\right)(\lambda_s + 2\mu_s)}{A_{00}h^2} \\
 f_{11}^{**} &= \frac{\left(\frac{gh^2}{3} + \frac{2h^3}{15}\right)(\lambda_s + 2\mu_s)}{A_{00}h^2} \quad , \quad h_{11}^{**} = \frac{\left(\frac{8gh^2}{27} + \frac{8h^3}{63}\right)(\lambda_s + 2\mu_s)}{A_{00}h^2} \\
 \{a_{11}^T, d_{11}^T, h_{11}^T\} &= \left\{ \frac{A_{11}^T \Delta T}{A_{00}}, \frac{D_{11}^T \Delta T}{A_{00}h}, \frac{H_{11}^T \Delta T}{A_{00}h} \right\} \quad , \quad \tau_s^* = \frac{2\tau_s(g+h)}{A_{00}} \quad (37)
 \end{aligned}$$

where $A_{00} = E_m g h$.

Consequently, the governing surface stress-based differential equations can be expressed in dimensionless form as indicated in Eqs. (38a), (38b), and (38c) below;

$$\begin{aligned}
 (a_{11}^* + a_{11}^{**}) \left(\frac{d^2 U}{dX^2} - \beta_1 \frac{dW}{dX} + \beta_2^2 \frac{dW}{dX} \frac{d^2 W}{dX^2} \right) + b_{11}^* \beta_2 \frac{d^2 \Psi}{dX^2} - c_{11}^* \beta_2 \left(\frac{d^2 \Psi}{dX^2} + \beta_2 \frac{d^3 W}{dX^3} \right) - \\
 \tau_s^* \beta_2^2 \frac{dW}{dX} \frac{d^2 W}{dX^2} = 0 \quad (38a)
 \end{aligned}$$

$$\begin{aligned}
& c_{11}^* \beta_2 \left[\frac{d^3 U}{dX^3} - \beta_1 \frac{d^2 W}{dX^2} + \beta_2^2 \frac{dW}{dX} \frac{d^3 W}{dX^3} + \beta_2^2 \left(\frac{d^2 W}{dX^2} \right)^2 \right] + (f_{11}^* + f_{11}^{**}) \beta_2^2 \frac{d^3 \psi}{dX^3} - \\
& (h_{11}^* + h_{11}^{**}) \beta_2^2 \left(\frac{d^3 \psi}{dX^3} + \beta_2 \frac{d^4 W}{dX^4} \right) + (a_{44}^* - d_{44}^*) \left(\frac{d\psi}{dX} + \beta_2 \frac{d^2 W}{dX^2} \right) + (a_{11}^* + a_{11}^{**}) \left[\beta_3 \frac{dU}{dX} + \right. \\
& \left. \beta_2 \frac{dU}{dX} \frac{d^2 W}{dX^2} - \beta_1 \beta_3 W - \beta_1 \beta_2 W \frac{d^2 W}{dX^2} + \frac{\beta_2^2 \beta_3}{2} \left(\frac{dW}{dX} \right)^2 + \frac{\beta_2^3}{2} \frac{d^2 W}{dX^2} \left(\frac{dW}{dX} \right)^2 \right] + b_{11}^* \beta_2 \left(\beta_3 + \right. \\
& \left. \beta_2 \frac{d^2 W}{dX^2} \right) \frac{d\psi}{dX} - c_{11}^* \beta_2 \left(\beta_3 + \beta_2 \frac{d^2 W}{dX^2} \right) \left(\frac{d\psi}{dX} + \beta_2 \frac{d^2 W}{dX^2} \right) + \tau_s^* \left[\beta_3 \left(1 - \frac{\beta_2^2}{2} \left(\frac{dW}{dX} \right)^2 \right) + \right. \\
& \left. 2\beta_2 \frac{d^2 W}{dX^2} - \frac{\beta_2^3}{2} \frac{d^2 W}{dX^2} \left(\frac{dW}{dX} \right)^2 \right] - a_{11}^T \left(\beta_3 + \beta_2 \frac{d^2 W}{dX^2} \right) + q^* = 0
\end{aligned} \tag{38b}$$

$$\begin{aligned}
& (b_{11}^* - c_{11}^*) \beta_2 \left(\frac{d^2 U}{dX^2} - \beta_1 \frac{dW}{dX} + \beta_2^2 \frac{dW}{dX} \frac{d^2 W}{dX^2} \right) + (d_{11}^* + d_{11}^{**} - f_{11}^* - f_{11}^{**}) \beta_2^2 \frac{d^2 \psi}{dX^2} - \\
& (f_{11}^* + f_{11}^{**} - h_{11}^* - h_{11}^{**}) \beta_2^2 \left(\frac{d^2 \psi}{dX^2} + \beta_2 \frac{d^3 \psi}{dX^3} \right) - (a_{44}^* - d_{44}^*) \left(\psi + \beta_2 \frac{dW}{dX} \right) - \\
& \tau_s^* \beta_2 \frac{dW}{dX} = 0
\end{aligned} \tag{38c}$$

In addition, the relevant discretization procedure can be employed via estimation of the β^{th} derivative for a one-dimensional function $\mathcal{F}(x)$ as a linear sum of its values as follows in Eq. (39);

$$\left. \frac{d^\beta \mathcal{F}(x)}{dx^\beta} \right|_{x=x_i} = \sum_{j=1}^n \mathcal{T}_{ij}^{(\beta)} \mathcal{F}(x_j) \tag{39}$$

where n denotes the grid points number putting in the relevant discretization procedure to use.

Now, the nodal values associated with the considered one-dimensional function $\mathcal{F}(x)$ can be expressed as a column vector of $\mathbf{g}$ as illustrated in Eq. (40);

$$\mathbf{g} = [g_j] = [\mathcal{F}(x_j)] = \begin{bmatrix} \mathcal{F}(x_1) \\ \mathcal{F}(x_2) \\ \vdots \\ \mathcal{F}(x_n) \end{bmatrix} \tag{40}$$

Accordingly, by using the introduced column vector, one can propose a differential matrix operator in the following form shown in Eq. (41);

$$\frac{d^\beta}{dx^\beta}(\mathcal{G}) = \mathcal{D}_x^{(\beta)} \mathcal{G} = \left[\mathcal{D}_x^{(\beta)} \right]_{i,j} \{\mathcal{G}_j\} \quad (41)$$

in which yields to Eq. (42);

$$\mathcal{D}_x^{(\beta)} = \left[\mathcal{D}_x^{(\beta)} \right]_{i,j} = \mathcal{T}_{ij}^{(\beta)} \quad i, j = 1, 2, \dots, n \quad (42)$$

as the relevant weighting coefficients can be derived as illustrated in Eq. (43) below;

$$\mathcal{T}_{ij}^{(\beta)} = \begin{cases} \mathbf{I}_x & \beta = 0 \\ \frac{\mathcal{P}(x_i)}{(x_i - x_j)\mathcal{P}(x_j)} & \beta = 1, \quad i \neq j, \quad i, j = 1, 2, \dots, n \\ \beta \left(\mathcal{T}_{ij}^{(1)} \mathcal{T}_{ii}^{(\beta-1)} - \frac{\mathcal{T}_{ij}^{(\beta-1)}}{x_i - x_j} \right) & \beta = 2, 3, \dots, n-1, \quad i \neq j, \quad i, j = 1, 2, \dots, n \\ -\sum_{j=1, j \neq i}^n \mathcal{T}_{ij}^{(\beta)} & \beta = 1, 2, \dots, n-1, \quad i = j, \quad i, j = 1, 2, \dots, n \end{cases} \quad (43)$$

where $\mathbf{I}_x$ stands for an identity matrix having dimensions of $n \times n$, and indicated below in Eq. (44);

$$\mathcal{P}(x_i) = \prod_{j=1, j \neq i}^n \{x_i - x_j\} \quad (44)$$

In this regard, the meshing process within the framework of the Chebyshev-Guass-Lobatto gridding pattern relevant to the dimensionless regime of $0 \leq X \leq 1$ can be written as shown in Eq. (45);

$$X_i = \frac{1}{2} \left[1 - \cos \left(\frac{(i-1)\pi}{n-1} \right) \right] \quad i = 1, 2, \dots, n \quad (45)$$

Consequently, the column vectors relevant to each discretized displacement component can be read as indicated in Eq. (46) below;

$$\mathbf{U} = \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{Bmatrix}, \quad \mathbf{W} = \begin{Bmatrix} W_1 \\ W_2 \\ \vdots \\ W_n \end{Bmatrix}, \quad \mathbf{\Psi} = \begin{Bmatrix} \Psi_1 \\ \Psi_2 \\ \vdots \\ \Psi_n \end{Bmatrix} \quad (46)$$

By inserting equations (39), (41), and (46) in dimensionless equations (38), the discretized form of the governing surface elastic-based nonlinear equations associated

with the thermomechanical in-plane stability of curved nanobeams can be extracted as indicated in Eqs. (47a), (47b), and (47c) below;

$$(a_{11}^* + a_{11}^{**}) \left(\mathcal{D}_X^{(2)} \mathbf{U} - \beta_1 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) + \beta_2^2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + b_{11}^* \beta_2 \left(\mathcal{D}_X^{(2)} \Psi \right) - c_{11}^* \beta_2 \left(\mathcal{D}_X^{(2)} \Psi + \beta_2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \right) - \tau_s^* \beta_2^2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) = 0 \quad (47a)$$

$$\begin{aligned} & c_{11}^* \beta_2 \left(\mathcal{D}_X^{(3)} \mathbf{U} - \beta_1 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) + \beta_2^2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(3)} \mathbf{W} \right) + \beta_2^2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + \\ & (f_{11}^* + f_{11}^{**}) \beta_2^2 \left(\mathcal{D}_X^{(3)} \Psi \right) - (h_{11}^* + h_{11}^{**}) \beta_2^2 \left(\left(\mathcal{D}_X^{(3)} \Psi \right) + \beta_2 \left(\mathcal{D}_X^{(4)} \mathbf{W} \right) \right) + \\ & (a_{44}^* - d_{44}^*) \left(\mathcal{D}_X^{(1)} \Psi + \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + (a_{11}^* + a_{11}^{**}) \left[\beta_3 \left(\mathcal{D}_X^{(1)} \mathbf{U} \right) + \right. \\ & \left. \beta_2 \left(\mathcal{D}_X^{(1)} \mathbf{U} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) - \beta_1 \beta_3 \mathbf{W} - \beta_1 \beta_2 \left(\mathbf{W} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) + \frac{\beta_2^2 \beta_3}{2} \left(\mathcal{D}_X^{(1)} \mathbf{U} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) + \right. \\ & \left. \frac{\beta_2^3}{2} \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \right] + b_{11}^* \beta_2 \left(\beta_3 + \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) \circ \left(\mathcal{D}_X^{(1)} \Psi \right) - \\ & c_{11}^* \beta_2 \left(\beta_3 + \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) \circ \left(\mathcal{D}_X^{(1)} \Psi + \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + \tau_s^* \left[\beta_3 \left(1 - \right. \right. \\ & \left. \left. \frac{\beta_2^2}{2} \left(\mathcal{D}_X^{(1)} \mathbf{U} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \right) + 2 \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) - \frac{\beta_2^3}{2} \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \right] - \\ & a_{11}^T \left(\beta_3 + \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + q^* = 0 \end{aligned} \quad (47b)$$

$$\begin{aligned} & (b_{11}^* - c_{11}^*) \beta_2 \left(\mathcal{D}_X^{(2)} \mathbf{U} - \beta_1 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) + \beta_2^2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \circ \left(\mathcal{D}_X^{(2)} \mathbf{W} \right) \right) + (d_{11}^* + d_{11}^{**} - \\ & f_{11}^* - f_{11}^{**}) \beta_2^2 \left(\mathcal{D}_X^{(2)} \Psi \right) - (f_{11}^* + f_{11}^{**} - h_{11}^* - h_{11}^{**}) \beta_2^2 \left(\mathcal{D}_X^{(2)} \Psi + \beta_2 \left(\mathcal{D}_X^{(3)} \mathbf{W} \right) \right) - \\ & (a_{44}^* - d_{44}^*) \left(\Psi + \beta_2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) \right) - \tau_s^* \beta_2 \left(\mathcal{D}_X^{(1)} \mathbf{W} \right) = 0 \end{aligned} \quad (47c)$$

in which $\circ$ refers to the Hadamard product.

Consequently, the related set of the surface elastic-based nonlinear equations can be expressed for the considered domain as illustrated in Eq. (48);

$$\mathfrak{A}: \mathbb{O}^{3n+1} \rightarrow \mathbb{O}^{3n}, \mathfrak{A}(N_{ss}^b, \mathbf{X}) = 0, \mathbf{X}^T = [\mathbf{U}^T, \mathbf{W}^T, \mathbf{\Psi}^T] \quad (48)$$

in which N_{ss}^b refers to the critical bifurcation load.

Thereafter, by putting the pseudo arc-length continuation approach to use, the solution of the achieved parametrized problem can be obtained with the aid of the proper residuals and considering the clamped end condition in a vectorized form relevant to the defined regime of $\mathfrak{A}(N_{ss}^b, \mathbf{X})$.

Chapter 4

RESULTS AND DISCUSSION

This section presents the experimental findings for the size-dependent nonlinear in-plane stability characteristics of functionally graded (FG) multilayer composite nanoarches subjected to uniform radial pressure together with temperature changes. In case 1, the nonlocal strain gradient (NSG) continuum elasticity is implemented in a higher-order shear flexible arch model to capture nonlocal stress tensor as well as strain gradient size dependencies. In case 2, the Gurtin-Murdoch continuum elasticity is applied within a higher-order shear flexible curved beam theory in the presence of geometric type of nonlinearity to capture surface elasticity and surface residual stress.

4.1 Validation of the Experiment

At first, the validity of the developed curved beam formulation is examined. Accordingly, after removing the expressions associated with the Gurtin-Murdoch continuum elasticity and the NSG continuum elasticity, the non-dimensional buckling loads of a clamped isotropic curved beam under a uniform compressive load in radial direction is calculated and compared with those presented by Timoshenko and Gere [359] based upon an exact analytical solution, Luu et al. [346] via an isogeometric method and Ozturk et al. [418] using the finite element technique. In accordance with Table 7, a very good agreement is revealed which confirms the accuracy of the constructed numerical curved beam formulations. Moreover, the non-dimensional buckling loads for thick clamped sandwich curved beams having various L/R ratios are tabulated in Table 8, and are compared with those given by Luu et al. [346] using

the isogeometric method. In Table 9, the dimensionless critical buckling loads of an axially compressed simply supported isotropic arch at nanoscale are presented corresponding to different values of the nonlocal parameter and are compared with those reported by Ganapathi and Polit [419] using a Naviers-based analytical solution. An excellent agreement is found again which represent the validity of the current curved beam model.

Table 7: Comparison representation for non-dimensional buckling load of a clamped circular isotropic curved beam ($L/R = \pi/3$, $q^* = q/(Eh)$, $L/h = 200$)

Dimensionless buckling load	Solution strategy
80.38	Exact analytical solution [81]
82.15	Isogeometric method [82]
82.19	Finite element method [83]
81.87	Current study

Table 8: Comparison representation for non-dimensional buckling loads of clamped sandwich curved beams having various L/R ratios ($L/h = 10$, $q^* = 12qL^3/(Egh^3)$)

L/R	Lamination scheme	Ref. [84]	Current study
$\pi/3$	[0/90/0]	359.75	358.92
	[0/90]	181.18	180.24
$2\pi/3$	[0/90/0]	375.45	374.59
	[0/90]	190.27	189.47
π	[0/90/0]	386.62	385.96
	[0/90]	197.84	197.02

Table 9: Comparison investigation on nonlocal-based dimensionless critical buckling loads of simply supported arches at nanoscale corresponding to various nonlocal parameters.

L/h	$\mu^2 \text{ (nm}^2\text{)}$	Present study	Ref. [64]
10	0	9.6109	9.6112
	1	8.7475	8.7478
	2	8.0264	8.0268
	3	7.4151	7.4155
	4	8.8902	6.8908
20	0	9.8032	9.8037
	1	8.9224	8.9230
	2	8.1873	8.1875
	3	7.5637	7.5641
	4	7.0286	7.0288

4.2 Numerical Results and Discussion for Case 1

After establishment the NSG-based arch model for FG multilayer composite micro/nano-arches, now the numerical results related to their thermomechanical nonlinear in-plane stability characteristics are presented. For the preceding numerical results, it is supposed that $h = g = 60 \text{ nm}$, $a = 5 \text{ nm}$, $b = 2.5 \text{ nm}$, $h_{GN} = 0.3 \text{ nm}$. Also, for the material properties, one will have $E_{GN} = 1010 \text{ GPa}$, $E_m = 3 \text{ GPa}$, $\nu_{GN} = 0.3$, $\nu_m = 0.25$, $\rho_{GN} = 1062.5 \text{ Kg/m}^3$, $\rho_m = 1200 \text{ Kg/m}^3$, $\alpha_{GN} = 5 \times 10^{-6} \text{ 1/K}$, $\alpha_m = 60 \times 10^{-6} \text{ 1/K}$ [166,420]. Also, it is assumed that $L/h = 30$, $R\theta^2/h = 8$ in which θ represents the semi-vertex angle of the micro/nano-arch.

In Figure 51, the NSG-based and conventional dimensionless nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths associated with the in-plane nonlinear stability characteristics of FG multilayer shallow micro/nano-arches are depicted corresponding to different small-scale parameters. It is detected that there are two limit points namely as upper and lower ones in the presented

nonlinear equilibrium paths. The upper limit load is associated with the maximum applied radial pressure point, and the lower limit load is related to the minimum point. It is shown that by taking the nonlocal stress tensor into account, the both upper and lower limit loads of radially loaded shallow micro/nano-arches decreases, while the strain gradient size dependency plays an opposite role and causes to enhance them. However, it is seen that these reduction and increment for the upper limit load are a bit more significant than those for the lower limit load.

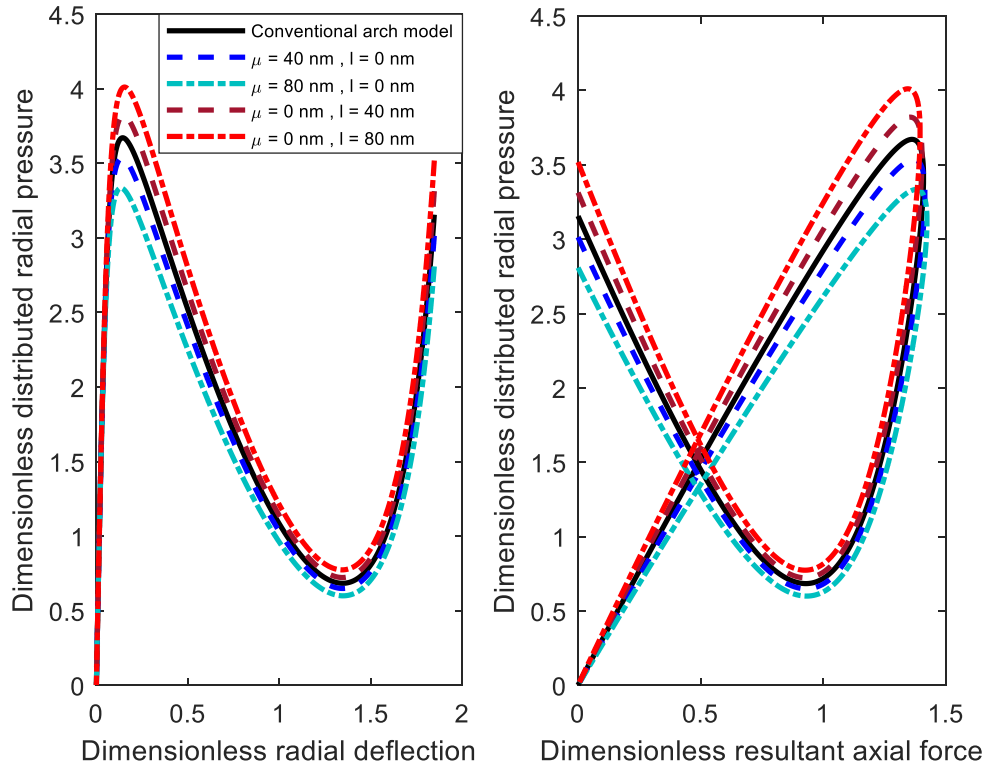


Figure 51: NSG-based and conventional dimensionless thermomechanical nonlinear in-plane stability curves of FG multilayer composite micro/nano-arches corresponding to various small-scale parameters ($a/b = 5, b/h_{GN} = 10^3, \Delta T = 0, \text{U-GNRC}$)

The influence of the FG multilayer pattern on the NSG-based dimensionless thermomechanical nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths of FG multilayer shallow micro/nano-arches is illustrated

in Figure 52. It is found that by changing the FG multilayer pattern from X-GNRC to O-GNRC one, the both upper and lower limit loads get larger, especially the first one. In addition, via considering this change for the FG multilayer pattern, the radial deflection associated with the upper limit load increases, while this prediction becomes vice versa for the lower limit load.

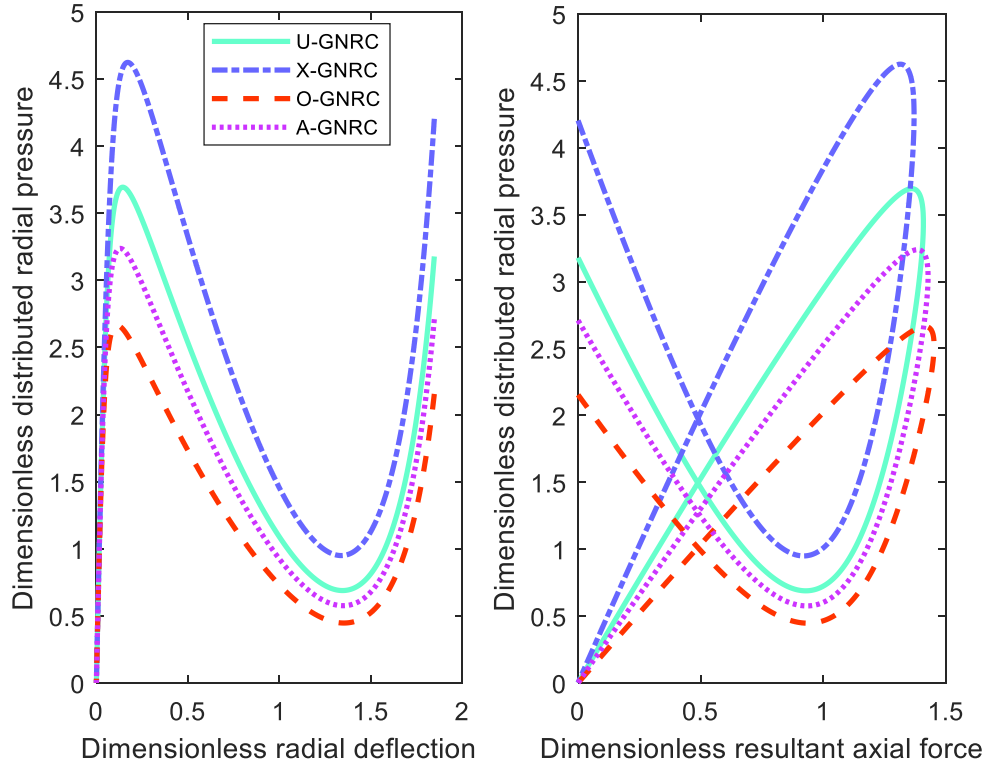


Figure 52: NSG-based dimensionless thermomechanical nonlinear in-plane stability curves of FG multilayer composite micro/nano-arches corresponding to various FG patterns ($a/b = 5$, $b/h_{GN} = 10^3$, $\Delta T = 0$, $\mu = l = 80 \text{ nm}$)

In Tables 10 and 11, the NSG-based dimensionless radial pressures and dimensionless resultant axial forces associated with the upper and lower limit loads are presented, respectively, corresponding to various FG multilayer patterns. It is found that both effects of the nonlocal stress tensor and strain gradient size dependency on the value of lower and upper limit radial pressures are more significant than those on the resultant axial forces associated with these limit points. In addition, it can be deduced

that for all FG multilayer patterns, the nonlocal stress tensor and strain gradient size dependency in order causes to reduce and enhance the both lower and upper limit radial loads. However, these small-scale features lead to, respectively, increase and decrease the resultant axial forces related to the lower and upper limits. Moreover, the nonlocality and strain gradient size effects for the lower limit radial pressure is more considerable than those for the upper limit radial pressure. Nevertheless, these effects are somehow more notable for upper limit resultant axial force than the lower limit one.

Table 10: NSG-based and conventional dimensionless lower and upper limit radial pressures for FG multilayer composite micro/nano-arches corresponding to various FG multilayer patterns ($\Delta T = 0, a/b = 5, b/h_{GN} = 10^3$)

FG multilayer scheme	Small scale parameters	Lower limit radial pressure	Upper limit radial pressure
U-GNRC	Classical	0.6855	3.6791
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.6509 (-5.05%)	3.5424 (-3.72%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.6014 (-12.27%)	3.3414 (-9.18%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.7245 (+5.69%)	3.8288 (+4.07%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.7758 (+13.17%)	4.0199 (+9.26%)
X-GNRC	Classical	0.9445	4.6059
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.8954 (-5.20%)	4.4416 (-3.57%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.8255 (-12.60%)	4.1992 (-8.83%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9999 (+5.86%)	4.7852 (+3.89%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	1.0729 (+13.59%)	5.0133 (+8.85%)
O-GNRC	Classical	0.4450	2.6535
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.4232 (-4.90%)	2.5504 (-3.88%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.3920 (-11.91%)	2.3994 (-9.58%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.4695 (+5.51%)	2.7667 (+4.27%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.5015 (+12.70%)	2.9119 (+9.74%)
A-GNRC	Classical	0.5734	3.2246
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.5448 (-4.99%)	3.1024 (-3.79%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.5040 (-12.10%)	2.9230 (-9.35%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.6055 (+5.60%)	3.3586 (+4.15%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.6477 (+12.96%)	3.5300 (+9.47%)

Table 11: NSG-based and convention dimensionless lower and upper limit resultant axial forces for FG multilayer micro/nano-arches corresponding to various FG multilayer patterns ($\Delta T = 0, a/b = 5, b/h_{GN} = 10^3$)

FG multilayer scheme	Small scale parameters	Lower limit resultant axial force	Upper limit resultant axial force
U-GNRC	Classical	0.9290	1.3615
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9301 (+0.12%)	1.3683 (+0.50%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9322 (+0.34%)	1.3784 (+1.24%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9275 (-0.16%)	1.3541 (-0.54%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9254 (-0.39%)	1.3443 (-1.26%)
X-GNRC	Classical	0.9195	1.3155
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9215 (+0.22%)	1.3234 (+0.60%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9240 (+0.49%)	1.3356 (+1.53%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9173 (-0.24%)	1.3066 (-0.68%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9150 (-0.49%)	1.2949 (-1.57%)
O-GNRC	Classical	0.9380	1.4145
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9385 (+0.05%)	1.4200 (+0.39%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9399 (+0.20%)	1.4281 (+0.96%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9370 (-0.11%)	1.4085 (-0.42%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9356 (-0.25%)	1.4004 (-1.00%)
A-GNRC	Classical	0.9330	1.3845
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9339 (+0.10%)	1.3910 (+0.47%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9356 (+0.28%)	1.4000 (+1.12%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9316 (-0.15%)	1.3775 (-0.51%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9299 (-0.33%)	1.3687 (-1.14%)

The NSG-based dimensionless radial pressures and dimensionless resultant axial forces associated with the upper and lower limit loads in order are given in Tables 12 and 13 corresponding to different temperature changes. It is observed that for all values of the temperature change, the enhancement in the upper and lower limit radial pressures due to the strain gradient size dependency is a bit higher than the reduction in these pressures caused by the nonlocality. This more significant effect of the strain gradient type of size effect than that of the nonlocal stress tensor is repeated for the resultant axial forces associated with the lower and upper limit points for all values of temperature changes. In addition, it can be seen that by increasing the value of temperature change, the effects of nonlocality and strain gradient size dependency on

the NSG-based lower limit radial pressures enhance, while these effects on the NSG-based resultant axial forces related to the lower limit point decrease.

Fig. 53 illustrates the influence of temperature change on the NSG-based dimensionless thermomechanical nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths of FG multilayer shallow micro/nano-arches. It is indicated that by increasing the value of temperature change, the upper limit radial pressure gets larger, while the lower limit one tends to decrease. Also, it is demonstrated that a higher temperature rise leads to enhance the resultant axial force associated with the upper limit point, but it causes to reduce the resultant axial force associated with the lower limit point.

Table 12: NSG-based and conventional dimensionless lower and upper limit radial pressures for FG multilayer micro/nano-arches corresponding to various temperature changes (U-GNRC, $a/b = 5$, $b/h_{GN} = 10^3$)

FG multilayer scheme	Small scale parameters	Lower limit radial pressure	Upper limit radial pressure
$\Delta T = 0$	Classical	0.6855	3.6791
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.6509 (-5.05%)	3.5424 (-3.72%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.6014 (-12.27%)	3.3414 (-9.18%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.7245 (+5.69%)	3.8288 (+4.07%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.7758 (+13.17%)	4.0199 (+9.26%)
$\Delta T = 50$	Classical	0.6475	3.8841
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.6144 (-5.11%)	3.7427 (-3.64%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.5676 (-12.34%)	3.5349 (-8.99%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.6845 (+5.71%)	4.0387 (+3.98%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.7333 (+13.25%)	4.2362 (+9.07%)
$\Delta T = 100$	Classical	0.6099	4.1184
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.5786 (-5.13%)	3.9739 (-3.51%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.5341 (-12.43%)	3.7618 (-8.66%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.6450 (+5.76%)	4.2767 (+3.84%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.6914 (+13.36%)	4.4790 (+8.76%)
$\Delta T = 150$	Classical	0.5727	4.4060
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.5431 (-5.17%)	4.2642 (-3.22%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.5010 (-12.52%)	4.0598 (-7.86%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.6060 (+5.81%)	4.5627 (+3.56%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.6499 (+13.48%)	4.7649 (+8.15%)

Table 13: NSG-based and conventional dimensionless lower and upper limit resultant axial forces for FG multilayer micro/nano-arches corresponding to various temperature changes ($\Delta T = 0, a/b = 5, b/h_{GN} = 10^3$)

FG multilayer scheme	Small scale parameters	Lower limit resultant axial force	Upper limit resultant axial force
$\Delta T = 0$	Classical	0.9290	1.3615
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9301 (+0.12%)	1.3683 (+0.50%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9322 (+0.34%)	1.3784 (+1.24%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9275 (-0.16%)	1.3541 (-0.54%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9254 (-0.39%)	1.3443 (-1.26%)
$\Delta T = 50$	Classical	0.9375	1.4051
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9384 (+0.10%)	1.4126 (+0.53%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9404 (+0.31%)	1.4240 (+1.35%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9361 (-0.15%)	1.3966 (-0.60%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9343 (-0.34%)	1.3862 (-1.35%)
$\Delta T = 100$	Classical	0.9455	1.4535
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9463 (+0.08%)	1.4627 (+0.62%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9482 (+0.29%)	1.4765 (+1.58%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9445 (-0.11%)	1.4439 (-0.66%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9426 (-0.31%)	1.4314 (-1.52%)
$\Delta T = 150$	Classical	0.9535	1.5125
	$\mu = 40 \text{ nm}, l = 0 \text{ nm}$	0.9541 (+0.06%)	1.5228 (+0.68%)
	$\mu = 80 \text{ nm}, l = 0 \text{ nm}$	0.9561 (+0.27%)	1.5376 (+1.66%)
	$\mu = 0 \text{ nm}, l = 40 \text{ nm}$	0.9527 (-0.08%)	1.5018 (-0.71%)
	$\mu = 0 \text{ nm}, l = 80 \text{ nm}$	0.9508 (-0.28%)	1.4862 (-1.74%)

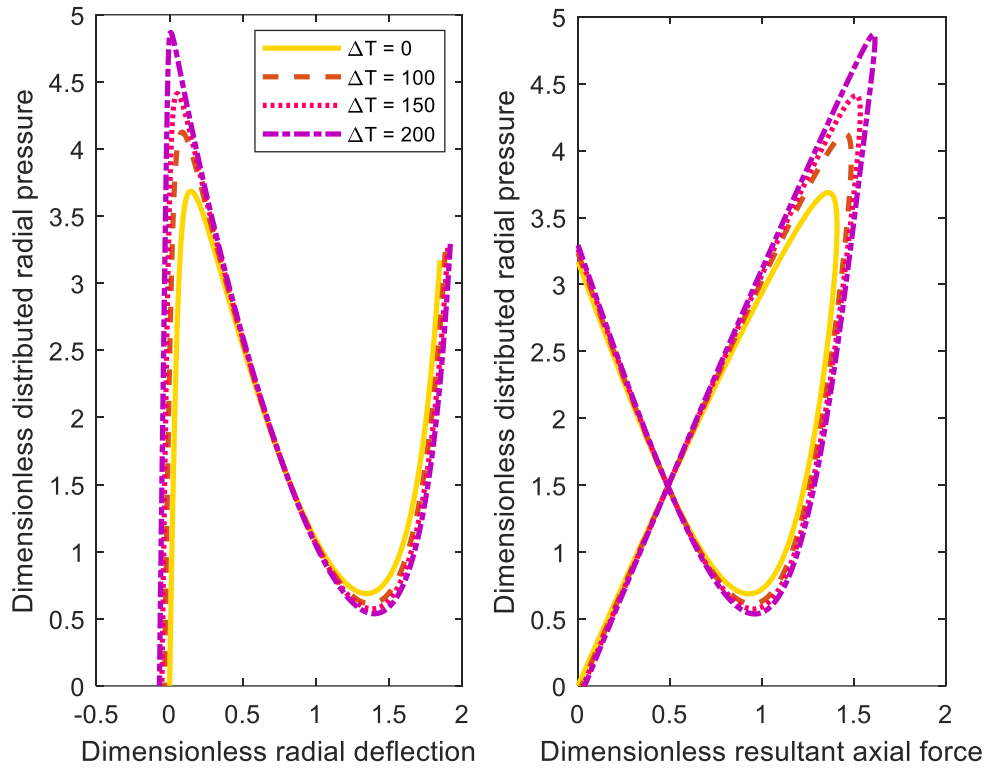


Figure 53: NSG-based dimensionless thermomechanical nonlinear in-plane stability curves of FG multilayer composite micro/nano-arches corresponding to various temperature changes ($a/b = 5$, $b/h_{GN} = 10^3$, $\mu = l = 80 \text{ nm}$, U-GNRC)

In Fig. 54, the NSG-based dimensionless thermomechanical nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths of FG multilayer shallow micro/nano-arches are plotted relevant to various length to width ratios of graphene nanofillers. It can be deduced that through increasing the slenderness ratio of graphene nanofillers, the stiffening character of them as reinforcements of FG multilayer micro/nano-arches enhances which results in higher values of the lower and upper limit radial pressures. Furthermore, it can be observed that this increment in the limit radial pressures is somehow more significant for the upper limit point than the lower one.

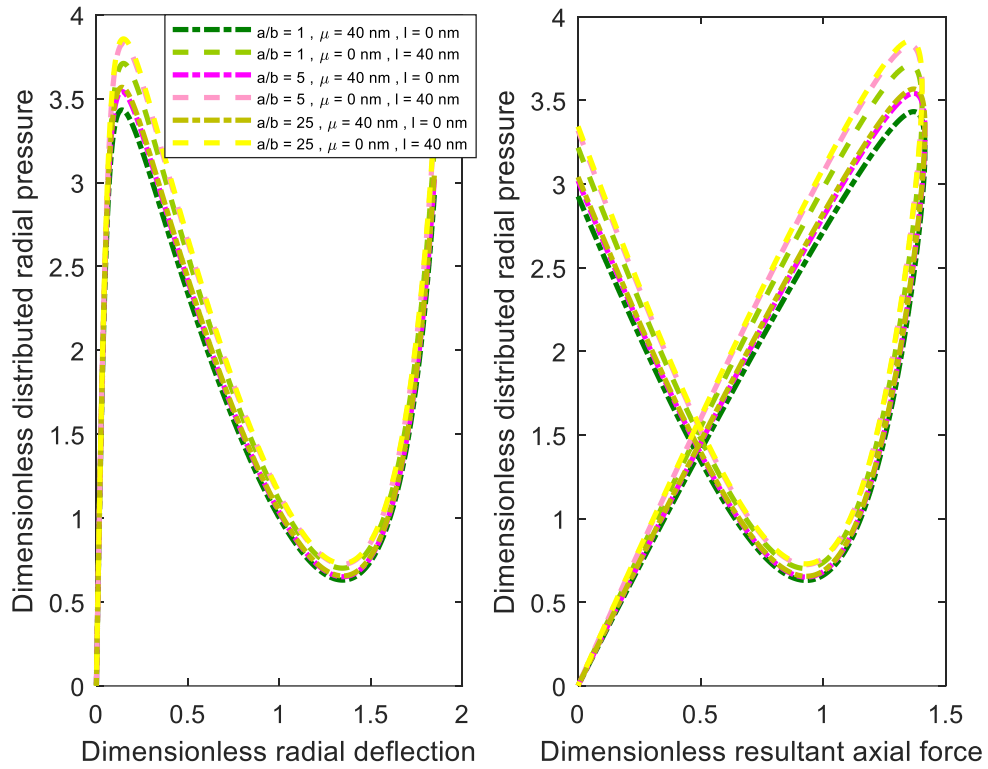


Figure 54: NSG-based dimensionless thermomechanical nonlinear in-plane stability curves of FG multilayer composite micro/nano-arches corresponding to various length to width ratios of nanofillers ($b/h_{GN} = 10^3, \mu = l = 80 \text{ nm}, \Delta T = 0$, U-GNRC)

Fig. 55 represents the influence of width to thickness ratio of nanofillers on the NSG-based dimensionless thermomechanical nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths of FG multilayer shallow micro/nano-arches. It is presented that by decreasing the width to thickness ratio of graphene nanofillers, both lower and upper limit radial pressures reduce, and this reduction is more prominent at the upper limit point in comparison with the lower limit one. Also, it can be seen that a higher value of width to thickness ratio of graphene nanofillers results in a small decrease in the resultant axial forces associated with the upper and lower limit loads.

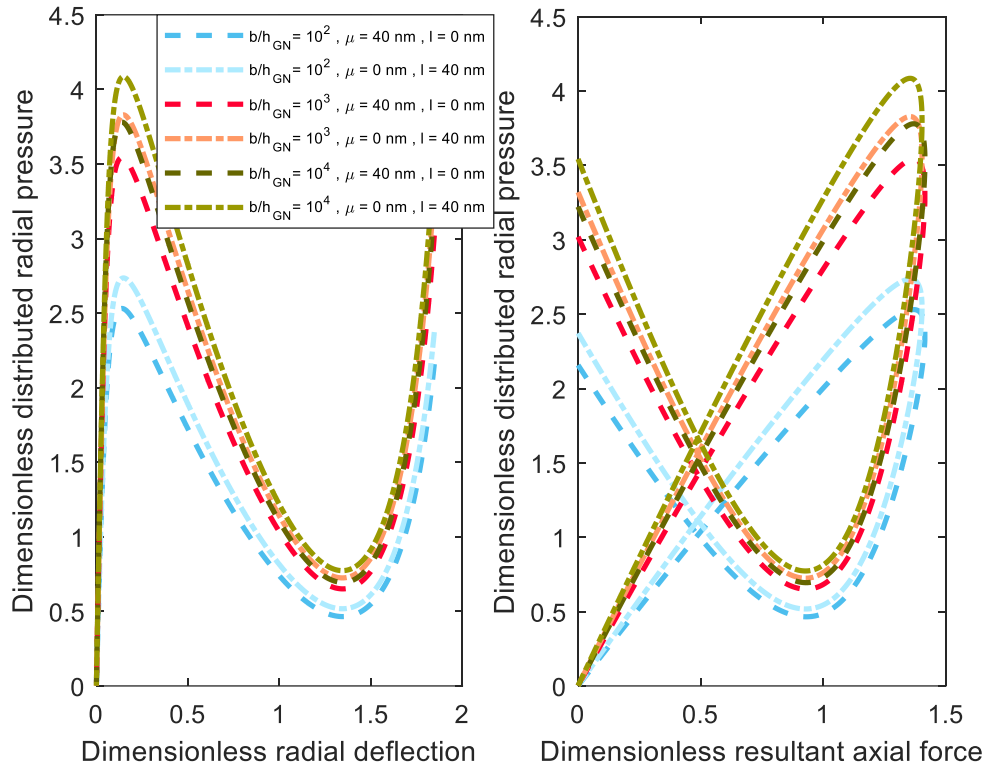
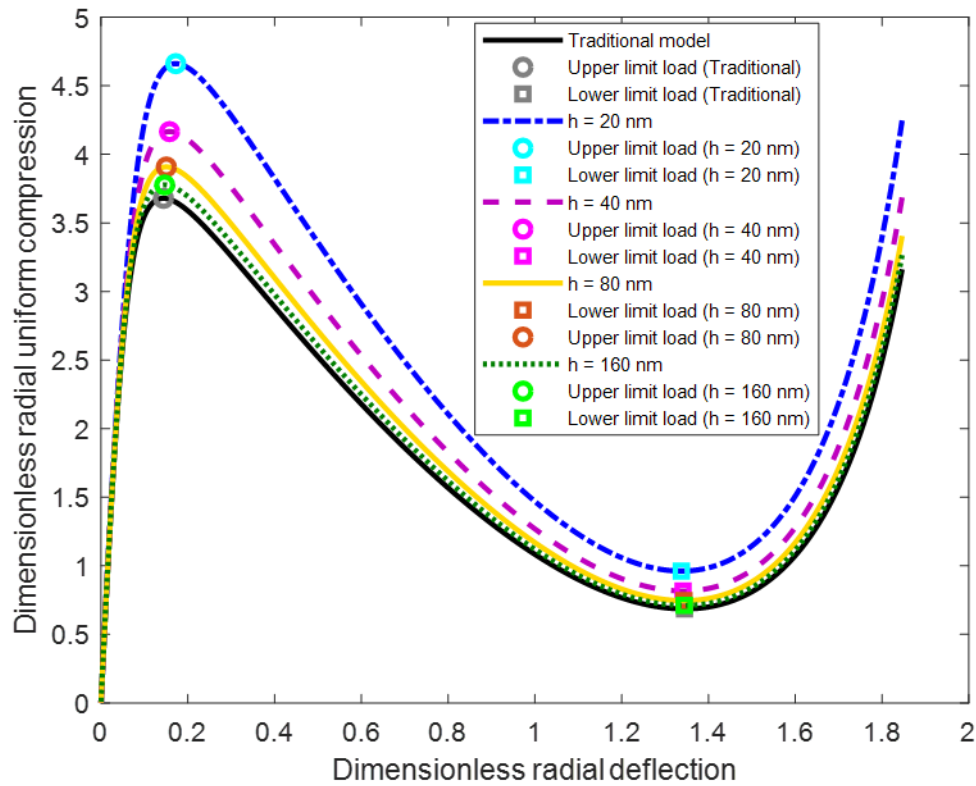


Figure 55: NSG-based dimensionless thermomechanical nonlinear in-plane stability curves of FG multilayer composite micro/nano-arches corresponding to various width to thickness ratios of nanofillers ($a/b = 5, \mu = l = 80 \text{ nm}, \Delta T = 0$, U-GNRC)

4.3 Numerical Results and Discussion for Case 2

Based upon the formulated surface elastic-based curved beam model, the numerical results associated with the nonlinear in-plane stability behavior of FG laminated curved nanobeams under thermomechanical loading conditions are given in this section. For the geometries and laminated nanocomposites considered herein, it is supposed that, $a = 2 \text{ nm}, b = 1.5 \text{ nm}, h_{GN} = 0.1 \text{ nm}, L/h = 30, R\theta^2/h = 8$ in which θ refers to the semi-vertex angle of the curved nanobeam. Additionally, one will have $E_{GN} = 1010 \text{ GPa}, E_m = 3 \text{ GPa}, \nu_{GN} = 0.3, \nu_m = 0.25, \rho_{GN} = 1062.5 \text{ Kg/m}^3, \rho_m = 1200 \text{ Kg/m}^3, \alpha_{GN} = 5 \times 10^{-6} \text{ 1/K}, \alpha_m = 60 \times 10^{-6} \text{ 1/K}$ [384,420]. Also, the surface elastic properties for the matrix part of the laminated nanocomposites are $\lambda_s = 5.188 \text{ N/m}, \mu_s = 4 \text{ N/m}$, and $\tau_s = 1.7 \text{ N/m}$ [421].

In Fig. 56, the traditional (in the absence of surface stress effects) and surface elastic-based radial load-deflection and radial load-axial load equilibrium curves are depicted showing the lower and upper limit points corresponding to different curved beam thicknesses. It can be found that the surface stress type of size dependency causes to enhance the radial load associated with the both lower and upper limit points, and this enhancement is together with a larger beam deflection for the upper limit point, but a smaller deflection for the lower one. In addition, the surface stress effect results in a reduction in the values of axial resultant loads relevant to the upper and lower limit points. It is observed that all of these features of surface stress size dependency are more prominent for a curved nanobeam having a lower thickness representing bigger gap between the traditional and size-dependent equilibrium curves.



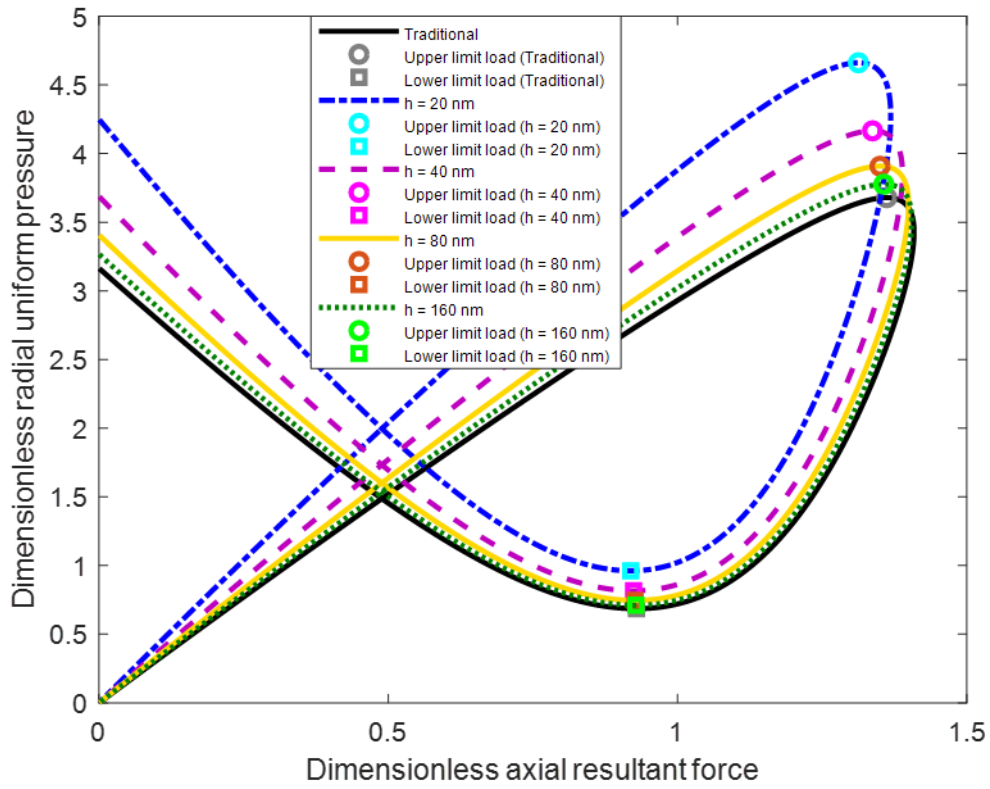


Figure 56: Traditional and surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including limit points for various thicknesses ($\Delta T = 0, a/b = 5, b/h_{GN} = 10, U\text{-GNRC}$)

Fig. 57 illustrates the traditional and surface elastic-based radial load-deflection and radial load-axial load equilibrium curves showing the bifurcation paths including the first and second bifurcation points corresponding to different curved beam thicknesses. It is demonstrated that by taking the surface stress effect into account, the values of the radial load related to the first and second bifurcation points increases. Also, it causes to increase the curved beam deflection associated with the first bifurcation point, while it leads to decrease the radial deflection for the second one. Accordingly, the effect of surface stress size dependency inspires a reduction in the slope of bifurcation path related to the nonlinear radial load-deflection equilibrium curve of a curved nanobeam. Moreover, it is obviously seen that the surface stress effect has no

considerable influence on the values of axial resultant loads associated with the first and second bifurcation points.

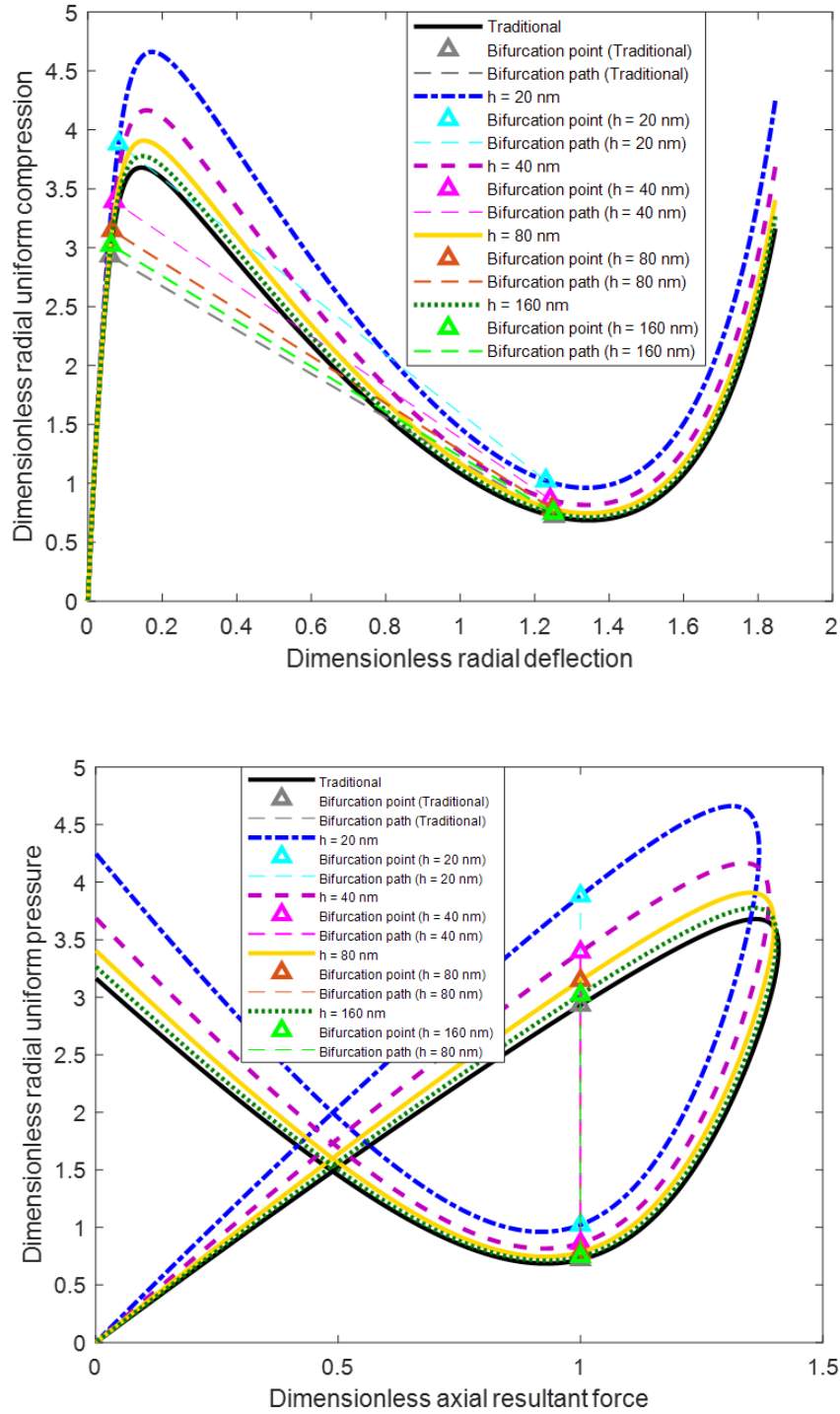
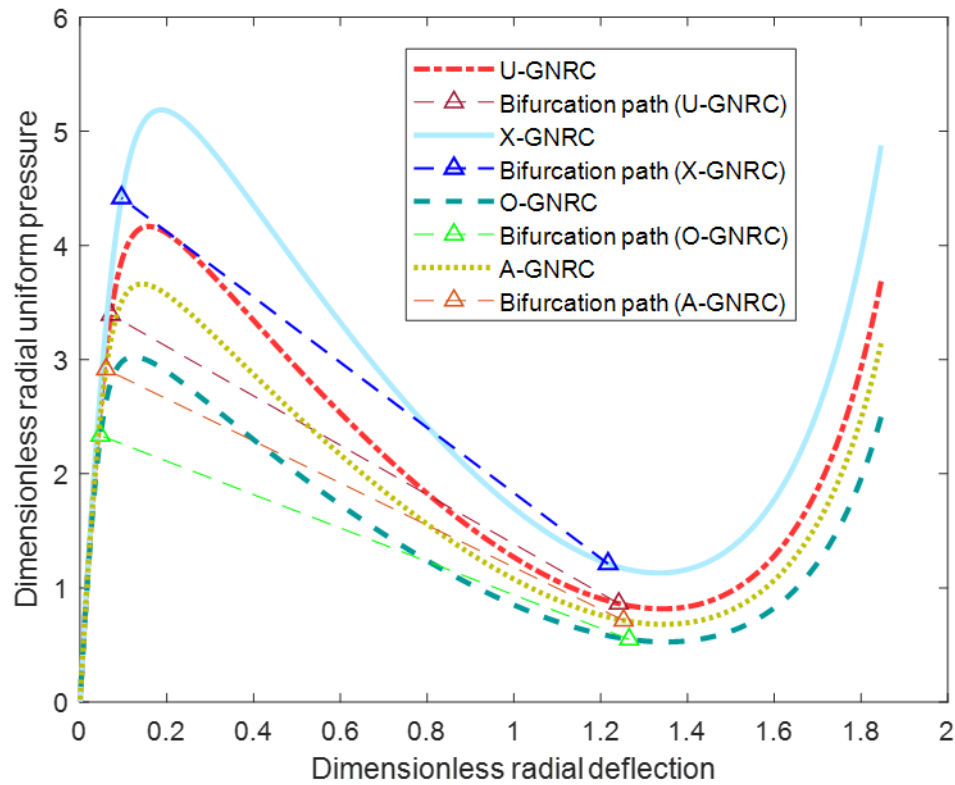


Figure 57: Traditional and surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including bifurcation points and paths for various thicknesses ($\Delta T = 0$, $a/b = 5$, $b/h_{GN} = 10$, U-GNRC)

Plotted in Fig. 58 are the surface elastic-based radial load-deflection and radial load-axial load equilibrium curves of FG laminated curved nanobeams showing the bifurcation paths including the first and second bifurcation points corresponding to different FG lamination schemes. It is revealed that the FG laminated curved nanobeams having X-GNRC and O-GNRC patterns of FG lamination have, respectively, the maximum and minimum values of radial loads associated with the limit points as well as the bifurcation points. In this regard, it can be seen that compared to the U-GNRC laminated curved nanobeam having uniform dispersion of nanofillers, the slope of bifurcation path related to the nonlinear radial load-deflection equilibrium curve of X-GNRC one is higher, but those of A-GNRC and O-GNRC ones are lower.



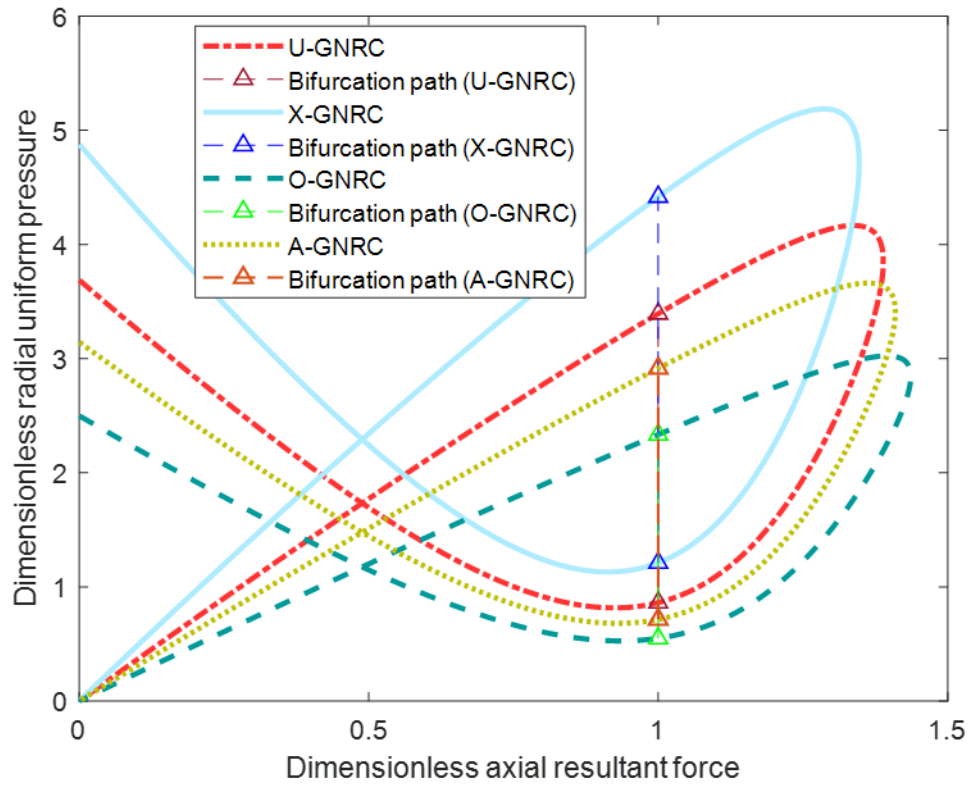


Figure 58: Surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including bifurcation points and paths for various FG lamination schemes ($\Delta T = 0, a/b = 5, b/h_{GN} = 10, h = 40 \text{ nm}$)

In Tables 14 and 15, the radial loads and axial resultant loads of FG laminated curved nanobeam are given corresponding to various FG lamination schemes, respectively. The percentages presented in parentheses refers to the difference between the traditional and surface elastic-based cases. It is found that the stiffening nature of surface stress effect related to the considered nanocomposite material is more significant for the radial load associated with the lower limit point in comparison with that related to the upper limit point. Similar prediction for the significance of the surface stress size-dependency is achieved related to the reduction in the values of axial resultant loads at the lower and upper limit points.

Table 14: Traditional and surface elastic-based dimensionless radial loads of FG laminated curved nanobeams at lower and upper limit points relevant to different lamination schemes ($\Delta T = 0, a/b = 5, b/h_{GN} = 20$)

FG lamination scheme	Curved nanobeam thickness	Lower limit radial pressure	Upper limit radial pressure
U-GNRC	Traditional	0.6855	3.6791
	$h = 20 \text{ nm}$	0.9615 (+40.26%)	4.6611 (+26.69%)
	$h = 40 \text{ nm}$	0.8161 (+19.05%)	4.1658 (+13.23%)
	$h = 80 \text{ nm}$	0.7455 (+8.75%)	3.9081 (+6.22%)
	$h = 160 \text{ nm}$	0.7108 (+3.69%)	3.7767 (+2.65%)
X-GNRC	Traditional	0.9445	4.6059
	$h = 20 \text{ nm}$	1.3391 (+41.78%)	5.7716 (+25.31%)
	$h = 40 \text{ nm}$	1.1306 (+19.70%)	5.1866 (+12.61%)
	$h = 80 \text{ nm}$	1.0298 (+9.03%)	4.8798 (+5.95%)
	$h = 160 \text{ nm}$	0.9804 (+3.80%)	4.7228 (+2.54%)
O-GNRC	Traditional	0.4450	2.6535
	$h = 20 \text{ nm}$	0.6166 (+38.56%)	3.4041 (+28.29%)
	$h = 40 \text{ nm}$	0.5266 (+18.31%)	3.0232 (+13.93%)
	$h = 80 \text{ nm}$	0.4827 (+8.47%)	2.8269 (+6.53%)
	$h = 160 \text{ nm}$	0.4609 (+3.57%)	2.7273 (+2.78%)
A-GNRC	Traditional	0.5734	3.2246
	$h = 20 \text{ nm}$	0.7999 (+39.50%)	4.1078 (+27.39%)
	$h = 40 \text{ nm}$	0.6809 (+18.75%)	3.6611 (+13.54%)
	$h = 80 \text{ nm}$	0.6228 (+8.62%)	3.4297 (+6.36%)
	$h = 160 \text{ nm}$	0.5943 (+3.65%)	3.3119 (+2.71%)

Tabulated in Table 16 are the traditional and surface elastic-based radial loads associated with the first and second bifurcation points corresponding to various FG lamination schemes. It is deduced that the role of the surface stress type of size dependency in the value of radial load related to the second bifurcation point is somehow more important than that of the one associated with the first bifurcation point. This observation is repeated for all schemes of the FG lamination. Fig. 59 displays the surface elastic-based radial load-deflection and radial load-axial load equilibrium curves of FG laminated curved nanobeams showing the bifurcation paths including the first and second bifurcation points corresponding to different temperature rises. It can be observed that the temperature rise causes to increase the radial loads associated with the upper limit point and the first bifurcation point, while

it leads to reduce the values of radial load related to the lower limit point and the second bifurcation point. In this regard, there is a unique intersection point as all equilibrium curves obtained for various temperature rises pass through that. Furthermore, it is seen that the slope of bifurcation path remains constant by changing the value of temperature rise.

Table 15: Traditional and surface elastic-based dimensionless axial resultant loads of FG laminated curved nanobeams at lower and upper limit points relevant to different lamination schemes ($\Delta T = 0, a/b = 5, b/h_{GN} = 20$)

FG lamination scheme	Curved nanobeam thickness	Lower limit point	Upper limit point
U-GNRC	Traditional	1.3615	0.9290
	$h = 20 \text{ nm}$	1.3125 (-3.60%)	0.9191 (-1.07%)
	$h = 40 \text{ nm}$	1.3371 (-1.79%)	0.9239 (-0.55%)
	$h = 80 \text{ nm}$	1.3500 (-0.84%)	0.9264 (-0.28%)
	$h = 160 \text{ nm}$	1.3566 (-0.36%)	0.9281 (-0.10%)
X-GNRC	Traditional	1.3155	0.9194
	$h = 20 \text{ nm}$	1.2574 (-4.42%)	0.9065 (-1.40%)
	$h = 40 \text{ nm}$	1.2865 (-2.20%)	0.9134 (-0.65%)
	$h = 80 \text{ nm}$	1.3020 (-1.03%)	0.9165 (-0.32%)
	$h = 160 \text{ nm}$	1.3096 (-0.45%)	0.9184 (-0.11%)
O-GNRC	Traditional	1.4144	0.9380
	$h = 20 \text{ nm}$	1.3755 (-2.75%)	0.9316 (-0.68%)
	$h = 40 \text{ nm}$	1.3951 (-1.36%)	0.9345 (-0.37%)
	$h = 80 \text{ nm}$	1.4049 (-0.67%)	0.9366 (-0.15%)
	$h = 160 \text{ nm}$	1.4105 (-0.28%)	0.9370 (-0.09%)
A-GNRC	Traditional	1.3845	0.9331
	$h = 20 \text{ nm}$	1.3400 (-3.21%)	0.9244 (-0.93%)
	$h = 40 \text{ nm}$	1.3626 (-1.58%)	0.9289 (-0.45%)
	$h = 80 \text{ nm}$	1.3741 (-0.75%)	0.9310 (-0.23%)
	$h = 160 \text{ nm}$	1.3799 (-0.33%)	0.9320 (-0.10%)

Table 16: Traditional and surface elastic-based dimensionless radial loads of FG laminated curved nanobeams at the first and second bifurcation points relevant to different lamination schemes ($\Delta T = 0, a/b = 5, b/h_{GN} = 20$)

FG lamination scheme	Curved nanobeam thickness	First bifurcation point	Second bifurcation point
U-GNRC	Traditional	2.9307	0.7203
	$h = 20 \text{ nm}$	3.8805 (+32.41%)	1.0224 (+41.94%)
	$h = 40 \text{ nm}$	3.3929 (+15.77%)	0.8623 (+19.71%)
	$h = 80 \text{ nm}$	3.1460 (+7.35%)	0.7854 (+9.04%)
	$h = 160 \text{ nm}$	3.0220 (+3.12%)	0.7476 (+3.79%)
X-GNRC	Traditional	3.8252	1.0036
	$h = 20 \text{ nm}$	5.0362 (+31.66%)	1.4485 (+44.33%)
	$h = 40 \text{ nm}$	4.4167 (+15.46%)	1.2113 (+20.70%)
	$h = 80 \text{ nm}$	4.1013 (+7.22%)	1.0984 (+9.45%)
	$h = 160 \text{ nm}$	3.9424 (+3.06%)	1.0434 (+3.97%)
O-GNRC	Traditional	2.0114	0.4629
	$h = 20 \text{ nm}$	2.6768 (+33.08%)	0.6460 (+39.55%)
	$h = 40 \text{ nm}$	2.3342 (+16.05%)	0.5496 (+18.73%)
	$h = 80 \text{ nm}$	2.1616 (+7.47%)	0.5028 (+8.62%)
	$h = 160 \text{ nm}$	2.0750 (+3.16%)	0.4797 (+3.63%)
A-GNRC	Traditional	2.5141	0.5997
	$h = 20 \text{ nm}$	3.3369 (+32.73%)	0.8446 (+40.84%)
	$h = 40 \text{ nm}$	2.9138 (+15.90%)	0.7152 (+19.26%)
	$h = 80 \text{ nm}$	2.7002 (+7.40%)	0.6527 (+8.84%)
	$h = 160 \text{ nm}$	2.5930 (+3.14%)	0.6220 (+3.72%)

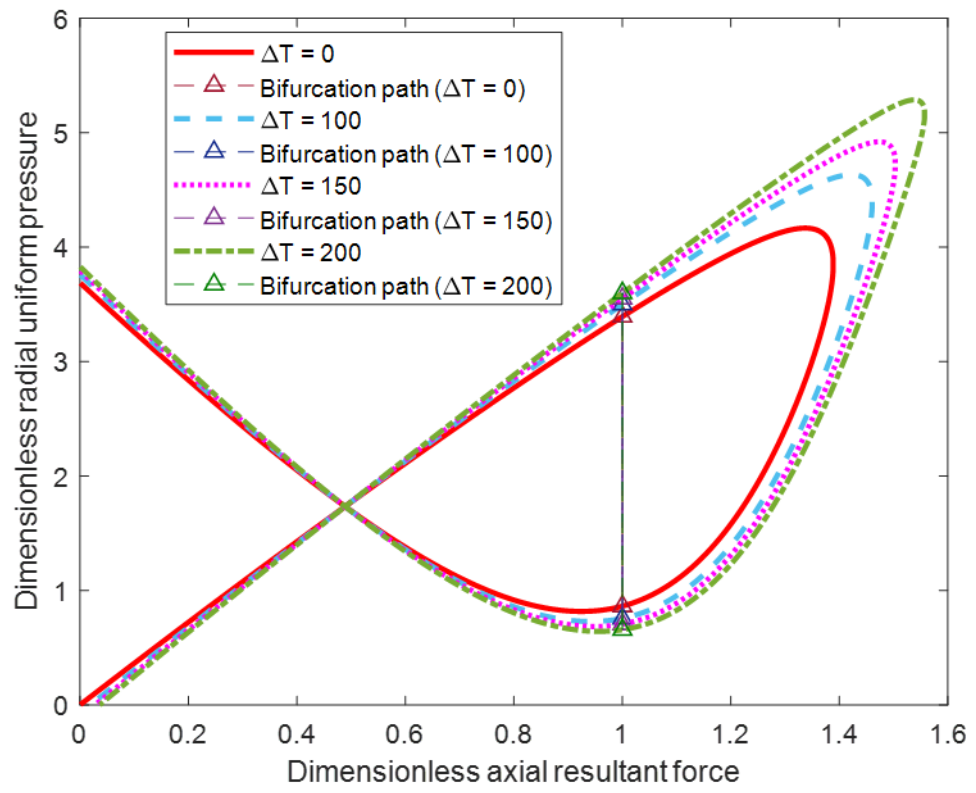
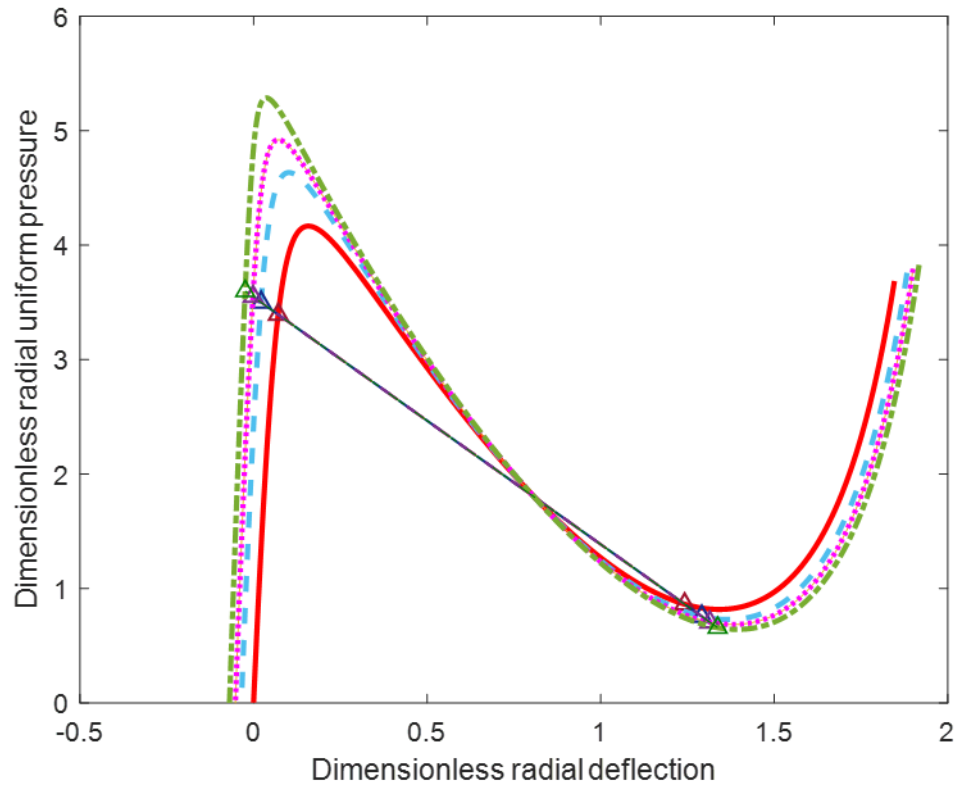


Figure 59: Surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including bifurcation points and paths for various temperature rises ($a/b = 5, b/h_{GN} = 10, h = 40 \text{ nm}$, U-GNRC)

Tables 17 and 18 presents, respectively, the radial loads and axial resultant loads of FG laminated curved nanobeam corresponding to various values of the temperature rise. It is disclosed that through rising the temperature, the significance of the surface stress effect resulting in as increment in the value of radial load and reduction in the value of axial resultant load associated with the lower limit point gets higher. However, the magnitude of surface stress effect related to these increment and reduction becomes smaller at the upper limit point. In Table 19, the traditional and surface elastic-based radial loads of FG laminated curved nanobeams associated with the first and second bifurcation points are given corresponding to different values of the temperature rise. It is found that for a higher temperature rise, the significance of the surface stress type of size dependency in radial loads associated with the first and second bifurcation points improves. However, this improvement is not so much, especially for the first bifurcation point.

Table 17: Traditional and surface elastic-based dimensionless radial loads of FG laminated curved nanobeams at lower and upper limit points relevant to different temperature changes (U-GNRC, $a/b = 5$, $b/h_{GN} = 20$)

Temperature change	Curved nanobeam thickness	Lower limit point	Upper limit point
$\Delta T = 0$	Traditional	0.6855	3.6791
	$h = 20 \text{ nm}$	0.9615 (+40.26%)	4.6611 (+26.69%)
	$h = 40 \text{ nm}$	0.8161 (+19.05%)	4.1658 (+13.23%)
	$h = 80 \text{ nm}$	0.7455 (+8.75%)	3.9081 (+6.22%)
	$h = 160 \text{ nm}$	0.7108 (+3.69%)	3.7767 (+2.65%)
$\Delta T = 100$	Traditional	0.6099	4.1184
	$h = 20 \text{ nm}$	0.8597 (+40.96%)	5.1585 (+25.25%)
	$h = 40 \text{ nm}$	0.7279 (+19.35%)	4.6335 (+12.51%)
	$h = 80 \text{ nm}$	0.6640 (+8.87%)	4.3606 (+5.88%)
	$h = 160 \text{ nm}$	0.6327 (+3.74%)	4.2216 (+2.51%)
$\Delta T = 150$	Traditional	0.5727	4.4060
	$h = 20 \text{ nm}$	0.8096 (+41.36%)	5.4509 (+23.72%)
	$h = 40 \text{ nm}$	0.6845 (+19.52%)	4.9201 (+11.67%)
	$h = 80 \text{ nm}$	0.6240 (+8.96%)	4.6464 (+5.46%)
	$h = 160 \text{ nm}$	0.5943 (+3.77%)	4.5081 (+2.32%)
$\Delta T = 200$	Traditional	0.5359	4.8707
	$h = 20 \text{ nm}$	0.7601 (+41.84%)	5.7962 (+19.00%)
	$h = 40 \text{ nm}$	0.6416 (+19.72%)	5.2863 (+8.53%)
	$h = 80 \text{ nm}$	0.5843 (+9.03%)	5.0426 (+3.53%)
	$h = 160 \text{ nm}$	0.5563 (+3.81%)	4.9339 (+1.30%)

Table 18: Traditional and surface elastic-based dimensionless axial resultant loads of FG laminated curved nanobeams at lower and upper limit points relevant to different temperature changes (U-GNRC, $a/b = 5$, $b/h_{GN} = 20$)

Temperature change	Curved nanobeam thickness	Lower limit resultant axial force	Upper limit resultant axial force
$\Delta T = 0$	Traditional	1.3615	0.9290
	$h = 20 \text{ nm}$	1.3125 (-3.60%)	0.9191 (-1.07%)
	$h = 40 \text{ nm}$	1.3371 (-1.79%)	0.9239 (-0.55%)
	$h = 80 \text{ nm}$	1.3500 (-0.84%)	0.9264 (-0.28%)
	$h = 160 \text{ nm}$	1.3566 (-0.36%)	0.9281 (-0.10%)
$\Delta T = 100$	Traditional	1.4535	0.9455
	$h = 20 \text{ nm}$	1.3930 (-4.16%)	0.9358 (-1.02%)
	$h = 40 \text{ nm}$	1.4224 (-2.14%)	0.9408 (-0.50%)
	$h = 80 \text{ nm}$	1.4385 (-1.03%)	0.9432 (-0.24%)
	$h = 160 \text{ nm}$	1.4471 (-0.44%)	0.9447 (-0.09%)
$\Delta T = 150$	Traditional	1.5125	0.9535
	$h = 20 \text{ nm}$	1.4385 (-4.89%)	0.9444 (-0.95%)
	$h = 40 \text{ nm}$	1.4736 (-2.57%)	0.9491 (-0.46%)
	$h = 80 \text{ nm}$	1.4934 (-1.26%)	0.9514 (-0.22%)
	$h = 160 \text{ nm}$	1.5040 (-0.56%)	0.9525 (-0.08%)
$\Delta T = 200$	Traditional	1.6100	0.9605
	$h = 20 \text{ nm}$	1.4915 (-7.36%)	0.9525 (-0.83%)
	$h = 40 \text{ nm}$	1.5379 (-4.48%)	0.9566 (-0.41%)
	$h = 80 \text{ nm}$	1.5696 (-2.51%)	0.9589 (-0.17%)
	$h = 160 \text{ nm}$	1.5899 (-1.25%)	0.9599 (-0.06%)

Table 19: Traditional and surface elastic-based dimensionless radial loads of FG laminated curved nanobeams at the first and second bifurcation points relevant to different temperature changes (U-GNRC, $a/b = 5$, $b/h_{GN} = 20$)

Temperature change	Curved nanobeam thickness	First bifurcation point	Second bifurcation point
$\Delta T = 0$	Traditional	2.9307	0.7203
	$h = 20 \text{ nm}$	3.8805 (+32.41%)	1.0224 (+41.94%)
	$h = 40 \text{ nm}$	3.3929 (+15.77%)	0.8623 (+19.71%)
	$h = 80 \text{ nm}$	3.1460 (+7.35%)	0.7854 (+9.04%)
	$h = 160 \text{ nm}$	3.0220 (+3.12%)	0.7476 (+3.79%)
$\Delta T = 100$	Traditional	3.0207	0.6303
	$h = 20 \text{ nm}$	4.0057 (+32.61%)	0.8972 (+42.34%)
	$h = 40 \text{ nm}$	3.4995 (+15.85%)	0.7557 (+19.89%)
	$h = 80 \text{ nm}$	3.2437 (+7.38%)	0.6877 (+9.11%)
	$h = 160 \text{ nm}$	3.1152 (+3.13%)	0.6544 (+3.82%)
$\Delta T = 150$	Traditional	3.0632	0.5878
	$h = 20 \text{ nm}$	4.0646 (+32.69%)	0.8383 (+42.62%)
	$h = 40 \text{ nm}$	3.5498 (+15.89%)	0.7054 (+20.01%)
	$h = 80 \text{ nm}$	3.2898 (+7.40%)	0.6416 (+9.15%)
	$h = 160 \text{ nm}$	3.1593 (+3.14%)	0.6104 (+3.84%)
$\Delta T = 200$	Traditional	3.1044	0.5466
	$h = 20 \text{ nm}$	4.1214 (+32.76%)	0.7815 (+42.97%)
	$h = 40 \text{ nm}$	3.5984 (+15.91%)	0.6568 (+20.16%)
	$h = 80 \text{ nm}$	3.3343 (+7.41%)	0.5970 (+9.22%)
	$h = 160 \text{ nm}$	3.2018 (+3.14%)	0.5678 (+3.88%)

Represented in Fig. 60 are the surface elastic-based radial load-deflection and radial load-axial load equilibrium curves of FG laminated curved nanobeams showing the bifurcation paths including the first and second bifurcation points corresponding to different length to width ratio of graphene nanofillers. It can be revealed that by increasing the slenderness ratio of graphene nanofillers, the radial loads related to the upper and lower limit points as well as the first and second bifurcation points enhances in such a way that the slope of the bifurcation path increases a bit. The increment in the axial resultant loads at the upper and lower limit point is also observed via reinforcing by graphene nanofillers having a higher slenderness ratio.

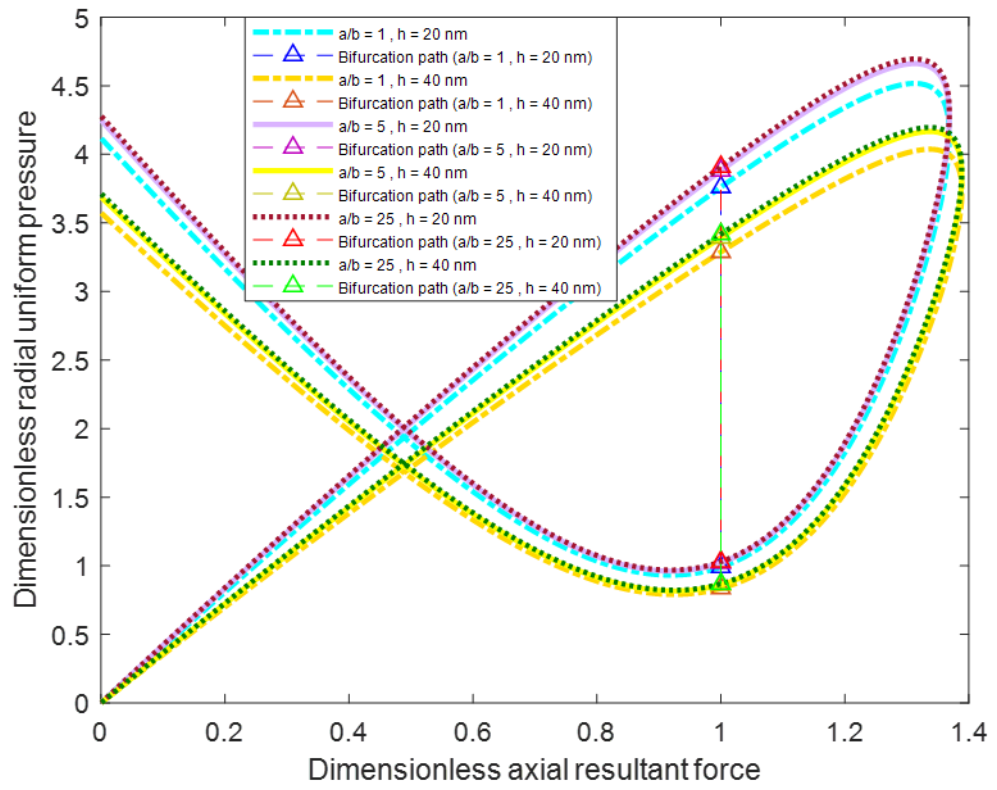
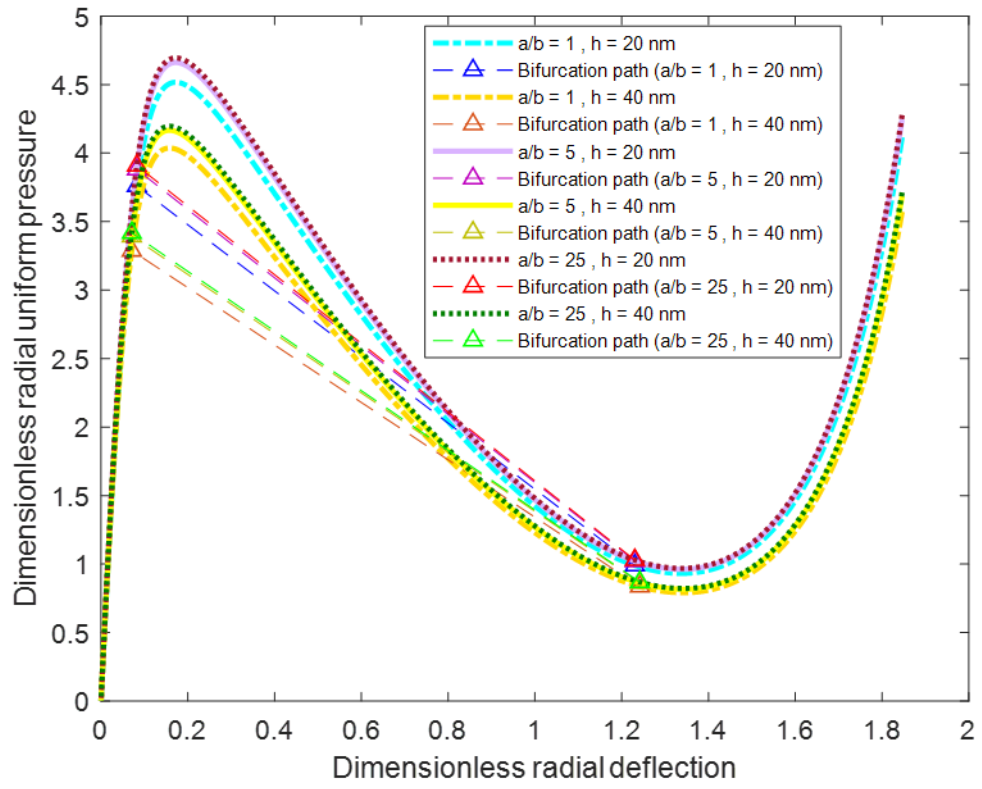
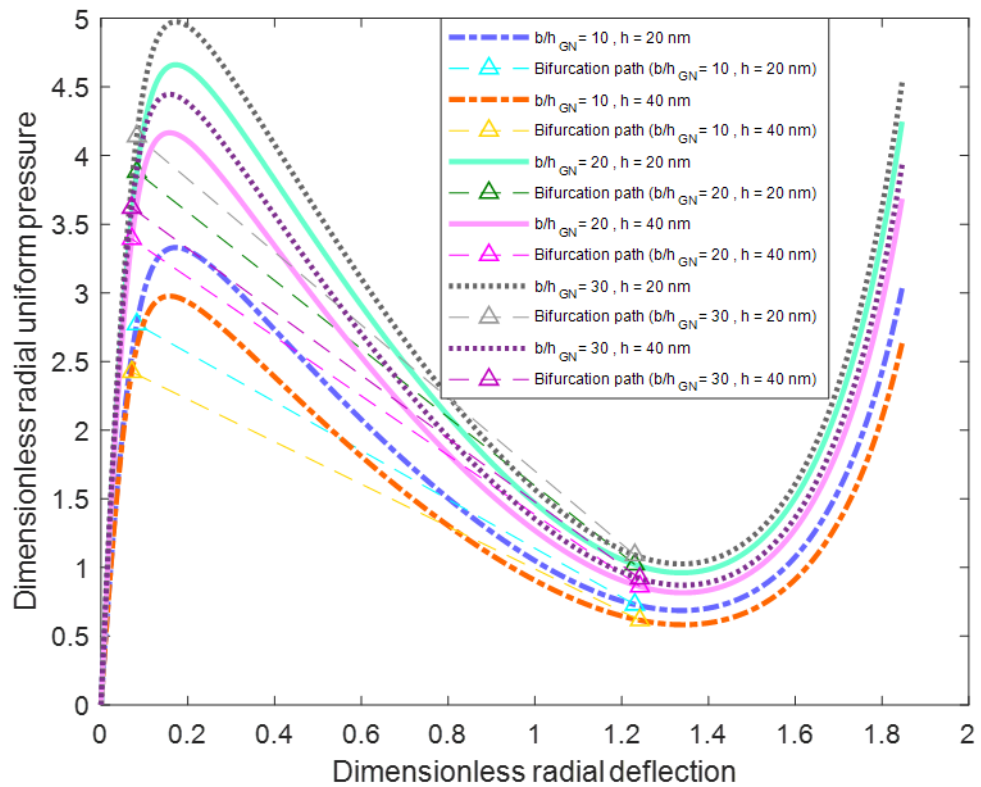


Figure 60: Surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including bifurcation points and paths for various ratios of a/b ($b/h_{GN} = 10, \Delta T = 0, U\text{-GNRC}$)

Fig. 61 shows the surface elastic-based radial load-deflection and radial load-axial load equilibrium curves of FG laminated curved nanobeams showing the bifurcation paths including the first and second bifurcation points corresponding to different width to thickness ratio of graphene nanofillers. It is deduced that for a FG laminated nanobeam reinforced with graphene nanofillers with high width to thickness ratio, the radial loads associated with the upper and lower limit points increases. In addition, the curved beam deflections at the upper limit point as well as the first bifurcation point somehow increase, while those of the lower limit and second bifurcation points decrease.



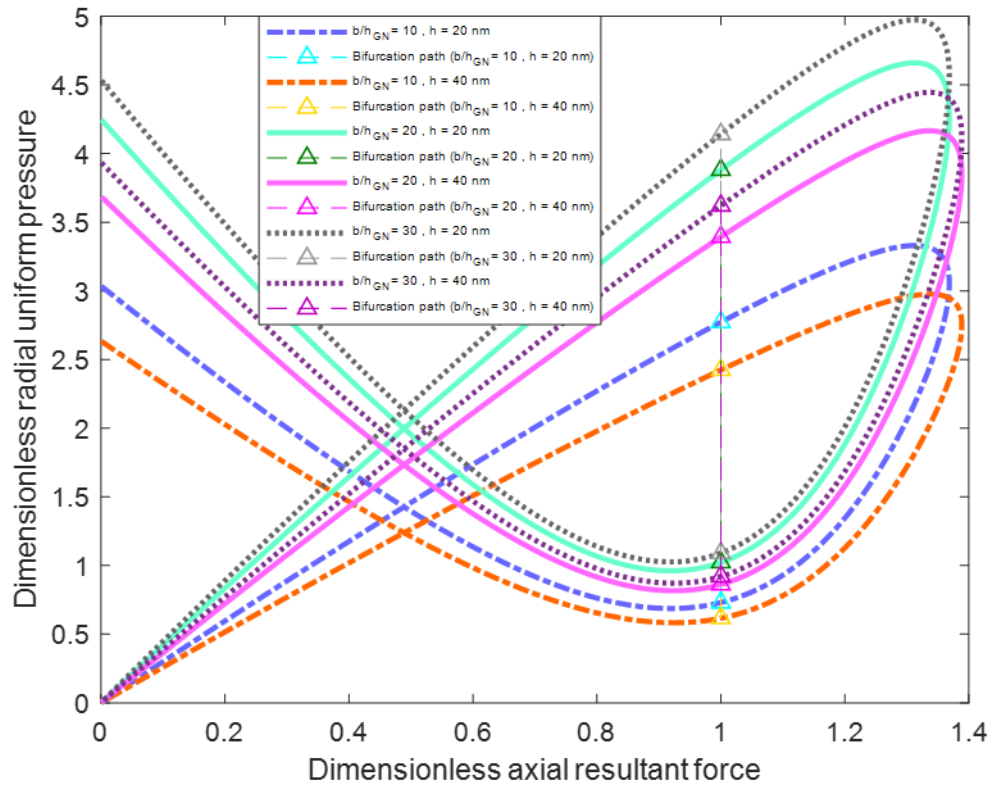


Figure 61: Surface elastic-based radial load-deflection and radial load-axial load equilibrium curves including bifurcation points and paths for various ratios of b/h_{GN} ($a/b = 5$, $\Delta T = 0$, U-GNRC)

Chapter 5

CONCLUSION

In the present research examination, a NSG-based arch model was developed to analyze the nonlinear in-plane thermomechanical stability characteristics of clamped FG multilayer composite shallow micro/nano-arches under uniform radial pressure together with a temperature change. An efficient numerical solving process based upon the generalized differential quadrature discretization technique was employed to trace the NSG-based nonlinear equilibrium curves including radial load-deflection and radial load-axial load paths corresponding to various parameters.

It was displayed that there are two limit points namely as upper and lower ones in the presented nonlinear equilibrium paths. The upper limit load is associated with the maximum applied radial pressure point, and the lower limit load is related to the minimum point. It was found that by taking the nonlocal stress tensor into account, the both upper and lower limit loads of radially loaded shallow micro/nano-arches decreases, while the strain gradient size dependency plays an opposite role and causes to enhance them. However, these reduction and increment for the upper limit load are a bit more significant than those for the lower limit load. It was deduced that both effects of the nonlocal stress tensor and strain gradient size dependency on the value of lower and upper limit radial pressures are more significant than those on the resultant axial forces associated with these limit points. Moreover, it was found that by decreasing the width to thickness ratio of graphene nanofillers, both lower and

upper limit radial pressures reduce, and this reduction is more prominent at the upper limit point in comparison with the lower limit one. Additionally, it was observed that by increasing the value of temperature change, the effects of nonlocality and strain gradient size dependency on the NSG-based lower limit radial pressures enhance, while these effects on the NSG-based resultant axial forces related to the lower limit point decrease.

The main objective of the current investigation, for the first time, the Gurtin-Murdoch surface elasticity theory was employed within the framework of the higher-order shear flexible curved beam model to analysis the nonlinear stability characteristics of thermomechanical loaded FG laminated curved nanobeams. Accordingly, with the aid of Halpin-Tsai homogenization model, the effective material properties were extracted relevant to different FG lamination schemes. GDQ numerical solution plan was then utilized to trace the nonlinear equilibrium curves.

It was deduced that the surface stress type of size dependency causes to enhance the radial load associated with the both lower and upper limit points, and this enhancement is together with a larger beam deflection for the upper limit point, but a smaller deflection for the lower one. In addition, the surface stress effect results in a reduction in the values of axial resultant loads relevant to the upper and lower limit points. Also, it was found that the FG laminated curved nanobeams having X-GNRC and O-GNRC patterns of FG lamination have, respectively, the maximum and minimum values of radial loads associated with the limit points as well as the bifurcation points. In this regard, it can be seen that compared to the U-GNRC laminated curved nanobeam having uniform dispersion of nanofillers, the slope of bifurcation path related to the nonlinear radial load-deflection equilibrium curve of X-GNRC one is higher, but those

of A-GNRC and O-GNRC ones are lower. Furthermore, it was observed that through rising the temperature, the significance of the surface stress effect resulting in as increment in the value of radial load and reduction in the value of axial resultant load associated with the lower limit point gets higher. However, the magnitude of surface stress effect related to these increment and reduction becomes smaller at the upper limit point. For a higher temperature rise, the significance of the surface stress type of size dependency in radial loads associated with the first and second bifurcation points improves. However, this improvement is not so much, especially for the first bifurcation point. Moreover, the increment in the axial resultant loads at the upper and lower limit point was revealed via reinforcing by graphene nanofillers having a higher slenderness ratio.

In future research on the size-dependent thermomechanical stability behavior of functionally graded multilayer shallow nano-arches, several key areas can be explored. Firstly, there is a need for experimental validation of the theoretical findings and predictions presented in the initial study. This involves fabricating and characterizing actual functionally graded multilayer shallow nano-arches under different thermal and mechanical loading conditions to understand their real-world behavior and performance. Secondly, researchers can focus on material design and optimization. By exploring various material combinations and compositions, they can enhance the thermomechanical stability of these nano-arches. Utilizing advanced materials like nanocomposites or metamaterials may improve their strength, stiffness, and resistance to thermal deformation. Numerical modeling is another important aspect that warrants attention. Refinement and improvement of computational models used to analyze the thermomechanical stability of these arches can lead to more accurate predictions. This

can involve incorporating better material models, accounting for nonlinear behavior, and considering complex geometries and loading conditions. Additionally, the study can delve deeper into the analysis of size effects on the thermomechanical stability of nano-arches. Investigating how varying dimensions such as thickness, width, and span influence critical buckling temperature, deformation modes, and failure mechanisms will contribute to a comprehensive understanding of these structures. Furthermore, researchers can explore the impact of environmental factors on the thermomechanical stability of functionally graded multilayer shallow nano-arches. Considering variables such as humidity, temperature gradients, and corrosive conditions will provide insights into the arches' long-term performance and durability in real-world operating environments. Ultimately, future research aims to advance our understanding of the size-dependent thermomechanical stability behavior of functionally graded multilayer shallow nano-arches and translate this knowledge into practical applications with improved performance and reliability.

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