A Second Chance for Failed Projects Using Data Envelopment Analysis Based on Project Attractiveness Factors

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Submitted to the
Institute of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Industrial Engineering

Eastern Mediterranean University
January 2025
Gazimağusa, North Cyprus

Approval of the Institute of Graduate Studies and Research

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ABSTRACT

Assessing a project's profitability using Net Present Value (NPV), Internal Rate of Return (IRR), and Modified Internal Rate of Return (MIRR) is a common practice for determining its viability. However, these traditional metrics often overlook the differences between new and existing projects, leading to data uncertainty. Additionally, they fail to account for the complex relationships between various factors that influence project outcomes.

In this paper, we propose using Data Envelopment Analysis (DEA) to provide projects with a second evaluation opportunity. DEA identifies the most efficient year within a project’s lifecycle, optimizes input and output values, and highlights the factors that most significantly impact the efficiency of Decision-Making Units (DMUs).

By applying DEA to optimize the input and output values, we can reassess the project’s NPV, IRR, and MIRR, allowing for a comparison with the original evaluation. If the results indicate enhanced profitability, the project becomes more attractive, with its revised input and output values serving as targets for project redesign.

This paper presents a case study of a non-viable road project to demonstrate the application of this approach. The findings reveal that, although the project's NPV, IRR, and MIRR values improve with DEA, the project remains unattractive even under optimal conditions. Additionally, a novel dataset was simulated based on the parameters of the case study. Analysis of the simulated data shows increased profitability, suggesting that the simulated data, with its modified inputs and outputs, now presents an improved case for viability.

Keywords: Project Attractiveness Factors, Data Envelopment Analysis, NPV, IRR, MIRR, Investment Appraisal, Efficiency

ÖZ

Bir projenin karlılığını Net Mevcut Değer (NPV), İç Getiri Oranı (IRR) ve Değiştirilmiş İç Getiri Oranı (MIRR) kullanarak değerlendirmek, uygulanabilirliğini belirlemek için yaygın bir uygulamadır. Ancak, bu geleneksel metrikler genellikle yeni ve mevcut projeler arasındaki farkları göz ardı ederek veri belirsizliğine yol açar. Ayrıca, proje sonuçlarını etkileyen çeşitli faktörler arasındaki karmaşık ilişkileri hesaba katmazlar. Bu makalede, projelere ikinci bir değerlendirme fırsatı sunmak için Veri Zarflama Analizi (DEA) kullanmayı öneriyoruz. DEA, bir projenin yaşam döngüsü içindeki en verimli yılı belirler, girdi ve çıktı değerlerini optimize eder ve Karar Alma Birimlerinin (DMU) verimliliğini en önemli ölçüde etkileyen faktörleri vurgular.

Giriş ve çıktı değerlerini optimize etmek için DEA uygulayarak, projenin NPV, IRR ve MIRR'sini yeniden değerlendirebilir ve orijinal değerlendirmeyle karşılaştırma yapabiliriz. Sonuçlar artan karlılığı gösteriyorsa, proje, revize edilmiş girdi ve çıktı değerlerinin proje yeniden tasarımı için hedef olarak hizmet etmesiyle daha çekici hale gelir. Bu makale, bu yaklaşımın uygulanmasını gösteren, uygulanabilir olmayan bir yol projesinin vaka çalışmasını sunmaktadır. Bulgular, projenin NPV, IRR ve MIRR değerlerinin DEA ile iyileşmesine rağmen, projenin en uygun koşullar altında bile çekici olmadığını ortaya koymaktadır. Ek olarak, vaka çalışmasının parametrelerine dayalı olarak yeni bir veri seti simüle edilmiştir. Simüle edilen verilerin analizi, artan karlılığı göstermektedir ve bu da simüle edilen verilerin, değiştirilmiş girdileri ve çıktılarıyla artık uygulanabilirlik için geliştirilmiş bir durum sunduğunu göstermektedir.

Anahtar Sözcükler: Proje Çekiciliği Faktörleri, Veri Zarflama Analizi, NPV, IRR, MIRR, Yatırım Değerlendirmesi, Verimlilik ÖZET Bir projenin karlılığını Net Mevcut Değer (NPV), İç Getiri Oranı (IRR) ve Değiştirilmiş İç Getiri Oranı (MIRR) kullanarak değerlendirmek, uygulanabilirliğini belirlemek için yaygın bir uygulamadır. Ancak, bu geleneksel ölçütler genellikle yeni ve mevcut projeler arasındaki farkları göz ardı ederek veri belirsizliğine yol açmaktadır. Ek olarak, proje sonuçlarını etkileyen çeşitli faktörler arasındaki karmaşık ilişkileri hesaba katmakta başarısız oluyorlar. Bu makalede, projelere ikinci bir değerlendirme fırsatı sunmak için Veri Zarflama Analizi (DEA) kullanmayı öneriyoruz.

DEA, bir projenin yaşam döngüsü içindeki en verimli yılı belirler, girdi ve çıktı değerlerini optimize eder ve Karar Alma Birimlerinin (DMU) verimliliğini en önemli ölçüde etkileyen faktörleri vurgular. Giriş ve çıktı değerlerini optimize etmek için DEA uygulayarak, projenin NPV, IRR ve MIRR'sini yeniden değerlendirebilir ve orijinal değerlendirmeyle karşılaştırma yapabiliriz. Sonuçlar artan karlılığı gösteriyorsa, proje, revize edilmiş girdi ve çıktı değerlerinin proje yeniden tasarımı için hedef olarak hizmet etmesiyle daha çekici hale gelir. Bu makale, bu yaklaşımın uygulanmasını gösteren, uygulanabilir olmayan bir yol projesinin vaka çalışmasını sunmaktadır.

Bulgular, projenin NPV, IRR ve MIRR değerlerinin DEA ile iyileşmesine rağmen, projenin en uygun koşullar altında bile çekici olmadığını ortaya koymaktadır. Ek olarak, vaka çalışmasının parametrelerine dayalı olarak yeni bir veri kümesi simüle edilmiştir. Simüle edilmiş verilerin analizi artan karlılığı göstermektedir, bu da simüle edilmiş verilerin, değiştirilmiş girdileri ve çıktılarıyla artık uygulanabilirlik için geliştirilmiş bir durum sunduğunu göstermektedir.

Anahtar Sözcükler: Proje Cazibe Faktörleri, Veri Zarflama Analizi, NPV, IRR, MIRR, Yatırım Değerlendirmesi, Verimlilik

ACKNOWLEDGMENT

I extend my heartfelt gratitude to my supervisor, Asst. Prof. Dr. Sahand Daneshvar, for his invaluable guidance and support throughout every phase of this thesis. His introduction to the fascinating field of data envelopment analysis, which was both new and intriguing to me, has greatly enriched my perspective. I am especially thankful for his patience, encouragement, and insightful ideas, which motivated and propelled me forward at every step. This knowledge has significantly contributed to my growth as a student and an aspiring engineer.

I would also like to express my sincere thanks to the members of the graduation committee for their insightful advice and guidance. Their wisdom and constructive feedback have been instrumental in enhancing this study. It has been a privilege to learn from their expertise, which has positively influenced my academic journey within my department.

Furthermore, I am deeply appreciative of Hasan Ulas Altiok for providing the essential data and information that were crucial for completing my paper. Their patience and support have been invaluable, and I am sincerely grateful for their assistance.

Lastly, I owe my deepest gratitude to my family and friends for their unwavering support and encouragement throughout this journey. Their help and understanding, especially during challenging times, have been a source of immense strength and inspiration

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LIST OF ABBREVIATIONS

ADB	Asian Development Bank
ADD-BCC	Additive Bunker Model
ADD-FDH	Additive Free Disposal Hull
AIRR	Average Internal Rate of Return
BCC	Banker, Charnes, and Cooper
BCR	Benefit-Cost Ratio
CCR	Charnes, Cooper and Rhodes Model
CF	Cash Flow
CRS	Constant Return to Scale
DEA	Data Envelopment Analysis
DMU	Decision Making Units
EOCK	Economic Opportunity Cost of Capital
ER	Enterprise Rate
FIIs	Foreign Institutional Investors
FV(B)	Future Value of Benefits
IBFS	Infeasibility-Based Flow-Store
IRR	Internal Rate of Return
JICA	Japan International Cooperation Agency
LP	Linear Programming
MIRR	Modified Internal Rate of Return
MMUES	Metro Manila Urban Expressway System

MNPV	Modified Net Present Value
MOILP	Multi-Objective Integer Linear Programming
MOLP	Multi-Objective Linear Programming
NCF	Net Cash Flow
NPV	Net Present Value
PPAP	Photovoltaic Poverty Alleviation Project
PPP	Public-Private Partnerships
PV(C)	Present Value of Costs
ROI	Return on Investment
VAT	Value-Added Tax

Chapter 1

INTRODUCTION

Investment appraisal is a crucial process used by investors and government officials to make informed decisions about allocating resources to projects, programs, or portfolios. The primary goals of investment appraisal are to accept beneficial projects and reject detrimental ones, manage and mitigate risks, and ensure the stability of project components.

As outlined by Jenkins, Yan Kuo, and Harberger (2011), accurate forecasting requires understanding how past projects compare to future ones, the size of the sample (with larger being preferable), the similarity of past events to current ones, the consistency of past outcomes, the projected longevity of the project, and historical correlations.

Net Present Value (NPV), introduced by Irving Fisher in 1907, (Jones & Smith, 1982) s defined as the sum of all projected cash flows during a project's life, discounted to present value. Essentially, NPV represents the change in wealth resulting from the project.

Internal Rate of Return (IRR), also introduced by Fisher in 1907, (Jones & Smith, 1982) measures the rate at which the initial investment is recovered. If the discount rate is lower than the IRR, the investor is generating wealth from the project. Modified Internal Rate of Return (MIRR), proposed by Findlay and Messner (1973), (Cary &

Dunn, 1997) refines the IRR calculation to address issues related to investment profitability.

Additional methods for evaluating project attractiveness include the Benefit-Cost Ratio (BCR), which assesses the relative advantages and costs of a project in monetary or qualitative terms, and the payback period, which indicates how long it takes to recoup the original investment. The debt service ratio is another critical measure of a company's ability to meet its financial obligations. Despite their limitations, these measures are essential for financial management in evaluating a project's viability. (Jenkins, Yan Kuo, & Harberger, 2019)

Data Envelopment Analysis (DEA), introduced in 1978, is a powerful method for measuring the efficiency of decision-making units (DMUs) by analyzing multiple inputs and outputs. DEA has applications across various sectors, including industry, healthcare, education, and banking. (Emrouznejad & Podinovski, 2004)

The first DEA model, the Charnes-Cooper-Rhodes (CCR) model, calculates efficiency by maximizing each DMU's performance while ensuring that none exceeds a value of one. Although CCR is straightforward, other methods like Super Efficiency, Cross Efficiency, and Weight Evaluation provide more nuanced insights into DMU performance. Efficiency values derived from the CCR model can help investors identify the most profitable and cost-effective periods. (Wenli, Ying-Ming, & Shulong, 2017)

This study aims to use DEA to estimate target values for the inputs and outputs of projects that are initially unattractive for investment, and then re-evaluate their attractiveness based on these target values. The study will also identify the most efficient DMUs and analyze metrics such as Efficiency, Cross Efficiency, Lambda (a comparison score between inefficient and efficient DMUs), Slack (differences between actual and optimal values), and Weights (importance of each input and output). The goal is to define optimal conditions for the project.

CCR models are utilized in this paper due to their advantages, such as effective linear programming with multiple inputs and outputs and their nonparametric nature, meaning they do not require normal distribution assumptions. However, DEA has limitations, such as sensitivity to input and output data, which can lead to uncertain results if data is unclear or if the number of DMUs is insufficient (generally at least three times the sum of inputs and outputs). (Tone & Tsutsui, 2009)

This paper employs the random variability method to generate a synthetic dataset, facilitating the application of DEA to explore how the project could become an attractive investment opportunity. This method is widely used in simulations, modeling, and statistical analyses to explore hypothetical scenarios and assess potential outcomes when actual data is limited or unavailable. (Buldygin & Kozachenko, 2000)

The structure of this paper includes four sections: after the introduction, the literature review, which features four tables of relevant articles; the methodology section, detailing the models used for analysis and data collection; the results section, presenting DEA analysis (efficiency, weights, cross efficiency, lambda, targets) and revised NPV, IRR, and MIRR values; and the conclusion, summarizing the findings comprehensively.

Chapter 2

LITERATURE REVIEW

This literature review covers various aspects related to the topics addressed in this paper. The following tables summarize relevant articles on: (i) investment appraisal, (ii) Modified Internal Rate of Return (MIRR), Net Present Value (NPV), and Internal Rate of Return (IRR), (iii) Linear Programming (LP) and Data Envelopment Analysis (DEA), and (iv) DEA models in the investment sector.

2.1 Articles Related to Investment Appraisal

Investment appraisal plays a crucial role in financial decision-making, helping businesses and investors evaluate the profitability and risks of potential ventures. This process involves analyzing projects, assets, or financial opportunities using various techniques to assess their viability and expected returns.

A wealth of literature exists on investment appraisal, covering topics such as capital budgeting, risk assessment, discounting methods, and real-world case studies. These articles explore both traditional approaches—like **Net Present Value (NPV), Internal Rate of Return (IRR), and Payback Period**—and advanced models such as **Real Options Analysis and Monte Carlo simulations.**

By reviewing investment appraisal literature, investors, financial analysts, and decision-makers can gain valuable insights into best practices, emerging trends, and industry applications. This knowledge helps businesses strategically allocate resources to **maximize returns while minimizing risks**, ensuring more informed and effective investment decisions.

Xue (2015): The paper argues that while share prices can significantly impact a company's financial management, they can also lead to misleading or incorrect decisions. Some commonly used criteria for making investment decisions may result in errors. The objective of this study is to analyze various aspects of the capital market and its efficiency, using decision-making criteria such as the Internal Rate of Return (IRR) and Payback Period. The paper provides general suggestions for financial managers on capital budgeting and corporate goal setting. However, these problems can still be addressed through further studies. (Xue, 2015)

Vajarothai, Al-Jibouri, & Halman (2015): Project selection is a process that involves decision-making by both organizations and individuals. Evidence shows that most evaluation methods used for assessing projects are based on the principles of cost-benefit analysis, regardless of the type of appraisal. The paper proposes a conceptual methodology that combines various decision-making and evaluation techniques to enhance the project selection and evaluation process. This methodology can be applied to address the environmental, social, and economic aspects of a project. The proposed methodology is demonstrated through a project selection and evaluation process in Thailand, which serves as a case study. The paper explores how this new approach addresses the limitations of the current system to improve its effectiveness. (Vajarothai, Al-Jibouri, & Halman, 2015)

Sims, Powell, & Vidgen (2015): The transformation of tacit knowledge into a more easily replicable form of knowledge could threaten the competitive advantage it supports. This study aims to investigate the effects of e-learning projects on higher education. The results of the study revealed little evidence of a formal evaluation or appraisal of investments in projects characterized by uncertainty. (Sims, Powell, & Vidgen, 2015)

Throsby (2016): This paper provides an overview of the appraisal procedure for projects aiming to adaptively reuse urban heritage assets. It also explores the scarcity of data on these types of assets in developing countries. The paper presents two case studies demonstrating the use of ex-post investment appraisal techniques for urban heritage projects in Jordan and Macedonia. The cultural significance of urban heritage is widely acknowledged in cities and towns across the developing world. This significance is reflected in the physical forms of buildings and structures, as well as in intangible elements such as rituals and services related to urban life. (Throsby, 2016)

Higham, Fortune, & Boothman (2016): This study explores appraisal methods that can be used to determine the sustainability of social developments in the United Kingdom. Experts suggest that such tools can help builders identify the best value by enabling them to make informed decisions. Additionally, the study examines the key elements of feasibility-phase assessments. A survey was conducted among 481 professionals working in the built environment sector of the UK's social housing industry. The survey achieved a 24% response rate. This study serves as a valuable guide for builders by providing a variety of tools and models to evaluate and implement sustainable practices in their projects. It also considers the various factors that affect their utilization. (Higham, Fortune, & Boothman, 2016)

Khan, Hashmi, Hussain, & Ilyas (2017): The purpose of this study is to examine the factors influencing investment appraisal decisions in non-financial firms listed on the Pakistan Stock Exchange. They utilized fixed-effect and common-effect models, along with OLS regression, to analyze panel data for 60 non-listed firms on the Pakistan Stock Exchange from 2003 to 2015. The study found that various factors, such as growth, leverage, and inflation, positively affect investment appraisal. Conversely, municipal finance and GDP have significant negative impacts on the procedure. This study provides investors with a framework for evaluating these variables before investing. (KHAN, HASHMI, HUSSAIN, & ILYAS, 2017)

Laird & Venables (2017): A framework capable of capturing all relevant impacts is required to effectively conduct an appraisal. It should also enable the identification of opportunities for cost-effective resource allocation. The four types of impacts within the major transport investment framework are productivity and user benefits, land use and investment, employment, and proximity. Various transport-economy mechanisms apply to each of these categories. Although some of these mechanisms are already widely used, further research is necessary to enhance their effectiveness. The evidence base requires continuous improvement. (Laird & Venables, 2017)

Bakri Bakri (2019): The thesis investigates the capital budgeting procedures of Lebanese companies. It addresses four main research questions: how capital projects are appraised in the country, how risk is incorporated into the process, whether Lebanese firms experience capital rationing, and whether political risk significantly affects the process. This thesis explores various contentious issues through an interpretive approach. The results of the study were gathered through surveys and interviews. The research aimed to gain insights into the capital budgeting procedures of Lebanese companies, conducted via a survey of 151 individuals from different industrial sectors. The study revealed that, when assessing potential projects, Lebanese businesses prioritize people and experience over financial considerations. It also found that they employ various approaches when making investment decisions. The research showed that while the payback method is preferred by most companies, more sophisticated approaches are becoming increasingly common. (Bakri Bakri, 2019)

Pawlak & Zarzecki (2020): The objective of this paper was to analyze both the theoretical and practical aspects of investment valuation in European Union countries. This analysis aims to identify areas of research that can be explored in the future. The goal of this study was to analyze the literature on the effectiveness of investments in European Union countries and to identify which scientific sources are relevant to this topic. The findings will be used to develop a new study. It was surprising to learn that, despite companies in Europe being more capital-efficient, issues with investment appraisal persist. Advanced techniques are not widely used by firms in the region. Additionally, there is a lack of research on how to apply appraisal methods to assess investment opportunities within the EU and Switzerland. (Pawlak & Zarzecki, 2020)

Liu, Tapani, Kristoffersson, Rydergren, & Jonsson (2021): The cost-benefit analysis of cycling infrastructure projects is typically not as thoroughly developed as those for public transport and private cars. This is mainly due to the lack of well-defined transport models for the sector. The transport model used to assess cycling infrastructure in Stockholm, Sweden, takes into account changes in the infrastructure that affect the choice of routes, destinations, and trip generation. It also considers the cycling flow at the link level. The results of the study suggest that although the cycling flow on the links may increase, only a small portion of the increase comes from modal shifts. The minimal external effects of this change on emissions and car congestion are not significant. For the three scenarios studied, almost all of the benefits were derived from the existing cyclists. (Liu, Tapani, Kristoffersson, Rydergren, & Jonsson, 2021)

2.2 Articles Related to NPV, IRR and MIRR

Net Present Value (NPV), Internal Rate of Return (IRR), and Modified Internal Rate of Return (MIRR) are fundamental financial metrics in investment appraisal. These techniques help investors and decision-makers evaluate a project's profitability and feasibility by analyzing cash flows, discount rates, and overall returns.

Numerous articles have explored the **application, advantages, and limitations** of these methods in real-world investment decisions. **NPV calculates the present value of future cash flows, offering a clear measure of a project's contribution to wealth.** IRR, on the other hand, determines the discount rate at which a project breaks even, while MIRR improves upon IRR by **accounting for reinvestment rates and eliminating multiple rate issues.**

By examining literature on NPV, IRR, and MIRR, **financial analysts and business leaders can deepen their understanding of capital budgeting techniques and make more informed investment decisions.** These articles provide valuable insights into theoretical foundations, comparative analyses, and practical applications across various industries.

Leyman & Vanhoucke (2016): The goal of this paper is to analyze the project scheduling issue concerning different payment models and cash flows. This strategy aims to enhance a project's value by repositioning its activities. It integrates this process with other steps to develop a set of tasks that can be relocated. The approach was formulated using a genetic algorithm capable of addressing both resource and deadline feasibility, depending on the situation. The method's components were evaluated using various datasets. The project's net present value is shown and analyzed in relation to the data parameters. The results of this study can be utilized as the baseline for future benchmarks for the other six models. (Leyman & Vanhoucke, 2016)

Eagan, et al. (2017): This study aims to assess the validity of the Internal Rate of Return (IRR) metric, a standard measure used by researchers to evaluate the reliability of two processes. Among various IRR measures, Cohen's kappa is the most commonly used. This study involved a Monte Carlo simulation to analyze Cohen's kappa. The results revealed that the method used to evaluate IRR produces unacceptable error rates. (Eagan, et al., 2017)

Mellichamp (2017): This paper addresses the limitations of parameters such as the enterprise rate or project finance rate in assessing opportunity cost, and how various metrics and criteria interact with Net Present Value (NPV) to evaluate potential returns. The earlier proposal involved combining metrics for NPV and NPV%, addressing constraints associated with these metrics. The IRR projection can be carried out by taking into account the NPV% and the enterprise rate (ER) plane, which provides insight into the IRR's sensitivity to risk in the operating environment. (Mellichamp, 2017)

Marchioni & Magni (2018): This study presents a method for evaluating how value drivers affect output by measuring the coherence between return rates and NPV. The method introduces an NPV-consistent metric to ensure value creation aligns with NPV rankings. Additionally, a coefficient of cohesion is computed using Spearman and Iman correlation coefficients. In this paper, they analyze the performance of the Average Internal Rate of Return (AIRR) class using the sensitivity analysis method. They find that the Return on Investment (ROI) is consistent with NPV and other methods. (Marchioni & Magni, 2018)

Dhavale & Sarkis (2018): This paper explores a methodology for analyzing cash flows generated by the carbon trading and emissions credit markets, focusing on the reduction of greenhouse gas emissions and the allocation of carbon allowances. This method employs a Bayesian model to calculate IRR. The study found that the IRR implied by the carbon credit market depends on the volatility of cash flows and the variability of their distribution. Ignoring these elements can lead to inaccurate outcomes. The findings contrast with those of the deterministic model, which suggests that variability in cash flow distributions can have either a positive or negative impact on the implied return from greenhouse gas emission reduction assets. (Dhavale & Sarkis, 2018)

Gaspars-Wieloch (2019): This paper explores methods for adjusting net present value under uncertainty, emphasizing the need to consider factors that affect unpredictable cash flows. This innovative approach combines decision rules from Bayes and

Hurwicz with sensitivity analysis, taking into account the decision-maker's attitude in addressing the problem at hand. Net Present Value can be adjusted to support decisions under economic uncertainty. (Gaspars-Wieloch, 2019)

Badhani & Kumar (2020): This study evaluates the market timing skills of foreign institutional investors (FIIs) in India. The method involves constructing a simulated portfolio that mimics the trading behavior of various FIIs. The performance of the simulated portfolio is compared with two benchmark portfolios. Based on the bootstrapped internal rate of return analysis, the results show that the FII portfolio underperforms compared to the benchmark portfolios, while the portfolios based on the investment strategies of other investors and domestic institutional investors (DIIs) also perform poorly. (Badhani & Kumar, 2020)

Singh, Anand, Shukla, & Sharma (2020): This paper investigates the environmental and techno-economic feasibility of using solar water heaters across various sectors in India, utilizing the RETScreen Expert software. This project evaluates the performance of solar water heating systems at three locations in India: Ludhiana, Amethi, and Leh. The study aims to develop case studies for commercial, industrial, and residential sectors. The three different commercial models exhibited average Solar Factor (SF) values of 74.64, 64.64, and 13. The project's overall greenhouse gas emissions have decreased by almost half over the last two decades. The financial viability of these solar power plants was evaluated using various system parameters. (Singh, Anand, Shukla, & Sharma, 2020)

Coppieters, Fricker, & Blondeau (2022): The goal of this study is to analyze the economic viability of passive and active condensation systems in a biomass [energy system or plant]. This study assesses the economic feasibility of a project by analyzing factors such as net present value, discount rate, and modified internal rate of return. An active plant can achieve a 66% higher initial investment than a passive one. It can also reduce its energy consumption by more than 50%. On the other hand, the difference between an active and a passive plant is due to how variable its return temperature is. For instance, the former's NPV can be overestimated by more than 150% if the annual heat demand is constant. (Coppieters, Fricker, & Blondeau, 2022)

Shou (2022): This article provides a comprehensive overview of the Net Present Value (NPV) method, including its advantages and disadvantages, to help investors choose the most suitable approach for their project management needs. This article compares various return methods, including the Internal Rate of Return (IRR). It aims to provide an extensive analysis of the NPV valuation method, which is commonly used by investors when conducting project analysis. It enables them to quickly determine and allocate their cash flows. Furthermore, the MIRR and IRR methods are introduced to provide investors with a better understanding of these returns. (Shou, 2022)

2.3 Articles Related to DEA and LP

Data Envelopment Analysis (DEA) and Linear Programming (LP) are widely used quantitative methods in operations research and decision-making. DEA is a non-parametric technique for evaluating the efficiency of Decision-Making Units (DMUs) based on multiple inputs and outputs, while LP is an optimization method used to maximize or minimize an objective function under given constraints.

Numerous articles have explored the **theoretical foundations, applications, and advancements** of DEA and LP. **DEA is commonly applied to performance**

evaluation in industries such as banking, healthcare, education, and energy. LP, on the other hand, is utilized in resource allocation, production planning, and financial optimization. Many studies also integrate DEA with LP to **enhance efficiency assessments and optimization processes.**

By reviewing articles on DEA and LP, **researchers, analysts, and decision-makers can gain deeper insights into these methodologies, their real-world applications, and ongoing advancements.** These articles contribute to the continuous improvement of efficiency analysis and optimization strategies across various sectors.

Keshavarz & Toloo (2015): There are two types of efficient solutions in multi-core integer linear programming problems: supported and unsupported. Although many researchers attempt to evaluate the efficiency of a given solution, they often do not focus on identifying the status of the solution's efficiency. After establishing the relationship between Multi-Objective Integer Linear Programming (MOILP) and DEA, the paper presents two practical procedures. The first procedure evaluates the efficiency of an arbitrary solution, while the second assesses the efficiency of an optimal solution. The paper also highlights the disadvantages of Chen and Lu's methodology. This concept is based on additive models known as the Additive Free Disposal Hull (ADD-FDH) and the Additive Banker Model (ADD-BCC). Two examples demonstrate the practical implications of this approach. (Keshavarz & Toloo, 2015)

Toloo, Masoumzadeh, & Barat (2015): This paper introduces a closed-form Infeasibility-Based Flow-Store (IBFS) method for DEA models. It eliminates the need for artificial variables and applies the proposed solution to a real-world dataset to demonstrate its practicality. The closed-form IBFS method can reduce the total number of computations by about 50%. (Toloo, Masoumzadeh, & Barat, Finding an Initial Basic Feasible Solution for DEA Models with an Application on Bank Industry, 2015)

Wu, Zhu, An, Chu, & Ji (2016): The paper presents a mechanism for allocating resources among Decision-Making Units (DMUs) within an organization, managed centrally. Data analysis and programming techniques are used to determine the effects of resource allocation and consumption. The concept of Multi-Objective Linear Programming (MOLP) is applied to optimize DMUs while reducing consumption. Factors affecting the growth rates of DMUs, such as equipment type and resource allocation, are also considered. The framework provides a method for resource allocation in decision-making environments where a central unit controls resources across multiple DMUs. (Wu, Zhu, An, Chu, & Ji, 2016)

Cavone, Dotoli, Epicoco, & Seatzu (2017): The goal is to develop a self-learning procedure to manage the effects of disturbances on real-time train timetables, applicable to mixed-track network scheduling. The first two steps are performed in real time: finding a suitable schedule for a given time horizon and merging various elements of the schedule. The third step involves identifying potential conflicts. The effectiveness of the rescheduling process is then evaluated using an offline method. The paper develops a procedure to help train service providers manage delays in real-time timetables. The method enhances efficiency and effectiveness in rescheduling.

The study, performed on real data, demonstrates its effectiveness. (Cavone, Dotoli, Epicoco, & Seatzu, 2017)

Haldar, et al. (2017): This paper proposes a framework to help organizations select the most appropriate third-party logistics (3PL) service provider based on vendor performance indicators. The project provides a comprehensive analysis of the variables affecting logistics provider selection. Data analysis is performed to find the ideal solution and rank vendors by efficiency. The proposed model for allocating 3PL quantities was rigorously tested, analyzing the factors influencing firm selection. The study evaluates the performance of 26 logistics providers using DEA, LP, and TOPSIS algorithms. It was found that Vendor V4 performed better than the others. (Haldar, et al., 2017)

Jianfeng, Linan, & Lizhi (2017): The paper presents DEA models for a two-stage system with three sub-DMUs and an additional input to the second stage. The proposed models consider the system's internal structure and use multiplicative and synthetically additive DEA techniques to estimate and decompose efficiency. A heuristic procedure is adopted to convert nonlinear programs into linear ones. The proposed models offer a deeper understanding of the system's efficiency by accounting for various inefficiency mechanisms. (Jianfeng, Linan, & Lizhi, 2017)

Mehdiloozad (2017): This study revisits a linear programming model proposed by Mirdehghan, Mehdiloozad, Sahoo, and Roshdi (2015) to identify the global reference set of a decision-making unit. The paper presents a revised version of this model. The LP relaxation method is used to transform the formulated models into equivalent LP models. The results indicate that the new model is comparable to the LP model presented by Mehdiloozad et al. (2015), maintaining similar performance. (Mehdiloozad, 2017)

Førsund (2018): This paper provides a basic overview of efficiency analysis. DEA is used by researchers who may not be familiar with efficiency analysis. The significance of shadow prices is acknowledged. (Førsund, 2018)

Lin & Li (2020): The paper proposes two super-efficient models to determine the efficiency of funds in the financial market. These models handle negative values in various risk and return calculations. Methods for calculating super-efficient scores for various types of mutual funds are proposed. The paper provides a comprehensive analysis of the practical aspects of these techniques, including their implementation in linear programming. The proposed super-efficient models can distinguish between efficient and inefficient funds, providing valuable insights for portfolio selection. (Lin & Li, 2020)

Lee (2021): The paper presents a linear programming model that integrates Super-SBM and SBM. The efficiency of inefficient DMUs can be differentiated from that of super-efficient DMUs at a later stage. The proposed model was compared with Tran et al.'s model and found to be faster, with a computation time approximately two-thirds of the latter. (Lee, 2021)

2.4 Articles Related to DEA and Investment

Data Envelopment Analysis (DEA) is a powerful tool for evaluating the efficiency of decision-making units (DMUs) across various sectors, including investment and financial decision-making. DEA helps investors, portfolio managers, and policymakers assess the performance of investment portfolios, mutual funds, stock markets, and financial institutions **by comparing multiple inputs (such as capital, risk, and expenses) with outputs (such as returns, profitability, and market value).**

Numerous articles have explored **DEA's application in investment analysis, particularly in efficiency measurement, risk-adjusted performance evaluation, and benchmarking.** Researchers have applied DEA to assess the performance of hedge funds, pension funds, real estate investments, and corporate finance decisions. Some studies integrate DEA with other techniques, such as stochastic frontier analysis, machine learning, and financial modeling, to enhance decision-making accuracy.

By reviewing **literature on DEA in investment,** financial analysts and researchers can gain valuable insights into how efficiency-based methods contribute to **optimizing investment strategies, improving resource allocation, and identifying top-performing assets in competitive markets.**

Tavana, Keramatpour, Santos-Arteaga, & Ghorbaniane (2015): The proposed method aims to select an optimal mix of projects, considering various organizational goals to achieve the final selection. The model is structured into three phases, each incorporating different procedural aspects. One phase involves a screening method using Data Envelopment Analysis (DEA), while the other two phases use methods for identifying the optimal solution through similarity and preference ordering. A case study is conducted to demonstrate the method's effectiveness. The method ensures that projects are selected based on the best alignment between their initial rankings and the final selection. (Tavana, Keramatpour, Santos-Arteaga, & Ghorbaniane, 2015)

Zhong & Zhao (2016): The DEA method is applied to develop an optimal model that accounts for the relationship between proven reserves, production costs, and investments. The DEA process is used to evaluate data and determine the appropriate funding required for production. Exponential and correlation models analyze expenditures on exploration and assess their correlation with proven reserves. The objective is to maximize the asset's present value while enhancing production and investment returns. A case study demonstrates that the optimal model, with its multi-stage structure, can be effectively implemented. It provides a scientific foundation for developing long-term strategies and investment plans, facilitating more organized and rational resource development in oil companies. (Zhong & Zhao, 2016)

Karasakal & Aker (2017): The study evaluates R&D projects using DEA and applies various criteria for assessment. Weight intervals are generated through the interval analytic process to develop threshold and assignment estimation models. The UTADIS Sorting method is applied in a case study to evaluate the performance of a grant program managed by an agency in 2009, resulting in a guide for project success and allocation. The developed methods demonstrate greater stability compared to UTADIS. (Karasakal & Aker, 2017)

Wu, Ke, Zhang, Liu, & Wang (2018): The paper addresses the Photovoltaic Poverty Alleviation Project (PPAP), which provides clean electricity to impoverished households. Current assessment tools fail to account for the social and economic benefits and do not offer recommendations for site selection. A three-phase approach is proposed to analyze project performance and identify influencing factors. This approach includes screening out outliers, minimizing the likelihood of errors, and implementing DEA to improve model accuracy. Tobit regression analysis is conducted to verify environmental conditions. The study finds that the performance of PPAPs in China is generally low due to excessive production scales and inadequate site selection. (Wu, Ke, Zhang, Liu, & Wang, 2018)

Fiala (2018): This paper presents a new approach for designing a project portfolio that integrates the De Novo optimization method with the DEA model. The concept aims to create an optimal project portfolio that balances cost-effectiveness and efficiency. The portfolio's performance is assessed based on its efficiency and output. Portfolio analysis experiments reveal that the proposed technique significantly reduces project completion time and aids in developing information technology strategies. It also helps identify ideal partners for such projects. (Fiala, 2018)

Toloo & Mirbolouki (2019): The paper introduces a project selection method that evaluates overall project performance rather than focusing solely on individual proposal performance. A comprehensive dataset of 21 proposals for an information system in Iran's e-commerce development project demonstrates the potential of the proposed method. The paper introduces a DEA method that considers both the inputs and outputs of submitted proposals. The results indicate that this approach leads to reduced time and enhanced management capabilities. (Toloo & Mirbolouki, 2019)

Ahbab, Daneshvar, & Çelik (2019): The study analyzes the factors influencing cost and time management in large road projects. A data analysis tool known as DEA is used to configure decision-making units for projects. This non-parametric mathematical method measures relative managerial effectiveness and efficiency, with results calculated and ranked by project importance. The study finds that design changes, additional work, and inaccurate project scope significantly impact time and cost management. (AHBAB, DANESHVAR, & ÇELIK, 2019)

Jiang, Lee, & Rah (2020): The paper examines the research effectiveness of Chinese universities by categorizing them into 'Project 211,' 'Project 985,' and 'general universities.' The DEA method, using EnPAS software, is applied to analyze the research efficiency of various projects, including 'Project 211' and 'Project 985.' The research reveals that 'Project 211' universities have lower research efficiency compared to 'Project 985' and general universities. Additionally, research output varies based on the type of institution and its region in China. (Jiang, Lee, & Rah, 2020)

Peykani, Mohammadi, & Emrouznejad (2021): The study presents a novel method for ranking and appraising DMUs with a two-stage structure, addressing the imprecise nature of the data. A new framework is developed to analyze data leakage in fuzzy networks using a two-stage DEA model. The framework addresses model issues, calculates efficiency scores, and plots results to calculate the volume under the efficiency score surface. An illustrative case study demonstrates that the proposed method is highly useful and effective in its application. (Peykani, Mohammadi, & Emrouznejad, 2021)

Ershadi, Taghizadeh, & Molana (2021): The paper establishes a set of performance criteria and indicators for evaluating general ledger software projects. General ledger projects are selected using a DEA approach, followed by an inference procedure to assess their success. Inputs and outputs from two projects are evaluated using the ANFIS model's statistical capabilities, including measures such as absolute error, root mean square error, and Nash-Sutcliffe efficiency. The evaluation of the indicators from the initial two categories demonstrates that the model accurately predicts the success of the general ledger software projects and provides a proper estimate of the observed values. The model's TRL component is crucial for assessing project performance. (Ershadi, Taghizadeh, & Molana, 2021)

Chapter 3

METHODOLOGY

This chapter outlines the **methodological framework** used to evaluate investment efficiency through Data Envelopment Analysis (DEA) and investment-related equations. The primary objective is to assess underperforming or failed investment projects and determine whether they **merit reconsideration** based on efficiency analysis and financial performance metrics.

DEA, a non-parametric technique, measures the relative efficiency of investment projects by comparing multiple inputs with outputs. This approach helps identify inefficient projects and provides insights into potential improvements. Additionally, financial equations—such as Net Present Value (NPV), Internal Rate of Return (IRR), and Modified Internal Rate of Return (MIRR)—are integrated into the analysis to assess the financial feasibility of reviving failed projects.

By combining DEA with investment appraisal techniques, this methodology aims to create a **decision-support model** that assists investors and decision-makers in determining whether a failed project can be **restructured, optimized, and reintroduced** with a higher probability of success. The following sections detail the DEA model formulation, selection of investment variables, data collection process, and computational procedures used to evaluate project viability.

3.1 Equations

3.1.1 DEA Standard Models

The efficiency of a **Decision-Making Unit (DMU)** can be assessed using the **CCR model**, introduced by **Charnes, Cooper, and Rhodes in 1978**. This model operates under the assumption of **constant returns to scale (CRS)**, meaning that if all inputs are multiplied by a factor of *k*, the outputs will also increase proportionally by *k*. This assumption is appropriate when DMUs are believed to be operating at an **optimal scale**, where efficiency remains unaffected by size.

The **CCR model** provides a clear and straightforward efficiency score, making it a valuable tool for **benchmarking** without the added complexity of scale efficiency adjustments. It is particularly useful for **initial DEA analyses**, as it identifies **technically efficient DMUs** before addressing potential scale-related inefficiencies. The efficiency scores derived from this model reflect both **technical efficiency and scale efficiency combined**.

Since this study aims to obtain a **comprehensive efficiency measure** without distinguishing between technical and scale inefficiencies, the **CCR model** is the most suitable choice. Based on **Farrell's efficiency measure**, this approach enables the **simultaneous evaluation of multiple inputs and outputs** through **mathematical programming**. Commonly referred to as the **Constant Returns to Scale (CRS) model** in **Data Envelopment Analysis (DEA)**, it is mathematically defined as follows:

$$Eff = \max_{U_r, V_i} \frac{\sum_r U_r y_{r0}}{\sum_i V_i x_{i0}} \tag{1}$$

s.t.

$$\frac{\sum_r U_r y_{rk}}{\sum_i V_i x_{ik}} \leq 1 \quad k = 1, \ldots, n \tag{2}$$

$$U_r, V_i \geq 0 \quad r = 1, \ldots, l \quad i = 1, \ldots, m \tag{3}$$

y_{r0}= amount of output r from DMU_0

U_r=the weight given to output r

x_{r0}= amount of input r from DMU_0

V_r= the weight given to input r.

y_{rk}: Output r for DMU k.

x_{ik}: Input i for DMU k.

where m and l are the numbers of inputs and outputs, respectively. This fractional linear model can be adapted into a linear programming (LP) model with either an input or output orientation. The input-oriented model focuses on reducing the proportion of inputs utilized while ensuring that the output level for each DMU remains unchanged. In contrast, the output-oriented model aims to maximize output production while maintaining the same input level. The input-oriented and output-oriented models for a given DMU, represented as DMUo where o=1,…,n, is expressed as follows:

For input-oriented model:

$$Eff = \min_{u_r, v_i} \sum_i V_i x_{ij0} \tag{4}$$

s.t.

$$\sum_r U_r y_{rj} - \sum_i V_i x_{ij} \leq 0 \; ; \forall j \tag{5}$$

$$\sum_r U_r y_{rj0} = 1 \tag{6}$$

$$U_r, V_i \geq 0 \; ; \forall r, \forall i. \tag{7}$$

x_{ij0}: Input I of the evaluated $DMUj_0$.

V_i: Weight assigned to input x_i.

y_{rj}: Output r for DMUj.

x_{ij}: Input i for DMU j

U_r: Weight assigned to output y_r.

y_{rj0}: Output r of the evaluated DMU j_0

For output-oriented model:

$$Eff = \max_{u_r, v_i} \sum_r U_r y_{rj0} \tag{8}$$

s.t.

$$\sum_r U_r y_{rj} - \sum_i V_i x_{ij} \leq 0 \; ; \forall j \tag{9}$$

$$\sum_i V_i x_{ij0} = 1 \tag{10}$$

$$U_r \, , V_i \geq 0 \;\; ; \forall r \, , \forall i. \tag{11}$$

y_{rj0}: Output r of the evaluated DMU j_0.

U_r: Weight assigned to output y_r.

y_{rj}: Output r for DMU j.

x_{ij}: Input i for DMU j.

x_{ij0}: Input i of the evaluated DMU j_0.

V_i: Weight assigned to input x_i.

These two models have pairs, leading to efficiency calculations while considering the **Production Possibility Set (PPS).** Assume that DMUs {(xj, yj) j=1,…, n} are defined in the same way as before. **the production possibility set follows five assumptions:**

assumption 1: Non-emptiness. If (xj, yj)∈**P** (∀ j=1,...,n) then **P** is non empty.

assumption 2: Constant Returns to Scale (CRS). If (xj, yj) ∈**P** then for any non-negative scalar α ≥ 0 (∝xj, ∝yj) ∈**P.**

assumption 3: Strong Disposability.

a) If (xj, yj) ∈**P** and xj1 ≥ xj then (xj1, yj) ∈**P** (Input Disposability).

b) If (xj, yj) ∈**P** and yj1 ≤ yj then (xj, yj1) ∈**P** (Output Disposability).

assumption 4: Convexity. P is a closed and convex set.

assumption 5: Minimum extrapolation. P is the smallest intersection of all production sets satisfying assumptions 1 to 4 and containing all observedDMUs.

If all these assumptions hold for **P** then **P** can be identified as:

P= {(x_{j0}, y_{j0}) s.t. $\sum_j \lambda_j x_j \leq x_{j0}$ and $\sum_j \lambda_j y_j \geq y_{j0}$, $\lambda_j \geq 0$; j=1, ..., n}.

The vector $\lambda = (\lambda 1, \lambda 2, \ldots, \lambda n) \in R^{+n}$ allows the minimization or maximization of a **single** DMU while ensuring a **practical and unbiased** evaluation.

Using the **Production Possibility Set (PPS)**, Farrell's **technical efficiency** and Shephard's **distance function** introduce a new **linear programming (LP) model** to measure the efficiency of DMU_{j0} known as the **Output-Oriented CRS Envelopment Model**:

$$\max_{\lambda, h, s_i^-, s_r^+} h \tag{12}$$

s.t.

$$\sum \lambda_j x_{ij} + S_i^- = x_{ij0} \quad \forall i \tag{13}$$

$$\sum \lambda_j y_{rj} - S_r^+ = h y_{rj0} \quad \forall r \tag{14}$$

$$S_i^+, S_r^- \geq 0 \quad \forall i, \forall r \tag{15}$$

$$\lambda_j \geq 0 \ \forall j. \tag{16}$$

h: Scalar that represents the proportional increase in outputs for the evaluated DMU.

$\mathbf{x_{ij}}$: Input i for DMU j.

$\boldsymbol{\lambda_j}$: Intensity variable (weight) for each DMU in the reference set.

$\mathbf{S_i^-}$: Input **slack** (excess input usage beyond the efficient frontier).

$\mathbf{x_{ij0}}$: Input i of the evaluated DMU j_0.

$\mathbf{y_{rj}}$: Output r for DMU j.

$\mathbf{S_r^+}$: Output **slack** (shortfall in output compared to the efficient frontier).

$\mathbf{y_{rj0}}$: Output r of the evaluated DMU j_0.

The model above is the other pair for output-oriented model that is mentioned before. A DMU_{j0} is **Pareto efficient** if two conditions are **fulfilled;** The optimal efficiency score h (h^*) is equal one and The **slack variables** Si^- & S_r^+ are **zero** for all i and r. This indicates that $DMUj_0$ is the most **efficient** unit, producing the **maximum output with minimum input** compared to other DMUs or mixed units of DMUs.

$S_{i,}$ S_r represent slack. In other words, if S_i^{-*} are not zero it suggests **inefficiency in input usage**., where if S_r^{+*} are not zero it suggests **inefficiency in output production**. The model mentioned can increase the output of DMU_{j0} with taking consider of CRS-PPS. In order to have n number of optimal sets of values (h^*, λ^*) the single DMU that is being evaluated must be calculated n times from the model. The efficiency expressed as $1/h_{j0}^*$ for the DMU_{j0}. if h_{j0}^* is larger than one then radial expansion can be done while if h_{j0}^* equal one there is no radial expansion.

If λ is bigger than zero this value is considered as the set that are dominating in DMUs which is on the constructed production frontier where DMU_{j0} is measured. The DMUs that have positive values are match to the $DMU_{j0.}$ the equation represents the **input-oriented envelopment form of the CCR DEA model**, where the goal is to **minimize inputs while keeping outputs unchanged**. By applying the efficiency frontier and the input-oriented CRS envelopment model outlined in equations (17) to (21), the slack values and lambda are determined as follows:

$$\min_{\lambda,h,s_i^-,s_r^+} \phi \tag{17}$$

s.t.

$\sum_j \lambda_j x_{ij} + S_i^{-} = \phi x_{i0} \quad i = 1, .., m$ (18)

$\sum_j \lambda_j y_{rj} - S_r^{+} = y_{r0} \;\; r = 1, \ldots, l$ (19)

$S_i^{+}, S_r^{-} \geq 0 \; r = 1, \ldots, l, i = 1, \ldots, m$ (20)

$\lambda_j \geq 0 \;\; \forall j \; j = 1, \ldots, l$ (21)

ϕ: Input contraction factor (efficiency score), where $0 < \phi \leq 1$.

x_{ij}: Input i for DMU j.

λ_j: Intensity variable (weight) for each DMU in the reference set.

S_i^{-}: Input **slack** (excess input usage beyond the efficient frontier).

x_{i0}: Input i of the evaluated DMU 0.

y_{rj}: Output r for DMU j.

S_r^{+}: Output slack (shortfall in output compared to the efficient frontier).

y_{r0}: Output r of the evaluated DMU 0.

In this context, ϕ^* denotes the optimal value of ϕ. Slack variables S_i^{-} and S_r^{+} reflect inefficiencies when they are non-zero, while $\lambda = (\lambda 1, \lambda 2\ldots, \lambda n) \in R_n^{+}$ is used to adjust the DMU to achieve efficiency. A DMU is considered Pareto efficient if the optimal (ϕ^*) equals one and all slack variables are zero.

Same as h^*, consider ϕ^* is optimum value of ϕ. The DMU_{j0} is Pareto efficient if two conditions are fellfield; first the optimal value of ϕ (ϕ^*) is equal one and S_i^{+} & S_r^{-} equal zero for all i and r. But the efficiency rate for the DMU_{j0} is ϕ^*.

DEA can be used to determine the values in which the DMU can reach its optimum wither it was inputs or outputs value these values can be found by:

1) Productive unit vector

Assuming λ^* is vector of optimal variable values the optimal values can be found by these equations:

$$\sum_j x^*_{ij0} = x_{ij}\ \lambda^* \tag{22}$$

$$\sum_j y^*_{rj0} = y_{rj}\ \lambda^* \tag{23}$$

x^*_{ij0}: Projected (efficient) input value for DMU j_0.

x_{ij}: Actual input value for DMU j.

λ^*: Scaling factor representing the intensity of the reference DMUs.

y^*_{rj0}: Projected (efficient) output value for DMU j_0.

y_{rj}: Actual output value for DMU j.

2) the efficiency rate and values of additional variables s– and s+

since h^* are efficiency rate for the input-oriented model and ϕ^* are efficiency rate for the output-oriented model, the optimal value can be found by:

a) input-oriented CCR model:

$$\sum_j x^*_{ij0} = x_{ij0}\, h^* - s_i^- \tag{24}$$

$$\sum_j y^*_{rj0} = y_{rj0} + s_r^+ \tag{25}$$

x^*_{ij0}: Projected (efficient) input for DMU j_0.

x_{ij} : Actual input for DMU j_0.

h^*: Scaling factor (input contraction ratio, $0 < h^* \leq 1$)

S_i^-: Input slack (excess input usage that should be eliminated).

y^*_{rj0}: Projected (efficient) output for DMU j_0.

y_{rj0} : Actual output for DMU j_0.

S_r^+: Output slack (shortfall in output that needs to be increased).

b) output-oriented CCR model:

$$\sum_j x^*_{ij0} = x_{ij0} - s_i^- \tag{26}$$

$$\sum_j y^*_{rj0} = y_{rj0}\,\phi^* + s_r^+. \tag{27}$$

x^*_{ij0}: Projected (efficient) input for DMU j_0.

x_{ij} : Actual input for DMU j_0.

S_i^-: Input slack (excess input usage that should be eliminated).

y^*_{rj0}: Projected (efficient) output for DMU j_0.

y_{rj0} : Actual output for DMU j_0.

ϕ^*: Scaling factor for outputs (output expansion ratio, $0 < \phi^* \leq 1$)

S_r^+: Output slack (shortfall in output that needs to be increased). (Kočišová, 2004).

Cross-efficiency is an approach where the efficiency of each DMU is evaluated using the optimal weights derived from other DMUs. This process is represented by the following equation:

$$Eff_{kj} = \frac{\sum_r u^*_{rk} y_{rj}}{\sum_i v^*_{ik} x_{ij}} \quad \forall j \; \{j = 1, \dots, n\} \tag{28}$$

Eff_{kj}: Efficiency score of DMU j when evaluated using the optimal weights of DMU k

u^*_{rk}: Optimal weight (multiplier) for output r in the evaluation of DMU k.

y_{rj}: Observed output r of DMU j.

v^*_{ik}: Optimal weight (multiplier) for input i in the evaluation of DMU k.

x_{ij}: Observed input i of DMU j.

Here u^*_{rk} and v^*_{ik} denote the optimal weights for DMU k. In the cross-efficiency matrix, DMU k corresponds to the columns, while DMU j represents the rows. (Zhu, 2014)

3.1.2 Investment Models

The Net Present Value (NPV) of a project is the sum of all expected cash inflows and outflows over the project's life, discounted to present value. Arshad (2012) defined NPV as the present value of all future cash flows, calculated by discounting both inflows and outflows.

In project management, NPV determines a project's worth by considering expected financial returns and associated risks, as noted by Archer et al. (1999):

$$NPV = -CF_0 + \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \cdots + \frac{CF_n}{(1+i)^n} \quad (29)$$

NPV (Net Present Value): The sum of the present values of all cash inflows and outflows.

CF_0 (Initial Cash Outflow): This is the **initial investment cost**, usually negative since it represents money spent at time **t=0**.

CF_t (Cash Flow at time t): Represents the **net cash inflow or outflow** at a given period t (year, month, etc.).

i (Discount Rate or Interest Rate): The rate at which future cash flows are discounted to account for the **time value of money**.

n (Total Number of Periods): The total number of years (or time periods) over which the investment generates returns.

Where CF(t) refers to the cash flow during period t, and i is the discount rate, a cash flow or net cash flow (NCF) are the profits from a certain period which means the revenues minus the cost of that period. A positive NPV indicates profitability, while a negative NPV suggests potential financial losses. A zero NPV requires further interpretation.

Tomaevi and Mackevi (2010) highlighted that investors might perceive projects differently due to minimal effects or market conditions. Therefore, ranking projects by profitability index and completing critical tasks before budget exhaustion is essential.

By identifying C_0 as initial investment and using Summation the NPV can be expressed as below:

$$\mathrm{NPV} = -\mathrm{I}_0 + \sum_1^n \frac{\mathrm{C_i}}{(1+\mathrm{r})^{\mathrm{i}}} \tag{30}$$

NPV (Net Present Value): The total present value of all cash inflows and outflows.

I0 (Initial Investment Cost): Represents the **initial capital outlay** or **cash outflow** at time t=0.

C_i (Cash Flow in Period i): Represents the **net cash inflow (or outflow)** for each period i.

r (Discount Rate or Required Rate of Return): The rate used to discount future cash flows to their present value.

n (Total Number of Periods): The total duration (years, months, etc.) over which cash flows are considered.

Where I_0 is the initial investment, C_i is the cash flow from periods i=1 to n, r is the interest rate, and n is the project life. (Osborne, 2010)

The Internal Rate of Return (IRR) is the discount rate at which NPV equals zero. If the IRR exceeds the discount rate, the project is not attractive; if it is less, the project is attractive. The IRR is found by solving:

$$\sum_{i=0}^{n} \frac{NCF_i}{(1+IRR)^i} = 0 \tag{31}$$

NCF_i (Net Cash Flow at time i): Represents the **net inflow or outflow of cash** at each time period i.

IRR (Internal Rate of Return): The discount rate that makes the sum of discounted cash flows **equal to zero**.

i (Time Period Index): Represents the time period (e.g., years, months, etc.).

n (Total Number of Periods): The total duration over which the investment generates cash flows.

This aligns with the concept of zero NPV, indicating a return on invested capital while recovering the initial investment. The IRR and NPV are interrelated, with IRR used by investors to assess a project's attractiveness.

However, IRR has limitations, such as its handling different project sizes, durations, and irregular cash flows. The Modified Internal Rate of Return (MIRR) addresses these issues by discounting negative cash flows at the firm's financing cost or the Economic Opportunity Cost of Capital (EOCK) and compounding positive cash flows at the same rate. The MIRR is calculated as:

$$MIRR = \left(\frac{FV(B_n)}{PV\ (C_n)} \right)^{\frac{1}{n}} - 1 \qquad (32)$$

MIRR (Modified Internal Rate of Return): The adjusted rate of return considering reinvestment and financing costs.

FV(B_n) (Future Value of Benefits): The future value of all **positive cash flows (inflows)**, compounded at the **reinvestment rate** until the final period n.

PV(C_n) (Present Value of Costs): The present value of all **negative cash flows (outflows)**, discounted at the **financing cost (or borrowing rate)** to time t=0.

n (Number of Periods): The total number of years (or periods) over which the investment takes place.

where $FV(B_n)$ is the future value of benefits, and $PV(C_n)$ is the present value of costs. MIRR helps determine the optimal scale of a project, assuming each investment increment has a unique IRR.

The relationship between the three components of this calculation—the IRR, the NPV, and the MIRR—is shown in Figure 1.

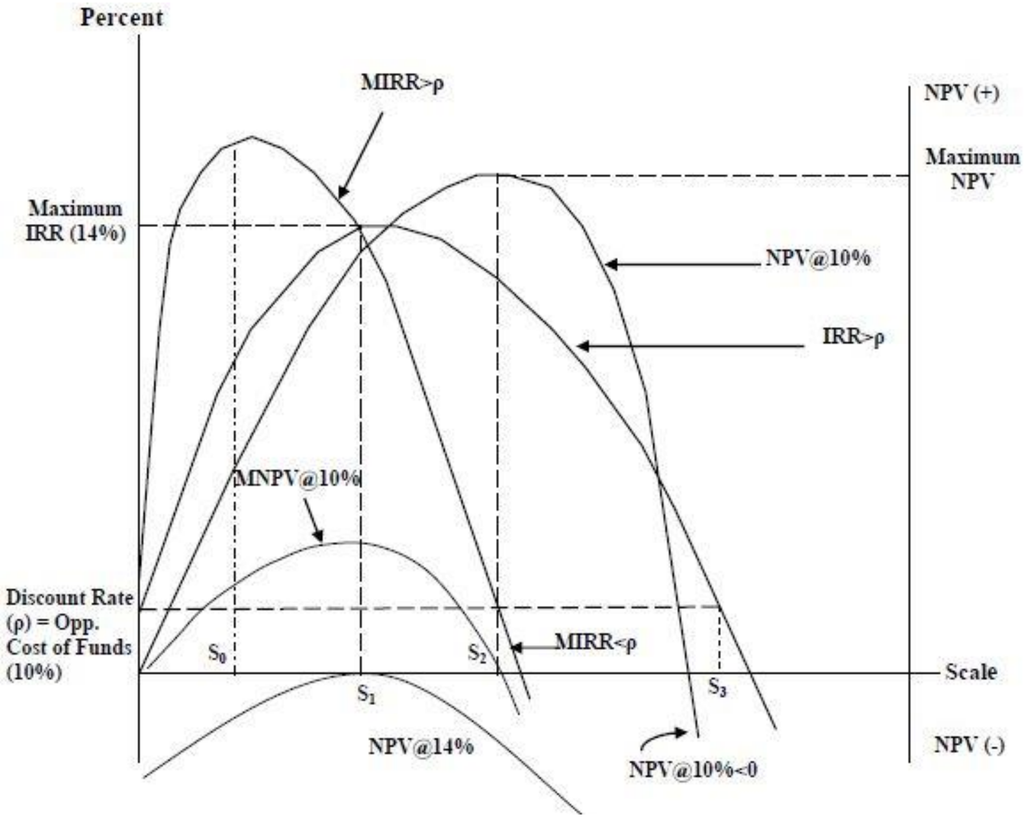

Figure 1: Relation between NPV, IRR and MIRR in Case of 10% and 14% Discount Rate[1]

Figure 1 illustrates the relationship between NPV, MNPV, IRR, and MIRR. **MNPV** typically stands for **Modified Net Present Value**. It is a variation of the standard **Net Present Value (NPV)** method used to evaluate investment projects. MNPV adjusts for certain factors like changing discount rates, reinvestment assumptions, or project-specific risks to provide a more accurate valuation. MIRR initially increases with project scale but eventually decreases, while IRR exhibit the same behavior. The relationship between IRR and MIRR is equal at the optimal scale (the maximum IRR).

In the given **NPV profile graph**, the points **S_0, S_1, S_2, and S_3** represent different **scale or size levels of investment**, where the project is evaluated at varying discount rates. The IRR curve peaks at a scale (S1), where IRR reaches its maximum value (14% in this case). The MIRR curve intersects with the IRR curve at S1, indicating that MIRR equals IRR at this scale.

NPVs are calculated at two different discount rates: 10% and 14%. At a discount rate of 10%, the maximum NPV occurs at a larger scale (S2), where MIRR equals the discount rate (10%). At a discount rate of 14%, the NPV becomes negative at smaller scales (e.g., beyond S3). The scale at which IRR is maximized (S1) is not necessarily where NPV is maximized. NPV depends on the chosen discount rate, and for a given rate, the optimal scale shifts (e.g., S2 for 10% and S3 for 14%).

Before S1, MIRR is greater than IRR, indicating that expanding the scale increases the project's IRR. After S1, MIRR is lower than IRR, suggesting that further expansion reduces MIRR. This graph provides a visual representation of how scale affects project performance metrics, emphasizing the trade-off between maximizing IRR and NPV based on the discount rate. (Jenkins, Yan Kuo, & Harberger, 2019)

3.1.3 Simulation method

Microsoft Excel provides powerful tools to generate random data, which can be useful for simulations, testing, or creating sample datasets. Excel's built-in functions like {RAND ()} and {RANDBETWEEN ()} allow users to create random numbers, while additional features and formulas enable the generation of various types of data, such as text, dates, and custom datasets. By using different methods, Excel serves as a useful tool for generating random datasets, aiding in analysis, testing, or demonstrations. Advanced users can even combine Excel's capabilities with macros or external data connections for more sophisticated data generation.

The random variability method, also known as random sampling, introduces randomness into data generation processes using the following steps:

1. **Define Parameters:** Determine the range or distribution for each variable.
2. **Random Sampling:** Use a random number generator to select values within the specified range or distribution.
3. **Repeat as Needed:** Continue sampling until the desired quantity of data is obtained.
4. **Optional Adjustment**: Apply constraints or adjustments as necessary.
5. **Analysis and Interpretation**: Analyze the generated data to derive insights. (Buldygin & Kozachenko, 2000)

Relationship Between Original and Simulated Data:

- **Preservation of Structure**: The simulated data maintains the original data's structure, including the number of columns and their names.
- **Use of Random Variability**: Random variability introduces randomness while preserving general trends.
- **Range and Distribution**: Simulated values remain within a similar range to the original data.
- **Consistency in Growth and Trends**: Simulated data retains general growth and trends, with added randomness.

- **Introduction of Noise**: Random noise simulates real-world fluctuations.

For example, administrative costs, toll revenues, and insurance costs in simulated data reflect similar trends to the original data but with added variability. This approach ensures realistic data for further analysis.

The relationship between the original data and the simulated data involves several key aspects, as outlined below:

Preservation of Structure: The original data's structure, such as the number of columns (variables) and their respective names, has been preserved in the simulated data. Both datasets contain the same categories of costs and revenues, ensuring comparability.

Use of Random Variability: The simulated data have been generated using random variability to introduce randomness while maintaining the original data's general trends. This method involves adding random variations to the original values to create a new dataset that reflects similar patterns but with added randomness.

Range and Distribution: The values in the simulated data are within a similar range to those in the original data, ensuring that the simulation remains realistic and relevant. Random sampling was performed within plausible ranges to reflect potential fluctuations in costs and revenues.

Consistency in Growth and Trends: The simulated data retain the general growth and trends observed in the original data. For instance, if a specific revenue stream was increasing in the original data, the simulated data also show a similar trend, albeit with random variations.

Introduction of Noise: Random noise has been introduced into the original values to simulate potential real-world fluctuations and uncertainties. This makes the simulated data useful for stress testing, scenario analysis, and exploring possible outcomes under varying conditions.

To illustrate, here are a few examples of the relationships between corresponding rows and columns in the original and simulated datasets:

Administrative Cost—Nominal Relationship: The simulated values are generated by adding random variability to the original values, leading to different but related figures.

Car Toll Revenue Relationship: The trend of increasing toll revenues is preserved, but the exact values vary due to the introduction of random noise.

Insurance Cost—Nominal (VAT Inclusive) Relationship: The simulated costs reflect similar increases over time, with variations added to each value.

Overall, the relationships between the original and simulated data are characterized by maintaining the same data structure and general trends while introducing random variability to create new, plausible data points. This approach ensures that the simulated data remain realistic and can be used for further analysis and scenario exploration.

The flowchart in Figure 2 illustrates the analysis process for projects that failed to meet attractiveness criteria.

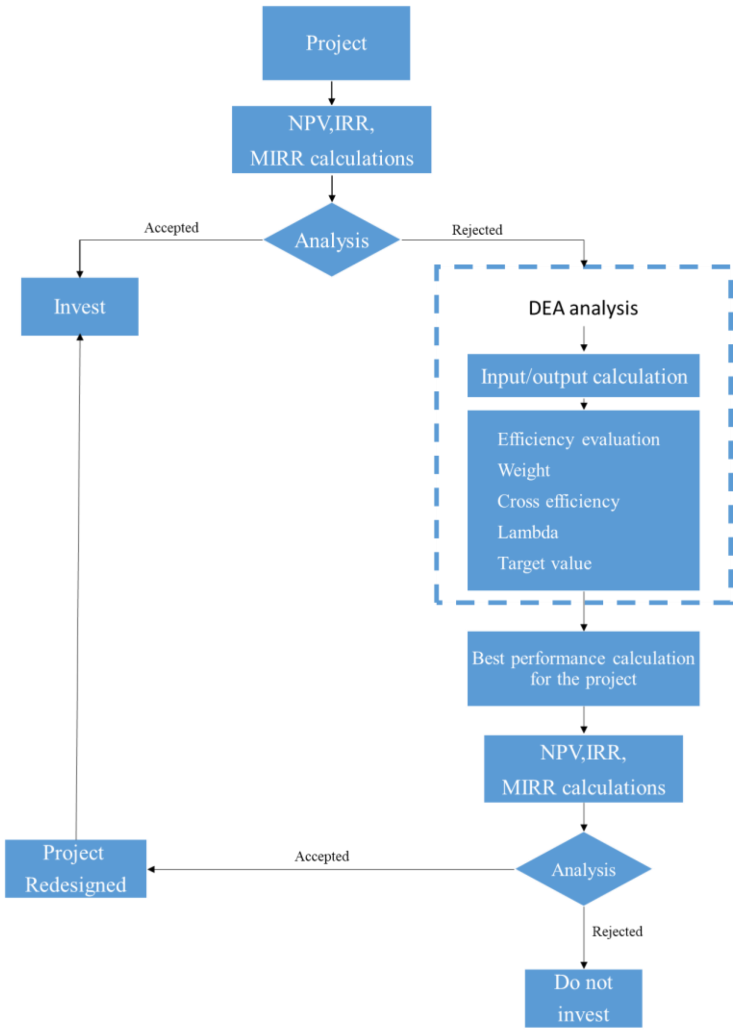

Figure 2: Flow Chart for the Method Implemented

3.2 Data Collection

There are several case studies and reports related to the Metro Manila Urban Expressway System, focusing on areas such as Traffic Management, Urban Development, Public-Private Partnerships (PPP), and Environmental Impact. One of the case studies includes Metro Manila Traffic and Road Network Studies, which are comprehensive studies conducted by organizations like JICA (Japan International Cooperation Agency) and ADB (Asian Development Bank). These studies often cover the entire Metro Manila expressway system as part of broader urban transport planning, these studies focus on:

- Traffic decongestion effects and simulation models predicting future demand
- Environmental and social impacts, including displacement issues and pollution reduction
- Integrated urban mobility strategies, combining expressways with mass transit systems

The data for this case study pertains to the development of a road project in the Philippines. Private investors have proposed a plan to design, build, finance, and operate three radial routes based on the Metro Manila Urban Expressway System (MMUES) alignment studies. The goal is to improve connectivity and reduce travel time between the eastern part of Metro Manila and the Makati area.

A contract will be established between the government and the private investors, ensuring that the government will not be responsible for the construction, operation, or maintenance of the road for 30 years. During this period, toll revenues will accrue to the investors.

The road construction is projected to take four years, and the residual land value will remain constant, with no anticipated increase or decrease over time.

Two toll plaza locations are identified: Fort Bonifacio and Manggahan. The proposal includes multiple inputs and outputs, measured in both United States Dollars and Philippine Pesos. For this analysis, the operational costs and tolls are expressed in Philippine Pesos. The final analysis includes three inputs and four outputs as follows:

- Inputs:
    - Administrative Cost – Nominal
    - Maintenance Cost - Nominal (a combination of Annual Maintenance Cost - Nominal and Periodic Maintenance Cost - Nominal)
    - Insurance Cost - Nominal (VAT Inclusive)
- Outputs:
    - Total Cars Toll Revenues
    - Total Jeepneys Toll Revenues
    - Total Buses Toll Revenues
    - Total Light Trucks Toll Revenues

The correlation between inputs and outputs is estimated and presented in Table 1.

Table 1:Correlation between Inputs and Outputs

Correlation	**Administrative Cost - Nominal**	**Maintenance cost - Nominal**	**Insurance Cost - Nominal (VAT Inclusive)**	**Total Cars Toll Revenues**	**Total Jeepneys Toll Revenues**	**Total Buses Toll Revenues**	**Total Light Trucks Toll Revenues**
Administrative Cost - Nominal	-	An increase in Maintenance Cost leads to an increase in Administrative Cost	No relation between Administrative Costs and Insurance Cost	No relation between Administrative Costs and Total Cars Toll Revenues	No relation between Administrative Costs and Total Jeepneys Toll Revenues	No relation between Administrative Costs and Total Buses Toll Revenues	No relation between Administrative Costs and Total Light Trucks Toll Revenues
Maintenance cost - Nominal	An increase in Administrative Costs leads to an increase in Maintenance Cost	-	An increase in Maintenance Cost leads to an increase in Insurance Cost	Increase in Total Cars Toll Revenues leads to an increase in Maintenance Cost	Increase in Total Jeepneys Toll Revenues leads to an increase in Maintenance Cost	An increase in Total Buses Toll Revenues leads to an increase in Maintenance Cost	Increase in Total Light Trucks Toll Revenues leads to an increase in Maintenance Cost

Insurance Cost - Nominal (VAT Inclusive)	No relation between Administrative Costs and Insurance Cost	An increase in Maintenance Cost leads to an increase in Insurance Cost	-	Increase in Total Cars Toll Revenues leads to an increase in Insurance Cost	Increase in Total Jeepneys Toll Revenues leads to an increase in Insurance Cost	An increase in Total Buses Toll Revenues leads to an increase in Insurance Cost	Increase in Total Light Trucks Toll Revenues leads to an increase in Insurance Cost
Total Cars Toll Revenues	No relation between Administrative Costs and Total Cars Toll Revenues	Increase in Maintenance Cost leads to a decrease in Total Cars Toll Revenues	No relation between Total Cars Toll Revenues and Insurance Cost	-	Increase in Total Jeepneys Toll Revenues leads to a decrease in Total Cars Toll Revenues	An increase in Total Buses Toll Revenues leads to a decrease in Total Cars Toll Revenues	An increase in Total Light Trucks Toll Revenues leads to a decrease in Total Cars Toll Revenues
Total Jeepneys Toll Revenues	No relation between Administrative Costs and Total Jeepneys Toll Revenues	An increase in Maintenance Cost leads to a decrease in Total Jeepneys Toll Revenues	No relation between Total Jeepneys Toll Revenues and Insurance Cost	An increase in Total Cars Toll Revenues leads to a decrease in Total Jeepneys Toll Revenues	-	An increase in Total Buses Toll Revenues leads to a decrease in Total Jeepneys Toll Revenues	An increase in Total Light Trucks Toll Revenues leads to a decrease in Total Jeepneys Toll Revenues
Total Buses Toll Revenues	No relation between Administrative Costs and Total Buses Toll Revenues	An increase in Maintenance Cost leads to a decrease in Total Buses Toll Revenues	No relation between Total Buses Toll Revenues and Insurance Cost	An increase in Total Cars Toll Revenues leads to a decrease in Total Buses Toll Revenues	Increase in Total Jeepneys Toll Revenues leads to a decrease in Total Buses Toll Revenues	-	An increase in Total Light Trucks Toll Revenues leads to a decrease in Total Buses Toll Revenues
Total Light Trucks Toll Revenues	No relation between Administrative Costs and Total Light Trucks Toll Revenues	Increase in Maintenance Cost leads to a decrease in Total Light Trucks Toll Revenues	No relation between Total Light Trucks Toll Revenues and Insurance Cost	An increase in Total Cars Toll Revenues leads to a decrease in Total Light Trucks Toll Revenues	Increase in Total Jeepneys Toll Revenues leads to a decrease in Total Light Trucks Toll Revenues	An increase in Total Buses Toll Revenues leads to a decrease in Total Light Trucks Toll Revenues	-

In order to obtain the distribution characteristics of the original data, **Descriptive Statistics Analysis** and the **Shapiro-Wilk Test for Normality** (p-value results) were conducted, as shown below:

Descriptive Statistics:

- **Administrative Cost - Nominal:** Mean = 320.65, Std Dev = 151.71
- **Annual Maintenance Cost - Nominal:** Mean = 579.00, Std Dev = 297.92

- **Periodic Maintenance Cost - Nominal:** Mean = 150.11, Std Dev = 318.54 (many zero values)
- **Insurance Cost - Nominal (VAT Inclusive):** Mean = 151.98, Std Dev = 68.94

Shapiro-Wilk Test for Normality (p-value interpretation):

- **Administrative Cost:** p = 0.135 → Data is normally distributed ($p > 0.05$).
- **Annual Maintenance Cost:** p = 0.039 → Data is not normally distributed ($p < 0.05$).
- **Periodic Maintenance Cost:** p < 0.00000001 → Data is highly non-normal (due to many zeros).
- **Insurance Cost:** p = 0.074 → Data is approximately normal ($p > 0.05$).

For the output variables (toll revenues for different vehicle types and locations), here's the distribution analysis:

Descriptive Statistics:

- **Cars Toll Revenues - Fort Bonifacio:** Mean ≈ 943.75, Std Dev ≈ 464.32
- **Cars Toll Revenues - Manggahan:** Mean ≈ 10,584.23, Std Dev ≈ 5,292.71
- **Jeepneys Toll Revenues - Fort Bonifacio:** Mean ≈ 1,715.12, Std Dev ≈ 1,012.89
- **Jeepneys Toll Revenues - Manggahan:** Mean ≈ 2,012.45, Std Dev ≈ 1,094.73
- **Buses Toll Revenues - Fort Bonifacio:** Mean ≈ 111.72, Std Dev ≈ 48.65
- **Buses Toll Revenues - Manggahan:** Mean ≈ 1,241.88, Std Dev ≈ 489.26
- **Light Trucks Toll Revenues - Fort Bonifacio:** Mean ≈ 34.82, Std Dev ≈ 15.12
- **Light Trucks Toll Revenues - Manggahan:** Mean ≈ 426.74, Std Dev ≈ 175.39

Shapiro-Wilk Test for Normality (p-value results):

- **Cars (Fort Bonifacio):** $p \approx 0.045$ → Not normally distributed (right-skewed).
- **Cars (Manggahan):** $p \approx 0.002$ → Not normally distributed (strong skew due to growth trend).
- **Jeepneys (Fort Bonifacio):** $p \approx 0.110$ → Approximately normal.
- **Jeepneys (Manggahan):** $p \approx 0.084$ → Approximately normal.
- **Buses (Fort Bonifacio):** $p \approx 0.060$ → Nearly normal.
- **Buses (Manggahan):** $p \approx 0.039$ → Mildly skewed (non-normal).
- **Light Trucks (Fort Bonifacio):** $p \approx 0.051$ → Borderline normal.
- **Light Trucks (Manggahan):** $p \approx 0.009$ → Not normally distributed (moderate skewness).

Table 2 presents the minimum, maximum, and average values for each input and output throughout the project's lifespan. These values show a consistent increase over time, which can be attributed to inflation as the project progresses.

Table 2: Values of Minimum, Maximum, and Average for Each Input and Output

Name	Administrative Cost - Nominal	Maintenance cost – Nominal	Insurance Cost - Nominal (VAT Inclusive)	
Max value	956.22	3804.04	303.15	
Min value	113.98	204.41	64.17	
Average value	400.79	947.84	154.94	
Name	Total Cars Toll Revenues	Total Jeepneys Toll Revenues	Total Buses Toll Revenues	Total Light Trucks Toll Revenues
Max value	20997.76	3578.05	2451.32	835.61
Min value	3884.49	579.15	339.01	138.95
Average value	11277.23	1843.23	1206.52	430.80

Table 2 highlights that the insurance cost consistently exhibits the lowest value, while the maintenance cost demonstrates the highest value among all costs throughout the project's duration. The observed trend, driven by inflation, increases the cost of each input as the project progresses.

Additionally, the toll revenue generated by light trucks is the lowest, whereas cars contribute significantly to the investors' income. Inflation causes an escalation in the road's cost to customers, which, in turn, leads to a rise in revenue each year as the project advances.

Table (3) presents the **Net Cash Flow After Financing** over a **34-year project life**, measured in **millions of PHP (M'PHP)**. The data outlines the annual cash outflows minus the inflows, starting with a significant negative cash flow in the early years due to initial investments, followed by positive cash flows as the project progresses.

Table 3: Financial Analysis Before DEA

Project Life	**Net Cash Flow After Financing**	**Project Life**	**Net Cash Flow After Financing**
Years	**M'PHP**	**Years**	**M'PHP**
year 0	-5086.490	year 18	2580.213
year 1	-9319.818	year 19	2636.446
year 2	-7148.593	year 20	2878.265
year 3	-2183.555	year 21	2748.489
year 4	1197.411	year 22	2945.016
year 5	1373.285	year 23	2590.074
year 6	1696.196	year 24	2839.944
year 7	1731.502	year 25	3131.110
year 8	1837.453	year 26	3290.367
year 9	2039.261	year 27	3121.997
year 10	2411.430	year 28	3060.430
year 11	2324.123	year 29	3072.922
year 12	2554.232	year 30	3271.340
year 13	2222.826	year 31	3103.311

year 14	2619.280	year 32	3261.588
year 15	2543.392	year 33	2873.224
year 16	2710.926	year 34	9993.502
year 17	2592.528		

Table 3 displays the net cash flow for the project following the acquisition of a loan from the bank and the repayment of that loan over the project's lifespan. The cash flow includes construction costs, residual values, loan amounts, non-operational values, and operational values.

Before applying Data Envelopment Analysis (DEA), the financial net present value, calculated using Equation (17) in Excel with a discount rate of 8%, stands at -866.911 million pesos. This negative value indicates that investing in the project would result in a loss of approximately 866.9 million pesos, making it financially unattractive. This conclusion is supported by the Internal Rate of Return (IRR) and Modified Internal Rate of Return (MIRR) calculations, performed using Equations (18) and (19), respectively. In Excel, the IRR and MIRR are determined to be 7.7% and 7.9%, respectively, both of which are below the discount rate of 8%.

By introducing random variability, which adds randomness to simulate data variations, a new dataset is generated. This dataset facilitates an exploration of how the project's viability might shift from rejection to acceptance.

Table 4:Values of Minimum, Maximum, and Average for Each Simulated Input and Output

Name	Administrative Cost - Nominal	Maintenance cost – Nominal	Insurance Cost - Nominal (VAT Inclusive)
Max value	665.93	3482.41	204.55
Min value	116.78	210.42	61.99

Average value	340.82	768.06	133.08	
Name	Total Cars Toll Revenues	Total Jeepneys Toll Revenues	Total Buses Toll Revenues	Total Light Trucks Toll Revenues
Max value	21110.02	3293.74	2226.8	779.83
Min value	3818.62	586.94	407.85	160.13
Average value	11542.71	1795.25	1222.29	408.11

Table 4 illustrates that the insurance cost shows the lowest value, whereas the maintenance cost is the highest among all costs throughout the project's lifespan. Additionally, revenue generated by light trucks using the road is the lowest compared to other vehicle types, while cars provide substantial income for the investors.

Table 5:Financial Analysis for Simulated Data Before DEA

Project Life	**Net Cash Flow After Financing**	**Project Life**	**Net Cash Flow After Financing**
Years	**M'PHP**	**Years**	**M'PHP**
year 0	-5086.49	year 18	2667.23
year 1	-9319.82	year 19	2826.20
year 2	-7148.59	year 20	2959.30
year 3	-2183.55	year 21	2974.96
year 4	1107.94	year 22	3038.13
year 5	1607.72	year 23	2885.47
year 6	1816.93	year 24	2900.97
year 7	1876.91	year 25	3339.80
year 8	1843.29	year 26	3321.22
year 9	2044.40	year 27	3398.41
year 10	2380.91	year 28	3308.91
year 11	2379.83	year 29	3310.11
year 12	2571.76	year 30	3248.25
year 13	2443.45	year 31	3178.31
year 14	2723.56	year 32	3129.56
year 15	2738.84	year 33	2842.07
year 16	2772.34	year 34	9994.66
year 17	2775.77		

Table 5 illustrates the net cash flow for the project after incorporating the simulated data. Before applying DEA, the financial net present value, calculated using Excel with an 8% discount rate, amounts to -24.39 million pesos. This negative value indicates that investing in the project would result in a loss of 24.39 million pesos for the investors, making it financially unattractive and advising against investment. Similar conclusions are supported by the calculations of the internal rate of return (IRR) and the modified internal rate of return (MIRR) using Equations (18) and (19). Excel yields IRR and MIRR values of 7.99157% and 7.99641%, respectively, both of which are below the 8% discount rate.

Consistency refers to the degree of alignment or similarity between two datasets, particularly in terms of their statistical properties such as mean, variance, distribution patterns, and correlation. In data simulation, consistency ensures that the simulated data accurately reflects the key characteristics of the original dataset, making it reliable for analysis, modeling, and decision-making.

The consistency between the original data and the simulated data can be evaluated based on several key factors that ensure the simulated data remains realistic and relevant to the original dataset such as:

- Structural Consistency
    - Same Variables: Both datasets have the same columns (e.g., Administrative Cost, Toll Revenues, Insurance Cost).
    - Data Format: The numerical format, units, and data types are consistent, ensuring compatibility for analysis.
- Trend Consistency

- Growth Patterns: If the original data shows increasing trends (e.g., rising toll revenues), the simulated data reflects similar upward trends with slight fluctuations due to random variability.
- Seasonality (if any): If there are periodic trends, the simulation preserves them while introducing randomness within those patterns.

▪ Range Consistency

- Value Ranges: Simulated data values are within a plausible range derived from the original data, avoiding unrealistic extremes.
- Bounded Variability: Random variability is applied carefully to prevent deviations that would distort economic reality.

▪ Statistical Consistency

- Mean and Variance: The average (mean) values and variability (standard deviation) in the simulated data are close to those in the original dataset, maintaining statistical integrity.
- Correlation Between Variables: Relationships between variables (e.g., administrative costs rising with maintenance costs) remain consistent.

▪ Proportional Consistency

- Relative Differences: The proportional differences between related variables (like revenues from different vehicle types) are preserved, even if the absolute numbers vary slightly.
- Ratio Maintenance: Ratios such as cost-to-revenue or maintenance-to-administrative costs remain within reasonable limits

Using statistical analysis, we can compare between the original data to make sure that the Individual data points fluctuate to simulate real-world randomness, but without disrupting the overall economic picture.

Using two statistical comparisons between the original and simulated data: Descriptive Statistics, such as mean, standard deviation, minimum, and maximum values for each column, and Correlation Analysis to check if the relationships between variables are preserved.

Descriptive statistics are summary measures that help describe, organize, and simplify large datasets. They provide key insights into the data's central tendency (mean, median, mode), dispersion (range, variance, standard deviation), and distribution shape. By summarizing the essential features of a dataset, descriptive statistics make it easier to identify patterns, trends, and outliers, serving as a foundation for more advanced statistical analysis. An example of descriptive statistics analysis is shown below for both the original and simulated data:

Original Data:

- Administrative Cost: Mean = 131.96, Std Dev = 18.39
- Toll Revenues (Fort Bonifacio): Mean = 389.61, Std Dev = 51.14
- Insurance Cost: Mean = 70.33, Std Dev = 8.41

Simulated Data:

- Administrative Cost: Mean = 136.55, Std Dev = 19.90
- Toll Revenues (Fort Bonifacio): Mean = 398.35, Std Dev = 56.26
- Insurance Cost: Mean = 66.01, Std Dev = 7.05

By comparing the results of the descriptive analysis, the means and standard deviations are relatively close, indicating that the simulated data maintains a similar distribution to the original, though slight variations exist due to random variability.

Correlation analysis is a statistical method used to measure the strength and direction of the relationship between two variables. It helps determine how changes in one variable are associated with changes in another. The correlation coefficient, typically ranging from -1 to 1, indicates the nature of this relationship: a value close to 1 signifies a strong positive correlation, -1 indicates a strong negative correlation, and 0 suggests no correlation. This analysis is essential for identifying patterns, predicting trends, and understanding the interdependence of variables in data.

A sample of the correlation analysis between the original and simulated data is shown below:

- Administrative Cost: 0.93
- Toll Revenues (Fort Bonifacio): 0.96
- Insurance Cost: 0.84

Interpretation: Strong positive correlations (> 0.8) indicate that the simulated data closely follows the trends and patterns of the original data, especially in Toll Revenues.

Chapter 4

RESULTS

This chapter presents the findings of the study based on the application of Data Envelopment Analysis (DEA) and investment equations. The results highlight the efficiency scores of investment projects, identifying both well-performing and inefficient projects. Additionally, financial metrics such as Net Present Value (NPV), Internal Rate of Return (IRR), and Modified Internal Rate of Return (MIRR) are analyzed to assess the financial feasibility of reconsidering failed projects.

The analysis includes key DEA outputs such as weight values, lambda values, cross-efficiency scores, and target values, which provide deeper insights into the efficiency and performance of each project. **Weight values indicate the relative importance of each input and output in the efficiency calculation, while lambda values identify benchmark projects influencing inefficient ones.** Cross-efficiency analysis further evaluates project performance by incorporating peer evaluations, and target values suggest the necessary adjustments for inefficient projects to become efficient.

The findings offer insights into the factors contributing to project inefficiency and propose possible improvements. The following sections detail the efficiency scores, weight distributions, benchmarking influences, financial performance outcomes, and implications for investment decision-making.

The original data was generated using **PIM-DEA version 3.2**, where inputs were categorized into **three input variables** and **four output variables**. The findings are presented in tabular format, detailing **efficiency scores, weight distributions, cross-efficiency values, lambda coefficients, slack variables, and target metrics**. Each **Decision-Making Unit (DMU)** represents a specific year in the road's operational lifespan.

PIM-DEA 3.2, the latest iteration of the DEA program, is the result of decades of expertise in research and education. Designed for **enhanced functionality**, it efficiently processes large datasets with advanced capabilities. Key features include **automated data categorization for batch processing, seamless export to Microsoft Excel, and real-time updates to the Production Possibility Set**, ensuring a robust and user-friendly experience.

4.1 Efficiency

Using equations (4–7), we can calculate efficiency. Table 6 presents the efficiency values for each year, aiding in the identification of the most efficient DMUs based on their specific inputs and outputs. These performance values, obtained through the CCR model in the PIM-DEA program, assist investors in determining which year achieved the highest profit at the lowest expense.

Table 6:DMUs Efficiencies

Title	Efficiency	Title	Efficiency	Title	Efficiency
Year01	93.17	Year11	100	Year21	99.16
Year02	100	Year12	99.4	Year22	97.22
Year03	100	Year13	99.77	Year23	98.64
Year04	98.73	Year14	98.07	Year24	95.11
Year05	100	Year15	100	Year25	98.68
Year06	100	Year16	95.98	Year26	96.56
Year07	100	Year17	99.73	Year27	98.32

Year08	100	Year18	96.2	Year28	97.21
Year09	100	Year19	99.42	Year29	94.55
Year10	100	Year20	99.73	Year30	98.14

Table 6 displays the efficiency percentages for each Decision-Making Unit (DMU). DMUs with 100% efficiency are considered fully efficient, such as Year02, for example, indicating that the project achieved maximum profit-cost ratio. Conversely, DMUs with less than 100% efficiency are considered inefficient. The inefficiency percentage represents the proportion of cost that could potentially be avoided, which corresponds to the difference between 100% and the DMU's inefficiency percentage.

Table 6 reveals that 10 DMUs (equivalent to 33% of the total) achieved 100% efficiency. The least efficient DMU is Year01, with an efficiency value of 93.17%.

Table 7 presents the efficiency values for each year, based on simulated data. This table helps in identifying efficient DMUs by assessing the coefficients given to every input and output. The efficiency values were calculated applying the CCR model within the PIM-DEA program.

Table 7:DMUs Efficiencies After Using The Simulated Data

Name	Efficiency	Name	Efficiency	Name	Efficiency
Year01	91.39	Year11	95.92	Year21	99.25
Year02	93.29	Year12	93.27	Year22	98.23
Year03	93.03	Year13	95.29	Year23	100
Year04	91.53	Year14	94.84	Year24	100
Year05	70.75	Year15	78.31	Year25	97.37
Year06	98.21	Year16	95.39	Year26	100
Year07	100	Year17	97.42	Year27	100
Year08	100	Year18	96.1	Year28	99.45
Year09	99.9	Year19	96.73	Year29	100
Year10	74.04	Year20	87.79	Year30	100

Table 7 presents efficiency as a percentage (%), where DMUs achieving 100% efficiency are considered to represent efficient years. For example, Year07 represents a year that achieved maximum profit-cost ratio. In contrast, inefficient DMUs reflect the proportion of cost that could potentially be avoided, representing the difference between efficient and inefficient DMUs.

In Table 7, 8 DMUs achieve 100% efficiency, accounting for 26.66% of the total DMUs. The least efficient DMU is Year05, with an efficiency value of 70.75%.

4.2 Weights

Using equations (4–7), we can calculate weights which is symbolized in equations as U_r for output weights and V_i for input weights. Table 8 displays the weights assigned to inputs and outputs for every DMU. These weights reflect the relative importance for every input and output in calculating the efficiency of DMU. Inputs or outputs with higher weights are more significant in determining efficiency, while those with lower weights contribute less.

Table 8:Weights of Inputs & Outputs

Name	Administrative Cost - Nominal	Maintenance cost - Nominal	Insurance Cost - Nominal (VAT Inclusive)	Total Cars Toll Revenues	Total Jeepneys Toll Revenues	Total Buses Toll Revenues	Total Light Trucks Toll Revenues
Year01	0	18.61	0	3.74	1.48	0	0
Year02	4.57	0	1.85	4.48	0	0	0
Year03	5.12	4.59	0	0.71	0.67	0	3.26
Year04	0	10.37	1.12	0.63	0.61	0	3.05
Year05	6.26	0	0	0	0	2.61	1.41
Year06	5.8	0.03	0	0	0	3.3	0.46
Year07	4.17	0	0.78	2.9	0.36	0	0
Year08	2.26	0.11	1.75	0	0	0	3.14
Year09	1.4	0	2.15	2.83	0	0	0
Year10	1.46	0	1.93	0.82	0	0	1.93
Year11	1.25	0	1.91	2.51	0	0	0
Year12	0	0	2.62	2.38	0	0	0

Year13	0	0	2.48	0.28	2.07	0	0
Year14	0	0	2.36	0	2.23	0	0
Year15	0.03	0	2.21	2.03	0	0	0
Year16	0	0.05	2.1	0	0	0	2
Year17	0	0	2.01	0	1.7	0	0.2
Year18	0	0	1.9	1.73	0	0	0
Year19	0	0	1.8	0.2	1.5	0	0
Year20	0	0	1.71	0	0	1.68	0
Year21	0	0.04	1.6	0	0	0	1.53
Year22	0	0.04	1.52	0	0	0	1.45
Year23	0	0	1.45	0	1.23	0	0.15
Year24	0	0	1.38	0	1.17	0	0.14
Year25	0	0	1.31	0.15	1.09	0	0
Year26	0	0	1.24	0	1.17	0	0
Year27	0	0	1.17	0.13	0.98	0	0
Year28	0	0	1.11	1.01	0	0	0
Year29	0	0	1.05	0.96	0	0	0
Year30	0	0	1	0	0	0.98	0

The weights assigned to inputs and outputs signify their contribution to the efficiency evaluation. For instance, Year01 has zero weight for the inputs (Administrative Cost - Nominal and Maintenance Cost - Nominal), indicating that these inputs did not influence the efficiency assessment for Year01. Conversely, the output Total Cars Toll Revenues has a high weight of 3.74, signifying its significant contribution to the efficiency measurement, more so than the Total Jeepneys Toll Revenues, which has a weight of 1.48. This pattern applies across all DMUs, any input or output with a weight greater than other contributes to the efficiency evaluation more.

Some DMUs, such as Year12, rely heavily on certain inputs or outputs, like Insurance Cost - Nominal (VAT Inclusive) and Total Cars Toll Revenues.

Table 9 displays the weights assigned to inputs and outputs for each DMU after incorporating the simulated data. These weights illustrate the importance of each input and output for the DMUs. The input or output that has the highest weight has the greatest impact on the calculation of the DMU's efficiency, while the one with the lowest weight has the least impact.

Table 9:Weights of Inputs & Outputs After Using Simulated Data

Name	Administrative Cost - Nominal	Maintenance cost - Nominal	Insurance Cost - Nominal (VAT Inclusive)	Total Cars Toll Revenues	Total Jeepneys Toll Revenues	Total Buses Toll Revenues	Total Light Trucks Toll Revenues
Year01	1.25	10.27	0	0	0	0	4.26
Year02	1.17	9.87	0	0	0	3.96	0
Year03	1.07	9.51	0	0	3.85	0	0
Year04	0.75	10.68	0	3.82	0	0	0
Year05	2.55	0.03	0	0	0	2.58	0
Year06	0.96	8.03	0	0	0	3.22	0
Year07	0.65	9.16	0	3.27	0	0	0
Year08	0.85	7.13	0	0	0	2.86	0
Year09	0.74	6.57	0	0	2.66	0	0
Year10	1.87	0.01	0	0	1.89	0	0
Year11	0.64	5.64	0	0	2.28	0	0
Year12	0.44	6.17	0	2.21	0	0	0
Year13	0.42	5.94	0	2.12	0	0	0
Year14	0.4	5.69	0	2.04	0	0	0
Year15	1.53	0.01	0	1.54	0	0	0
Year16	0.36	5.16	0	1.84	0	0	0
Year17	0.34	4.87	0	1.74	0	0	0
Year18	0.33	4.61	0	1.65	0	0	0
Year19	0.33	4.29	0	1.41	0.16	0	0
Year20	1.33	0.01	0	0	1.34	0	0
Year21	0.4	3.54	0	0	1.43	0	0
Year22	0.41	3.23	0	0.02	0.49	0.84	0
Year23	0.39	3.11	0	0	0.3	0.99	0
Year24	0.24	3.45	0	1.24	0	0	0
Year25	1.13	0.01	0	1.14	0	0	0
Year26	0.24	3.12	0	1.03	0.11	0	0
Year27	0.23	3.02	0	1	0.09	0	0.02
Year28	0.53	1.82	0	1.06	0	0	0
Year29	1.25	10.27	0	0	0	0	4.26
Year30	1.17	9.87	0	0	0	3.96	0

The weights assigned to inputs and outputs indicate their contribution to the efficiency evaluation of each DMU. For example, Year01 has value approximate to zero for the input "Insurance Cost - Nominal," meaning this input does little to no contribution to the efficiency evaluation for Year01. Conversely, the output 'Total Light Trucks Toll Revenues' has a weight of 4.26, signifying its significant contribution to the efficiency evaluation for Year01. This pattern is consistent across all DMUs, where inputs or outputs with nonzero weights contribute to the evaluation of efficiency. Notably, no DMUs depend on the "Insurance Cost - Nominal (VAT Inclusive)" input.

4.3 Cross Efficiency

Using equation (44), we can calculate cross-efficiency. The table in the appendix displays the efficiency of DMUs calculated after applying the weights assigned to each DMU. This table is produced by solving each DMU iteratively, using weights obtained through linear programming. This method addresses the issue of weight flexibility, which could otherwise neglect certain inputs and outputs. Cross-efficiency also assists in identifying DMUs that can serve as reference units among the efficient ones, providing guidance to investors towards optimal production processes and outcomes.

The cross-efficiency evaluation uses the weights of DMUs in the rows to calculate the efficiency of DMUs in the columns. Appendix A presents the cross-efficiency scores, where DMUs achieving 100% efficiency are considered fully efficient. Higher efficiency scores across multiple cells indicate stronger overall performance.

Among the evaluated units, Year09 emerges as the most efficient DMU, serving as a key benchmark for improving the performance of other units. It achieves 100% efficiency in 19 cells, reinforcing its role as a reference model. In contrast, Year05 and

Year06 demonstrate relatively weak efficiency. Meanwhile, Year11, Year02, and Year10, with 18, and 5(for both, Year02 and Year10) efficient cells respectively, exhibit performance levels approaching that of Year09.

Similarly, Appendix B presents the DMUs evaluated using cross-efficiency with simulated data. The efficient DMUs identified in this case align with those from the CRS input model, confirming our previous results. DMUs with more efficient cells, such as Year07, are the most efficient and serve as critical reference units for improving the performance of other DMUs. Other units, such as Year24 and Year29, also demonstrate significant efficiency scores and potential as reference units.

4.4 Lambda

Using equations (17–21), we can calculate lambda. The lambda values (λ) in Table 10 are obtained from the dual CCR model. These values reflect the degree to which an inefficient DMU compares itself to efficient DMUs which is taken from the cross efficiency analysis in order to achieve optimal performance. This comparison helps identify efficient DMUs that can serve as benchmarks for improving the performance of inefficient ones, thus enhancing overall annual efficiency.

As shown in Table 10, lambda values range from 0 to 3.02. A lambda value of zero indicates that the inefficient DMU does not use the efficient DMU in that column as a benchmark which indicate that only the efficient DMUs are presented in the columns. On the other hand, a positive lambda value signifies that the inefficient DMU is using the efficient DMU in that column for comparison. Higher lambda values suggest that the corresponding efficient DMUs are more appropriate as reference units for the inefficient DMUs.

Table 10: Lambda Values

Name	Year02	Year03	Year05	Year06	Year07	Year08	Year09	Year10	Year11	Year15
Year01	0.46	0.37	0	0	0	0	0	0	0	0
Year02	1	0	0	0	0	0	0	0	0	0
Year03	0	1	0	0	0	0	0	0	0	0
Year04	0.2	0.6	0	0	0.1	0.1	0	0	0	0
Year05	0	0	1	0	0	0	0	0	0	0
Year06	0	0	0	1	0	0	0	0	0	0
Year07	0	0	0	0	1	0	0	0	0	0
Year08	0	0	0	0	0	1	0	0	0	0
Year09	0	0	0	0	0	0	1	0	0	0
Year10	0	0	0	0	0	0	0	1	0	0
Year11	0	0	0	0	0	0	0	0	1	0
Year12	0	0	0	0	0	0	0	0	1.04	0
Year13	0	0	0	0	0	0	0.86	0	0.34	0
Year14	0	0	0	0	0	0	1.28	0	0	0
Year15	0	0	0	0	0	0	0	0	0	1
Year16	0	0	0	0	0	0	0	0.01	1.25	0
Year17	0	0	0	0	0	0	0.61	0	0.83	0
Year18	0	0	0	0	0	0	0	0	1.39	0
Year19	0	0	0	0	0	0	0.57	0	1.01	0
Year20	0	0	0	0	0	0	1.8	0	0	0
Year21	0	0	0	0	0	0	0	0.01	1.69	0
Year22	0	0	0	0	0	0	0	0.01	1.74	0
Year23	0	0	0	0	0	0	1.67	0	0.38	0
Year24	0	0	0	0	0	0	0.03	0	1.88	0
Year25	0	0	0	0	0	0	0.73	0	1.43	0
Year26	0	0	0	0	0	0	2.4	0	0	0
Year27	0	0	0	0	0	0	1.12	0	1.31	0
Year28	0	0	0	0	0	0	0	0	2.41	0.01
Year29	0	0	0	0	0	0	0	0	2.47	0.01
Year30	0	0	0	0	0	0	3.02	0	0	0

Year11 has the highest number of references, with 14, optimizing it to be the most efficient year, with a lambda value greater than zero, in relation to the inefficient DMUs listed in the rows, followed by Year09 with 11 references, among others.

Table 11 presents the lambda values after including simulated data, with values ranging from 0 to 1 for each DMU. A lambda value of 0 means that the inefficient DMU does not use the efficient DMU in that column for comparison. In contrast, a positive lambda value indicates that the inefficient DMU is using the efficient DMU in that column as a benchmark. Overall, higher lambda values imply that the corresponding efficient DMU is more appropriate as a reference for the inefficient DMU.

Table 11: Lambda Values After Using Simulated Data

Name	Year07	Year08	Year23	Year24	Year26	Year27	Year29	Year30
Year01	0.52	0	0	0	0	0	0.05	0
Year02	0.64	0	0	0.03	0	0	0	0
Year03	0.7	0	0	0	0	0	0.02	0
Year04	0.73	0	0	0.02	0	0	0	0
Year05	0	0	0	0	0	0	0.12	0.16
Year06	0.88	0	0	0.02	0	0	0	0
Year07	1	0	0	0	0	0	0	0
Year08	0	1	0	0	0	0	0	0
Year09	0.92	0	0	0	0	0	0.09	0
Year10	0	0	0	0	0	0	0.2	0.2
Year11	0.95	0	0	0	0	0	0.13	0
Year12	0.88	0	0	0.19	0	0	0	0
Year13	0.94	0	0	0.2	0	0	0	0
Year14	0.94	0	0	0.22	0	0	0	0
Year15	0	0	0	0	0	0	0.2	0.32
Year16	0.82	0	0	0.33	0	0	0	0
Year17	0.73	0	0	0.42	0	0	0	0
Year18	0.59	0	0	0.5	0	0	0	0
Year19	0.58	0	0	0.26	0.26	0	0	0
Year20	0	0	0	0	0	0	0.44	0.23
Year21	0.61	0	0	0	0	0	0.52	0
Year22	0.23	0	0.46	0.04	0.3	0	0	0
Year23	0	0	1	0	0	0	0	0
Year24	0	0	0	1	0	0	0	0
Year25	0	0	0	0	0	0	0.58	0.29
Year26	0	0	0	0	1	0	0	0
Year27	0	0	0	0	0	1	0	0
Year28	0	0	0	0	0.53	0	0.49	0
Year29	0	0	0	0	0	0	1	0
Year30	0	0	0	0	0	0	0	1

Based on the lambda values in Table (11), Year07 is the most suitable reference, with 16 inefficient DMUs referencing it. Following Year07, Year24 and Year29 are also strong preference units, each being referenced by 16 and 11(for both year24 and year 29) DMUs, demonstrating their effectiveness among the efficient DMUs.

4.5 Target

Using equations (17–21), we can calculate slack, and using equations (24–25), we can determine target values. Slack values are utilized to establish target values aimed at improving the efficiency of the DMUs. Table 12 presents the target values for input and output variables required to enhance the efficiency of each DMU.

Table 12:Target Values

Name	Target of Administrative Cost - Nominal	Target of Maintenance cost - Nominal	Target of Insurance Cost - Nominal (VAT Inclusive)	Target of Total Cars Toll Revenues	Target of Total Jeepneys Toll Revenues	Target of Total Buses Toll Revenues	Target of Total Light Trucks Toll Revenues
Year01	105.1837008	190.2017589	57.59894497	3779.596652	572.4877267	392.2116158	142.0534004
Year02	124.30801	228.2421107	66.69351523	4619.507019	608.2682096	490.2645197	167.1216475
Year03	133.8701646	228.2421107	72.75656207	4829.484611	822.9511071	490.2645197	175.4777299
Year04	143.4323192	266.2824625	75.78808549	5039.462203	822.9511071	539.2909717	192.1898946
Year05	152.9944739	951.0087947	78.81960891	5459.417386	858.73159	612.8306497	208.9020594
Year06	162.5566285	304.3228143	84.88265575	5669.394978	966.0730387	661.8571016	217.2581417
Year07	181.6809377	342.3631661	87.91417917	6509.305345	1073.414487	686.3703276	242.3263889
Year08	191.2430923	380.4035179	93.97722601	6719.282937	1109.19497	710.8835536	267.394636
Year09	200.8052469	418.4438697	97.00874943	7349.215712	1216.536419	808.9364576	284.1068007
Year10	219.9295562	1293.371961	103.0717963	7559.193304	1216.536419	808.9364576	309.1750479
Year11	239.0538654	494.5245733	109.1348431	8399.103671	1359.658351	882.4761355	317.5311302
Year12	248.61602	494.5245733	115.1978899	8819.058855	1431.219317	931.5025875	334.243295
Year13	258.1781746	494.5245733	121.2609368	9239.014038	1502.780282	1005.042265	350.9554597
Year14	258.1781746	532.5649251	127.3239836	9448.99163	1574.341248	1029.555491	359.3115421
Year15	315.5511023	1711.815831	136.4185539	10288.902	1681.682697	1103.095169	392.7358716
Year16	296.4267931	608.6456286	136.4185539	10498.87959	1717.46318	1127.608395	401.091954
Year17	325.1132569	646.6859804	151.576171	11338.78996	1860.585112	1225.661299	434.5162835
Year18	334.6754116	684.7263322	154.6076944	11758.74514	1896.365595	1250.174525	442.8723659
Year19	353.7997208	722.766684	166.7337881	12598.65551	2075.268009	1372.740655	484.6527777
Year20	372.92403	722.766684	175.8283583	13228.58828	2218.389941	1446.280333	509.7210249
Year21	401.6104939	836.8877394	184.9229286	14068.49865	2325.73139	1519.820011	543.1453544
Year22	411.1726485	836.8877394	190.9859754	14698.43142	2397.292355	1568.846463	559.8575191
Year23	430.2969577	874.9280912	206.1435925	15538.34179	2576.19477	1691.412593	593.2818486
Year24	449.4212669	912.968443	209.1751159	15958.29697	2611.975253	1691.412593	609.9940134
Year25	487.6698854	989.0491465	230.3957799	17428.14012	2862.438633	1863.005175	660.1305076
Year26	487.6698854	989.0491465	236.4588267	17848.0953	2933.999599	1936.544853	676.8426724
Year27	545.0428131	1103.170202	254.6479672	19317.93844	3148.682497	2083.624209	735.335249
Year28	573.729277	1179.250905	263.7425375	20157.84881	3291.804428	2157.163887	768.7595785
Year29	592.8535862	1217.291257	272.8371078	20787.78159	3363.365394	2206.190339	793.8278256
Year30	621.54005	1217.291257	297.0892951	22467.60232	3721.170223	2451.322599	852.3204022

Table 12 helps identify the target values required to achieve optimal efficiency for the DMUs. For instance, in Year01, the Administrative Cost – Nominal has an actual value of 113.98 and a target value of 105.18. The difference of 8.8 between the actual and target values represents the slack value. This suggests that Year01 must reduce its Administrative Cost – Nominal by 8.8 to achieve optimal efficiency.

In summary, efficient DMUs have actual values equal to their target values, showing they have achieved optimal efficiency and require no input or output adjustments. Table 13 lists the target values for simulated input and output variables necessary to achieve optimal efficiency.

Table 13:Target Values Using Simulated Data

Name	Target of Administrative Cost - Nominal	Target of Maintenance cost - Nominal	Target of Insurance Cost - Nominal (VAT Inclusive)	Target of Total Cars Toll Revenues	Target of Total Jeepneys Toll Revenues	Target of Total Buses Toll Revenues	Target of Total Light Trucks Toll Revenues
Year01	186.4604	208.9447177	57.274	4433.1042	691.6854	489.896	163.7643
Year02	206.4383	208.9447177	63.4105	4644.2044	757.5602	534.432	179.3609
Year03	219.7569	208.9447177	67.5015	4855.3046	790.4976	556.7	187.1592
Year04	226.4162	208.9447177	69.547	5066.4048	823.435	556.7	194.9575
Year05	186.4604	661.6582727	57.274	5699.7054	889.3098	601.236	210.5541
Year06	266.372	243.7688373	81.82	6121.9058	988.122	668.04	233.949
Year07	293.0092	278.5929569	90.002	6544.1062	1053.9968	734.844	249.5456
Year08	299.6685	313.4170766	92.0475	6755.2064	1086.9342	779.38	265.1422
Year09	326.3057	348.2411962	100.2295	7810.7074	1251.6212	846.184	296.3354
Year10	259.7127	905.4271101	79.7745	8232.9078	1284.5586	868.452	304.1337
Year11	359.6022	383.0653158	110.457	8655.1082	1383.3708	957.524	327.5286
Year12	366.2615	383.0653158	112.5025	8866.2084	1416.3082	979.792	327.5286
Year13	386.2394	417.8894354	118.639	9499.509	1482.183	1046.596	350.9235
Year14	399.558	417.8894354	122.73	9921.7094	1548.0578	1091.132	358.7218
Year15	339.6243	1288.492426	104.3205	10766.1102	1679.8074	1135.668	397.7133
Year16	426.1952	487.5376747	130.912	10977.2104	1712.7448	1202.472	397.7133
Year17	446.1731	522.3617943	137.0485	11821.6112	1844.4944	1291.544	421.1082
Year18	452.8324	557.1859139	139.094	12243.8116	1910.3692	1336.08	436.7048
Year19	472.8103	592.0100335	145.2305	13088.2124	2009.1814	1402.884	467.898
Year20	439.5138	1218.844187	135.003	13932.6132	2173.8684	1447.42	506.8895

Year21	512.7661	661.6582727	157.5035	14565.9138	2272.6806	1558.76	538.0827
Year22	526.0847	696.4823924	161.5945	14988.1142	2338.5554	1647.832	530.2844
Year23	552.7219	766.1306316	169.7765	15621.4148	2470.305	1759.172	545.881
Year24	566.0405	800.9547512	173.8675	17099.1162	2569.1172	1825.976	584.8725
Year25	572.6998	1601.909502	175.913	17943.517	2799.679	1892.78	662.8555
Year26	599.337	870.6029905	184.095	18576.8176	2865.5538	1959.584	655.0572
Year27	619.3149	905.4271101	190.2315	19210.1182	2931.4286	1981.852	686.2504
Year28	632.6335	940.2512297	194.3225	19843.4188	3063.1782	2070.924	709.6453
Year29	645.9521	975.0753493	198.4135	20476.7194	3194.9278	2159.996	756.4351
Year30	665.93	3482.411962	204.55	21110.02	3293.74	2226.8	779.83

Table 13 lists the target values for simulated input and output variables required to achieve optimal efficiency. For instance, Year01's actual Administrative Cost – Nominal is 199.779, with a target value of 186.4604. The difference of 13.3186 between these values represents the slack value. For efficient DMUs, when the actual value equals the target value, it signifies that no adjustments are required, as optimal efficiency has been achieved.

4.6 Financial Analysis

Using equations (29 or 30), we can calculate NPV, and using equation (31), we can determine IRR. Additionally, using equation (32), we can calculate MIRR. Table 14 displays the net cash flow (NCF) values before and after the DEA analysis. It also highlights the differences in net present value (NPV), internal rate of return (IRR), and modified internal rate of return (MIRR). These differences underscore the potential for improved project profitability through redesigns aimed at achieving optimal efficiency.

Table 14: The Difference between Financial Analysis Values Before and After DEA Analysis

Project life	**Net Cash Flow Before DEA**	**Project life**	**Net Cash Flow After DEA**
Years	**M'PHP**	**Years**	**M'PHP**
year 0	-5086.490	year 0	-5086.490
year 1	-9319.818	year 1	-9319.818
year 2	-7148.593	year 2	-7148.593
year 3	-2183.555	year 3	-2183.555
year 4	1197.411	year 4	1064.660
year 5	1373.285	year 5	1643.265
year 6	1696.196	year 6	1810.454
year 7	1731.502	year 7	1859.372
year 8	1837.453	year 8	1787.265
year 9	2039.261	year 9	2045.643
year 10	2411.430	year 10	2373.298
year 11	2324.123	year 11	2301.603
year 12	2554.232	year 12	2488.121
year 13	2222.826	year 13	2234.367
year 14	2619.280	year 14	2623.435
year 15	2543.392	year 15	2721.142
year 16	2710.926	year 16	2721.263
year 17	2592.528	year 17	2681.035
year 18	2580.213	year 18	2574.242
year 19	2636.446	year 19	2710.280
year 20	2878.265	year 20	2862.966
year 21	2748.489	year 21	2838.264
year 22	2945.016	year 22	2939.030
year 23	2590.074	year 23	2976.090
year 24	2839.944	year 24	2891.798
year 25	3131.110	year 25	3277.075
year 26	3290.367	year 26	3296.169
year 27	3121.997	year 27	3199.380
year 28	3060.430	year 28	3304.744
year 29	3072.922	year 29	3227.447
year 30	3271.340	year 30	3292.040
year 31	3103.311	year 31	3256.774
year 32	3261.588	year 32	3178.717
year 33	2873.224	year 33	3275.925
year 34	9993.502	year 34	10043.340
Financial Net Present Value - FNPV	-866.911	**Financial Net Present Value - FNPV**	-398.795
Internal Rate of Return	7.697%	**Internal Rate of Return**	7.862%
Modified Internal Rate of Return	7.870%	**Modified Internal Rate of Return**	7.941%

The project currently has a negative net present value (NPV) of **(-866.91) million Philippine Pesos**, rendering it a financially unattractive investment. If investors were to fund this project, they would incur losses compared to the returns achievable by investing in a bank account with an annual interest rate of 8%. After applying Data Envelopment Analysis (DEA) and adjusting the input and output values, the estimated NPV improves to **(-398.8) million Philippine Pesos**. While this represents an enhancement in cash flow, it remains insufficient to make the project appealing to investors.

With a financial real discount rate of 8%, Table 14 shows that the Internal Rate of Return (IRR) and Modified Internal Rate of Return (MIRR) after the initial financial analysis are **7.70%** and **7.87%**, respectively. These figures confirm that the project is not viable under the current discount rate, leading to its rejection. After applying DEA and making input-output adjustments, the IRR and MIRR are estimated to increase by **0.165%** and **0.071%**, respectively, resulting in values of **7.86% for IRR** and **7.94% for MIRR**. However, these improvements remain insufficient to surpass the financial real discount rate of 8%. The differences in IRR and MIRR percentages indicate that they are approaching the discount rate, with a small gap of **0.14%** and **0.06%**, respectively.

Using the difference between the calculated NPV and its optimal value. It is assumed that if the project achieves a net present value of **-468.11 million Philippine Pesos**, it will break even and reach optimality. Achieving an NPV above this threshold would result in a positive NPV, assuming the project reaches its full potential.

Table 15 presents the net cash flow (NCF) values for the simulated data both before and after the DEA analysis. Additionally, it highlights the differences in net present value (NPV), internal rate of return (IRR), and modified internal rate of return (MIRR).

Table 15: The Difference between Financial Analysis Values Before and After DEA Analysis Using Simulated Data

Project life	**Net Cash Flow Before DEA**	**Project life**	**Net Cash Flow After DEA**
Years	**M'PHP**	**Years**	**M'PHP**
year 0	-5086.49	year 0	-5086.49
year 1	-9319.82	year 1	-9319.82
year 2	-7148.59	year 2	-7148.59
year 3	-2183.55	year 3	-2183.55
year 4	1107.94	year 4	1486.21
year 5	1607.72	year 5	1750.59
year 6	1816.93	year 6	1818.74
year 7	1876.91	year 7	1867.64
year 8	1843.29	year 8	1995.95
year 9	2044.40	year 9	2249.81
year 10	2380.91	year 10	2389.47
year 11	2379.83	year 11	2313.83
year 12	2571.76	year 12	2655.28
year 13	2443.45	year 13	2604.35
year 14	2723.56	year 14	2759.13
year 15	2738.84	year 15	2744.39
year 16	2772.34	year 16	2785.83
year 17	2775.77	year 17	2808.90
year 18	2667.23	year 18	2800.44
year 19	2826.20	year 19	2855.44
year 20	2959.30	year 20	2988.16
year 21	2974.96	year 21	2971.15
year 22	3038.13	year 22	3038.31
year 23	2885.47	year 23	3028.99
year 24	2900.97	year 24	2981.71

year 25	3339.80	year 25	3338.21
year 26	3321.22	year 26	3300.42
year 27	3398.41	year 27	3389.52
year 28	3308.91	year 28	3311.56
year 29	3310.11	year 29	3314.59
year 30	3248.25	year 30	3254.94
year 31	3178.31	year 31	3194.68
year 32	3129.56	year 32	3135.59
year 33	2842.07	year 33	2842.48
year 34	9994.66	year 34	9994.66
Financial Net Present Value - FNPV	-24.394	**Financial Net Present Value - FNPV**	708.681
Internal Rate of Return	7.992%	**Internal Rate of Return**	8.247 %
Modified Internal Rate of Return	7.996%	**Modified Internal Rate of Return**	8.103%

The project, utilizing simulated data, initially yields a negative net present value (NPV) of **(-24.394) million Philippine Pesos**, making it unattractive for investors, as it signals a potential loss compared to alternative investments with an annual interest rate of 8%. However, following Data Envelopment Analysis (DEA) and adjustments to input and output values, the estimated NPV increases to **708.681 million Philippine Pesos**, reflecting a significant improvement in the project's cash flow and making it attractive for investment.

Given the project's financial real discount rate of 8%, Table 15 shows that the initial Internal Rate of Return (IRR) and Modified Internal Rate of Return (MIRR) values are **7.992%** and **7.996%**, respectively. These figures fall short of the discount rate, leading to the project's rejection. However, after DEA and adjustments to the input-output values, the estimated IRR and MIRR percentages increase by **0.255%** and **0.107%**, resulting in IRR and MIRR values of **8.247%** and **8.103%**, respectively. These exceed the financial real discount rate of 8%, indicating the project's potential attractiveness for investment.

Chapter 5

CONCLUSION AND RECOMANDATION

Determining the profitability of a project to ensure it is a worthwhile investment can be challenging. Various tools exist to estimate project attractiveness, including Net Present Value (NPV), Internal Rate of Return (IRR), and Modified Internal Rate of Return (MIRR). These methods rely on estimated data to value the project, which might overlook its full potential.

Several challenges are encountered when appraising projects: profits are often calculated over periods that may be influenced by unexpected events; data collection can be costly; there is a high tendency to avoid bad projects while trying to accept good ones; and forecasts based on past project records may not apply to new concepts. To address these issues, a deterministic probability distribution is employed, assigning a fixed value to inputs, which can create different scenarios with varied outcomes. These inputs have infinite combinations, which are useful for risk analysis and diversification. However, these solutions often overlook significant inter-relationships among project variables, potentially leading to incorrect conclusions.

Data Envelopment Analysis (DEA) models have emerged as powerful nonparametric methods for computing the efficiency of multiple inputs and outputs in Decision-Making Units (DMUs). Standard DEA models can be adapted to reflect uncertainty in real-world evaluation problems and assess returns to scale, providing insights into investing in under-evaluated DMUs to enhance their efficiency.

Numerous studies have employed DEA in the investment sector, including firm selection and ranking, maximizing asset present value, evaluating R&D projects, improving model accuracy for poverty reduction projects, creating cost-effective and efficient project portfolios, analyzing factors influencing large road project costs and time management, and ranking and performance appraisal of DMUs. However, few papers incorporate NPV as an input for DEA.

This paper aims to give a second chance to projects that initially failed to attract investors by maximizing their efficiency through DEA. The first step involves calculating the project's attractiveness using NPV and determining its acceptability. DEA models are then applied to maximize the project's efficiency by minimizing inputs and maximizing outputs, estimating the project's full potential. The project is re-evaluated using the same attractiveness factors (NPV, IRR, MIRR) to determine its viability. If attractive, the project can proceed, provided it is redesigned to achieve similar target values, using efficiency, cross-efficiency, weights, and lambda to identify the most impactful inputs and outputs and the best reference DMUs. If the project remains unattractive post-DEA, it is rejected or re-evaluated after adjusting investment values and revenues.

A case study involving a road project that initially failed to attract interest was used to apply DEA, improving the inputs and outputs used to calculate NPV and maximizing them to realize the project's full potential. The project's NPV was initially negative at nearly 900 million pesos, leading to its rejection. After DEA, the NPV was estimated to improve to approximately negative (398.8) million pesos, though this was still insufficient to make the project attractive.

The IRR and MIRR were initially calculated at 7.70% and 7.87%, respectively, indicating unattractiveness. After DEA, these figures improved by 0.071% and 0.165%, but still failed to surpass the 8% financial real discount rate. A simulated data analysis using random variability further validated the methodology, highlighting "Input 2" (Maintenance Cost - Nominal, VAT Inclusive) and "Output 1" (Total Cars Toll Revenues) as influential factors, with Year07 emerging as the primary reference for efficiency improvement. Post-analysis, the project's NPV saw a substantial improvement, making it an attractive investment. While IRR and MIRR exceeded the 8% threshold post-DEA, suggesting that further improvements could enhance overall project attractiveness.

To further assess the potential of DEA in making a non-attractive project viable, a simulated dataset modeled after the road project was created. The results demonstrate that if the project achieves the target values identified by DEA, it can become financially attractive. This highlights the potential for optimization techniques like DEA to enhance the efficiency of infrastructure projects. However, in this case, significant improvements would still be required to ensure long-term financial viability.

Closing the gap needed to make the project attractive could involve decreasing the initial investment cost or increasing tolls on invested roads. Future research might explore using Tobit Regression and the Malmquist Index, applying a fuzzy DEA model for more accurate values under uncertainty, or utilizing other optimization tools to compare methods.

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APPENDICES

Appendix A: Cross Efficiency

Table 16: Cross Efficiency Table

Name	Year01	Year02	Year03	Year04	Year05	Year06	Year07	Year08	Year09	Year10	Year11	Year12	Year13
Year01	93.17	88.39	91.18	91.1	79.8	76.77	89.75	80.93	84.07	80.81	84.07	79.41	73.39
Year02	100	100	100	100	99.39	99.64	100	91.56	95.7	91.76	95.7	90.88	75.11
Year03	100	95.24	100	100	95.97	95.82	97.47	90.43	91.71	89.91	91.71	87.57	91.47
Year04	96.79	95.53	98.69	98.73	94.3	93.54	96.89	91.44	92.57	91.02	92.57	88.88	88.54
Year05	29.05	94.75	59.59	37.83	100	100	95.76	92.5	92.4	93.2	92.4	89.14	88.67
Year06	91.07	92.27	97.38	97.44	99.12	100	93.51	93.68	90.54	92.24	90.54	87.91	91.84
Year07	95.27	99.28	100	100	98.98	99.07	100	96.69	98.04	96.77	98.04	95.74	97.58
Year08	90.71	97.05	100	100	97.5	96.22	97.22	100	96.44	98.74	96.44	94.72	94.75
Year09	91.93	100	99.74	99.64	100	100	100	100	100	100	100	98.8	100
Year10	29.23	94.17	61.58	40.1	95.8	93.49	93.86	100	94.77	100	94.77	94.12	95.15
Year11	86.87	98.74	96.2	95.94	95.88	95.3	97.99	99.5	100	100	100	100	100
Year12	84.66	96.93	90.99	91.66	89.83	89.28	95.45	94.23	98.79	96.1	98.79	99.4	95.51
Year13	85.54	95.28	90.83	92.76	88.98	88.53	94.07	94.07	97.72	95.81	97.72	98.94	99.77
Year14	83.3	91.28	88.79	91.95	87.55	87.3	89.98	92.97	94.22	94.11	94.22	96	97.96
Year15	30.48	93.95	58.39	39.52	87.96	87.12	92.04	93.22	97.59	96.74	97.59	100	99.51
Year16	82.48	88.68	85.9	91.47	83.05	82.46	86.67	91.09	92.71	92.61	92.71	95.69	95.59
Year17	84.36	88.83	87.41	94.4	83.57	82.77	86.83	93.21	93.46	94.47	93.46	97.1	99.46
Year18	82.75	86.87	83.46	91.4	79.32	78.63	84.34	89.16	91.97	91.29	91.97	96.2	95.49
Year19	85.48	88.24	84.69	94.06	79.82	79.13	85.54	90.61	94.03	93.08	94.03	99.01	99.42
Year20	29.1	83.97	53.59	37.98	79.41	79.65	81.1	85.59	90.04	89.58	90.04	95.4	95.14
Year21	84.59	85.23	82.48	94.28	77.3	76.62	82.08	89.54	91.98	91.99	91.98	98.2	97.88
Year22	81.55	81.06	79.33	92	73.96	73.18	77.86	86.9	88.04	89.04	88.04	94.66	94.42
Year23	83.97	81.83	79.16	93.13	73.59	73.13	78.57	86.3	89.44	89.17	89.44	96.86	98.54
Year24	81.65	78.8	75.93	90.64	70.41	69.99	75.34	83.25	86.68	86.27	86.68	94.54	95.05
Year25	30.03	80.82	51.7	38.46	69.64	68.99	77.05	81.67	89.47	87.16	89.47	98.23	98.68
Year26	82.13	76.79	73.94	90.88	67.53	67.08	73.14	81.51	85.55	84.94	85.55	94.69	96.48
Year27	84.47	78.23	74.45	92.87	67.4	66.9	74.19	82.29	87.71	86.29	87.71	97.81	98.32
Year28	83.17	76.68	71.54	90.57	64.61	64.21	72.27	79.31	86.51	83.89	86.51	97.21	94.78
Year29	81.27	73.54	69.63	89.48	63.25	63.04	69.26	77.74	83.5	81.96	83.5	94.55	93.63
Year30	28.06	69.68	47.77	37.41	64.22	64.41	65.84	76.33	79.61	80.82	79.61	90.78	94.39

Year28	Year27	Year26	Year25	Year24	Year23	Year22	Year21	Year20	Year19	Year18	Year17	Year16	Year15	Year14
79.41	73.39	72.43	73.39	72.74	72.74	74.48	74.48	64.12	73.39	79.41	72.74	74.48	79.53	72.43
90.88	75.11	72.79	75.11	74.19	74.19	84.97	84.97	86.84	75.11	90.88	74.19	84.97	91.01	72.79
87.57	91.47	91.87	91.47	91.2	91.2	84.63	84.63	84.98	91.47	87.57	91.2	84.63	87.68	91.87
88.88	88.54	88.34	88.54	88.24	88.24	86.32	86.32	84.33	88.54	88.88	88.24	86.32	88.98	88.34
89.14	88.67	88.45	88.67	88.73	88.73	88.53	88.53	93.1	88.67	89.14	88.73	88.53	89.29	88.45
87.91	91.84	92.23	91.84	92.11	92.11	89.99	89.99	94.66	91.84	87.91	92.11	89.99	87.98	92.23
95.74	97.58	97.68	97.58	97.38	97.38	93.7	93.7	95.26	97.58	95.74	97.38	93.7	95.8	97.68
94.72	94.75	94.6	94.75	95.07	95.07	97.78	97.78	93.66	94.75	94.72	95.07	97.78	94.77	94.6
98.8	100	100	100	100	100	98.67	98.67	100	100	98.8	100	98.67	98.84	100
94.12	95.15	95.13	95.15	95.97	95.97	100	100	94.79	95.15	94.12	95.97	100	94.21	95.13
100	100	99.83	100	100	100	100	100	98.84	100	100	100	100	100	99.83
99.4	95.51	94.81	95.51	95.04	95.04	95.58	95.58	94.44	95.51	99.4	95.04	95.58	99.38	94.81
98.94	99.77	99.72	99.77	99.51	99.51	96.32	96.32	95.56	99.77	98.94	99.51	96.32	98.91	99.72
96	97.96	98.07	97.96	98	98	96.1	96.1	96.23	97.96	96	98	96.1	95.94	98.07
100	99.51	99.26	99.51	99.4	99.4	97.6	97.6	98.22	99.51	100	99.4	97.6	100	99.26
95.69	95.59	95.42	95.59	95.62	95.62	95.98	95.98	94.34	95.59	95.69	95.62	95.98	95.6	95.42
97.1	99.46	99.62	99.46	99.73	99.73	99.17	99.17	96.44	99.46	97.1	99.73	99.17	96.99	99.62
96.2	95.49	95.23	95.49	95.43	95.43	95.8	95.8	93.49	95.49	96.2	95.43	95.8	96.07	95.23
99.01	99.42	99.31	99.42	99.35	99.35	98.33	98.33	95.97	99.42	99.01	99.35	98.33	98.85	99.31
95.4	95.14	94.93	95.14	95.15	95.15	94.05	94.05	99.73	95.14	95.4	95.15	94.05	95.3	94.93
98.2	97.88	97.68	97.88	97.98	97.98	99.16	99.16	96.65	97.88	98.2	97.98	99.16	98	97.68
94.66	94.42	94.23	94.42	94.69	94.69	97.22	97.22	94.07	94.42	94.66	94.69	97.22	94.44	94.23
96.86	98.54	98.61	98.54	98.64	98.64	97.55	97.55	96.1	98.54	96.86	98.64	97.55	96.61	98.61
94.54	95.05	94.96	95.05	95.11	95.11	95.09	95.09	93.84	95.05	94.54	95.11	95.09	94.28	94.96
98.23	98.68	98.56	98.68	98.43	98.43	94.42	94.42	94.66	98.68	98.23	98.43	94.42	98.01	98.56
94.69	96.48	96.56	96.48	96.55	96.55	95.09	95.09	93.52	96.48	94.69	96.55	95.09	94.38	96.56
97.81	98.32	98.23	98.32	98.25	98.25	97.04	97.04	95.07	98.32	97.81	98.25	97.04	97.47	98.23
97.21	94.78	94.28	94.78	94.45	94.45	94.54	94.54	93.15	94.78	97.21	94.45	94.54	96.85	94.28
94.55	93.63	93.35	93.63	93.52	93.52	93.69	93.69	93.42	93.63	94.55	93.52	93.69	94.17	93.35
90.78	94.39	94.73	94.39	94.85	94.85	93.06	93.06	98.14	94.39	90.78	94.85	93.06	90.46	94.73

Year30	Year29
64.12	79.41
86.84	90.88
84.98	87.57
84.33	88.88
93.1	89.14
94.66	87.91
95.26	95.74
93.66	94.72
100	98.8
94.79	94.12
98.84	100
94.44	99.4
95.56	98.94
96.23	96
98.22	100
94.34	95.69
96.44	97.1
93.49	96.2
95.97	99.01
99.73	95.4
96.65	98.2
94.07	94.66
96.1	96.86
93.84	94.54
94.66	98.23
93.52	94.69
95.07	97.81
93.15	97.21
93.42	94.55
98.14	90.78

Appendix B: Cross Efficiency After Using the Simulated Data

Table 17: Cross Efficiency Table After Using Simulated Data

Name	Year01	Year02	Year03	Year04	Year05	Year06	Year07	Year08	Year09	Year10	Year11	Year12	Year13	Year14	Year15
Year01	91.39	76.11	78.61	79.05	60.98	76.11	79.05	76.13	78.61	61.04	78.61	79.05	79.05	79.05	60.05
Year02	83.18	93.29	72.61	89.85	72.02	93.29	89.85	93.28	72.61	54.29	72.61	89.85	89.85	89.85	65.32
Year03	83.49	90.53	93.03	90.49	69.61	90.53	90.49	90.52	93.03	69.27	93.03	90.49	90.49	90.49	65.47
Year04	86.47	90.13	89.74	91.53	68.74	90.13	91.53	90.13	89.74	66.26	89.74	91.53	91.53	91.53	65.58
Year05	32.58	34.26	33.46	31.32	70.75	34.26	31.32	34.27	33.46	67.9	33.46	31.32	31.32	31.32	68.2
Year06	87.38	98.21	96.58	91.1	74.92	98.21	91.1	98.21	96.58	71.34	96.58	91.1	91.1	91.1	65.29
Year07	100	100	100	100	75.12	100	100	100	100	72.7	100	100	100	100	70.36
Year08	99.91	100	95.98	97.79	78.04	100	97.79	100	95.98	72.56	95.98	97.79	97.79	97.79	72.01
Year09	93.92	87.95	99.9	97.22	70.36	87.95	97.22	87.96	99.9	77.46	99.9	97.22	97.22	97.22	73.72
Year10	36.57	35.64	39.15	35.68	68.64	35.64	35.68	35.65	39.15	74.04	39.15	35.68	35.68	35.68	72.13
Year11	86.18	82.78	95.92	95.23	67.38	82.78	95.23	82.79	95.92	75.7	95.92	95.23	95.23	95.23	73.71
Year12	86.24	82.74	90.68	93.27	67.76	82.74	93.27	82.74	90.68	72.01	90.68	93.27	93.27	93.27	72.71
Year13	87.77	88.07	92.98	95.29	72.09	88.07	95.29	88.07	92.98	73.81	92.98	95.29	95.29	95.29	74.25
Year14	90.08	85.65	93.54	94.84	70.5	85.65	94.84	85.65	93.54	74.67	93.54	94.84	94.84	94.84	74.38
Year15	35.13	35.32	34.92	34.71	75.4	35.32	34.71	35.33	34.92	73.28	34.92	34.71	34.71	34.71	78.31
Year16	88.52	90.29	94.03	95.39	76.89	90.29	95.39	90.29	94.03	77.71	94.03	95.39	95.39	95.39	77.85
Year17	84.9	94.22	89.73	97.42	82.1	94.22	97.42	94.21	89.73	75.91	89.73	97.42	97.42	97.42	81.67
Year18	87.94	91.91	93.87	96.1	82.14	91.91	96.1	91.91	93.87	81.49	93.87	96.1	96.1	96.1	82.97
Year19	88.28	94.47	95.85	96.62	85.7	94.47	96.62	94.47	95.85	84.49	95.85	96.62	96.62	96.62	84.87
Year20	49.19	52.71	54.39	49.78	86.81	52.71	49.78	52.72	54.39	87.79	54.39	49.78	49.78	49.78	85.01
Year21	91.16	96.86	99.25	96.04	90.23	96.86	96.04	96.86	99.25	89.88	99.25	96.04	96.04	96.04	86.99
Year22	89.87	97.62	97.08	96.23	92.74	97.62	96.23	97.62	97.08	89.68	97.08	96.23	96.23	96.23	89.14
Year23	91.49	99.9	97.89	96.08	96.04	99.9	96.08	99.9	97.89	91.54	97.89	96.08	96.08	96.08	90.23
Year24	93.28	100	97.38	100	97.99	100	100	100	97.38	92.85	97.38	100	100	100	95.99
Year25	56.94	59.54	59.76	57.45	97.36	59.54	57.45	59.55	59.76	95.77	59.76	57.45	57.45	57.45	97.37
Year26	96.01	98.4	99.95	99.69	98.9	98.4	99.69	98.41	99.95	97.78	99.95	99.69	99.69	99.69	98.5
Year27	97.73	96.84	99.41	99.72	97.07	96.84	99.72	96.84	99.41	97	99.41	99.72	99.72	99.72	98.23
Year28	97.43	95.96	98.13	98.68	97.5	95.96	98.68	95.97	98.13	97.07	98.13	98.68	98.68	98.68	98.71
Year29	100	97.37	100	98.76	100	97.37	98.76	97.38	100	100	100	98.76	98.76	98.76	100
Year30	36.95	35.84	36.37	33.4	100	35.84	33.4	35.85	36.37	100	36.37	33.4	33.4	33.4	100

Year30	Year29	Year28	Year27	Year26	Year25	Year24	Year23	Year22	Year21	Year20	Year19	Year18	Year17	Year16
60.98	80.96	71.01	79.28	79.06	60.05	79.05	76.8	77.25	78.61	61.04	79.06	79.05	79.05	79.05
72.02	89.45	79.11	88.33	88.12	65.32	89.85	88.69	85.91	72.61	54.29	88.12	89.85	89.85	89.85
69.61	89.31	79.49	90.62	90.78	65.47	90.49	91.15	91.51	93.03	69.27	90.78	90.49	90.49	90.49
68.74	90.38	80.04	91.32	91.36	65.58	91.53	90.08	90.07	89.74	66.26	91.36	91.53	91.53	91.53
70.75	33.08	39.86	31.76	31.73	68.2	31.32	34.64	34.81	33.46	67.9	31.73	31.32	31.32	31.32
74.92	91.94	79.68	91.52	91.68	65.29	91.1	97.88	97.58	96.58	71.34	91.68	91.1	91.1	91.1
75.12	100	86.71	100	100	70.36	100	100	100	100	72.7	100	100	100	100
78.04	98.81	86.61	97.72	97.64	72.01	97.79	99.19	98.67	95.98	72.56	97.64	97.79	97.79	97.79
70.36	95.01	87.26	97.47	97.56	73.72	97.22	90.79	92.61	99.9	77.46	97.56	97.22	97.22	97.22
68.64	36.87	44.65	36.24	36.24	72.13	35.68	37.01	37.84	39.15	74.04	36.24	35.68	35.68	35.68
67.38	91.39	86.27	95.25	95.39	73.71	95.23	85.92	87.96	95.92	75.7	95.39	95.23	95.23	95.23
67.76	90.2	84.76	93.05	93.09	72.71	93.27	84.71	86.04	90.68	72.01	93.09	93.27	93.27	93.27
72.09	92.82	86.58	95.09	95.15	74.25	95.29	89.36	90.24	92.98	73.81	95.15	95.29	95.29	95.29
70.5	92.5	86.42	94.78	94.81	74.38	94.84	87.62	88.93	93.54	74.67	94.81	94.84	94.84	94.84
75.4	35.98	44.53	35.02	34.97	78.31	34.71	35.82	36.11	34.92	73.28	34.97	34.71	34.71	34.71
76.89	93.65	88.46	95.33	95.38	77.85	95.39	91.41	92.15	94.03	77.71	95.38	95.39	95.39	95.39
82.1	95.14	91.39	96.77	96.8	81.67	97.42	93.53	93.17	89.73	75.91	96.8	97.42	97.42	97.42
82.14	94.51	91.28	96	96.05	82.97	96.1	92.73	93.28	93.87	81.49	96.05	96.1	96.1	96.1
85.7	95.48	92.44	96.66	96.73	84.87	96.62	95.2	95.66	95.85	84.49	96.73	96.62	96.62	96.62
86.81	51.56	59.69	50.48	50.51	85.01	49.78	53.84	54.46	54.39	87.79	50.51	49.78	49.78	49.78
90.23	96.23	93.05	96.48	96.57	86.99	96.04	97.88	98.48	99.25	89.88	96.57	96.04	96.04	96.04
92.74	96.35	94.09	96.48	96.54	89.14	96.23	98.02	98.23	97.08	89.68	96.54	96.23	96.23	96.23
96.04	97.05	94.46	96.44	96.49	90.23	96.08	100	100	97.89	91.54	96.49	96.08	96.08	96.08
97.99	99.93	99.18	100	100	95.99	100	100	100	97.38	92.85	100	100	100	100
97.36	59.27	68.75	58.03	58	97.37	57.45	60.42	60.91	59.76	95.77	58	57.45	57.45	57.45
98.9	100	100	100	100	98.5	99.69	99.39	100	99.95	97.78	100	99.69	99.69	99.69
97.07	100	99.91	100	99.96	98.23	99.72	98.04	98.81	99.41	97	99.96	99.72	99.72	99.72
97.5	99.16	99.45	98.96	98.91	98.71	98.68	97.1	97.81	98.13	97.07	98.91	98.68	98.68	98.68
100	100	100	99.23	99.17	100	98.76	98.64	99.42	100	100	99.17	98.76	98.76	98.76
100	35.66	45.27	33.99	33.93	100	33.4	36.64	37.08	36.37	100	33.93	33.4	33.4	33.4